Bend, Steer, Focus

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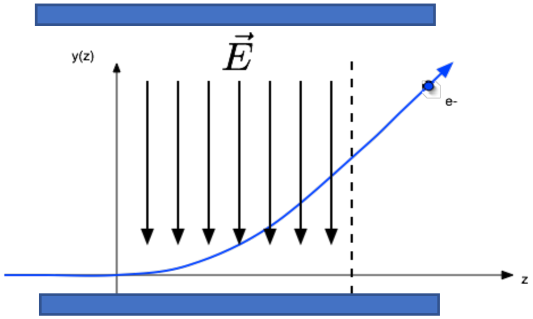
Spring 2018

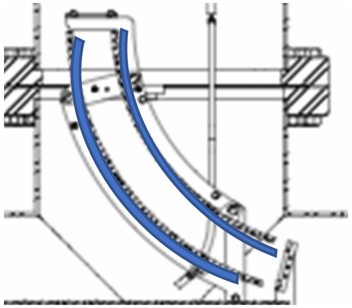
Need for steering and focusing

Lorentz Force:

$$\boldsymbol{F} = q\,(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

- Motion through static Electric Field (B=0)

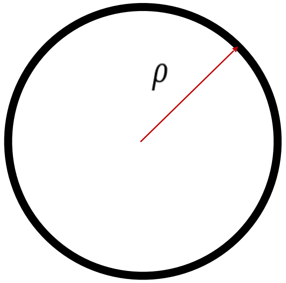




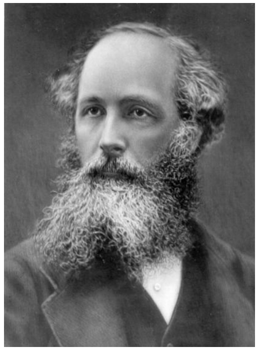
- Motion through Magnetic Field (E=0)

  -Magnetic Rigidity

$$B\rho = \frac{p}{q}$$



Maxwell's Equations



(in vacuum)

Gauss's law

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_o} \qquad \oiint \boldsymbol{E} \cdot d\boldsymbol{A} = \frac{Q}{\varepsilon_o}$$

$$\nabla \cdot \boldsymbol{B} = 0 \qquad \oiint \boldsymbol{B} \cdot d\boldsymbol{A} = 0$$

Faraday's law

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\iint \frac{\partial \boldsymbol{B}}{\partial t} \cdot d\boldsymbol{A}$$

Ampere's law

$$\nabla \times \boldsymbol{B} = \mu_o \boldsymbol{J} + \mu_o \varepsilon_o \frac{\partial \boldsymbol{E}}{\partial t} \qquad \oint \boldsymbol{B} . d\boldsymbol{l} = \mu_o \boldsymbol{I} + \mu_o \varepsilon_o \iint \frac{\partial \boldsymbol{E}}{\partial t} . d\boldsymbol{A}$$

(in magnetic material)

$$\boldsymbol{B} = \mu_0 \boldsymbol{H} + \boldsymbol{M}$$

Ampere's law

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \varepsilon_o \frac{\partial \boldsymbol{E}}{\partial t} \qquad \oint \boldsymbol{H} \cdot d\boldsymbol{l} = \boldsymbol{I} + \varepsilon_o \iint \frac{\partial \boldsymbol{E}}{\partial t} \cdot d\boldsymbol{A}$$

Magnet Types

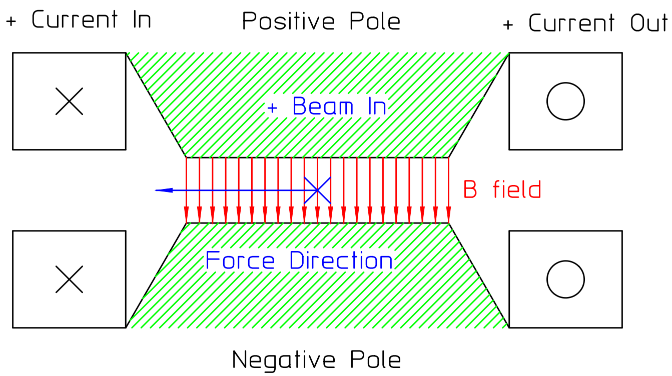
- Dipole
- Quadrupole
- Sextupole
- N-pole
- Combined function
- Solenoid
- …

These can all be:

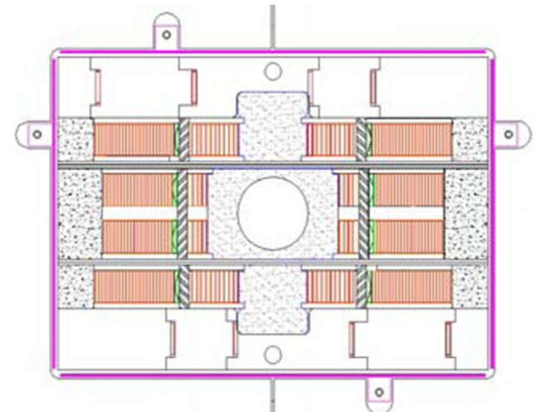
  - Iron-dominated- shaped by the iron
  - Superconducting- shaped by coil placement
  - Superferric-combination of both

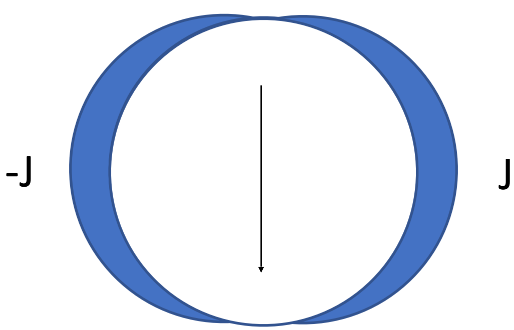
Dipoles

Two poles, a constant field, and steers particle beam

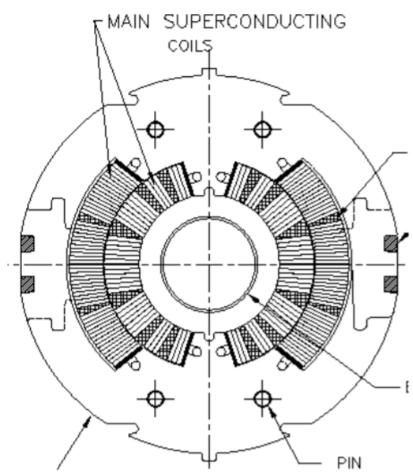


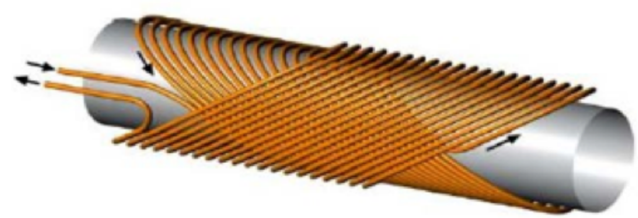

Dipole, FNAL


Block dipole



Cosine θ

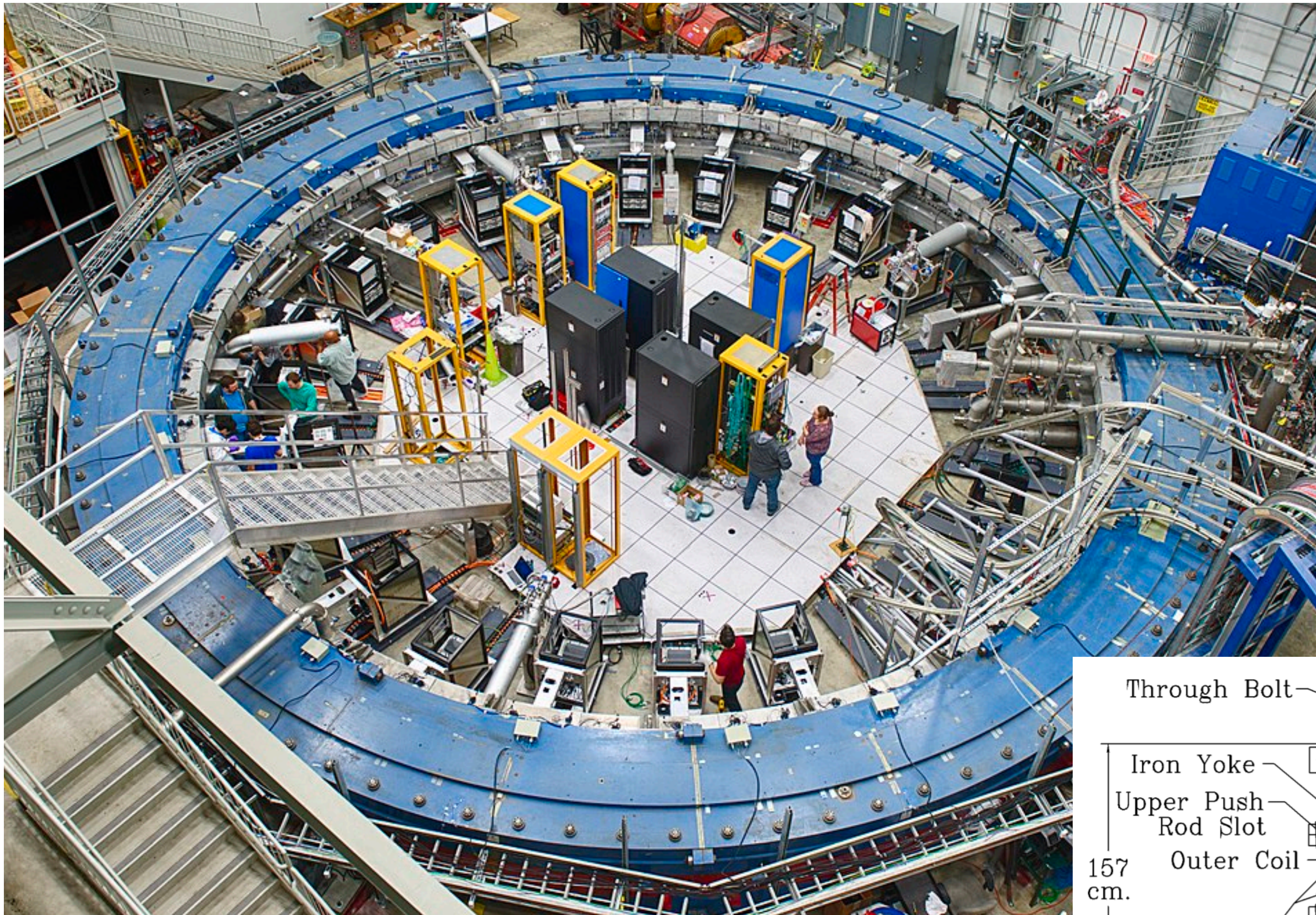


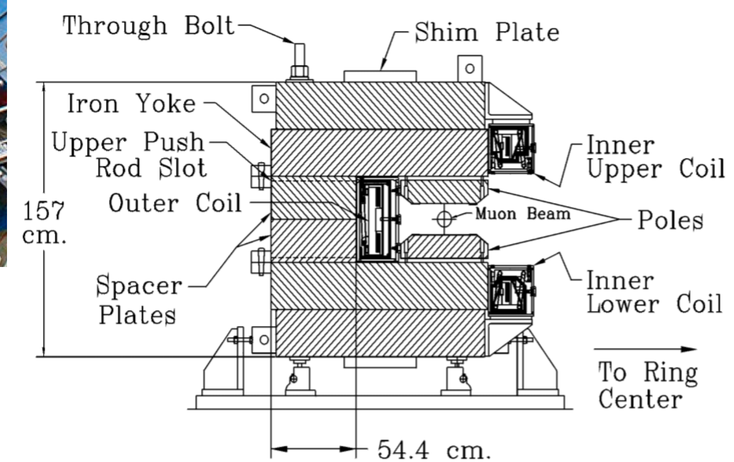

Canted cosine

Strength in iron-dominated dipole

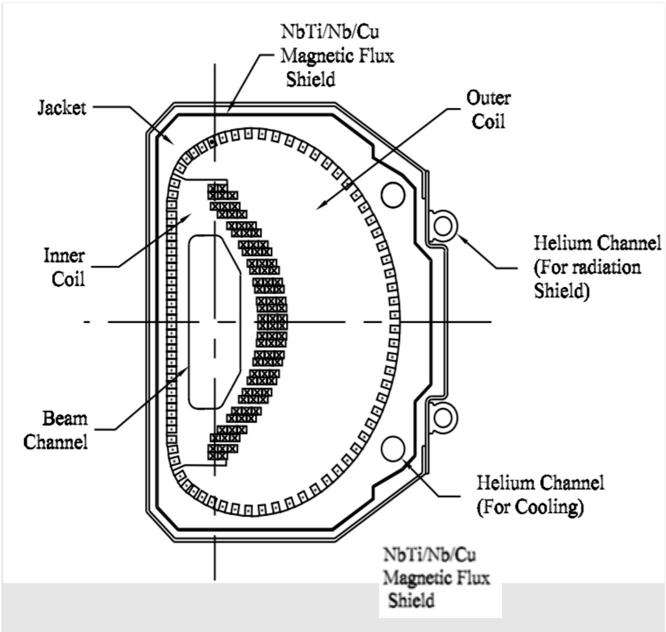
$$B = \frac{2\mu_0 IN}{d}$$

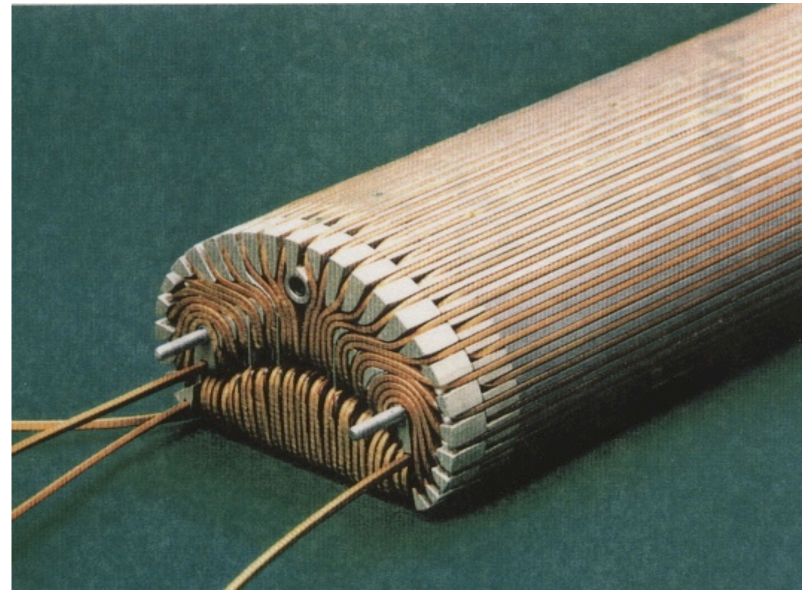
Large dipole for g-2

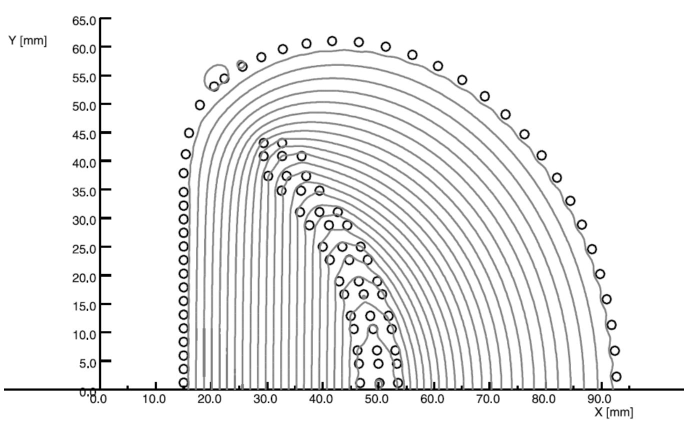


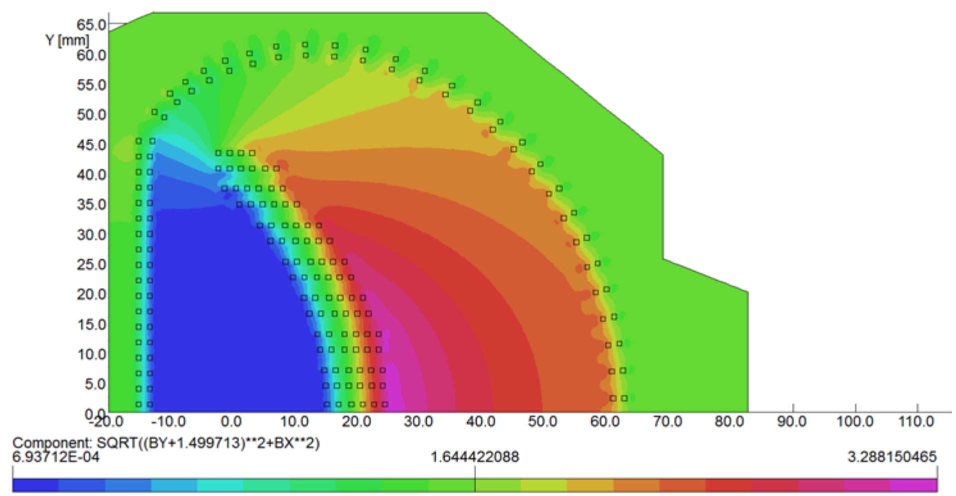


Truncated double cosine magnet



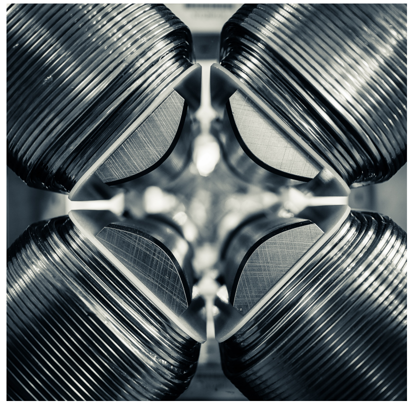


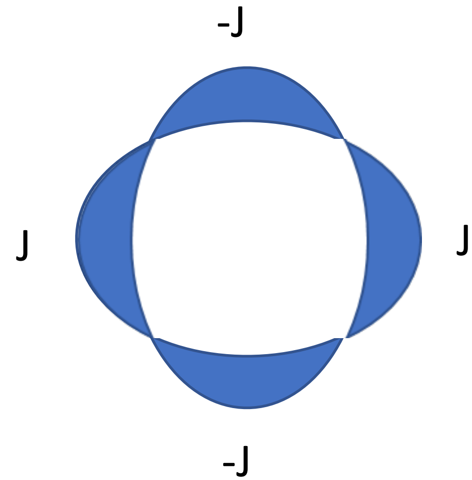




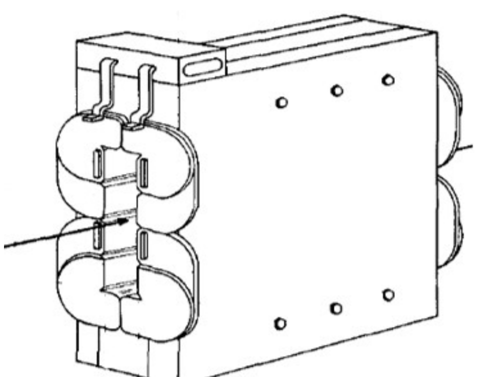
Quadrupole

Four poles, focuses in one plane, defocus in the other

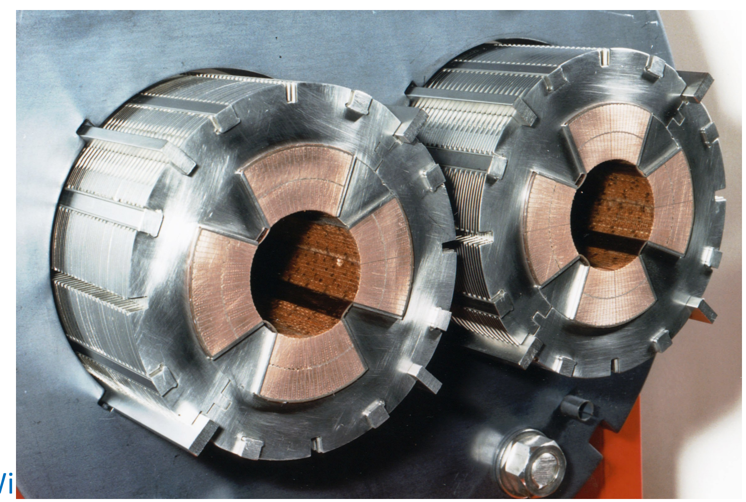




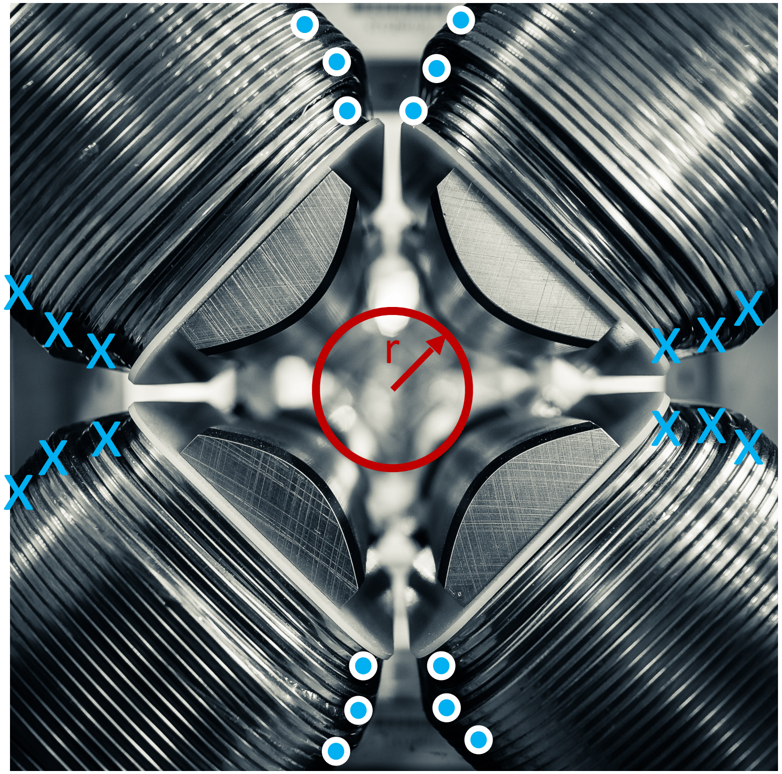
Cosine 2θ



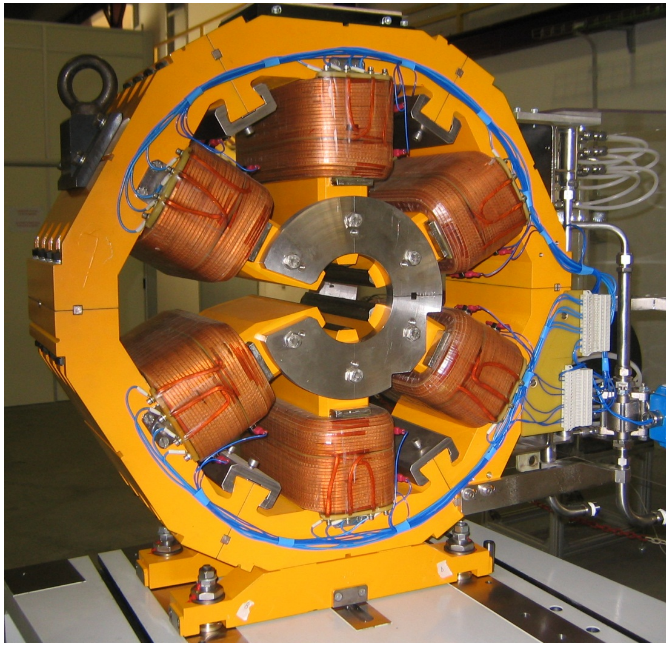
Panofsky Quad



Strength in iron-dominated quadrupole

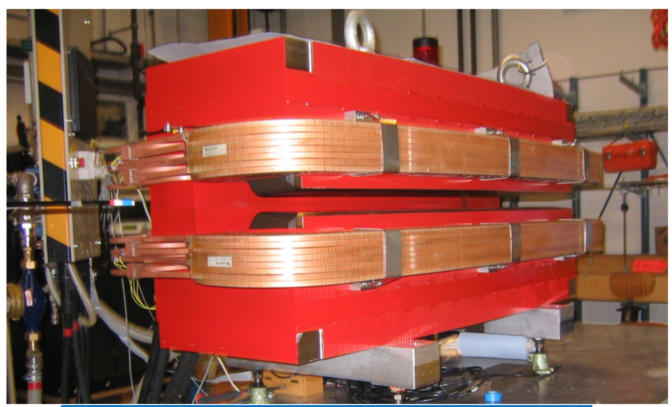


$$B' = \frac{2\mu_0 NI}{r^2} \quad [\frac{T}{m}]$$



Sextupole- six poles

Octupole- eight poles

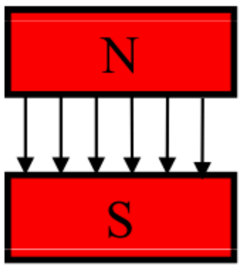


Combined function dipole

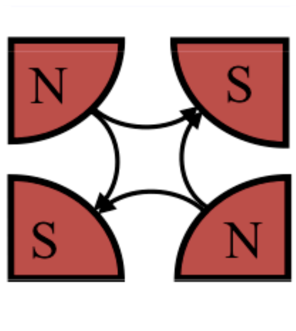
2018

Normal

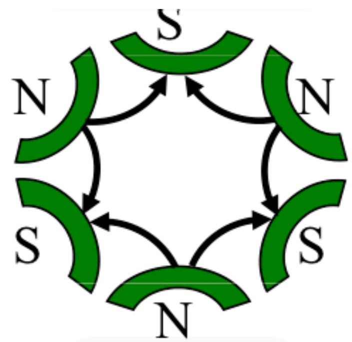
Dipole



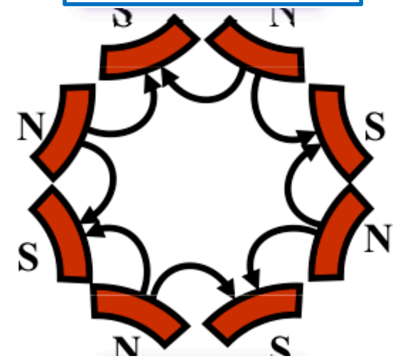
Quadrupole



Sextupole

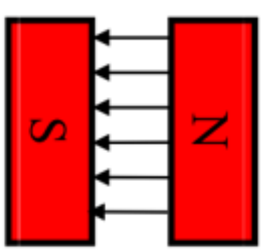


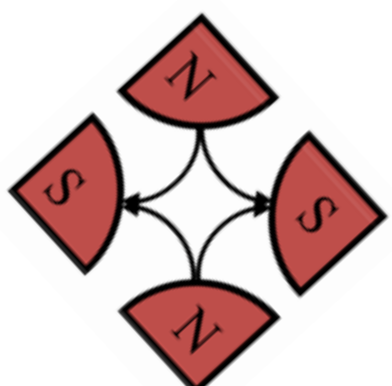
Octupole

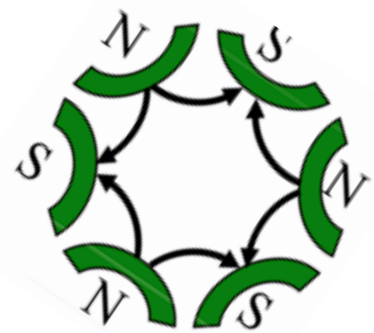


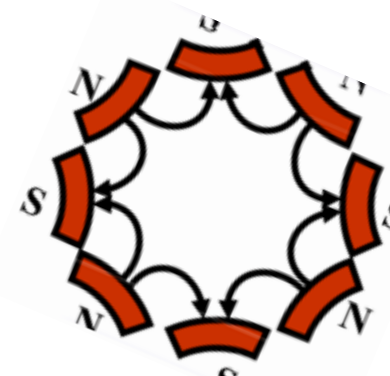
Magnets are called real when the magnetic field is vertical along the centerline, and skew when horizontal along the centerline.

Skew









Multipole Expansion

- The magnetic field can be written as:

$$B(z) = B_y + iB_x = B_0 \sum_n (b_n + ia_n)\,(x + iy)^n$$

where B_0 is the reference field and $z = x + iy$.

The coefficients b_n and a_n correspond to normal and skew terms, and n gives the order of the pole.

n=0 corresponding to a dipole, n=1 a quadrupole, n=2 a sextupole…

Terms of the expansion

- **Dipole (n=0):**

  $B_y + iB_x = B_0(b_0 + ia_0)$, so $B_y = B_0 b_0$ and $B_x = B_0 a_0$. If an ideal normal dipole, $b_0 = 1, a_0 = 0$

  $$B_y = B_0$$
  $$B_x = 0$$

  Quadrupole(n=1):

  $B_x = B_0(b_1 y + a_1 x)$, $B_y = B_0(b_1 x - a_1 y)$. Solve for b_1 by taking the derivative with respect to x:

  $$b_1 = \frac{1}{B_0}\frac{\partial B_y}{\partial x}$$

  $$B_x = B'y$$
  $$B_y = B'x$$

Terms of the expansion

Sextupole (n=2):

$$B_x = B''xy$$

$$B_y = \frac{B''}{2}(x^2 - y^2)$$

Magnetic field quality

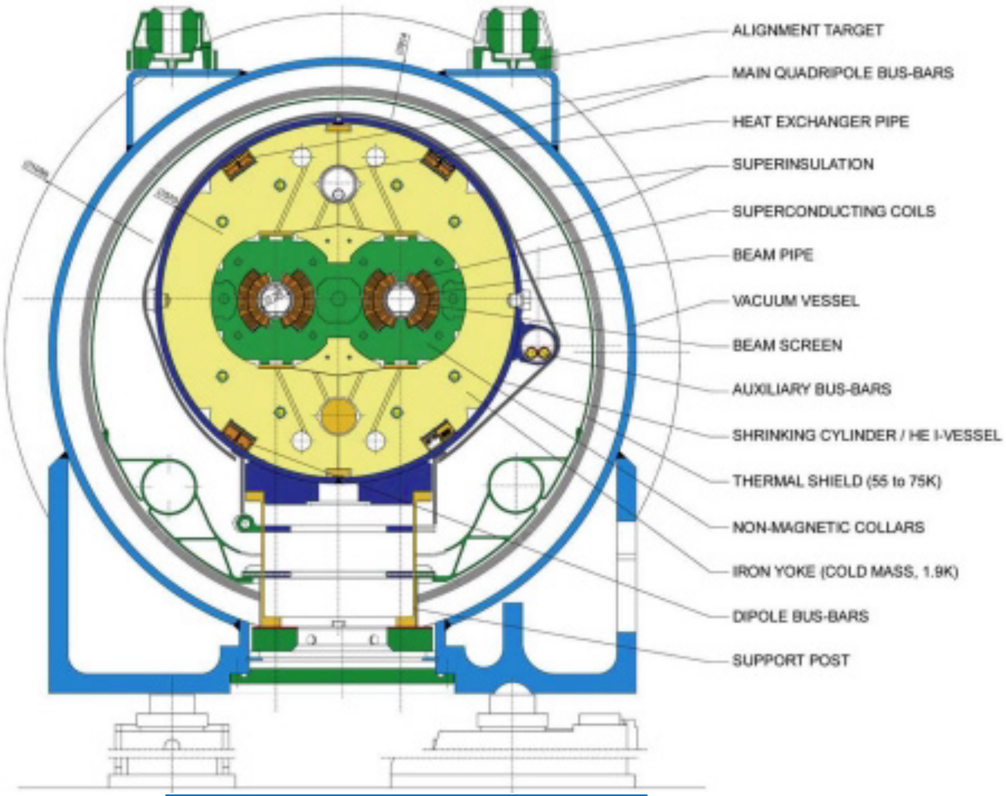
$$B(z) = B_y + iB_x = B_0 \sum_n (b_n + ia_n)\,(x+iy)^n$$

- Typically want “good” field up to ~75% of the beam aperture (R_{ref}).

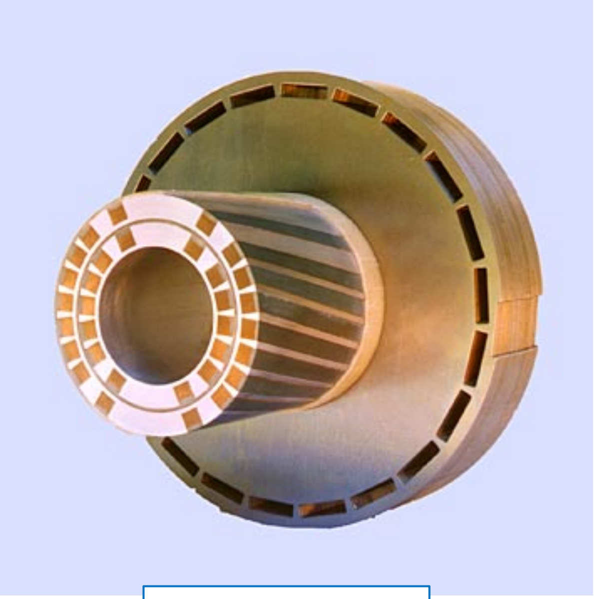
$$b_n = \frac{B_n}{B_0}(R_{ref})^n \qquad a_n = \frac{A_n}{B_0}(R_{ref})^n$$

“Good” if these unwanted multipoles are less than 10^{-4}

More exotic



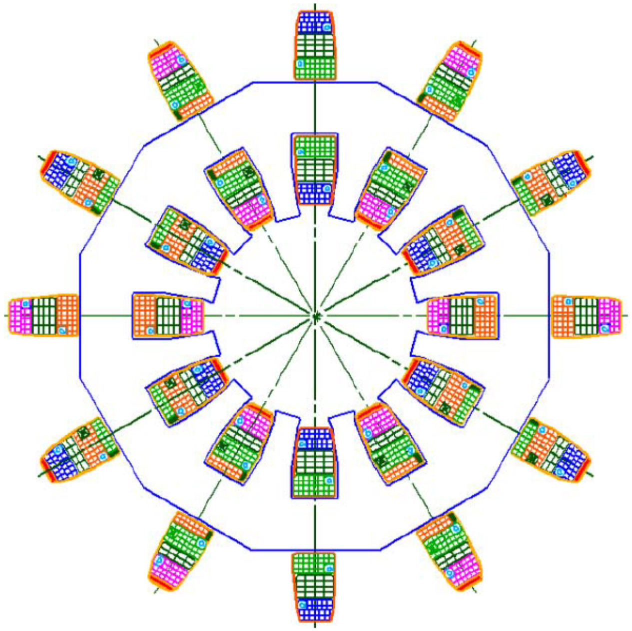
LHC double bore dipole

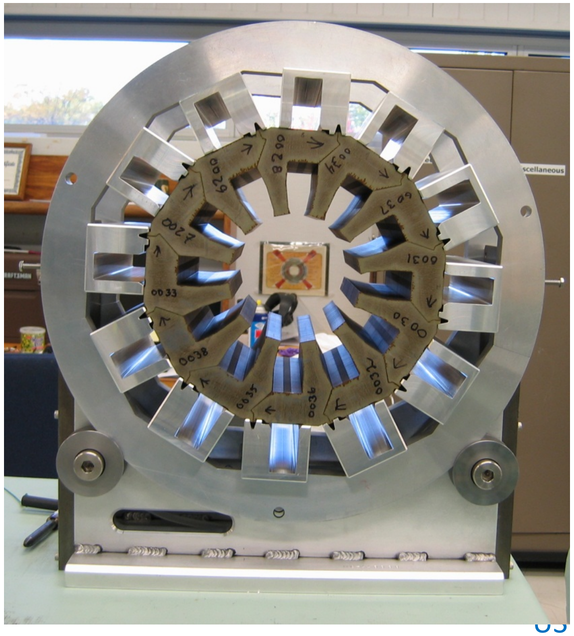


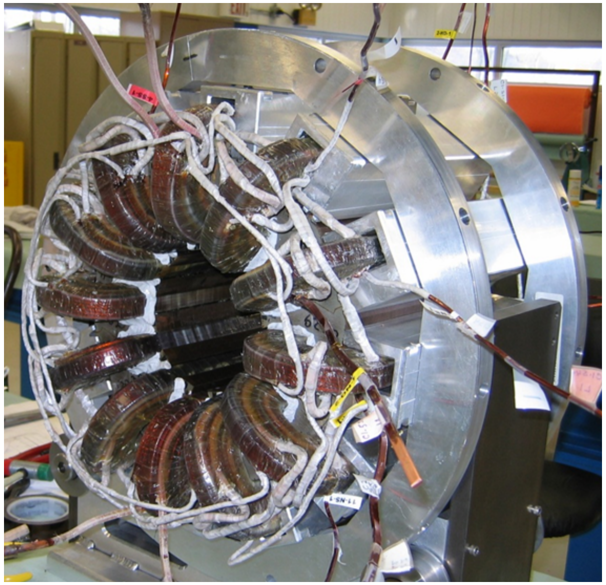
Helical dipole

FNAL Corrector

- Vertical dipole
- Horizontal dipole
- Normal Quadrupole
- Skew Quadrupole
- Normal Sextupole
- Skew Sextupole

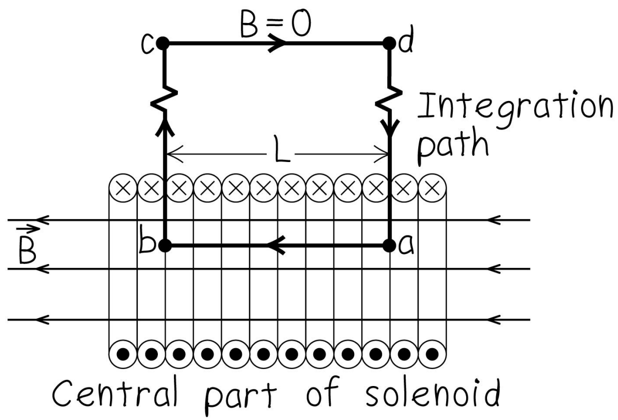


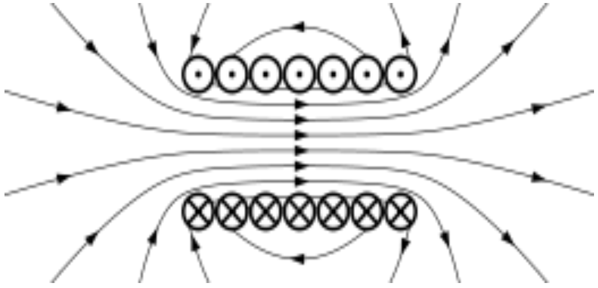




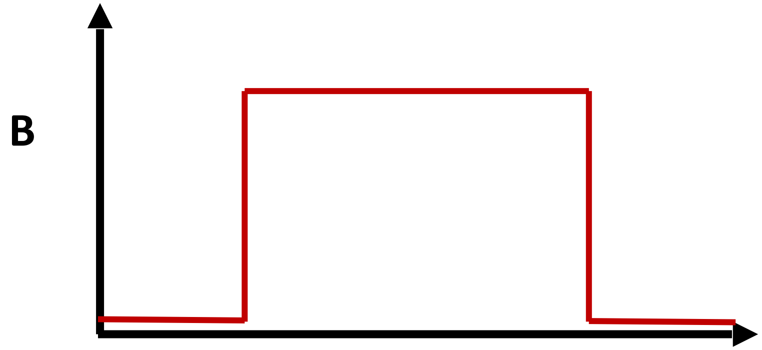
Solenoid

$$B = \mu_0 \frac{N}{L} I$$

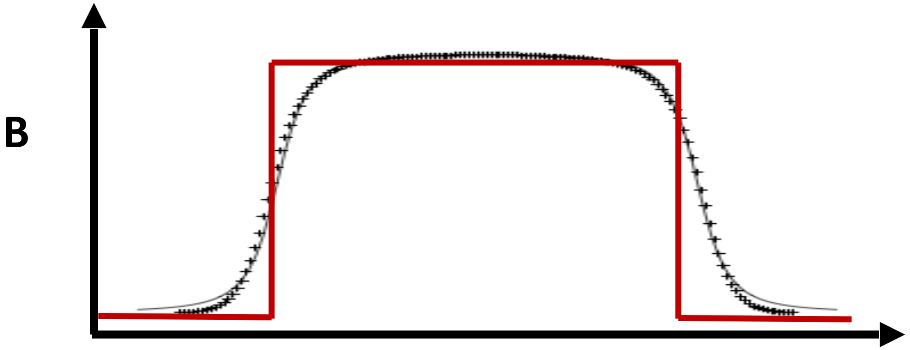




Hard edge

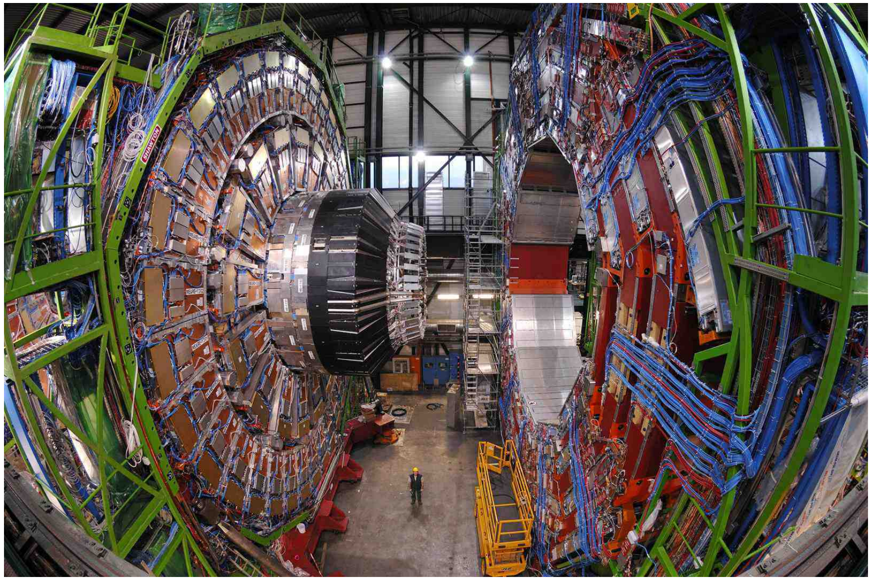


Real

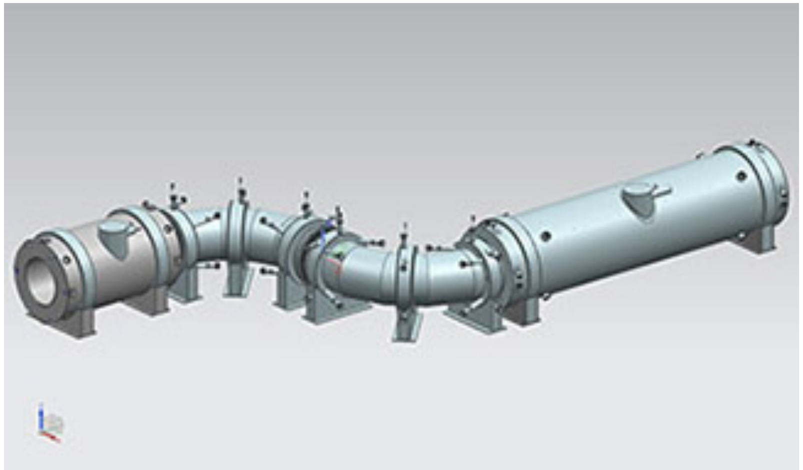


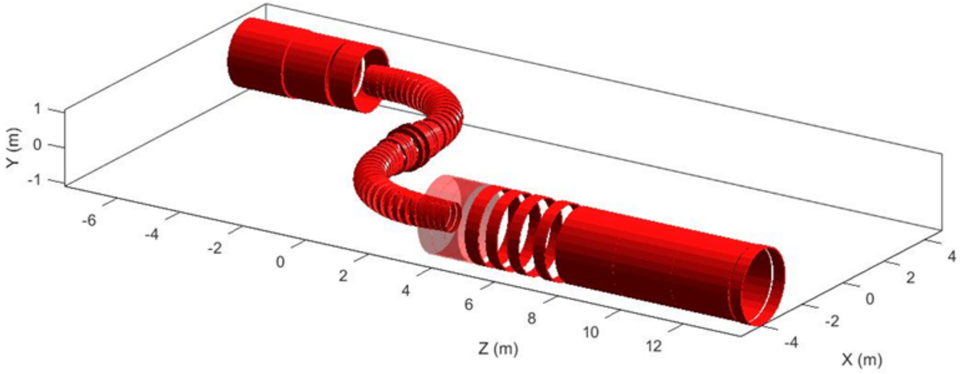
Solenoids in experiments

CMS



Mu2e





Muon Collider?