Fundamentals of Accelerator Physics and Technology with Simulations and Measurements Lab

U.S. Particle Accelerator School — Syphers, Harms
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Computer Session - Introduction to ***Mathcad***

Mathcad[1] is a powerful software calculational tool. It allows the user to create, solve and analyze a variety of mathematical problems. In this session some of the basic capabilities of Mathcad are introduced and then applied to calculate some real accelerator parameters.

Exercise 1 - Basic equation editing, calculations, and graphing

a) Type **ctrl-shift p*7^2 space space /13=**

Mathcad should respond with an equation and solution like this

$$\frac{\pi \cdot 7^2}{13} = 11.841$$

b) Let's create two variables and solve the same equation. Copy the equation you just created to another location in the worksheet. You can do this by clicking anywhere in the equation field, press the space bar until the entire equation is within the blue editing lines, and then use the copy and paste functions. Replace the '7' with a variable, say 'a', and '13' with another such as 'β' in the equation by typing over them. One way to insert the greek character *beta* is to select **Toolbars** from the **View** menu, hold down the mouse button and release it when the cursor is on **Greek**, then select the character you wish. Next, somewhere above the equation, type **a:7** and **β:13**. The ensuing result should look something like this:

$$a := 7$$
$$\beta := 13$$
$$\frac{\pi \cdot a^2}{\beta} = 11.841$$

Try typing in different values for a and β.
c) Mathcad does iterative calculations using *Range Variables*. Whenever

[1] Mathsoft, Inc.

Mathcad encounters a Range Variable it calculates the equation for every value of this special variable.

Copy our existing equation and variable '🚲' and paste them into another area. Now turn 'a' into a range variable by typing **a:7,14;70**. Matchcad should respond with

$$a := 7, 14 .. 70$$

Mathcad has now defined 'a' to have the values 7,14,21 ,...70. Our equation is now calculated for each value of 'a' and the results are listed below. Double clicking in the table of answers opens a window of display options. Experiment with these to see what Mathcad can do.

$$\frac{\pi \cdot a^2}{\beta} =$$

11.841
47.366
106.572
189.462
296.035
426.29
580.228
757.849
959.152
$1.184 \cdot 10^3$

d) With Mathcad you can define a function and solve for it. Once again let's use our exþression from above. Start by defining a function by typing **A(a):**.

A placeholder appears to the right of what has been typed. Either type or paste in the equation we have been using. We also need to select some values for a, so we could again type **a:7,1 4;**70. Finally we need to calculate some solutions to our function so type A(a)=. You should see something like:

$$A(a) := \frac{\pi \cdot a^2}{\beta}$$

$$a := 7, 14 .. 70$$

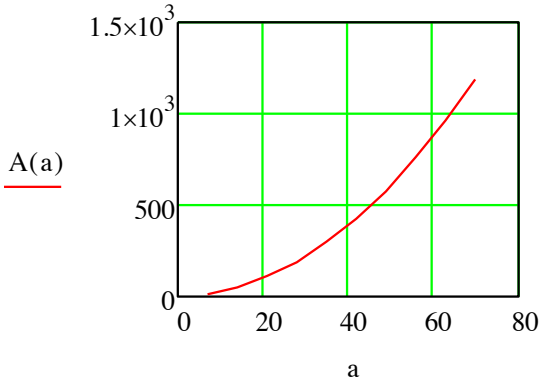
$A(a) =$

11.841
47.366
106.572
189.462
296.035
426.29
580.228
757.849
959.152
$1.184 \cdot 10^3$

Another way to solve for this function is simply to place the value for 'a' that you wish to solve for within the parentheses:

$$A(84) = 1.705 \times 10^3$$

e) Mathcad has graphing capabilities as well. Let's graph the value of A(a) as a function of a. From the **Insert** menu select **Graph** and choose **X-Y Plot**. Placeholders are displayed for the X and Y axes. Type or copy/paste 'a' for the X one and 'A(a)' for the Y axis. A plot such as the one below should result:



Double clicking in the field of the graph opens a window of customizing options. Check them out. Experiment with other features of the plot field.

Exercise 2 – Matrices

a) Click on **View > Toolbars > Matrix** to open the Matrix Toolbar. To create a matrix either click on the matrix icon in the toolbar (upper left corner) or type **Ctrl-M**. Create the following 2X2 matrix:

$$\begin{pmatrix} 7 & 9 \\ 14 & 12 \end{pmatrix}$$

b) Mathcad can do a variety of operations on a matrix. Open the Matrix and Symbolic toolbars. To **Transpose** a matrix use the space bar to enclose the entire matrix within the editing bars. Select the *Symbotic Matrix Transpose* icon (lower left corner) from the Symbolic Toolbar then click **Enter**. You should get:

$$\begin{pmatrix} 7 & 9 \\ 14 & 12 \end{pmatrix}^{T} \rightarrow \begin{pmatrix} 7 & 14 \\ 9 & 12 \end{pmatrix}$$

c) To **Invert** the matrix, follow the same procedure once more. This time select the *Symbolic Matrix Inverse* icon (lower right corner) from the Symbolic toolbar. Mathcad should produce:

$$\begin{pmatrix} 7 & 9 \\ 14 & 12 \end{pmatrix}^{-1} \rightarrow \begin{pmatrix} -\frac{2}{7} & \frac{3}{14} \\ \frac{1}{3} & -\frac{1}{6} \end{pmatrix}$$

d) To find the **Determinant**, follow the same procedure but select the *Symbolic Matrix Determinant* icon (lower right corner) from the Symbolic toolbar. Mathcad should produce:

$$\left|\begin{pmatrix} 7 & 9 \\ 14 & 12 \end{pmatrix}\right| \rightarrow -42$$

e) Create another matrix, M, and perform arithmetic operations on the two. For example:

$$M := \begin{bmatrix} 1 & 0 \\ -\left(\frac{1}{20}\right) & 1 \end{bmatrix}$$

$$M \cdot \begin{pmatrix} 7 & 9 \\ 14 & 12 \end{pmatrix} = \begin{pmatrix} 7 & 9 \\ 13.65 & 11.55 \end{pmatrix}$$

f) Finally, *Mathcad* also allows one to create and perform math functions on vectors. To create a vector, simply make a single column matrix:

$$\begin{pmatrix} 2x \\ y \end{pmatrix}$$

Set up Mathcad to do the following:

$$x := 6$$

$$y := -3$$

$$\frac{4}{\begin{pmatrix} 2x \\ y \end{pmatrix}} = \begin{pmatrix} 0.333 \\ -1.333 \end{pmatrix}$$

Exercise 3 – Extra Credit

Suppose we have a circular particle accelerator which can generate beam energies up to 10 TeV. It can operate as both a Collider as well as deliver beams for Fixed Target experiments at the same energy. The center of mass energy of a Collider can be given by: $E_{Collider} = 2*E_{beam}$
and the center of mass energy for fixed target by: $E_{FT} = \sqrt{(2*M*E_{beam})}$

Using Mathcad, generate a plot comparing the center of mass energy as a function of the beam energy. For a beam energy of **9 TeV**, what is the resulting center of mass energy for each mode?
Each mode has its pros and cons. What might some of those be?