Assignment: HW3 [25 points]

Assigned: 2010/02/22
Due: 2010/03/01

**P3.1** [4 points]
Find the inverse of each element of the permutation group of 3 labelled objects using the $3 \times 3$ matrix representation.

**P3.2** [3 points]
Show that topologically speaking, the group $SU(2)$ is a 3-sphere of unit radius embedded in the real Euclidian 4-dimensional space.

**P3.3** [3 + 3 = 6 points]
A bounded operator $P$ on a Hilbert space $\mathcal{H}$ is a projection operator if
\[ P^\dagger = P \quad \text{and} \quad P = P^2. \] (1)

(a) Show that $(1 - P)$ is also a projection operator (where $1$ is the identity operator).
(b) Find the eigenvalues of $P$.

Note: the first condition implies that $P$ is hermitian.

**P3.4** [6 points]
Give a pictorial representation of the $SU(3)$ raising and lowering operators $I_\pm, U_\pm, \text{and} V_\pm$ on the $t_3, t_8$ plane.

**P3.5** [6 points]
If a theory is to be invariant under a unitary local gauge (or phase) transformation $\psi' = U \psi$, where $\psi$ represents a fermion wave function, then the general expression for the transformation of the gauge field is given by
\[ A'^\mu = -\frac{i}{g} (\partial^\mu U) U^{-1} + U A^\mu U^{-1}. \] (2)
Show that this gives the expected answer for the electromagnetic field (i.e. the electromagnetic field tensor $F^{\mu \nu}$ is unaffected by such a transformation), where $U = e^{-ig\psi(x)}$ and $g = -e$. 