Fundamentals of Particle Accelerators

U.S. Particle Accelerator School — Old Dominion University, Winter 2018

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Course Syllabus – 2018 January 15-26

Wk 1 *Day*	9:00 - 9:50	10:00 - 10:50	11:00 - 11:50	14:00 - 14:50 15:15 - 16:45	>19:00
Mon	Introduction & Prerequisites	Accelerators & Particle Beams	Steering & Focusing	Accelerator Components LAB Intro	*LAB/ Study*
Tues	Transverse Motion	Phase Space Distributions	Courant-Snyder Parameterization	Lab/Study Session	*LAB/ Study*
Wed	Accelerating Structures	Longitudinal Focusing	Transverse Effects and Dispersion	Lab/Study Session	*LAB/ Study*
Thu	Motion through Periodic Systems	Repetitive Systems: Transverse Stability	Repetitive Systems: Acceleration	Lab/Study Session	*LAB/ Study*
Fri	Collection of Beam Optics Modules	Longitudinal Manipulations	A synchrotron and a Linac	Lab/ Study	

Wk 2 *Day*	9:00 - 9:50	10:00 - 10:50	11:00 - 11:50	14:00 - 16:45	>19:00
Mon	Linear Errors & Adjustments - I	Linear Errors & Adjustments - II	Nonlinear Motion and Resonances	Lab/Study Session	*LAB/ Study*
Tues	Synchrotron Radiation	Storage Ring Light Sources	X-Ray Free Electron Lasers	Lab/Study Session	*LAB/ Study*
Wed	Emittance Dilution	Intro to Intensity Dependent Effects	Beam Instrumentation and Diagnostics	Lab/Study Session	*LAB/ Study*
Thu	Overview of an Accelerator Facility	****** Spare Topic ******	Outlook for the Accelerator Field	**Review** (13:00-13:50) Finish Labs	*LAB/ Study*
Fri	**Final Exam (9:30 a.m.)**				

Constants and Other Useful Information

$c = 3.00 \times 10^8$ m/sec	$\epsilon_0 = 8.85 \times 10^{-12}$ C^2/N-m^2	$\mu_0 = 4\pi \times 10^{-7}$ N/A^2
$G = 6.67 \times 10^{-11}$ N-m^2/kg^2	$e = 1.60 \times 10^{-19}$ C	$\hbar = h/2\pi = 1.06 \times 10^{-34}$ J-sec
$m_p = 1.67 \times 10^{-27}$kg $= 938$ MeV/c^2	$N_A = 6.02 \times 10^{23}$/mol	$\hbar c = 197$ MeV fm
$m_e = 9.11 \times 10^{-31}$kg $= 0.511$ MeV/c^2	$m_u = 931$ MeV/c^2	$k = 1.38 \times 10^{-23}$ J/K$^\circ$
$M_{sun} = 2 \times 10^{30}$ kg	$M_{earth} = 6 \times 10^{24}$ kg	$R_{earth} = 6400$ km

$$r_0 \equiv \frac{e^2}{4\pi\epsilon_0 mc^2} \quad \Rightarrow \quad r_e = 2.82 \times 10^{-15} \text{ m}, \qquad r_p = 1.54 \times 10^{-18} \text{ m}$$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J} \qquad 1 \text{ TeV} = 10^3 \text{ (GeV} = 10^3 \text{ (MeV} = 10^3 \text{ (keV} = 10^3 \text{ eV)))}$$

1 barn $= 10^{-28}$ m^2	1 Angstrom $= 0.1$ nm	1 in $= 0.0254$ m
1 atm $= 760$ torr $= 10^5$ N/m^2	1 yr $= 3.16 \times 10^7$ sec	0° C $= 273^\circ$ K
1 au $= 1.5 \times 10^{11}$ m	1 pc $= 3.09 \times 10^{16}$ m $= 3.26$ ly	1 ly $= 0.946 \times 10^{16}$ m
$(\rho)_{Carbon}$: 2.265 gm/cm^3	$(dE/dx)_{Carbon}$: 1.745 MeV/(gm/cm^2)	

$$\langle \sin\theta \rangle = \langle \cos\theta \rangle = 0 \qquad \langle \sin^2\theta \rangle = \langle \cos^2\theta \rangle = \tfrac{1}{2}$$

$$\sin x \approx x - x^3/6, \quad \text{for } |x| \ll 1 \qquad \cos x \approx 1 - x^2/2, \quad \text{for } |x| \ll 1$$

$$e^x = \sum \tfrac{x^n}{n!} = 1 + x + x^2/2 + ... \qquad (1+x)^n \approx 1 + nx, \quad \text{for } |x| \ll 1$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \qquad \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin^2 A = \frac{1 - \cos 2A}{2} \qquad \cos^2 A = \frac{1 + \cos 2A}{2}$$

$$\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} e^{-x^2/2\sigma^2} dx = 1, \quad \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} x\, e^{-x^2/2\sigma^2} dx = 0, \quad \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} x^2 e^{-x^2/2\sigma^2} dx = \sigma^2$$

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad M^T = \begin{pmatrix} a & c \\ b & d \end{pmatrix}, \quad M^{-1} = \frac{1}{\det M}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}, \quad \det M = ad - bc$$

Week 1

Problem Set 1 — due Tuesday, 2018 January 16, 9:00 a.m.

1. **Particle Velocities.**

   (a) Two parallel metal plates are connected to the plus and minus terminals of a 9 volt battery. An electron is released at rest near the center of the "minus" surface. Estimate the speed, as a fraction of the speed of light $\beta = v/c$, of the electron when it reaches the "plus" surface.

   (b) Estimate the speed, $\beta = v/c$, of an electron in a CRT video monitor, assuming a 12,000 volt accelerating voltage.

   (c) Estimate the speed of a carbon atom, relative to the speed of light, at the end of a medical accelerator where the kinetic energy per nucleon is 200 MeV/u. (Note: The mass of the carbon atom is 12 m_u.)

   (d) A small synchrotron of average radius 10 m accepts protons with kinetic energy 100 MeV and can accelerate the particles to final kinetic energies of 1 GeV. Plot the revolution frequency as a function of kinetic energy for protons circulating the synchrotron.

2. **Charge-to-Mass Selection.** Positively charged ions are emitted from a source, emerging through a potential difference of V_0. The kinetic energy of each particle is thus qV_0, where q is the charge of the ion. The particles are then directed through a "velocity selector", a device made up of electric and magnetic fields, E and B, at right angles to each other such that when the values of these fields are tuned properly, particles travel through the device without deflection of their trajectory.

   (a) If the electric field of the Selector is $E_0 = 75$ kV/m and the magnetic field of the Selector is $B_0 = 0.05$ T, what is the velocity relative to the speed of light ($\beta = v/c$) of the particles that travel *straight* through the selector?

   (b) Next, the magnetic field of the Selector is turned off. The particles entering are now deflected solely by the uniform electric field, E_0. If the length of the field region is ℓ, show that upon exit from the Selector the particles leave at an angle of deflection given by

   $$\tan\theta = \frac{1}{2}\frac{E_0\ell}{V_0}.$$

   (c) If $\ell = 0.15$ m, and $V_0 = 60$ kV, at what angle do the particles emerge at the end of the Selector?

   (d) If, instead, the electric field is turned off and the magnetic field is left on at its original value of B_0, the particle trajectory through the field region will be circular. Show that the radius of curvature of the trajectory will be

   $$R = \frac{2V_0}{E_0}.$$

   (e) What is the radius of curvature for the parameters of our system?

(f) Show that the ratio of charge-to-mass of the particles in our system must be:

$$q/m = \frac{E_0^2}{2V_0 B_0^2}.$$

Note that if the ions have charge state $Q = q/e$ and atomic mass $A = m/m_u$, where $m_u c^2 = 931$ MeV is the rest mass of the nucleon, then one can write

$$\frac{Q}{A} = \frac{m_u c^2}{2eV_0}\left(\frac{E_0}{cB_0}\right)^2 = \frac{m_u c^2}{2eV_0}\,(v/c)^2.$$

(g) For our parameters above, what must be the value of Q/A for the ions? If we believe the ions to be those of Nitrogen atoms, what charge state most likely comprises the beam?

3. **Relativity, Forces, and Circles.**

(a) For what value of $\gamma = 1/\sqrt{1-(v/c)^2}$ is a particle's speed equal to 99.5% of the speed of light? What kinetic energy does this correspond to for (a) an electron, and (b) a proton?

(b) Consider an electron moving at a speed of $0.995c$. What is the magnitude of the force on the electron produced by a magnetic field of strength $B = 1.0$ T? What electric field, E, would be required to balance this force? How would the two fields need to be oriented with respect to each other?

(c) Compute the magnetic rigidity, $B\rho$, for this electron. What magnetic field strength, B, is required to bend the particle in a trajectory with radius of 0.12 m?

4. **Bending Magnets.** Suppose that we want to design a proton synchrotron to accelerate to a total energy of 200 GeV from a total injection energy of 15 GeV. The circumference is $C = 4000$ m, composed of 120 sections of equal length. Bending magnets are left out of 20 of these sections to provide space for injection, extraction, acceleration, and other major utilities. The remaining sections each contain 8 bending magnets and 2 quadrupole magnets. The bending magnets in the remaining sections each have an effective length of 3.25 m.

(a) Evaluate the necessary range of magnetic field in a bending magnet.

(b) If the magnet gap is 10 cm between the two poles, and each pole has 10 turns of conductor wrapped around it, what is the maximum current required for the magnet?

Problem Set 2 — due Wednesday, 2018 January 17, 9:00 a.m.

1. **Solenoid Lenses.** Superconducting solenoid magnets are employed to focus beams as they are accelerated through different types of beamlines. Consider two different beams:

   - (i) A Uranium-238 beam of ions, where electrons have been stripped away to give an average charge state of +78 and particles have a kinetic energy of 150 MeV/u.
   - (ii) A beam of protons with kinetic energy of 8 GeV.

   (a) Estimate the magnetic rigidity for the beams (i) and (ii) at the solenoid location.

   (b) Under thin lens approximation, estimate the solenoid magnetic field needed to set the focal length for beams (i) and (ii) equal to 15 m. Consider a solenoid length of 0.3 m.

   (c) Suppose superconducting cable with an average current density of 500 A/mm^2 is wrapped around the beam pipe to make the solenoid field. Estimate the thickness of the cable, in cm, needed to attain the magnetic fields found in (b). Are the two cases realistic?

2. **Doublet Channel – I.** Consider a periodic set of quadrupole doublets, made up of pairs of alternately focusing and defocusing magnets of strengths $\pm q \equiv \pm 1/f$ and the distance between the reference trajectory and the pole tips is 5 cm. The two magnets (considered to be thin lenses) are separated by a distance $d = 1.5$ m. The repeat distance between doublet pairs is $L + d = 8$ m, as shown in the figure below:

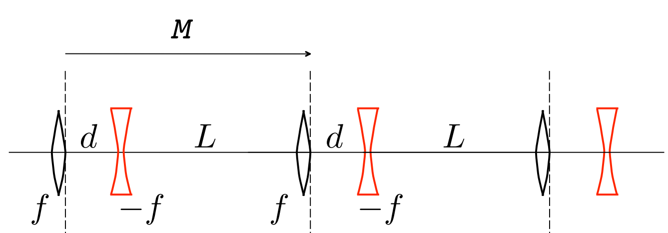


Figure 1: Quadrupole Doublet arrangement.

   (a) Compute the matrix M from the exit of one focusing quadrupole to the exit of the next in terms of f (in meters).

   (b) Calculate the magnetic field at the pole tip of the thin quadrupoles necessary to produce a focal length equal to 20 m, where the quadrupole's length is 30 cm and the beam going through travels with momentum of 10 GeV/c.

   (c) If the horizontal beam properties at the entrance of one quadrupole doublet were determined to be $\sigma_x = 20$ mm, $\sigma_{x'} = 3$ mrad and $\epsilon_x = 20\ \pi$ mm-mrad, calculate the same variables at the end of the doublet. Use the focal length indicated in (b).

3. **Emittance Measurement.** A beam line contains a region with no magnetic elements which is used for emittance measurement instrumentation for a beam of non-interacting particles (i.e., no space charge, for instance). The optics is arranged so that in the middle of the drift region the beam is focused to a waist. Two profile monitors are

present in the region, one located exactly at the beam waist, and the other a distance $L = 0.50$ m downstream. The beam spot size at the location of the waist has an rms value of $\sigma_1 = \sqrt{\langle x_1^2 \rangle}$. The second monitor is then used to make a simultaneous measurement of the beam size, giving $\sigma_2 = \sqrt{\langle x_2^2 \rangle}$. A measurement is made using a proton beam accelerated through 300 MeV, and we find that $\sigma_1 = 2.5$ mm, and $\sigma_2 = 3.0$ mm. What is the rms emittance (normalized) of the particle beam, in units of "π mm-mrad"?

Problem Set 3 — due Thursday, 2018 January 18, 9:00 a.m.

1. **Transit Time.** Consider a set of 2-gap, π-mode accelerating cavities, each designed for an optimal velocity, β_{opt}. (Assume constant E_z, *etc.*) For each, assume the length of each gap is $g = \beta_{opt}\lambda/8$, and the distance between gap centers is $d = \beta_{opt}\lambda/2$. The transit time factor for a cavity is thus

$$T(\beta) = \frac{\sin(\pi g/\beta\lambda)}{\pi g/\beta\lambda} \cdot \sin(\pi d/\beta\lambda).$$

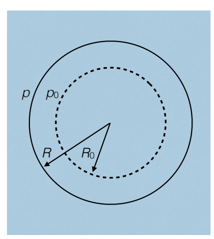
   Plot the transit time factors $T(\beta)$ *vs.* β for the following 4 cases:

   (a) for λ corresponding to 80.5 MHz: (i) $\beta_{opt} = 0.041$, and (ii) $\beta_{opt} = 0.085$; and

   (b) for λ corresponding to 322 MHz: (i) $\beta_{opt} = 0.29$, and (ii) $\beta_{opt} = 0.53$.

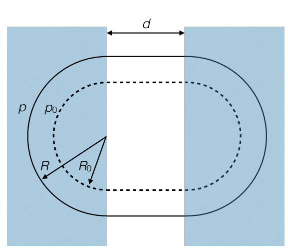
   For each case, estimate the range of β for which $T(\beta)$ is greater than 65%.

2. **Slip Factor.** Consider a circular accelerator, or storage ring, which is composed of a uniform magnetic field, B_0, everywhere within a square region as depicted in the first figure below. Our ideal path for a particle of ideal momentum, p_0, will be a circular orbit of radius R_0 which lies in a plane perpendicular to B_0. A particle with momentum $p = p_0 + \Delta p$ will have a different closed path with a different radius, R, as indicated in the figure.



   (a) Determine the slip factor, η for this simple accelerator.

   (b) Now split the magnetic region into two pieces and separate them by a distance d, as indicated in the new figure below. The ideal path for momentum p_0 is now a "racetrack" shape as indicated, entering and exiting perpendicular to the field regions. A closed path of a particle with momentum p will be a trajectory of similar shape. Find the new slip factor, this time in terms of the ratio d/R_0.



   (c) For $d = 2R_0$, plot η as a function of γ and note where it changes sign. This is the "transition gamma". What speed ($\beta = v/c$) would this correspond to?

3. **Momentum Separator.** Dispersion is created through a bending element (assumed "thin") which bends by an angle of $\theta = 20°$. A distance $L = 2$ m downstream of the bending magnet is a quadrupole of strength $q_1 = -1/F$ followed by a second quadrupole located at a distance $d = 4$ m further downstream, which has strength $q_2 = 1/F$. (*i.e.*, the first quad is defocusing and the second quad is focusing, with respect to the bend plane of the dipole magnet.)

   (a) Determine the absolute value of the common focal length F needed to make the dispersion function have zero slope upon exit from the second quadrupole magnet. Neglect any chromatic distortions due to the quadrupoles.

   (b) For this value of F, find the value of the dispersion function, $\hat{D}$, that emerges from the second quadrupole magnet.

   (c) If a mono-energetic electron beam would have an rms beam size of 1.7 mm at the exit of the second quadrupole, what would be the rms beam size of this same beam if the particles were given a momentum distribution with $(\Delta p/p)_{rms} = 0.25\%$?

Problem Set 4 — due Friday, 2018 January 19, 9:00 a.m.

1. **Doublet Channel – II.** Consider a periodic set of quadrupole doublets, made up of pairs of alternately focusing and defocusing magnets of strengths $\pm q \equiv \pm 1/f$ where the two magnets (considered to be thin lenses) are separated by a distance $d = 1.5$ m. The repeat distance between doublet pairs is $L + d = 8$ m, as shown in the figure.

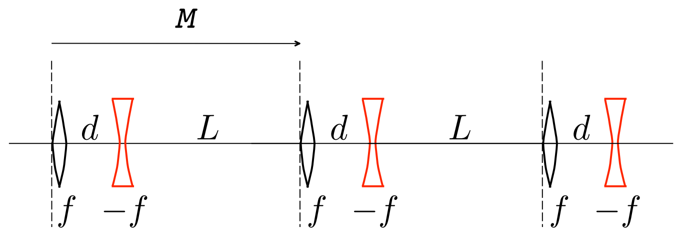


Figure 2: Quadrupole Doublet arrangement.

   (a) Compute the matrix M from the entrance of one focusing quadrupole to the entrance of the next in terms of f (in meters).

   (b) For what values of f will the system be stable or unstable?

2. **Phase Space.** Consider a particle beam at a location where Courant-Snyder parameters are known to be $\alpha = 0.55$, and $\beta = 14.3$ m. A collection of particles lies on a common elliptical trajectory in phase space, which has a maximum transverse excursion of 7.5 mm at this location.

   (a) Determine the Courant-Snyder parameter γ.
   (b) Determine the constant of integration A for the ellipse in question.
   (c) Compute the emittance of the phase space ellipse in mm mrad.
   (d) What is the maximum angle x' these particles could acquire at this location?

3. **Proton synchrotron acceleration.** A set of acceleration parameters for a large synchrotron might run as follows. A ring with a circumference of 60 km accelerates protons from 2 to 10 TeV in 800 seconds. The acceleration system has a 'voltage' amplitude of 10 MV, and the harmonic number is 72000.

   (a) Assuming $\gamma \gg \gamma_t$, what synchronous phase is appropriate during acceleration?
   (b) What RF system frequency (in Hz) is required to produce the 72000 buckets?
   (c) When beam is injected into this accelerator, only one out of every six buckets is filled. If the buckets with beam contain 2×10^{10} particles, what is the average beam current (in mA)?
   (d) If $\gamma_t = 110$, what is the synchrotron frequency (in Hz) at injection?

4. **Energy Acceptance.** A proton synchrotron with harmonic number $h = 80$, slip factor $\eta = -0.05$ and effective voltage amplitude $V = 1$ MV accepts protons with kinetic energy $W = 200$ MeV while the magnetic field is held constant. What is the maximum relative energy spread $\Delta W/W = \left(\frac{\gamma}{\gamma-1}\right) \Delta E/E$ that can be accepted by the synchrotron if the incoming particle has the correct synchronous phase?

Week 2

Problem Set 5 — due Tuesday, 2018 January 23, 9:00 a.m.

1. **Chromatic Light Source.** The following measurements are performed in an electron synchrotron operating at an energy of 9 GeV. The RF frequency (nominally 390 MHz) is increased by 3 kHz from its original value, and the horizontal tune is measured to be 20.428. Next, the RF frequency is decreased from its original value by 3 kHz and this time the tune is found to be 20.442.

   (a) What relative change in momentum, $\Delta p/p$, did the beam experience between these two measurements? Note that $1/\gamma_t^2 = 3 \times 10^{-3}$ in this ring.

   (b) If the horizontal dispersion function has a value of 0.6 m at a particular beam position monitor (BPM), what relative horizontal displacement would be detected at this location between these two measurements?

   (c) At what value of chromaticity is the accelerator operating?

2. **Gradient Errors.** An accelerator has 100 quadrupole magnets powered in series, so that each has the same magnetic field. The focal length of the quadrupoles is 20 m and the amplitude functions at their locations have values $\beta_x = 80$ m and $\beta_y = 20$ m.

   (a) If the magnetic gradient of all the quadrupoles is increased by 0.2%, what tune shift would this generate in each degree of freedom (horizontal and vertical)?

   (b) If, instead, each magnet is independently powered, and the setting of each power supply is accurate to 0.5%, estimate the accuracy to which the horizontal tune can be adjusted.

3. **Beam injection.** A beam made of 1 million protons with $p_0 = 3.9$ GeV/c and uniform momentum spread within the range $-0.01 < \delta p/p < 0.01$ is intended to be injected into a storage ring with constant dispersion function $D_x = 8.3$ m, and bending radius $\rho = 7.112$ m. The horizontal betatron phase space (in coordinates x, $\beta_x x' + \alpha_x x$) of the incoming beam is initially filled uniformly out to a maximum radius of 10 cm.

   (a) If the beam pipe inside the storage ring has a horizontal aperture of 4.5 cm, what is the range of momentum offsets $\Delta p/p$ that can be sustained within the ring? Ignore vertical losses.

   (b) After many, may turns, estimate the percentage of the injected proton beam that will be lost due to collimation with the beam pipe.

Problem Set 6 — due Wednesday, 2018 January 24, 9:00 a.m.

1. **Chasman-Green.** Modern light sources have strong achromatic sections used to control the dispersion function through the bending magnets in order to generate small beam sizes. On the other hand, such strong focusing systems produce high natural chromaticity which must be compensated using sextupole magnets. Suppose a light source is made up of 24 identical sections, and in each section a sextupole magnet is present at locations where the dispersion function has values of $D = 0.5$ m and the amplitude function has value $\beta_x = 25$ m. The natural chromaticity in the horizontal plane is $\xi_x = -96$.
   (a) What value of the sextupole strength $S = B''l/2B\rho$ is required to bring ξ_x back to zero?
   (b) If the vertical amplitude function at this location is $\beta_y = 11$ m, how is the natural chromaticity in the vertical plane affected?
   (c) What would you suggest be done to the accelerator system in order to bring both ξ_x and ξ_y to zero simultaneously?

2. **Synchrotron Radiation Power.** A storage ring synchrotron light source stores a 3 GeV electron beam with 200 mA stored current. The ring has 48 bending magnets with a uniform bending field of 1.2 T.
   (a) Calculate the total synchrotron radiation power (in kW) for one revolution of the machine.
   (b) What is the synchrotron radiation power (in kW) emitted at a single bending magnet?

3. **Energy Loss per Turn.** A $3\tilde{G}$eV electron beam is stored in an electron storage ring (bending radius 10 m, revolution period 1.87 μs). Because electrons lose a significant amount of energy to synchrotron radiation each turn, RF voltage is required to replenish energy lost each turn.
   (a) What is the energy loss per turn (in MeV) for a single electron due to the bending magnets?
   (b) Calculate the total gap voltage (in MV) required to store electrons with a synchronous phase of 20° from the zero-crossing of the RF.
   (c) Assume the RF trips. If the maximum horizontal dispersion $\eta_x = 0.6$ m and the radius of the vacuum chamber is 5 cm, how many turns will it take for the beam to hit the vacuum chamber? Assume a constant energy loss per turn for a beam at 3 GeV.

4. **Synchrotron Radiation Spectrum.** Beams of lead nuclei ($^{208}\mathrm{Pb}^{82+}$) in the Large Hadron Collider produce synchrotron radiation. Hint: assume $m_u = 0.931$ MeV$/c^2$.
   (a) What is the critical photon energy (in eV) of a lead nucleus at collision energy (2.563 TeV/U), if the bending radius is 6 km?
   (b) What is the critical photon energy in eV of a lead nucleus at injection energy (177.4 GeV/U), if the bending radius is 6 km?

(c) A short, two period undulator with field 5 T and period 28 cm is used to produce undulator radiation. If lead nuclei are at injection energy (177.4 GeV/U), what is the photon energy in eV of the first undulator harmonic on axis?

Problem Set 7 — due Thursday, 2018 January 25, 9:00 a.m.

1. **Vacuum Window.** An injection line leads into an antiproton storage ring. The storage ring requires a much lower vacuum pressure than does the beam line, and hence a thin metal "window" is inserted across the beam pipe to separate the two evacuated systems. The window is made of Titanium (L_{rad} = 3.56 cm.) and has a thickness of 127 μm. At the location of the window, the vertical amplitude function has value $\beta_0 = 50$ m, and the corresponding $\alpha_0 = 0.5$; the 8 GeV (kinetic energy) antiproton beam arriving at the window has an initial emittance of $\epsilon_0 = 1.0\ \pi$ mm-mr (un-normalized).

   (a) What is the rms scattering angle created by the window on the antiproton beam?

   (b) Find the new vertical emittance ϵ_1 and new values of β_1 and α_1 for the antiproton beam immediately after the window.

   (c) If we assume that in the absence of the window the optics of the beam line would be matched to the periodic optics of the storage ring, then the new values of the Courant-Snyder parameters after the window will be "mis-matched" to these values. Once the particles circulate the ring many times the particle distribution will filament and dilute. Estimate the final emittance reached after this dilution process.

   (d) Why do we not need to know the details of the storage ring Courant-Snyder parameters to answer the previous question?

2. **Space Charge Tune Shift.** At one time the AGS synchrotron at Brookhaven National Lab could generate proton beams with typical intensities up to about 7×10^{13} protons per pulse. Its circumference is about 800 m, and its original injection kinetic energy was 200 MeV, with beam being fed from a linac. Today, the linac delivers its beam to a Booster synchrotron which has a circumference of about 200 m, and the Booster delivers 1.5 GeV kinetic energy protons to the AGS.

   (a) Assuming un-bunched beam, estimate the incoherent tune spread due to space charge at injection into the AGS using the present Booster while at the above stated total intensity.

   (b) Using the same intensity, estimate the tune shift the AGS would have had with its old injection energy.

   (c) Assuming the same linear charge density as in the present-day AGS, estimate the incoherent tune spread due to space charge at injection into the Booster synchrotron.

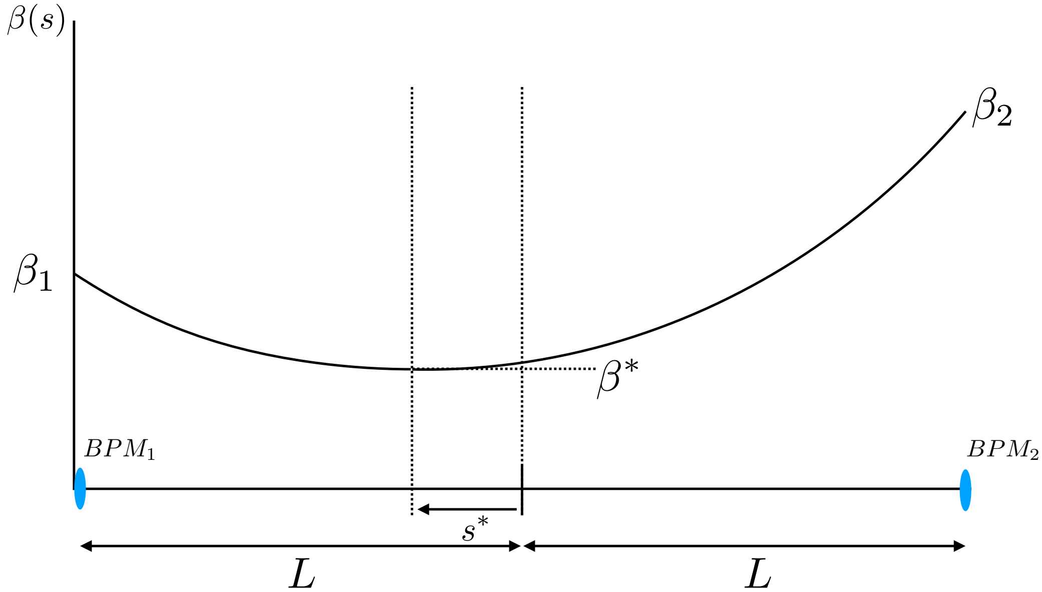
   In each case, assume a (normalized) rms transverse emittance of 8 π mm-mrad.

3. β^* **Measurement.** A long straight section in an electron storage ring is outfitted with beam position monitors (BPMs) at each end capable of measuring the average beam position each revolution for many revolutions. The local optics is tuned to generate a waist, ideally located at the middle of the straight section (at $s = 0$). The BPMs are located at $s = \pm L = \pm$ 10 m from the center of the straight section. In an attempt to ascertain the value of the amplitude function at its minimum, β^*, as well as the location s^* of the minimum, the following set of measurements are conducted:

- With the beam circulating in equilibrium, the average positions are determined at each of the two BPMs.
- A fast kicker magnet is then fired to induce a betatron oscillation about the ring, and the positions are recorded at the two BPMs over a large number of revolutions, enough such that very many oscillation periods are recorded.
- The new turn-by-turn position data is subtracted from the equilibrium data to produce sets of measured values x_1 and x_2 at the two BPMs.

Averaging over the large number of revolutions, the data produce the follow results: $\langle x_1^2 \rangle$ = 25 mm^2, $\langle x_2^2 \rangle$ = 49 mm^2, and $\langle x_1 x_2 \rangle$ = -18 mm^2. From these results, estimate the values of β^* and s^*.

[Hint: The slope of an individual particle through this region will be $x' = (x_2 - x_1)/2L$.]
[Hint: Relate the averages of x, x', and xx' for a free betatron oscillation to values of the Courant-Snyder parameters.]



FINAL EXAM — Friday, 2018 January 26, 9:30 a.m.