Physics Review

- Newtonian Mechanics
  - Gravitational vs. Electromagnetic forces
  - Lorentz Force
- Maxwell's Equations
  - Integral vs. Differential
- Relativity (Special)

Newtonian Mechanics

- $v = dx/dt$
- $p = mv$
- $F = dp/dt$
- $dW = F\, dx$
- $F_g = G\, Mm/r^2$ [$F_e = 1/4\pi\epsilon_0\, Qq/r^2$ $F_b = qv \times B$, etc.]
- The Simple Harmonic Oscillator + Phase Space

Simple Harmonic Motion

$$\ddot{x} = -kx \qquad\qquad \ddot{x} + kx = 0$$

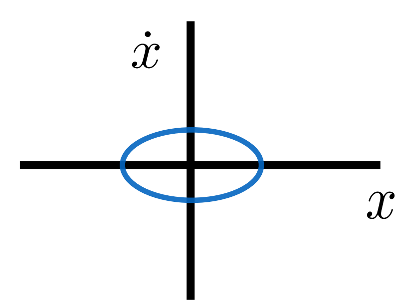
$$x = a\sin(\omega t) + b\cos(\omega t) = c\sin(\omega t + \delta)$$

$$\dot{x} = c\,\omega\cos(\omega t + \delta)$$

$$\ddot{x} = -c\,\omega^2\sin(\omega t + \delta) = -\omega^2 x$$

$$\omega = \sqrt{k}$$

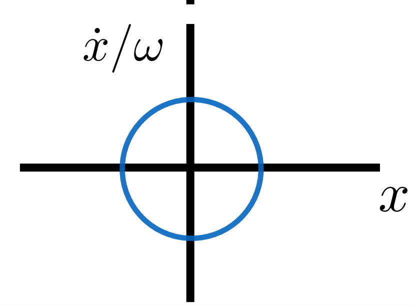
$$x^2 + \frac{1}{\omega^2}\dot{x}^2 = c^2$$



ellipse

phase space diagrams

$$x^2 + (\dot{x}/\omega)^2 = c^2$$



circle

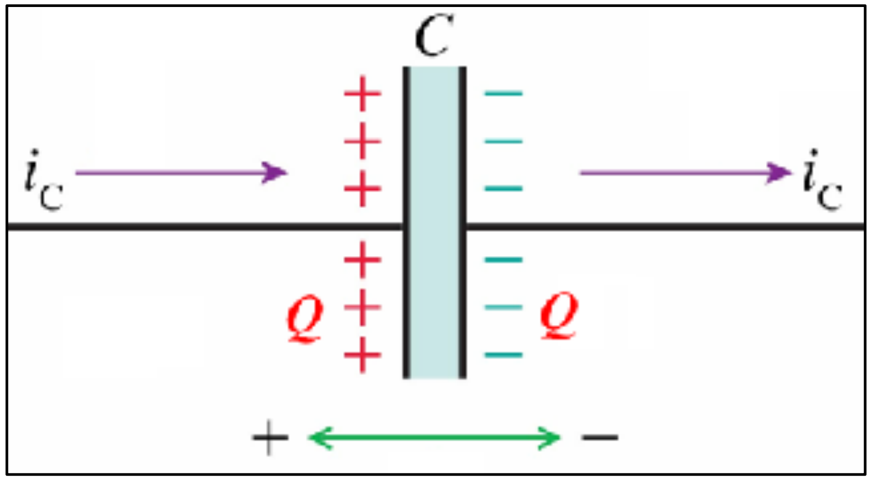
Maxwell's Equations

- Integral Form
- Differential Form
- One Consequence: EM Waves
  - speed of waves given by $c = (\mu_0 \epsilon_0)^{-1/2}$
- Another Consequence:
  - If μ_0 , ϵ_0 are fundamental quantities, same in all reference frames, then so should be the speed of light!

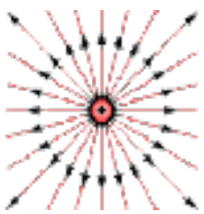
Flux:

$$\Phi_B \equiv \oint_{surface} \vec{B} \cdot d\vec{A}$$

$$\Phi_E \equiv \oint_{surface} \vec{E} \cdot d\vec{A}$$



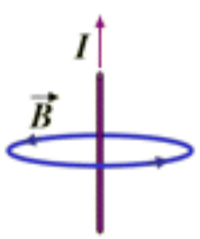
Maxwell's Equations:



$$(\Phi_E)_{closed\ \ surface} = \frac{Q_{encl}}{\epsilon_0}$$ Gauss' Law

Completed by Maxwell

$$(\Phi_B)_{closed\ \ surface} = 0$$



$$\oint_{loop} \vec{B} \cdot d\vec{s} = \mu_0 \left(I_{enclosed} + \epsilon_0 \left(\frac{d\Phi_E}{dt} \right)_{through\ \ loop} \right)$$ Ampere's Law

$$\oint_{loop} \vec{E} \cdot d\vec{s} = -\left(\frac{d\Phi_B}{dt} \right)_{through\ \ loop}$$ Faraday's Law

Differential Relationships

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \hat{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - \hat{j}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\begin{aligned}
(\Phi_E)_{closed\ \ surface} &= \frac{Q_{encl}}{\epsilon_0} \\
(\Phi_B)_{closed\ \ surface} &= 0 \\
\oint_{loop} \vec{B} \cdot d\vec{s} &= \mu_0 \left(I_{enclosed} + \epsilon_0 \left(\frac{d\Phi_E}{dt}\right)_{through\ \ loop} \right) \\
\oint_{loop} \vec{E} \cdot d\vec{s} &= -\left(\frac{d\Phi_B}{dt}\right)_{through\ \ loop}
\end{aligned}$$

$$\nabla \cdot \vec{E} = \rho/\epsilon_0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Stoke's Theorem:

$$\iint_S \nabla \times \vec{A} \cdot d\vec{S} = \oint_{\partial S} \vec{A} \cdot d\vec{r}$$

Wave Equation and the Speed of Propagation

Suppose in free space, no current sources…

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad\qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

in general:
$$\nabla \times \nabla \times \vec{f} = \nabla(\nabla \cdot \vec{f}) - \nabla^2 \vec{f}$$

so,
$$\nabla \times \nabla \times \vec{B} = \nabla(\nabla \cdot \vec{B}) - \nabla^2 \vec{B} = -\nabla^2 \vec{B}$$

$$-\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial (\nabla \times \vec{E})}{\partial t} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

thus,
$$\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$ and, likewise, $$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Wave Equation and the Speed of Propagation

$$\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \quad \text{and} \quad \nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

wave equation

Example: let $B = b\cos(\omega t - kx) = b\cos(2\pi f t - 2\pi x/\lambda)$

$$d^2B/dx^2 = -k^2 B$$

$$d^2B/dt^2 = -\omega^2 B$$

$$d^2B/dx^2 = -k^2 B = \mu_0\epsilon_0(-\omega^2 B)$$

$$\mu_0\epsilon_0 = (k/\omega)^2 = 1/(\lambda f)^2 = 1/v_{wave}^2$$

$$speed = 1/\sqrt{\mu_0\epsilon_0} \equiv c$$

$c = 1/(4\pi \times 10^{-7} \;\; \times \;\; 8.8\times 10^{-12})^{1/2}$ m/s = 3.0×10^{8} m/s

Maxwell's Equations

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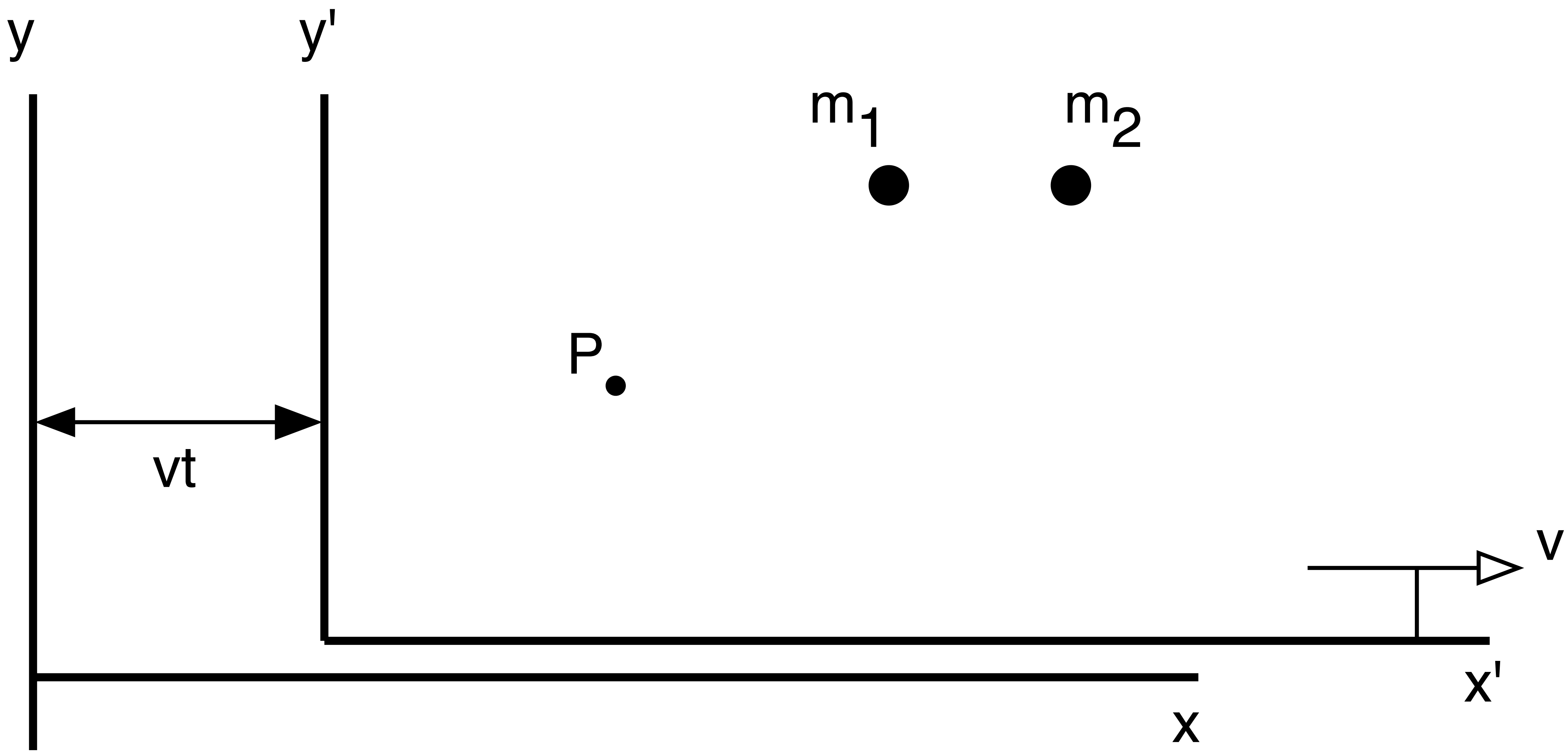
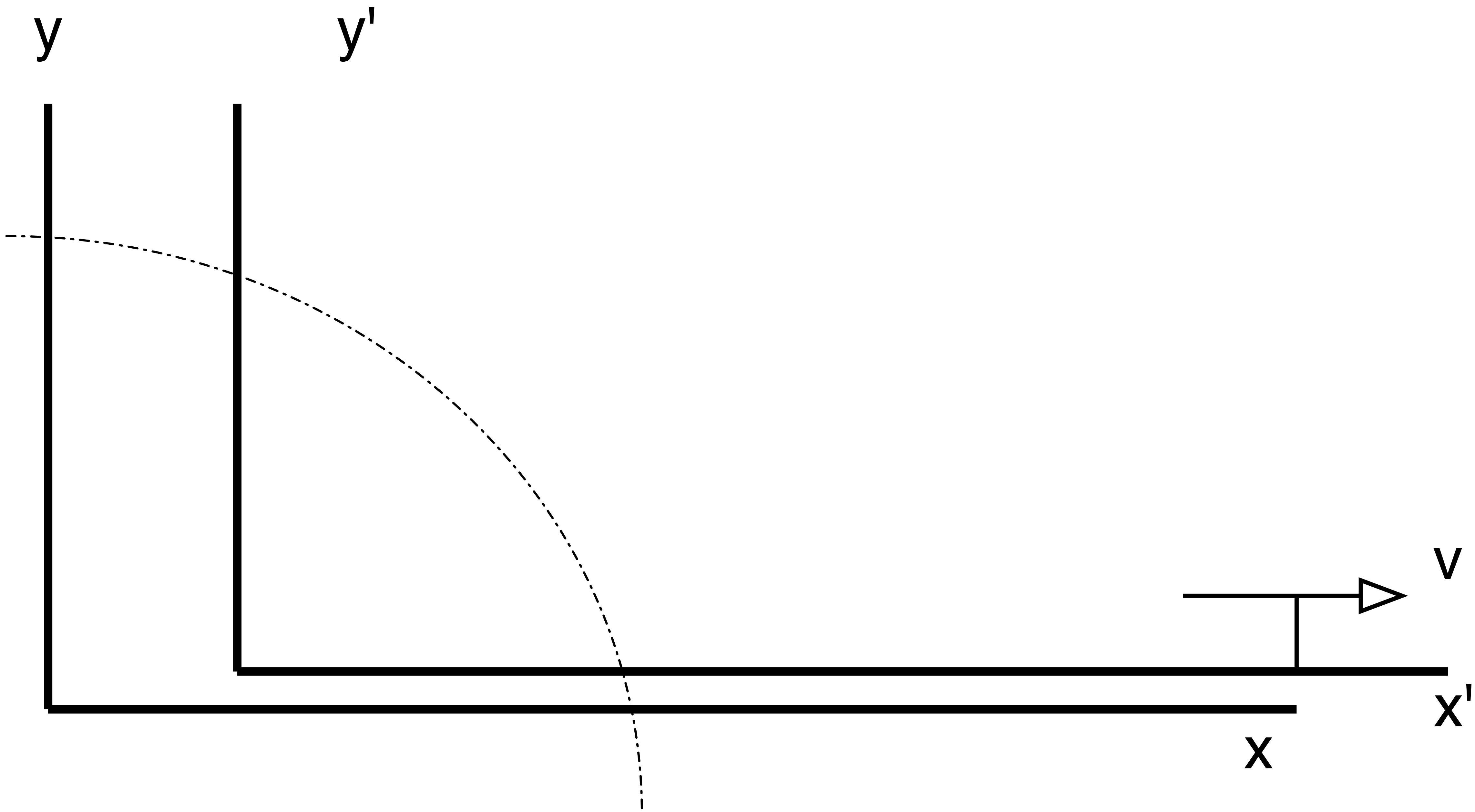
Special Relativity

- The Principle of Relativity
  - The Laws of Physics same in all inertial reference frames
- The Problem of the Velocity of Light
- Simultaneity
- Lengths and Clocks
- $E=mc^2$
- Differential Relationships


y
y'
m1
m2
P
vt
v
x'
x

Simultaneity


y
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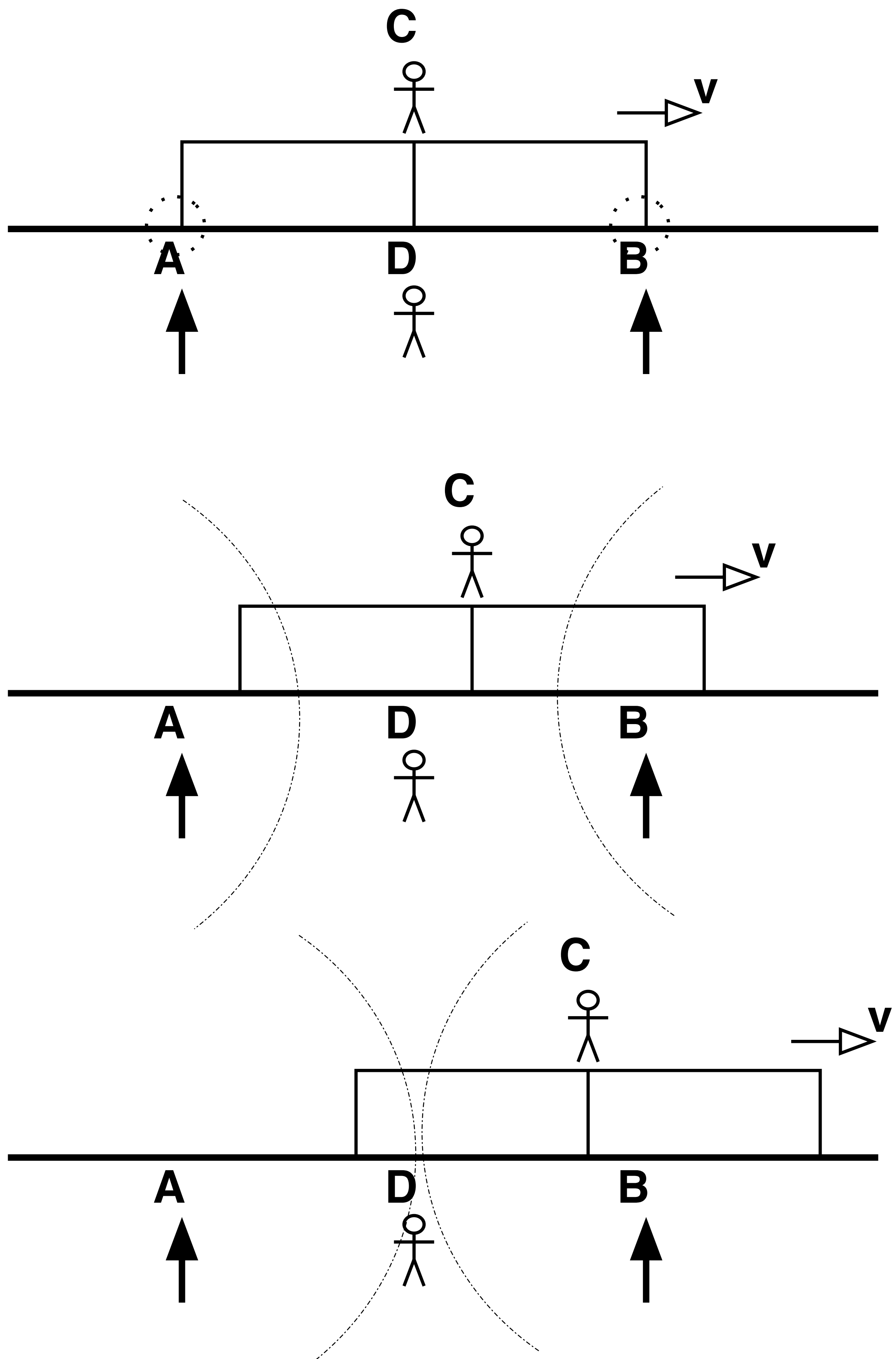

C
v
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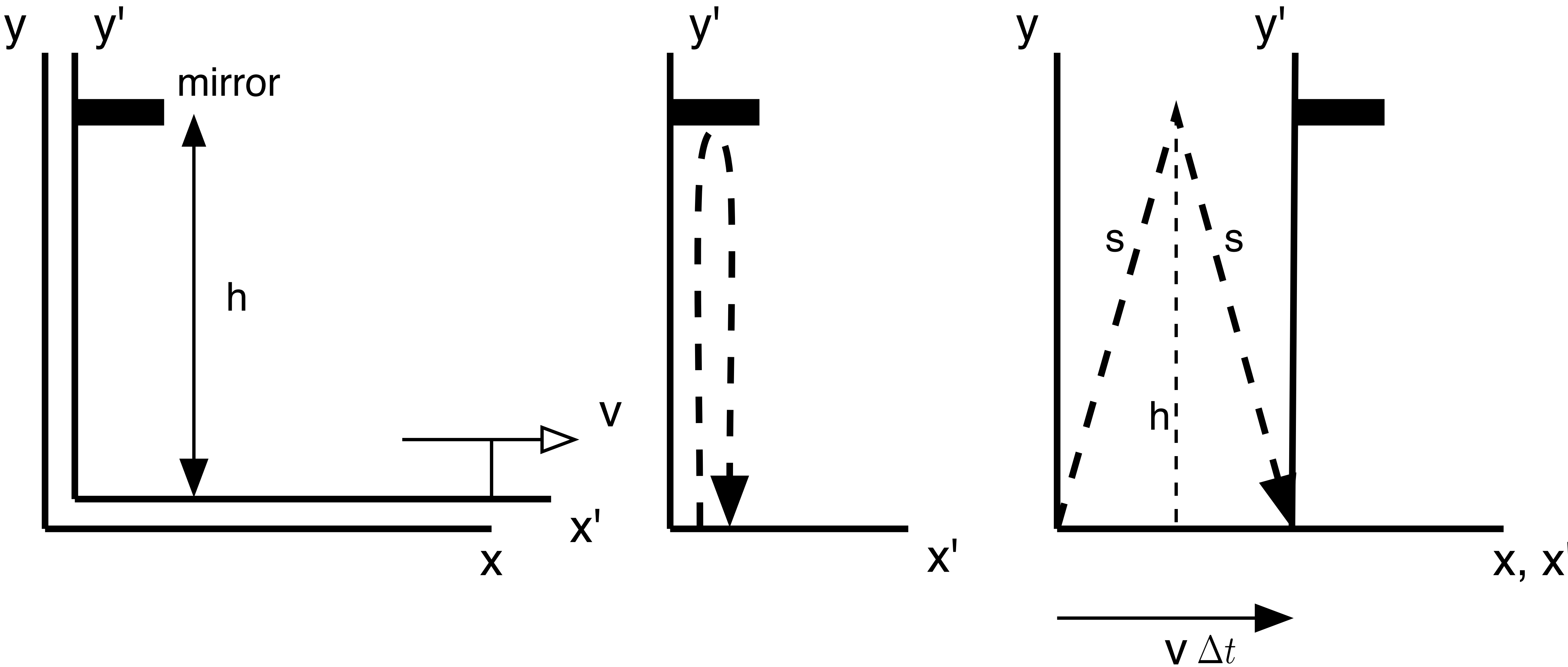
Lengths and Clocks



(a) (b) (c)

Relativistic Momentum

Principal of relativity: All the laws of physics (not just Newton's laws) are the same in all inertial reference frames.

Ex:

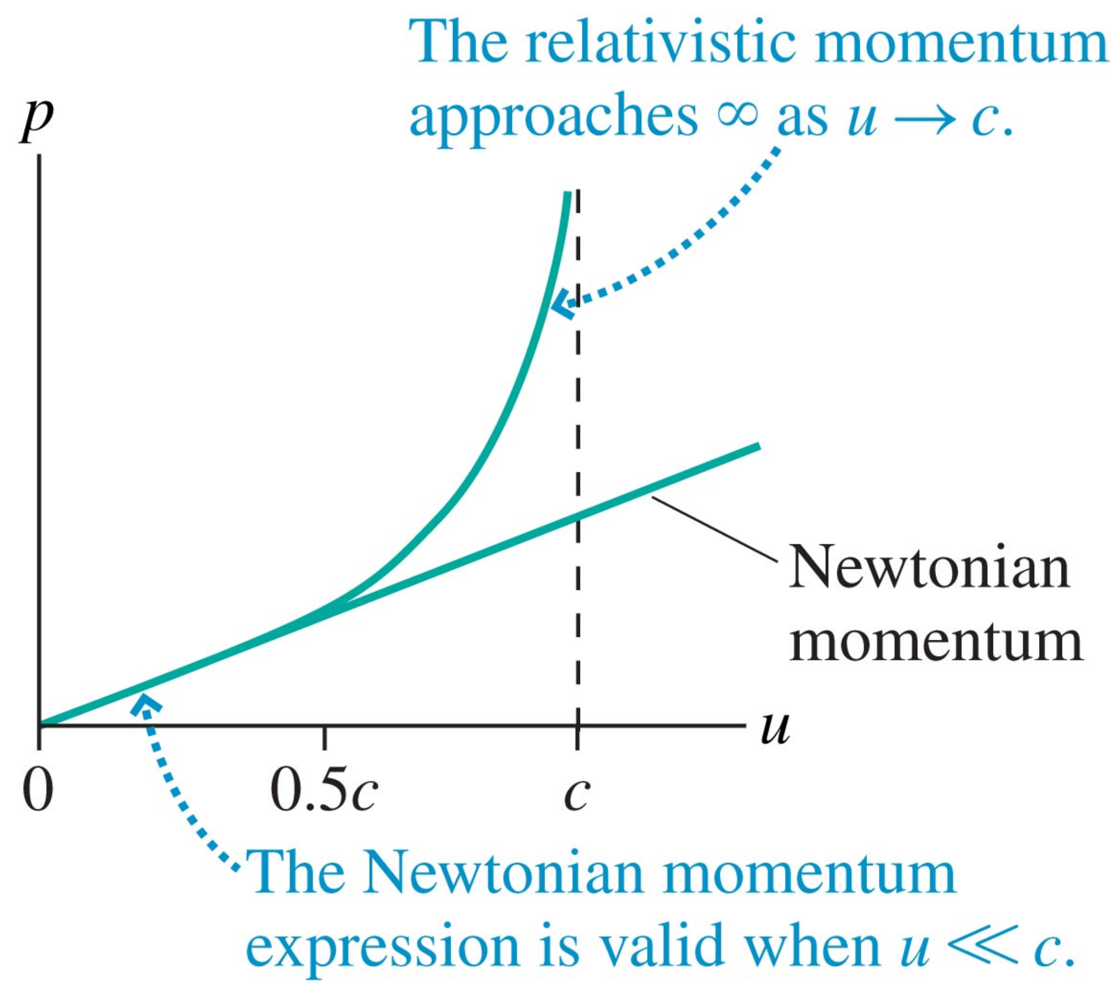
$F = \Delta p/\Delta t$

The law of conservation of momentum is valid in all inertial reference frames ***if*** the momentum of each particle (with mass *m* and speed *u*) is ***re-defined*** by:

$$p = \gamma m u$$

where

$$\gamma = \frac{1}{\sqrt{1-u^2/c^2}}$$



Relativity Summary

Time dilation: $$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}} = \gamma \Delta t'$$

Length contraction: $$L = L'\sqrt{1 - v^2/c^2} = L'/\gamma$$

Lorentz transformation of velocities: $$u' = \frac{u - v}{1 - uv/c^2} \quad \text{or} \quad u = \frac{u' + v}{1 + u'v/c^2}$$

Relativistic Momentum: $$p = \frac{mu}{\sqrt{1 - u^2/c^2}} = \gamma m u$$

$E = mc^2$

- The Laws of Physics, and redefining the momentum
- What about Energy?
- Energy-momentum relationship

Work done by a Force Acting on a Mass

- The work done on a particle is given by

$$\Delta W = \int F \cdot ds = \int dp/dt \cdot ds = \int (ds/dt) dp = \int v \cdot dp.$$

  Check: if $p = mv$ then, starting from rest,
  $\Delta W = \int v dp = \int v \; m dv = \frac{1}{2} m v^2$.
- But, using our new definition of momentum, $p = \gamma m v$, then

$$\Delta W = \int v \; d(\gamma m v) = \int (v/c) \; m \; d(\gamma v/c) c^2 = mc^2 \int \beta d(\beta\gamma)$$

$$\gamma^2 = 1 + (\beta\gamma)^2 \qquad \longrightarrow \qquad d\gamma = \beta d(\beta\gamma)$$

- So finally, our original integral becomes,

$$\Delta W = mc^2 \int \beta d(\beta\gamma) = mc^2 \int d\gamma = (\gamma_{final} - \gamma_{initial}) mc^2$$

- The previous equation tells us that as we do work on a particle its energy will change by an amount $\Delta E = \Delta W = \Delta\gamma mc^2$. Thus, the energy of a particle should be defined as

$$E = \gamma\, mc^2.$$

- If the particle starts from rest, then $\gamma_{initial} = 1$, and its energy is $E = mc^2$. As it speeds up its kinetic energy will be

$$KE = \Delta W = (\gamma - 1)mc^2, \text{where here } \gamma \equiv \gamma_{final}.$$

- So we see that the energy is a combination of a "rest energy" and a "kinetic energy":

$$E = \gamma mc^2 = mc^2 + (\gamma - 1)mc^2.$$

If no work were done ($\Delta W = 0$), and the particle were still at rest, the particle would *still* have energy (rest energy):

$$E_0 = mc^2 \quad \rightarrow \quad \text{mass is energy!}$$

Relativity Summary

Time dilation: $\Delta t = \dfrac{\Delta t'}{\sqrt{1-v^2/c^2}} = \gamma \Delta t'$

Length contraction: $L = L'\sqrt{1-v^2/c^2} = L'/\gamma$

Lorentz transformation of velocities: $u' = \dfrac{u-v}{1-uv/c^2}$ or $u = \dfrac{u'+v}{1+u'v/c^2}$

Relativistic Momentum: $p = \dfrac{mu}{\sqrt{1-u^2/c^2}} = \gamma mu$

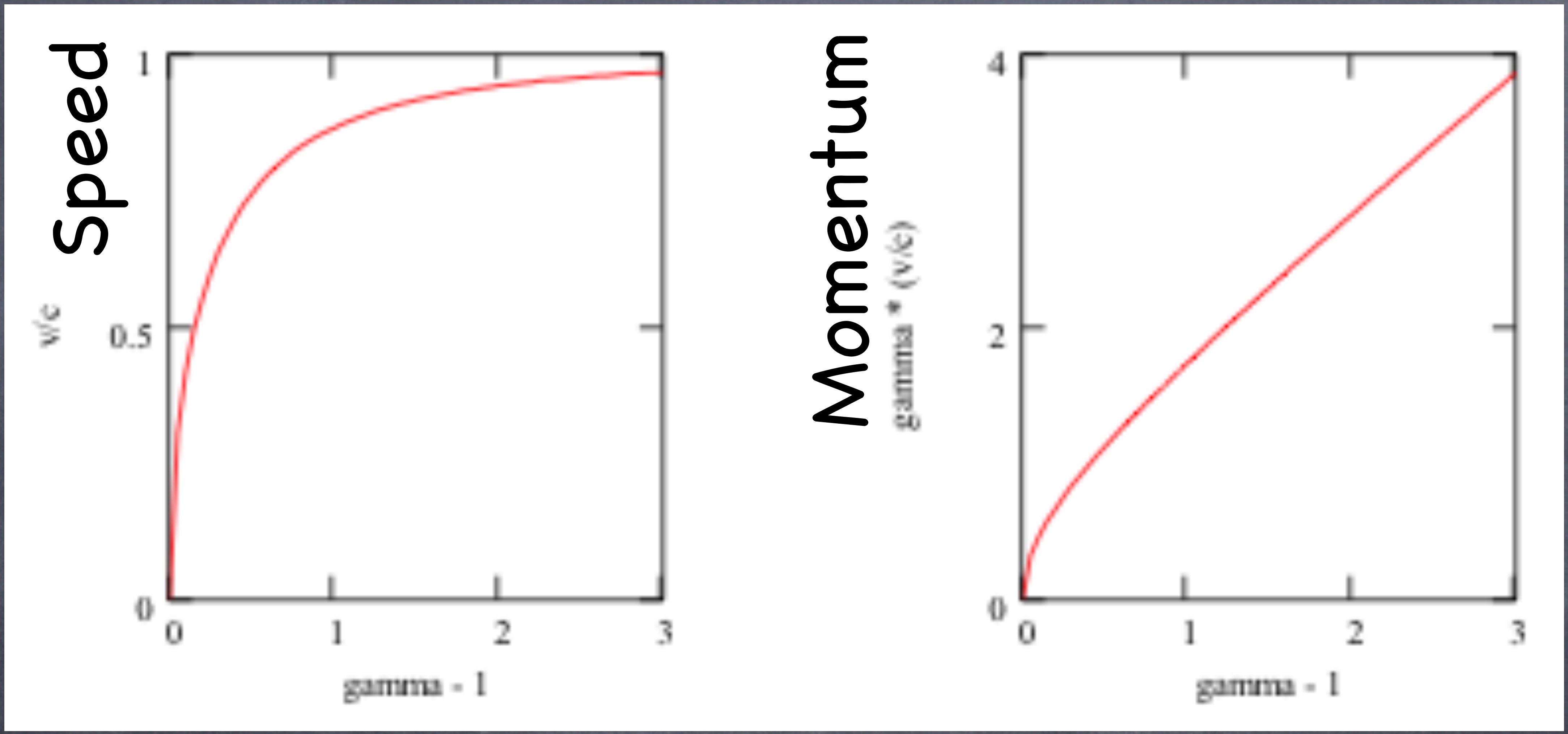
Relativistic Energy: $E = \dfrac{mc^2}{\sqrt{1-u^2/c^2}} = \gamma mc^2$

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$$

The total energy is made up of two contributions

$$E = \underbrace{mc^2}_{\text{Rest energy } E_0} + \underbrace{(\gamma - 1)mc^2}_{\text{Kinetic energy } K}$$

Speed, Momentum, vs. Energy



Kinetic Energy Kinetic Energy

Electron:	0	0.5	1.0	1.5	MeV
Proton:	0	1000	2000	3000	MeV

Relationships Differential, and Otherwise

- β = v/c $\qquad\qquad$ $\gamma = 1/(1-\beta^2)^{1/2}$
- F = dp/dt $\qquad\qquad$ $p = \gamma mv = \beta\gamma mc$
- $E = \gamma mc^2 = mc^2 + W$ $\qquad\qquad$ $W = (\gamma-1)mc^2$
- $\gamma = 1/(1-\beta^2)^{1/2}$ $\qquad$ $\beta^2 = 1 - 1/\gamma^2$ $\qquad$ $\gamma^2 = 1 + (\beta\gamma)^2$
- given E, find p; given p, find W; given W, find v
- find: $d\beta/\beta$ in terms of dp/p; dE/E in terms of dp/p
- find: $d\beta/\beta$ in terms of dW/W; dW/W in terms of dE/E
  - (coefficients only to contain factors of β, γ)

Quantization

- Accelerator and Beam Physics are typically "classical" physics; occasionally (esp. wrt synchrotron radiation and light sources), quantum physics comes into play
- Photon: $E = h\nu = hc/\lambda = pc$ (i.e., $m = 0$)
- De Broglie Wavelength: $\lambda = h/p$
- $\Delta x \cdot \Delta p_x = h/4\pi$ (Uncertainty Principle)