Problem Set 7 — due Thursday, 2018 January 25, 9:00 a.m.

1. **Vacuum Window.** An injection line leads into an antiproton storage ring. The storage ring requires a much lower vacuum pressure than does the beam line, and hence a thin metal "window" is inserted across the beam pipe to separate the two evacuated systems. The window is made of Titanium (L_{rad} = 3.56 cm.) and has a thickness of 127 μm. At the location of the window, the vertical amplitude function has value β_0 = 50 m, and the corresponding α_0 = 0.5; the 8 GeV (kinetic energy) antiproton beam arriving at the window has an initial emittance of $\epsilon_0 = 1.0\ \pi$ mm-mr (un-normalized).

   (a) What is the rms scattering angle created by the window on the antiproton beam?

   (b) Find the new vertical emittance ϵ_1 and new values of β_1 and α_1 for the antiproton beam immediately after the window.

   (c) If we assume that in the absence of the window the optics of the beam line would be matched to the periodic optics of the storage ring, then the new values of the Courant-Snyder parameters after the window will be "mis-matched" to these values. Once the particles circulate the ring many times the particle distribution will filament and dilute. Estimate the final emittance reached after this dilution process.

   (d) Why do we not need to know the details of the storage ring Courant-Snyder parameters to answer the previous question?

2. **Space Charge Tune Shift.** At one time the AGS synchrotron at Brookhaven National Lab could generate proton beams with typical intensities up to about 7×10^{13} protons per pulse. Its circumference is about 800 m, and its original injection kinetic energy was 200 MeV, with beam being fed from a linac. Today, the linac delivers its beam to a Booster synchrotron which has a circumference of about 200 m, and the Booster delivers 1.5 GeV kinetic energy protons to the AGS.

   (a) Assuming un-bunched beam, estimate the incoherent tune spread due to space charge at injection into the AGS using the present Booster while at the above stated total intensity.

   (b) Using the same intensity, estimate the tune shift the AGS would have had with its old injection energy.

   (c) Assuming the same linear charge density as in the present-day AGS, estimate the incoherent tune spread due to space charge at injection into the Booster synchrotron.

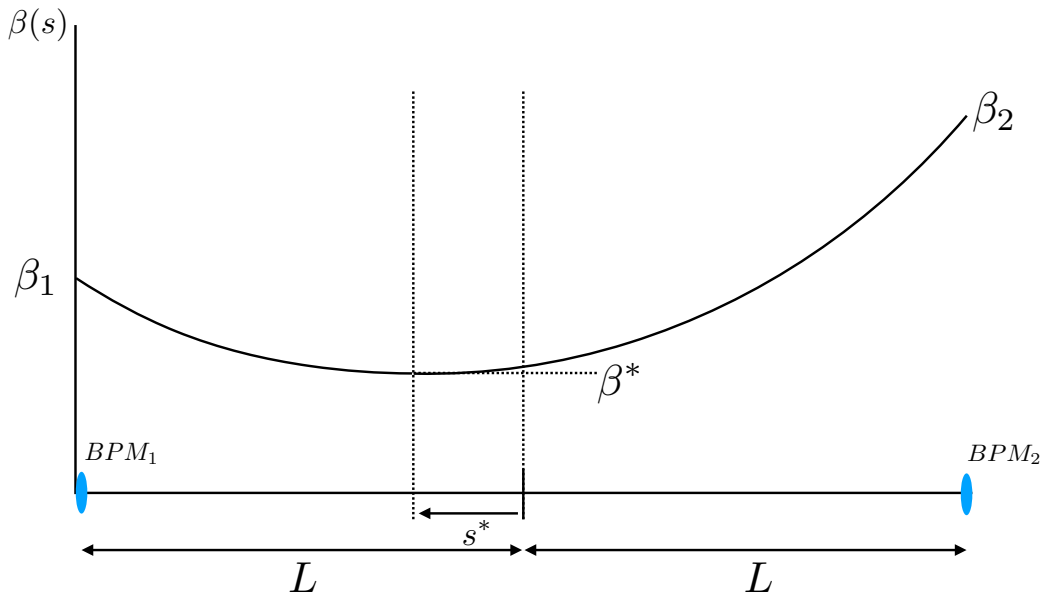
   In each case, assume a (normalized) rms transverse emittance of 8 π mm-mrad.

3. β^* **Measurement.** A long straight section in an electron storage ring is outfitted with beam position monitors (BPMs) at each end capable of measuring the average beam position each revolution for many revolutions. The local optics is tuned to generate a waist, ideally located at the middle of the straight section (at $s = 0$). The BPMs are located at $s = \pm L = \pm$ 10 m from the center of the straight section. In an attempt to ascertain the value of the amplitude function at its minimum, β^*, as well as the location s^* of the minimum, the following set of measurements are conducted:

- With the beam circulating in equilibrium, the average positions are determined at each of the two BPMs.
- A fast kicker magnet is then fired to induce a betatron oscillation about the ring, and the positions are recorded at the two BPMs over a large number of revolutions, enough such that very many oscillation periods are recorded.
- The new turn-by-turn position data is subtracted from the equilibrium data to produce sets of measured values x_1 and x_2 at the two BPMs.

Averaging over the large number of revolutions, the data produce the follow results: $\langle x_1^2 \rangle$ = 25 mm^2, $\langle x_2^2 \rangle$ = 49 mm^2, and $\langle x_1 x_2 \rangle$ = -18 mm^2. From these results, estimate the values of β^* and s^*.

[Hint: The slope of an individual particle through this region will be $x' = (x_2 - x_1)/2L$.] [Hint: Relate the averages of x, x', and xx' for a free betatron oscillation to values of the Courant-Snyder parameters.]



FINAL EXAM — Friday, 2018 January 26, 9:30 a.m.