Week 1

Problem Set 1 — due Tuesday, 2018 January 16, 9:00 a.m.

1. **Particle Velocities.**

   (a) Two parallel metal plates are connected to the plus and minus terminals of a 9 volt battery. An electron is released at rest near the center of the "minus" surface. Estimate the speed, as a fraction of the speed of light $\beta = v/c$, of the electron when it reaches the "plus" surface.

   (b) Estimate the speed, $\beta = v/c$, of an electron in a CRT video monitor, assuming a 12,000 volt accelerating voltage.

   (c) Estimate the speed of a carbon atom, relative to the speed of light, at the end of a medical accelerator where the kinetic energy per nucleon is 200 MeV/u. (Note: The mass of the carbon atom is 12 m_u.)

   (d) A small synchrotron of average radius 10 m accepts protons with kinetic energy 100 MeV and can accelerate the particles to final kinetic energies of 1 GeV. Plot the revolution frequency as a function of kinetic energy for protons circulating the synchrotron.

2. **Charge-to-Mass Selection.** Positively charged ions are emitted from a source, emerging through a potential difference of V_0. The kinetic energy of each particle is thus qV_0, where q is the charge of the ion. The particles are then directed through a "velocity selector", a device made up of electric and magnetic fields, E and B, at right angles to each other such that when the values of these fields are tuned properly, particles travel through the device without deflection of their trajectory.

   (a) If the electric field of the Selector is $E_0 = 75$ kV/m and the magnetic field of the Selector is $B_0 = 0.05$ T, what is the velocity relative to the speed of light ($\beta = v/c$) of the particles that travel *straight* through the selector?

   (b) Next, the magnetic field of the Selector is turned off. The particles entering are now deflected solely by the uniform electric field, E_0. If the length of the field region is ℓ, show that upon exit from the Selector the particles leave at an angle of deflection given by

   $$\tan\theta = \frac{1}{2}\frac{E_0\ell}{V_0}.$$

   (c) If $\ell = 0.15$ m, and $V_0 = 60$ kV, at what angle do the particles emerge at the end of the Selector?

   (d) If, instead, the electric field is turned off and the magnetic field is left on at its original value of B_0, the particle trajectory through the field region will be circular. Show that the radius of curvature of the trajectory will be

   $$R = \frac{2V_0}{E_0}.$$

   (e) What is the radius of curvature for the parameters of our system?

(f) Show that the ratio of charge-to-mass of the particles in our system must be:

$$q/m = \frac{E_0^2}{2V_0 B_0^2}.$$

Note that if the ions have charge state $Q = q/e$ and atomic mass $A = m/m_u$, where $m_u c^2 = 931$ MeV is the rest mass of the nucleon, then one can write

$$\frac{Q}{A} = \frac{m_u c^2}{2eV_0}\left(\frac{E_0}{cB_0}\right)^2 = \frac{m_u c^2}{2eV_0}\,(v/c)^2.$$

(g) For our parameters above, what must be the value of Q/A for the ions? If we believe the ions to be those of Nitrogen atoms, what charge state most likely comprises the beam?

3. **Relativity, Forces, and Circles.**

(a) For what value of $\gamma = 1/\sqrt{1-(v/c)^2}$ is a particle's speed equal to 99.5% of the speed of light? What kinetic energy does this correspond to for (a) an electron, and (b) a proton?

(b) Consider an electron moving at a speed of $0.995c$. What is the magnitude of the force on the electron produced by a magnetic field of strength $B = 1.0$ T? What electric field, E, would be required to balance this force? How would the two fields need to be oriented with respect to each other?

(c) Compute the magnetic rigidity, $B\rho$, for this electron. What magnetic field strength, B, is required to bend the particle in a trajectory with radius of 0.12 m?

4. **Bending Magnets.** Suppose that we want to design a proton synchrotron to accelerate to a total energy of 200 GeV from a total injection energy of 15 GeV. The circumference is $C = 4000$ m, composed of 120 sections of equal length. Bending magnets are left out of 20 of these sections to provide space for injection, extraction, acceleration, and other major utilities. The remaining sections each contain 8 bending magnets and 2 quadrupole magnets. The bending magnets in the remaining sections each have an effective length of 3.25 m.

(a) Evaluate the necessary range of magnetic field in a bending magnet.

(b) If the magnet gap is 10 cm between the two poles, and each pole has 10 turns of conductor wrapped around it, what is the maximum current required for the magnet?