Pitch correction

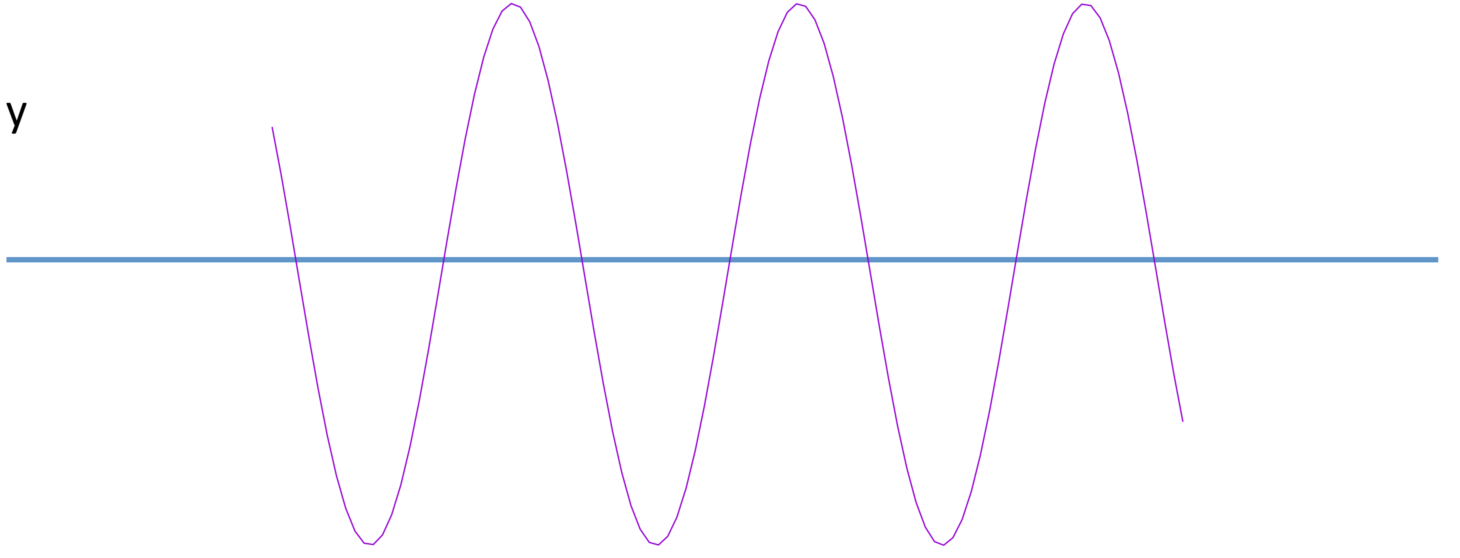
D. Rubin

USPAS January 2019

Suppose the muon is oscillating in a vertical plane at the magic radius

$$\psi = \psi_0 \sin \omega_p t$$

$$\omega_a = \frac{ae}{mc\gamma} B^*$$ In the rest frame of the muon



F. Farley

$$\mathbf{B}' = \gamma \mathbf{B} - \frac{\gamma^{\mathbf{2}}}{\gamma + \mathbf{1}} \beta(\beta \cdot \mathbf{B})$$

$$B_z^{eff} = B_z(1 - \frac{\gamma - 1}{\gamma}\psi^2)$$

$$B_x = -B_z \left(\frac{\gamma - 1}{\gamma} \psi \right)$$

$$\omega_z = \omega_0 \left(1 - \frac{\gamma - 1}{\gamma} \psi^2 \right)$$

$$\omega_x = -\omega_0 \left(\frac{\gamma - 1}{\gamma} \psi \right)$$

Where $\omega_0 = a_\mu (e/mc) B_z$

The vertical restoring force (E or B) provides a third axes

$$\omega_y = -f \omega_p \psi_0 \cos \omega_p t$$

where $f = (1 + \beta^2 \gamma a - \gamma^{-1})$

$$
\begin{aligned}
x &= \cos\theta\cos\phi \\
y &= \cos\theta\sin\phi \\
z &= \sin\theta
\end{aligned}
$$

$$
\dot{\theta} = \omega_x \sin\phi - \omega_y \cos\phi = -\omega_0\psi_0 \frac{\gamma - 1}{\gamma} \sin\phi \sin\omega_p t + \omega_p \psi_0 f \cos\phi \cos\omega_p t
$$

$$
\begin{aligned}
\dot{\phi} &= \omega_z - \omega_x \tan\theta\cos\phi - \omega_y \tan\theta\sin\phi \\
&= \omega_0 \left(1 - \frac{\gamma - 1}{\gamma}\psi_0^2 \sin^2\omega_p t\right) + \omega_0\psi_0 \frac{\gamma - 1}{\gamma} \tan\theta\cos\phi\sin\omega_p t + \omega_p\psi_0 f \tan\theta\sin\phi\cos\omega_p t
\end{aligned}
$$

Substitute the lowest order time dependence for $\phi = (\omega t + \xi)$

And integrate

$$
\theta = \frac{A_1}{\omega_0 + \omega_p} \sin((\omega_0 + \omega_p)t + \xi) - \frac{A_2}{\omega_0 - \omega_p} \sin((\omega_0 - \omega_p)t + \xi)
$$

$$\dot{\phi} = \omega_0 \left(1 - \frac{\gamma - 1}{2\gamma}\psi_0^2\right) + \frac{A_1^2}{2(\omega_0 + \omega_p)} + \frac{A_2^2}{2(\omega_0 - \omega_p)}$$

$$A_1 = \frac{1}{2}\psi_0(\omega_0\gamma^{-1}(\gamma - 1) + f\omega_p)$$
$$A_2 = \frac{1}{2}\psi_0(\omega_0\gamma^{-1}(\gamma - 1) - f\omega_p)$$

$$\dot{\phi} = \omega_0 \left(1 - \frac{\gamma - 1}{2\gamma}\psi_0^2\right) + \frac{A_1^2}{2(\omega_0 + \omega_p)} + \frac{A_2^2}{2(\omega_0 - \omega_p)}$$

This is what we measure

At the magic momentum $\dot{\phi} = \omega_0(1 - C)$

$$C = \frac{1}{4}\psi_0^2(1 - (\omega_0^2 + 2\pi\gamma^2)/\gamma^2(\omega_0^2 - \omega_p^2))$$

Resonance when $\omega_p \sim \omega_0$

Otherwise $C \sim \frac{1}{4}\psi_0^2$

Assume that polarization is in the plane

where ω_v is the vertical betatron frequency and ψ is the pitch angle. Define θ so that $\hat{\beta}\cdot\mathbf{s} = s\cos\theta$ and $\hat{\beta}\times\mathbf{s} = s\sin\theta\hat{\mathbf{s}}_\perp$. Then $\hat{\mathbf{s}}_\perp\cdot\mathbf{B} = B\cos\psi$. ($B$ in the vertical direction). Now

$$\begin{aligned}
\frac{d(\hat{\beta}\cdot\mathbf{s})}{dt} &= -\frac{e}{mc}\mathbf{s}_\perp\cdot\left[\left(\frac{g}{2}-1\right)(\hat{\beta}\times\mathbf{B}) + \left(\frac{g}{2}-1-\frac{1}{\gamma^2-1}\right)\mathbf{E}\right] \\
\frac{sd(\cos\theta)}{dt} &= -\frac{e}{mc}\mathbf{B}\cdot\left(\frac{g}{2}-1\right)(\mathbf{s}\times\hat{\beta}) \\
-\sin\theta\frac{d\theta}{dt} &= -\frac{e}{mc}B\left(\frac{g}{2}-1\right)\cos\psi(s\sin\theta) \\
\rightarrow\omega_a &= -\frac{e}{mc}B\left(\frac{g}{2}-1\right)\cos\psi
\end{aligned}$$

Then $\beta\times\mathbf{B} = \hat{\mathbf{i}}\psi B\cos\omega_v t$

$\hat{\beta} = (\cos\alpha\hat{\mathbf{k}} + \sin\alpha\hat{\mathbf{j}})$. Note that $\mathbf{s}_\perp\cdot(\hat{\beta}\times\mathbf{B}) = \mathbf{s}\cdot(\hat{\beta}\times\mathbf{B})$. Then 5 becomes

$$\frac{d(\hat{\beta}\cdot\mathbf{s})}{dt} = \frac{d}{dt}\left(s(\sin\alpha\sin\theta\sin\phi + \cos\alpha\cos\theta)\right) \sim \frac{d}{dt}\left(s(\cos\alpha\cos\theta)\right) \tag{10}$$

The pitch angle α oscillates with the vertical betatron frequency which is much greater than the precession frequency with average value of zero. Then

$$\begin{aligned}
\frac{d(\hat{\beta}\cdot\mathbf{s})}{dt} &= -s(\cos\alpha\sin\theta)\frac{d\theta}{dt} = sB\sin\theta\cos\phi\cos\alpha\left(\frac{g}{2}-1\right) \\
&\rightarrow\quad \omega = -B\cos\phi\left(\frac{g}{2}-1\right)
\end{aligned}$$

Pedestrian estimate of pitch correction

$$\frac{d}{dt}(\hat{\beta}\cdot\mathbf{s}) = -\frac{e}{mc}\mathbf{s}_\perp \cdot \left[\left(\frac{g}{2}-1\right)\hat{\beta}\times\mathbf{B} + \left(\frac{g\beta}{2}-\frac{1}{\beta}\right)\mathbf{E}\right]$$

$$\hat{\beta}\cdot\mathbf{s} = \cos\phi \qquad\qquad \hat{\beta}\times\mathbf{B} = B\cos\psi\hat{x}$$

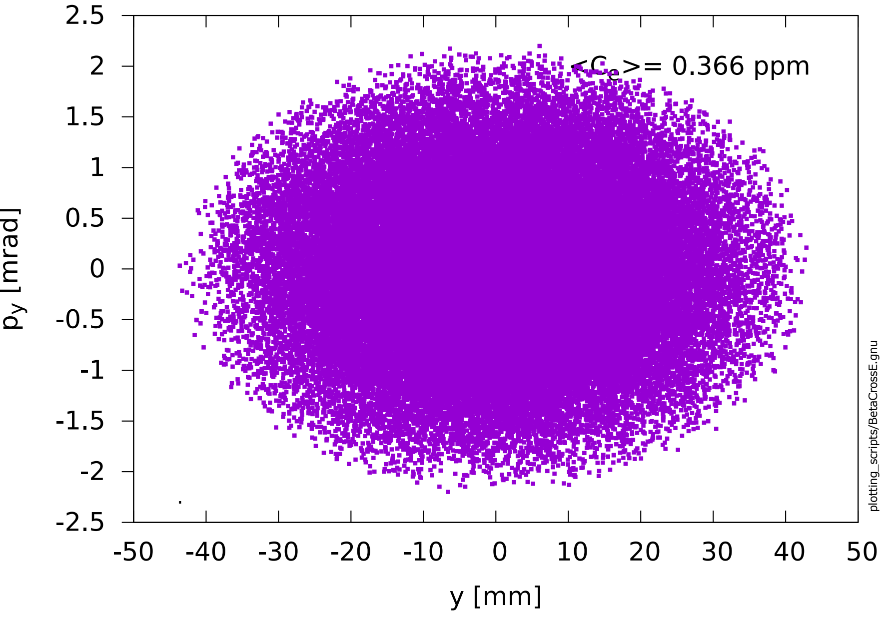
$$\frac{d}{dt}\cos\phi = -\frac{e}{mc}\sin\phi(a_\mu B)\cos\psi$$

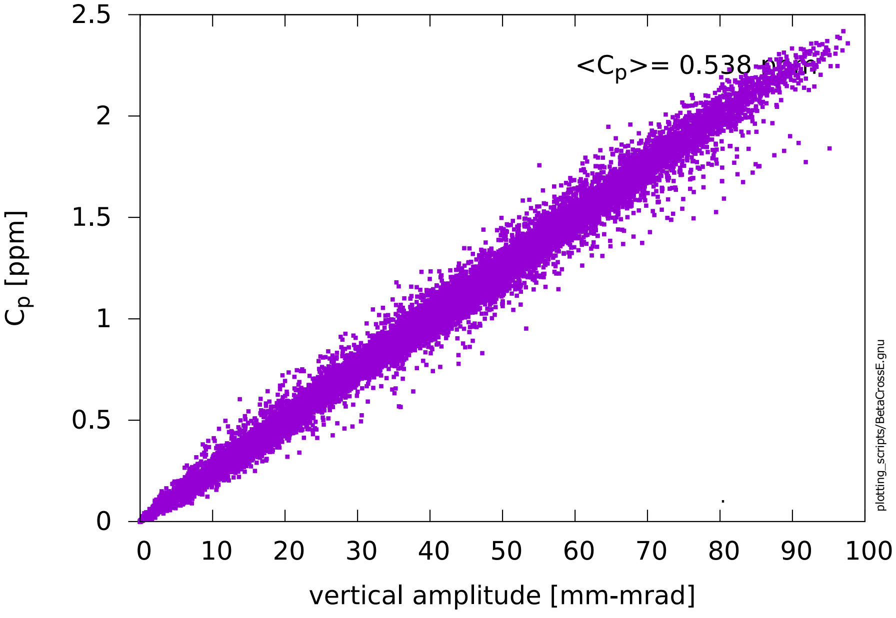
$$\begin{aligned}\dot{\phi} &= -\frac{e}{mc}a_\mu B(1-\frac{1}{2}\psi^2)\\ &= \omega_0(1-\frac{1}{2}\psi^2)\\ \rightarrow C &= -\frac{1}{2}\langle\psi^2\rangle = -\frac{1}{4}\psi_0^2\end{aligned}$$

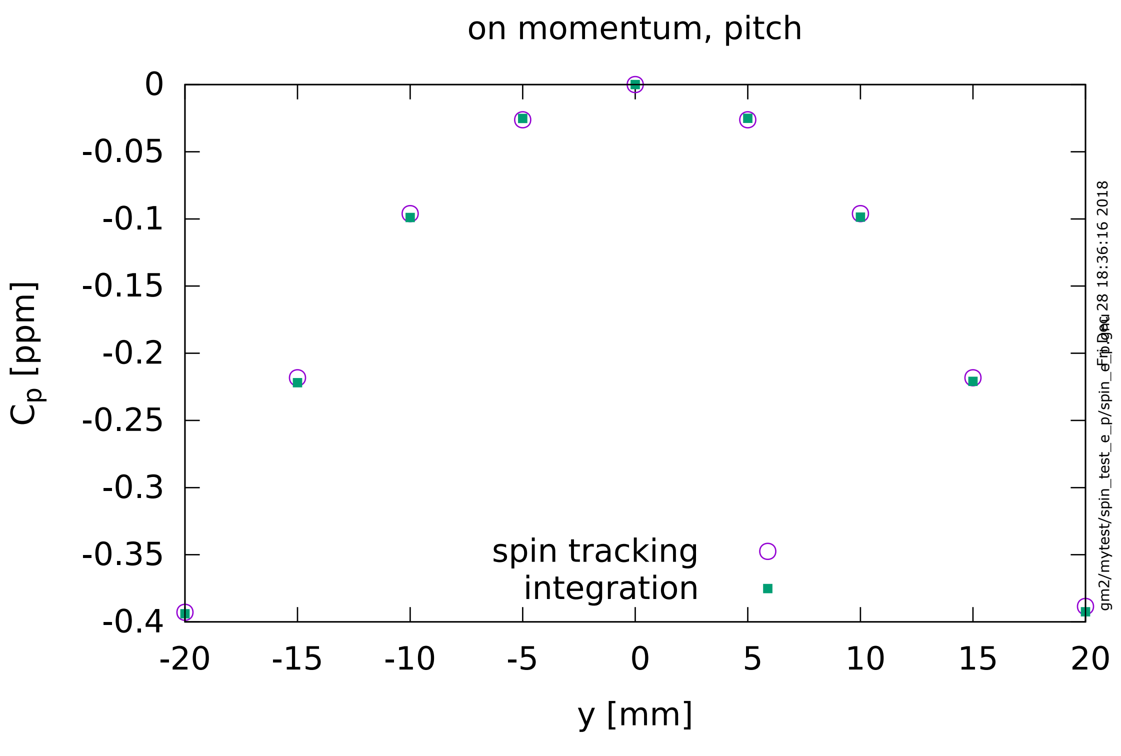
Pitch correction

$$\vec{C}_p(T) = \frac{1}{B}\frac{1}{T}\int_0^T (\vec{\beta}\cdot\vec{B})\vec{\beta}dt$$

1. Generated and track a distribution and compute
   - Momentum and frequency distribution
   - E-field correction for each particle
   - Vertical phase space distribution and pitch correction






on momentum, pitch
C_p [ppm]
y [mm]
spin tracking
integration
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
-0.4
-20
-15
-10
-5
0
5
10
15
20
gm2/mytest/spin_test_e_p/spin_e_p.gnu
Fri Dec 28 18:36:16 2018