Student Name: KEY

Fundamentals of Particle Accelerators

U. S. Particle Accelerator School
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Instructions: Read the questions carefully, and mark the **best** answer, one answer per question. Answer all questions. Any notes you've taken, papers handed out in class, the class textbook, and a calculator may be used during the exam. Access to materials on the class web site is allowed, but **NOT** internet searches. Total time for the exam is two hours.

1. In an accelerator made up of FODO cells, the emittance at a defocusing quadrupole is smaller than the emittance at a focusing quadrupole.

   (a) TRUE
   (b) FALSE

   ϵ = constant

2. In a **linear** accelerator the slip factor, η, is always positive.

   (a) TRUE
   (b) FALSE

   $\eta = \alpha_p - \frac{1}{\gamma^2} = -\frac{1}{\gamma^2} < 0$

3. A particle near the separatrix of an RF bucket has the same synchrotron tune as a particle with small synchrotron oscillation amplitude.

   (a) TRUE
   (b) FALSE

   lower ν_s

4. The matrix describing a particle's motion through a periodic section of an accelerator is given by

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 0 & 25 \text{ m} \\ -\frac{1}{25 \text{ m}} & 0 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

   The periodic β function for this matrix is $\beta = 25$ m.

   (a) TRUE
   (b) FALSE

   $a = d = 0 \rightarrow \alpha = 0$ + $\mu = 90°$
   $\therefore b = \beta \sin\mu = \beta = 25\,\text{m}$

5. Indicate which of the following statements concerning the tune shift due to space charge for a round beam is correct:

   (a) The electric and magnetic forces are pointed in opposite directions.
   (b) The tune shift is larger when the particles are at at higher energies.
   (c) The tune shift is independent of energy.
   (d) None of the above.

6. A closed orbit distortion produced by a single dipole steering error changes the tune of the accelerator to exactly an integer value.

   (a) TRUE
   (b) FALSE

7. Synchrotron radiation from charged particles in a circular accelerator

   (a) is generally more of an issue in electron rings than in proton rings.
   (b) is produced at a rate proportional to the fourth power of the particle energy.
   (c) is emitted primarily within a forward angle of total width $2/\gamma$.
   (d) (a) and (b) above only.
   (e) (a) and (c) above only.
   (f) (a), (b), and (c) above.

8. The ideal energy gain per turn in a proton synchrotron is given by

$$\Delta E_s = eV \sin \phi_s.$$

   For $\gamma > \gamma_t$, to achieve an energy gain per turn of 2 MeV for the ideal particle, what synchronous phase is required if the total RF voltage that can be delivered by the system is 4 MV?

   (a) 30°
   (b) 90°
   (c) 150°
   (d) 210°
   (e) 300°

$\gamma > \gamma_t$
$\to \eta > 0$
$\to \cos\phi_s < 0$
$\to \phi_s = 150°$

$\sin 150° = \sin 30° = \frac{1}{2}$

9. An electron beam is accelerated from pc = 10 GeV up to pc = 250 GeV in a linear collider. In a region with no dispersion, by what factor is the physical beam size changed during this process?

   (a) 1/5
   (b) 1/240
   (c) 1/25
   (d) 5
   (e) 25

$x_{rms} \propto \frac{1}{\sqrt{p}}$, $\epsilon \propto \frac{1}{p}$

10. If the horizontal betatron tune for an ideal synchronous particle is 12.710, and the horizontal chromaticity of the accelerator is $\xi = -12$, then a particle with $\Delta p/p = 0.002$ will have a tune of

(a) 12.470
(b) 12.686
(c) 12.710
(d) 12.734
(e) 12.830

$$\nu = \nu_0 + \xi \frac{\Delta p}{p}$$
$$= 12.710 - 12 \cdot 0.002$$
$$= 12.686$$

11. A particle of charge e with momentum $p = 3.6$ GeV/c is traveling in a magnetic field of strength 2 T for a total distance of 20 cm. What is the particle's bend angle, θ?

(a) 3 mrad
(b) 11 mrad
(c) 33 mrad
(d) 150 mrad
(e) 330 mrad

$$B\rho = \frac{10}{3} \cdot 3.6 = 12 \text{ T.m}$$
$$\theta = \frac{2\text{T} \cdot 0.2\text{m}}{12 \text{ T}\cdot\text{m}} = .033$$

12. A 20 GeV proton beam with normalized rms emittance $\epsilon_N = 3\pi$ mm-mrad and rms momentum spread $(\Delta p/p)_{rms} = 0.002$ is observed at a location where $\beta = 80$ m and the dispersion function has value $D = 2$ m. The rms beam size, σ_{tot}, at this location will be

(a) 3 mm
(b) 5 mm
(c) 7 mm
(d) 10 mm

$$\sigma_t = \sqrt{\frac{\epsilon_N \beta}{\pi(\beta\gamma)} + D^2 \sigma_p^2} = \sqrt{\frac{3 \cdot 80}{20} + 2^2 \cdot 2^2}$$
$$= \sqrt{12 + 16} = \sqrt{28}$$
$$= 5.\sim \text{ mm}$$

13. For a synchrotron to be stable, the harmonic number, h, should never be an integer.

(a) TRUE
(b) FALSE

$h \equiv$ integer

14. The frequency of the accelerating voltage of an RF cavity

(a) is the same thing as the synchrotron frequency.
(b) is always the only resonant mode of the structure.
(c) depends on the cavity geometry.
(d) depends on the level of the peak RF voltage.

15. A proton circulating in a synchrotron oscillates longitudinally and arrives at the RF cavities within ± 10 nanoseconds relative to the ideal synchronous particle of energy 2 GeV. Once accelerated to 24 GeV, the same proton will still be oscillating but the maximum $|\Delta t|$ relative to the synchronous particle

   (a) will be $>$ 10 nanoseconds.
   (b) will be $<$ 10 nanoseconds.
   (c) will be $=$ 10 nanoseconds.
   (d) will be sometimes $>$ 10 nanoseconds, and sometimes $<$ 10 nanoseconds.

(other parameters being constant)
(didn't emphasize in class...)

16. A proton has speed of $v = 2.7 \times 10^8$ m/s. Its kinetic energy is approximately

   (a) 97 MeV
   (b) 210 MeV
   (c) 380 MeV
   (d) 940 MeV
   (e) 1200 MeV

$\beta = 2.7/3 = 0.9$

$\gamma = \frac{1}{\sqrt{1-0.81}} = 2.29$

$W = (\gamma - 1) mc^2 = 1.29 (938) = 1.21$ GeV

17. A thin lens quadrupole of focal length $F = 5$ m is required for a 3 GeV electron beam line. A quadrupole magnet design, which has a pole tip radius of $a = 2$ cm, can produce a pole tip field of 0.25 T. The required length of the magnet needed to produce the desired focal length is

   (a) 8 cm
   (b) 16 cm
   (c) 0.48 m
   (d) 1.6 m

$B\rho = 3 \cdot \frac{10}{3} = 10$ T.m

$\frac{B'l}{B\rho} = \frac{1}{F} \longrightarrow l = \frac{B\rho}{FB'} \cdot \frac{a}{a}$

$= \frac{10 \text{ T.m} \cdot 0.02 \text{ m}}{5 \text{ m} \cdot \frac{1}{4} \text{ T}} = 0.16$ m

18. Imagine we have two quadrupole magnets in a circular accelerator which are initially unpowered. The first is located where $\beta_x = 10$ m and $\beta_y = 20$ m. At the second quadrupole, $\beta_x = 20$ m and $\beta_y = 10$ m. The two magnets are now powered to produce strengths $q_1 = 1/f_1 = 0.02$/m and $q_2 = 1/f_2 = -0.04$/m. (A positive value focuses in the horizontal direction.) The resulting changes in the tunes $\Delta\nu_x$ and $\Delta\nu_y$ will be

   (a) $\Delta\nu_x = 0.05$, and $\Delta\nu_y =$ -0.05
   (b) $\Delta\nu_x = 0$, and $\Delta\nu_y = 0.08$
   (c) $\Delta\nu_x =$ -0.05, and $\Delta\nu_y = 0$
   (d) $\Delta\nu_x =$ -0.08, and $\Delta\nu_y = 0.08$
   (e) $\Delta\nu_x = 0.6$, and $\Delta\nu_y =$ -0.6

$\Delta\nu = \frac{1}{4\pi}(\beta_1 q_1 + \beta_2 q_2)$

$\Delta\nu_x = \frac{1}{4\pi}(10 \cdot 0.02 - 20 \cdot 0.04) = -0.05$

$\Delta\nu_y = -\frac{1}{4\pi}(20 \cdot 0.02 - 10 \cdot 0.04) = 0$

For the following three questions, consider a circular accelerator composed of a focusing system made up of quadrupole magnets spaced apart by a distance $L < F$. **Each** quadrupole has an identical gradient, B', which produces the same **positive** focal length, F, in the horizontal plane. Since the "cell" repeats itself after each quadrupole, the matrix M_x for the horizontal motion through a cell is given by

$$M_x = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/F & 1 \end{pmatrix} = \begin{pmatrix} 1 - L/F & L \\ -1/F & 1 \end{pmatrix}$$

and the horizontal motion through N such cells is given by M_x^N.

19. The horizontal motion through this system is stable because

 (a) $\det M_x = 1$ $0 < L/F < 1$

 (b) $\cos(2\pi\nu_x) = 1/2$

 (c) $|2 - L/F| < 2$

 (d) horizontal motion in this system is not stable

20. The matrix through one cell of this system for vertical motion is

 (a) $M_y = \begin{pmatrix} 1 - L/F & L \\ -1/F & 1 \end{pmatrix}$

 (b) $M_y = \begin{pmatrix} 1 - L/F & L \\ 1/F & 1 \end{pmatrix}$ $F \to -F$

 (c) $M_y = \begin{pmatrix} 1 + L/F & L \\ 1/F & 1 \end{pmatrix}$

 (d) does not exist

21. The vertical motion through this system is stable because

 (a) $\det M_y = 1$

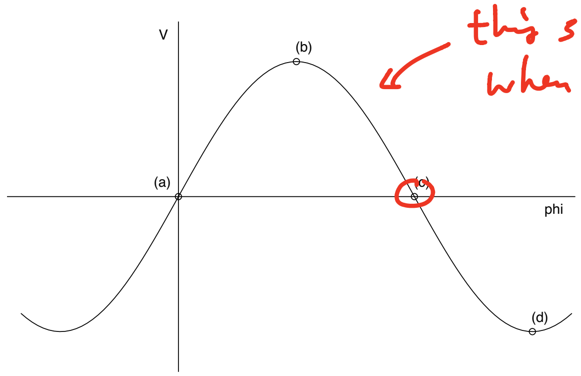
 (b) $\cos(2\pi\nu_y) = 1/2$ $\mathrm{tr} M = 2 + L/F > 2$

 (c) $|2 - L/F| < 2$

 (d) vertical motion in this system is not stable

22. A synchrotron is operating **above** its transition energy and the voltage waveform of the RF is a sine wave. For stable synchrotron motion with no average acceleration what synchronous phase is required? Choose from the options in the figure below.

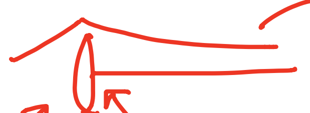
   (a)
   (b)
   (c)
   (d)



this side when $\gamma > \gamma_t$

23. Suppose β_x = 22 m and α_x = -2 when entering a horizontally focusing thin lens quadrupole of focal length 11 m. The quadrupole is followed by 44 m of drift space. The value of β_x at the end of the drift will be

   (a) 12 m.
   (b) 22 m.
   (c) 26 m.
   (d) 88 m.
   (e) 110 m.



$\beta = \beta_0 + s^2/\beta_0 \quad (\alpha = 0)$

$\Longrightarrow \beta = 22 + 44^2/22 = 110\,m$

$\Delta\alpha = \beta/F = 22/11 = 2$

$\therefore \alpha = -2 + 2 = 0$

$\alpha_0 = -2$

$\beta_0 = 22\,m$

24. During the operation of the Large Electron-Positron collider (LEP) at CERN, the energy of the machine was upgraded from 50 GeV to 100 GeV. The power generated by synchrotron radiation went up by a factor of

   (a) 0.5
   (b) 1 (stayed the same)
   (c) 2
   (d) 4
   (e) 8
   (f) 16
   (g) 100

$P = f_0 U_0 \sim E^4$

$\therefore \uparrow 2^4 = 16$

25. What does the Q-factor tell you about an RF cavity?

   (a) Its resonant frequency
   (b) The surface roughness of the material
   (c) The operating temperature of the superconducting cavity
   (d) That it is a quarter-wave resonator
   (e) None of the above

maybe, if $Q \sim 10^9$!

26. What is the wakefield of the beam in a particle accelerator?

   (a) An electromagnetic field generated by the beam in the structure
   (b) A beam instability
   (c) The effect of gas scattering
   (d) The effect of gravity on the beam motion
   (e) The beam abort system

27. Particle bunches traveling at $v = 0.8c$ are spaced every 2.4 m. The minimum RF frequency required to maintain this bunch spacing would be

   (a) 50 kHz
   (b) 100 kHz
   (c) 50 MHz
   (d) 100 MHz
   (e) 1 GHz

$$f_{RF} = \frac{v}{\Delta s} = \frac{0.8 \cdot 3 \cdot 10^8 \, m/s}{2.4 \, m} = 100 \text{ MHz}$$

Extra Credit Problem (2 pt each part)

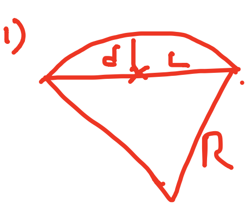
A linear accelerator is designed to deliver an average of 50 MeV/m along its length to a beam of electrons.

1. If two *straight* linacs are aimed toward each other to create an *electron collider* with 5 TeV electrons in each beam, estimate the depth of the collision point below the earth's surface. Assume that the terrain at the surface above the linacs is continuously level with respect to gravity. $d = 780$ m

2. In the above configuration the linacs cannot easily be upgraded to higher energies by making them longer. Thus, suppose our "linacs" are actually constructed to follow the curvature of the earth's surface. (This should also help mitigate construction costs as well.) However, as the electrons follow the curvature of the earth, they will lose energy due to synchrotron radiation. Estimate the ultimate energy of each electron beam in such an accelerator. $\hat{E} = 20$ TeV

Note: The radius of the earth is approximately 6.4×10^6 m.

1)



$$L = \frac{5 \cdot 10^6 \, MeV}{50 \, MeV/m} = 100 \text{ km}$$

$$R^2 = L^2 + (R-d)^2 \rightarrow d = R - \sqrt{R^2 - L^2} = R\left[1 - \sqrt{1 - (L/R)^2}\right]$$

$$= 6400 \text{ km}\left[1 - \sqrt{1 - (100/6400)^2}\right]$$

$$= \boxed{780 \text{ m}}$$

2) When $E_{gain} = E_{loss}$, then $\Delta E = 0$

$$E' = C_\gamma \frac{E^4}{R} \cdot \frac{1}{2\pi R} \Rightarrow E = \left[\frac{2\pi E' R^2}{C_\gamma}\right]^{1/4} = \left[\frac{2\pi (0.05)(6.4 \cdot 10^6)^2}{8.85 \cdot 10^{-5}}\right]^{1/4} \text{GeV}$$

$$= \boxed{20 \text{ TeV}}$$