Problem Set 2 — due Wednesday, 2018 January 17, 9:00 a.m.

1. **Solenoid Lenses.** Superconducting solenoid magnets are employed to focus beams as they are accelerated through different types of beamlines. Consider two different beams:

   - (i) A Uranium-238 beam of ions, where electrons have been stripped away to give an average charge state of +78 and particles have a kinetic energy of 150 MeV/u.
   - (ii) A beam of protons with kinetic energy of 8 GeV.

   (a) Estimate the magnetic rigidity for the beams (i) and (ii) at the solenoid location.

   (b) Under thin lens approximation, estimate the solenoid magnetic field needed to set the focal length for beams (i) and (ii) equal to 15 m. Consider a solenoid length of 0.3 m.

   (c) Suppose superconducting cable with an average current density of 500 A/mm^2 is wrapped around the beam pipe to make the solenoid field. Estimate the thickness of the cable, in cm, needed to attain the magnetic fields found in (b). Are the two cases realistic?

2. **Doublet Channel – I.** Consider a periodic set of quadrupole doublets, made up of pairs of alternately focusing and defocusing magnets of strengths $\pm q \equiv \pm 1/f$ and the distance between the reference trajectory and the pole tips is 5 cm. The two magnets (considered to be thin lenses) are separated by a distance $d = 1.5$ m. The repeat distance between doublet pairs is $L + d = 8$ m, as shown in the figure below:

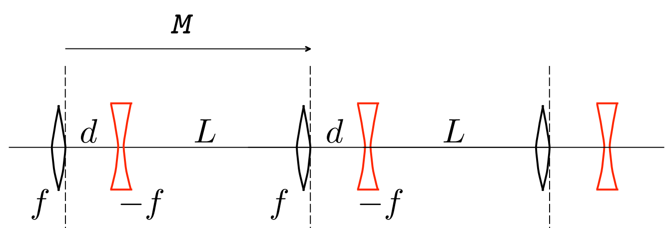

Figure 1: Quadrupole Doublet arrangement.

   (a) Compute the matrix M from the exit of one focusing quadrupole to the exit of the next in terms of f (in meters).

   (b) Calculate the magnetic field at the pole tip of the thin quadrupoles necessary to produce a focal length equal to 20 m, where the quadrupole's length is 30 cm and the beam going through travels with momentum of 10 GeV/c.

   (c) If the horizontal beam properties at the entrance of one quadrupole doublet were determined to be $\sigma_x = 20$ mm, $\sigma_{x'} = 3$ mrad and $\epsilon_x = 20\ \pi$ mm-mrad, calculate the same variables at the end of the doublet. Use the focal length indicated in (b).

3. **Emittance Measurement.** A beam line contains a region with no magnetic elements which is used for emittance measurement instrumentation for a beam of non-interacting particles (i.e., no space charge, for instance). The optics is arranged so that in the middle of the drift region the beam is focused to a waist. Two profile monitors are

present in the region, one located exactly at the beam waist, and the other a distance $L = 0.50$ m downstream. The beam spot size at the location of the waist has an rms value of $\sigma_1 = \sqrt{\langle x_1^2 \rangle}$. The second monitor is then used to make a simultaneous measurement of the beam size, giving $\sigma_2 = \sqrt{\langle x_2^2 \rangle}$. A measurement is made using a proton beam accelerated through 300 MeV, and we find that $\sigma_1 = 2.5$ mm, and $\sigma_2 = 3.0$ mm. What is the rms emittance (normalized) of the particle beam, in units of "π mm-mrad"?