Spin

D. Rubin
USPAS
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BMT $$\frac{dS}{dt} = -\frac{e}{\gamma m}\left[(1+G\gamma)(\mathbf{B}_\perp + (1+G)\mathbf{B}_\parallel\right] \times \mathbf{s})$$

$$\frac{d\beta}{dt} = \frac{e}{\gamma m}\beta \times \mathbf{B}$$

Polarization in lab frame rotates $1 + G\gamma$ times faster than velocity

where $G = g/2 - 1$

$$\frac{dS}{ds}\frac{ds}{dt} \quad = \quad \frac{dS}{ds}v(1-\frac{x}{\rho})^{-1}$$

$$\rightarrow \frac{dS}{ds} \quad = \quad -\frac{e}{\gamma m v}(1+\frac{x}{\rho})\left((1+G\gamma)\mathbf{B}_\perp + (G+1)\mathbf{B}_\parallel\right)$$

$$\rightarrow \mathbf{\Omega_p} \quad = \quad -\frac{e}{p_0}\frac{1+x/\rho}{1+\delta}\left((1+G\gamma)\mathbf{B}_\perp + (G+1)\mathbf{B}_\parallel\right)$$

$$\frac{d\beta}{ds} \quad = \quad \frac{e}{\gamma m v}\beta \times \mathbf{B}$$

[1/L]

$$\rightarrow \mathbf{\Omega_c} \quad = \quad -\frac{e}{\gamma_0 m v}\mathbf{B}_\perp$$

Relative precession per unit length

$$\rightarrow \mathbf{\Omega} \quad = \quad \mathbf{\Omega_p} - \mathbf{\Omega_c}$$

$$R_{eff}\Omega = 2\pi\nu = 2\pi G\gamma$$

Spin tune

Propagation through finite length at B

$$d\theta = \frac{e}{p_0}\nu B_\perp ds$$

In longitudinal field

$$d\phi = \frac{e}{p_0}(1+G)B_{||}ds$$

Propagation of spin is a succession of rotations

$$\vec{S}_f = R\vec{S}_i$$

The product of rotations through each element

$$\begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}_f = \begin{pmatrix} \alpha^2(1-C)+C & \alpha\beta(1-C)-\gamma S & \alpha\gamma(1-C)+\beta S \\ \alpha\beta(1-C)+\gamma S & \beta^2(1-C)-C & \beta\gamma(1-C)-\alpha S \\ \alpha\gamma(1-C)-\beta S & \beta\gamma(1-C)+\alpha S & \gamma^2(1-C)+C \end{pmatrix} \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}_i$$

$$\begin{aligned} \alpha &= \hat{\Omega}\cdot\hat{x} \\ \beta &= \hat{\Omega}\cdot\hat{y} \\ \gamma &= \hat{\Omega}\cdot\hat{z} \end{aligned} \qquad \begin{aligned} S &= \sin(\Omega s) \\ C &= \cos(\Omega s) \end{aligned}$$

For example

$$\begin{aligned} Horizontal\ bend &: \frac{B_y}{B\rho_0}[\delta\hat{y}+\nu_0 y'\hat{z}] \\ Vertical\ bend &: \frac{B_x}{B\rho_0}[\delta\hat{x}+\nu_0 x'\hat{z}] \\ Quadrupole &: -\frac{1+\nu_0}{B\rho_0}\frac{\partial B_y}{\partial x}[y\hat{x}+x\hat{z}] \end{aligned}$$

The rotation matrix R is orthogonal with eigenvalues ± 1

The eigenvector is a closed spin trajectory.

Accelerating beam hits a spin resonance whenever $G\gamma = N$

Every 523 MeV for protons
441 MeV for electrons
91.3 GeV for muons

Snake

Longitudinal field rotate spin about velocity vector

Full snake $\phi = \pi \to \Delta\nu = \frac{1}{2} - \nu$

$$\nu \to \frac{1}{2}$$

Spin tune in g-2 ring

At the magic momentum

$$\begin{aligned} G &= \frac{1}{\gamma^2 - 1} = 0.001166 \\ \gamma &= \left(\frac{1}{G} + 1\right)^{1/2} \\ \nu &= G\gamma = (G^2 + G)^{1/2} = 0.034 \end{aligned}$$

Spin resonance in g-2ring

Skew octupole multipole

$$B_x \propto y^3 \cos(s/R)$$

$$y = A_y \cos\left(\frac{Q_y s}{R}\right)$$

$$y^3 = \frac{A_y^3}{4}(3\cos\left(\frac{Q_y s}{R}\right) + \cos\left(\frac{3Q_y s}{R}\right))$$

$$y^3 \cos(s/R) = A_y^3 \cos\left(\frac{3Q_y s}{R}\right)\cos\left(\frac{s}{R}\right) + \ldots$$

Spin resonance in g-2ring

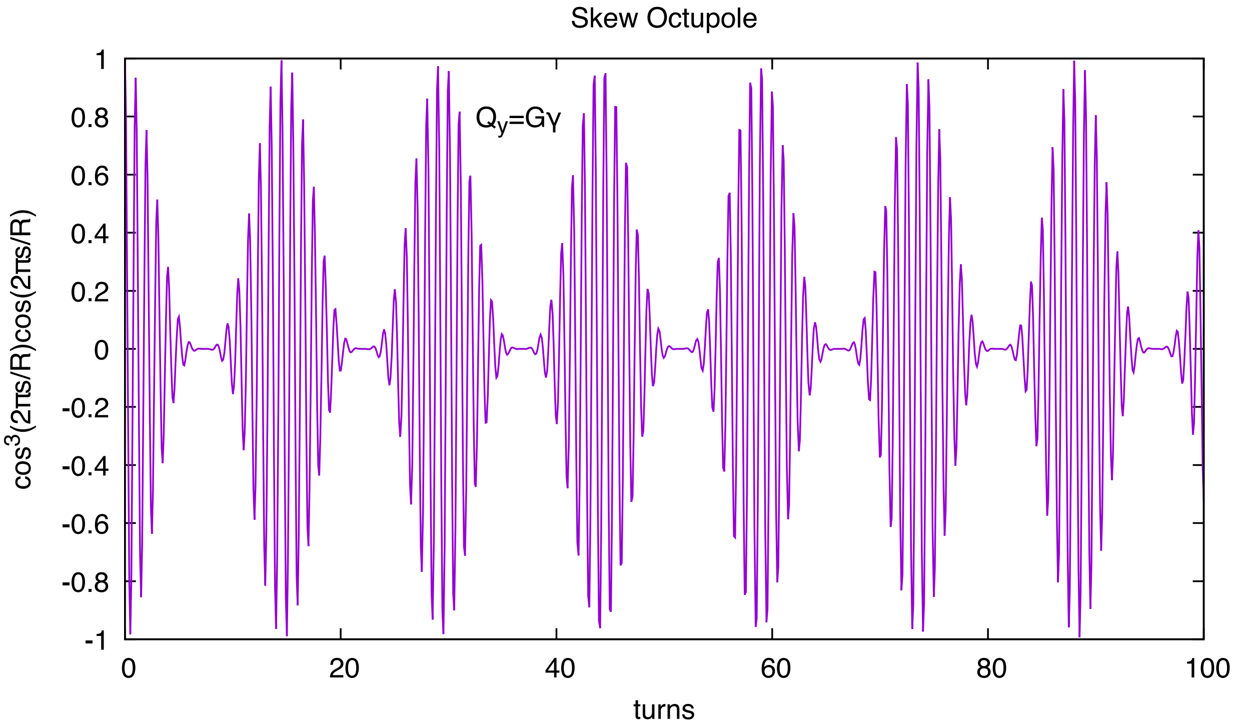
Skew octupole multipole

$$y^3 \cos(s/R) = A_y^3 \cos\left(\frac{3Q_y s}{R}\right) \cos\left(\frac{s}{R}\right) + \ldots$$

$$y^3 \cos(s/R) \sim A_y^3 \cos\left(\frac{(3Q_y - 1)s}{R}\right) + \ldots$$

Spin resonance

$$3Q_y - 1 = G\gamma = 0.034$$



Skew Octupole
Q_y=Gγ
cos^3(2πs/R)cos(2πs/R)
turns
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
20
40
60
80
100