Problem Set 4 — due Friday, 2018 January 19, 9:00 a.m.

1. **Doublet Channel – II.** Consider a periodic set of quadrupole doublets, made up of pairs of alternately focusing and defocusing magnets of strengths $\pm q \equiv \pm 1/f$ where the two magnets (considered to be thin lenses) are separated by a distance $d = 1.5$ m. The repeat distance between doublet pairs is $L + d = 8$ m, as shown in the figure.

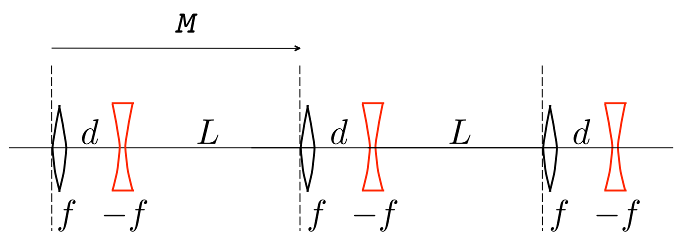

Figure 2: Quadrupole Doublet arrangement.

   (a) Compute the matrix M from the entrance of one focusing quadrupole to the entrance of the next in terms of f (in meters).

   (b) For what values of f will the system be stable or unstable?

2. **Phase Space.** Consider a particle beam at a location where Courant-Snyder parameters are known to be $\alpha = 0.55$, and $\beta = 14.3$ m. A collection of particles lies on a common elliptical trajectory in phase space, which has a maximum transverse excursion of 7.5 mm at this location.

   (a) Determine the Courant-Snyder parameter γ.
   (b) Determine the constant of integration A for the ellipse in question.
   (c) Compute the emittance of the phase space ellipse in mm mrad.
   (d) What is the maximum angle x' these particles could acquire at this location?

3. **Proton synchrotron acceleration.** A set of acceleration parameters for a large synchrotron might run as follows. A ring with a circumference of 60 km accelerates protons from 2 to 10 TeV in 800 seconds. The acceleration system has a 'voltage' amplitude of 10 MV, and the harmonic number is 72000.

   (a) Assuming $\gamma \gg \gamma_t$, what synchronous phase is appropriate during acceleration?
   (b) What RF system frequency (in Hz) is required to produce the 72000 buckets?
   (c) When beam is injected into this accelerator, only one out of every six buckets is filled. If the buckets with beam contain 2×10^{10} particles, what is the average beam current (in mA)?
   (d) If $\gamma_t = 110$, what is the synchrotron frequency (in Hz) at injection?

4. **Energy Acceptance.** A proton synchrotron with harmonic number $h = 80$, slip factor $\eta = -0.05$ and effective voltage amplitude $V = 1$ MV accepts protons with kinetic energy $W = 200$ MeV while the magnetic field is held constant. What is the maximum relative energy spread $\Delta W/W = \left(\frac{\gamma}{\gamma-1}\right) \Delta E/E$ that can be accepted by the synchrotron if the incoming particle has the correct synchronous phase?