1 Lecture 1: Introduction, Outline and Motivation

“The most incomprehensible thing about the world is that it is comprehensible.”

Albert Einstein

Astrophysics is the branch of astronomy that deals with the physics of the Universe, including the physical properties (luminosity, density, temperature, chemical structure) of celestial objects such as stars, galaxies and the interstellar medium, as well as their interactions. Astrophysics is a very broad subject: it includes mechanics, statistical mechanics, thermodynamics, electromagnetism, relativity, particle physics, high energy physics, nuclear physics, and others.

Cosmology is theoretical astrophysics at its largest scales, where general relativity plays a major role. It deals with the Universe as a whole — its origin, distant past, evolution, structure. When looking at the world at such grand scales, locally “flat” and “slow” approximation — the realm of the Newtonian mechanics — is no longer justified.

Because its subject matter involves such important and overarching questions, such as: ‘How did we get here?’, ‘Was there a beginning?’, ‘Are we special?’, thus heavily flirting with philosophy and theology, the modern cosmology has proven to be a dynamical battleground for competing ideas. In this arena where greatest scientific minds (and egos!) battled, we have many instances of drama, thrills, twists, and, of course, mystery:

- a priest-scientist breaking with the church cannons to interpret his solutions as having “a day without yesterday” (Fr. Georges Lemaître), a progenitor term to the “Big Bang”;
- one scientist’s mockery of the opposing camp’s view immortalized (term “Big Bang” was coined by a steady-state theory proponent Fred Hoyle);
- a “fudge factor” introduced, then discarded in embarrassment, then later reintroduced as our only hope to get our cosmic books to balance (Einstein’s cosmological constant);
- the greatest experimental evidence for the Big Bang coming about by sheer accident! (cosmic microwave background radiation);
- finally, we are still searching for answers so as to what comprises about 96% of the content of the Universe. Over 70% of the mass-energy content of the Universe is in form of the unknown vacuum energy called “dark energy”. Over 80% of the mass is in the form of the mysterious “dark matter”.

Course Outline

This course will be composed of three parts:

1. General relativity as the foundation of cosmology

Overview of the basic concepts of the theory of general relativity (GR) and the formalism it provides for studying the evolution of the Universe:

(a) Spacetime: time and space treated on equal footing.
(b) GR uses tools of differential geometry: metrics, covariant and contravariant tensors, invariants. When the equations of motion are written in tensor form, they are invariant under metric transformation.
(c) Geodesic equation: how particles move in curved spacetime.
(d) Einstein’s equations: how matter curves spacetime.
(e) Solutions: Friedmann-Lemaître-Robertson-Walker Universe.
(f) The horizon problem leads to inflation theory. Inflation theory also explains the observed flatness of the Universe. De Sitter Universe.

2. Interpreting the Universe

Implications of solutions to Einstein’s equations:

(a) Brief history of time: from the Big Bang to present day.
(b) Cosmic Microwave Background (CMB) radiation.
(c) Dark matter: possible candidates and the current search.

3. Black holes, stars and galaxies:

(a) Black holes: singularities of Einstein’s equations.
(b) Stars: structure, evolution and mathematical models.
(c) Galaxies: classification, evolution and mathematical models.

Motivation: Newton vs. Einstein

Newtonian mechanics is an approximation which works quite well for most our “earthly” needs, at least when the velocity $v \ll c$, where $c$ is the speed of light. The basic differences and analogies between Newtonian and Einsteinian physics are presented in Table 1.

<table>
<thead>
<tr>
<th>Newton</th>
<th>Einstein</th>
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<tbody>
<tr>
<td>absolute time and absolute space</td>
<td>spacetime</td>
</tr>
<tr>
<td>Galilean invariance of space (simultaneity)</td>
<td>Lorentz invariance of spacetime (time-dilation, length-contraction, no simultaneity)</td>
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<tr>
<td>existence of preferred inertial frames (at rest or moving with constant velocity wrt the absolute space)</td>
<td>no preferred frames (physics is the same everywhere)</td>
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<tr>
<td>infinite speed of light $c$ (instantaneous action at the distance)</td>
<td>finite and fixed speed of light $c$ (nothing propagates faster than $c$)</td>
</tr>
<tr>
<td>gravity is a force</td>
<td>gravity as a distortion of the fabric of spacetime</td>
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<tr>
<td>Newton’s Second Law</td>
<td>geodesic equation</td>
</tr>
<tr>
<td>Poisson equation</td>
<td>Einstein’s equations</td>
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Newtonian mechanics quickly runs into problems which cannot be explained within its realm:

- All observers measure the same speed of light $c$ (in a vacuum), as demonstrated by Michelson-Morley experiment.
- Electromagnetism does not respect Galilean invariance.
• Why do all bodies experience the same acceleration regardless of their mass, i.e., why is the inertial and gravitational mass the same (as measured experimentally throughout history)?

Einstein’s theory of *special relativity* (SR) introduced some revolutionary concepts:

• “Abolished” absolute time — introduced 4D spacetime as an inseparable entity.
• Finite and fixed speed of light $c$.
• Established equivalence between energy and mass (massless photons are subject to gravity).
• However, the 4D spacetime considered in SR is still flat — Minkowski metric.

Einstein’s theory of *general relativity* continued the revolution:

• *Equivalence principle*: Established equivalence between the inertial and gravitational mass.
• *Cosmological principle*: Our position is “as mundane as it can be” (on large spatial scales, the Universe is homogeneous and isotropic).
• *Relativity*: Laws of physics are the same everywhere.
• New definition of gravity: Gravity is the distortion of the structure of spacetime as caused by the presence of matter and energy. The paths followed by matter and energy in spacetime are governed by the structure of spacetime. This great feedback loop is described by Einstein’s field equations. So, the 4D spacetime considered in GR is no longer flat.

After establishing GR as the way to describe the Universe and learning its mathematical formalism, we will finally embark on a journey of expressing mathematically the world around us on largest scales, physically interpreting the implications and reconciling them with the the observations.

Many of the phenomena for which we now have overwhelming evidence — the Big Bang, expanding Universe, CMB radiation, black holes, among others — have been *first* predicted by the solutions of Einstein’s equations. Therefore, it is the mathematics that holds the keys to unlocking the mysteries of the Universe, so let us begin acquiring required mathematical skills!