Problem Set 3 — due Thursday, 2018 January 18, 9:00 a.m.

1. **Transit Time.** Consider a set of 2-gap, π-mode accelerating cavities, each designed for an optimal velocity, β_{opt}. (Assume constant E_z, *etc.*) For each, assume the length of each gap is $g = \beta_{opt}\lambda/8$, and the distance between gap centers is $d = \beta_{opt}\lambda/2$. The transit time factor for a cavity is thus

$$T(\beta) = \frac{\sin(\pi g/\beta\lambda)}{\pi g/\beta\lambda} \cdot \sin(\pi d/\beta\lambda).$$

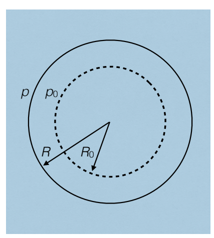
Plot the transit time factors $T(\beta)$ *vs.* β for the following 4 cases:

(a) for λ corresponding to 80.5 MHz: (i) $\beta_{opt} = 0.041$, and (ii) $\beta_{opt} = 0.085$; and

(b) for λ corresponding to 322 MHz: (i) $\beta_{opt} = 0.29$, and (ii) $\beta_{opt} = 0.53$.

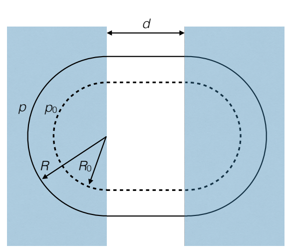
For each case, estimate the range of β for which $T(\beta)$ is greater than 65%.

2. **Slip Factor.** Consider a circular accelerator, or storage ring, which is composed of a uniform magnetic field, B_0, everywhere within a square region as depicted in the first figure below. Our ideal path for a particle of ideal momentum, p_0, will be a circular orbit of radius R_0 which lies in a plane perpendicular to B_0. A particle with momentum $p = p_0 + \Delta p$ will have a different closed path with a different radius, R, as indicated in the figure.



(a) Determine the slip factor, η for this simple accelerator.

(b) Now split the magnetic region into two pieces and separate them by a distance d, as indicated in the new figure below. The ideal path for momentum p_0 is now a "racetrack" shape as indicated, entering and exiting perpendicular to the field regions. A closed path of a particle with momentum p will be a trajectory of similar shape. Find the new slip factor, this time in terms of the ratio d/R_0.



(c) For $d = 2R_0$, plot η as a function of γ and note where it changes sign. This is the "transition gamma". What speed ($\beta = v/c$) would this correspond to?

3. **Momentum Separator.** Dispersion is created through a bending element (assumed "thin") which bends by an angle of $\theta = 20°$. A distance $L = 2$ m downstream of the bending magnet is a quadrupole of strength $q_1 = -1/F$ followed by a second quadrupole located at a distance $d = 4$ m further downstream, which has strength $q_2 = 1/F$. (*i.e.*, the first quad is defocusing and the second quad is focusing, with respect to the bend plane of the dipole magnet.)

   (a) Determine the absolute value of the common focal length F needed to make the dispersion function have zero slope upon exit from the second quadrupole magnet. Neglect any chromatic distortions due to the quadrupoles.

   (b) For this value of F, find the value of the dispersion function, $\hat{D}$, that emerges from the second quadrupole magnet.

   (c) If a mono-energetic electron beam would have an rms beam size of 1.7 mm at the exit of the second quadrupole, what would be the rms beam size of this same beam if the particles were given a momentum distribution with $(\Delta p/p)_{rms} = 0.25\%$?