Beam Cooling (Emittance Reduction)

Northern Illinois University

- Electron Cooling

- Stochastic Cooling

- Ionization Cooling

Electron Cooling*

Northern Illinois University

The appeal of electron cooling is easy to illustrate. In conventional kinetic theory, the gas temperature is related to the mean energy of the molecules by

$$\frac{\langle p^2 \rangle}{2m} = \frac{3}{2}kT.$$

So for an ion beam, one can define a "temperature" for each degree of freedom by

$$\frac{\langle p_{x0}^2 \rangle}{m}, \quad \frac{\langle p_{y0}^2 \rangle}{m}, \quad \frac{\langle p_{s0}^2 \rangle}{m},$$

where the Boltzmann constant has been supressed. The subscript "0" implies that the momenta are measured with respect to the rest frame of the beam centroid.

$$T_x = mc^2\left(\frac{v}{c}\right)^2 \gamma^2 (\sigma')^2 = mc^2\left(\frac{v}{c}\right)\gamma\frac{\epsilon_N}{\pi\beta},$$

$$T_s = mc^2\left(\frac{v}{c}\right)^2 \sigma_p^2, \qquad \sigma_p^2 \equiv \left\langle \left(\frac{\Delta p}{p}\right)^2 \right\rangle.$$

In electron cooling, an electron beam with very small emittance is made to travel at the same speed as the proton/ion beam that has a relatively high emittance (transverse temperature).

The cool electrons then exchange transverse energy with the hot protons/ions, and then the electrons are discarded (and new electrons re-generated).

Electron Cooling Time

M. Steck CAS 2011 Chios Greece

first estimate: (Budker 1967) $\tau = \frac{3}{8\sqrt{2\pi} n_e Q^2 r_e r_i c L_C}\left(\frac{k_B T_e}{m_e c^2} + \frac{k_B T_i}{m_i c^2}\right)^{3/2}$

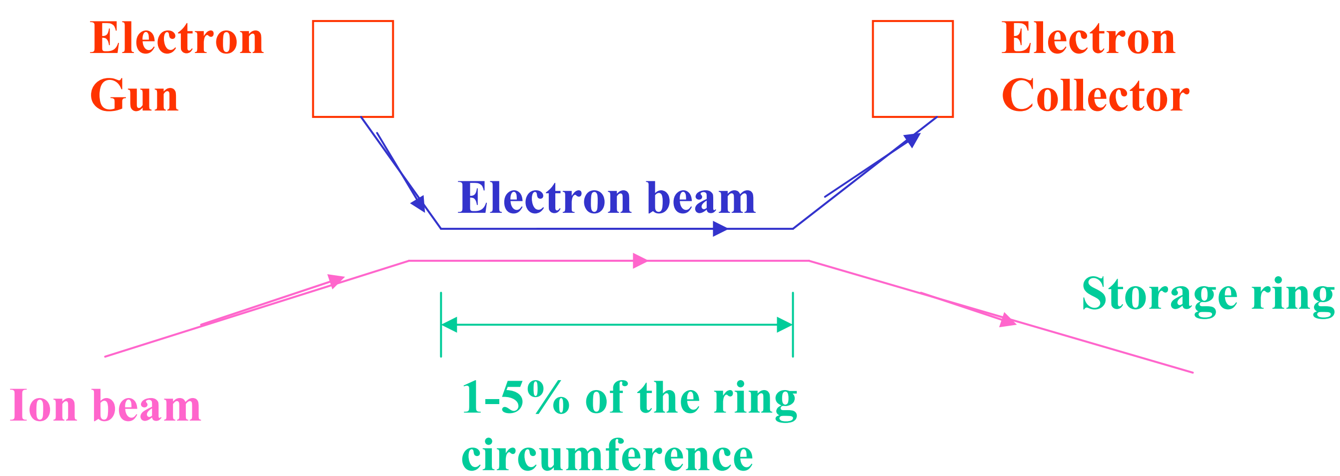
for large relative velocities

cooling time $\tau_z \propto \frac{A}{Q^2}\frac{1}{n_e \eta}\beta^3\gamma^5\theta_z^3 \quad \left\{ \begin{array}{l} \theta_{x,y} = \frac{v_{x,y}}{\gamma\beta c} \\ \theta_{\parallel} = \frac{v_{\parallel}}{\gamma\beta c} \end{array} \right.$

LEIR ring, CERN

*G.1. Budker, ***Proc. Intl. Symp. on Electron and Positron Storage Rings,*** Saclay, 1966, p. 11-1-1

Electron Cooling



To cool W = 8 GeV proton beam requires an electron beam with W = 4.3 MeV
must have same speeds:

$$\gamma_p = \gamma_e$$

$$W_p m_e / m_p = W_e$$

Electron cooling system setup at Fermilab

Pelletron

(MI-31 building)

Cooling section solenoids

(MI-30 straight section)

S. Nagaitsev – Electron Cooling 67

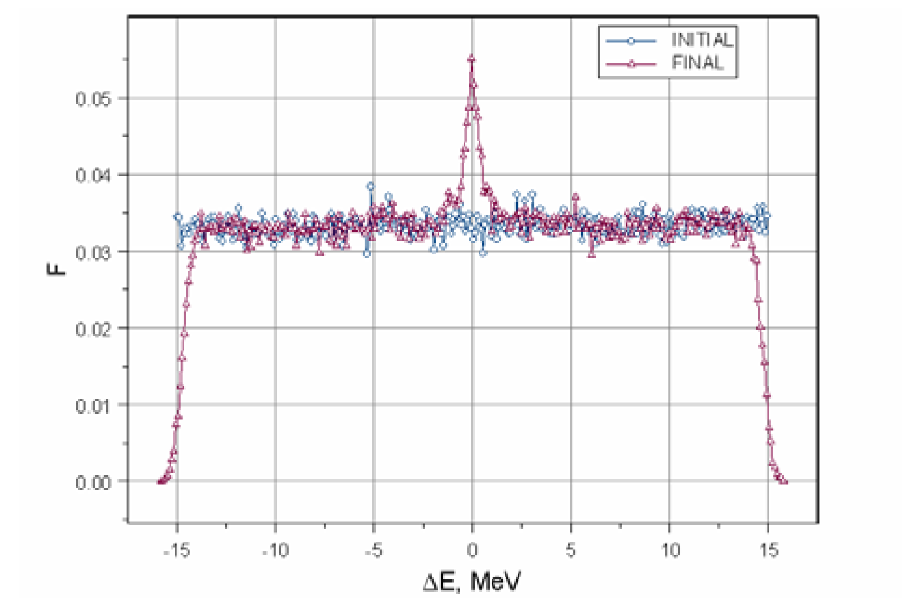


Figure 5: The momentum distribution (arb. units) as a function of antiproton energy deviation (simulation by MOCAC code [5]). The initial distribution is uniform in energy. The final distribution is plotted after 30 minutes.

Transverse Stochastic Cooling*

- *Transverse Cooling*

k samples in the ring

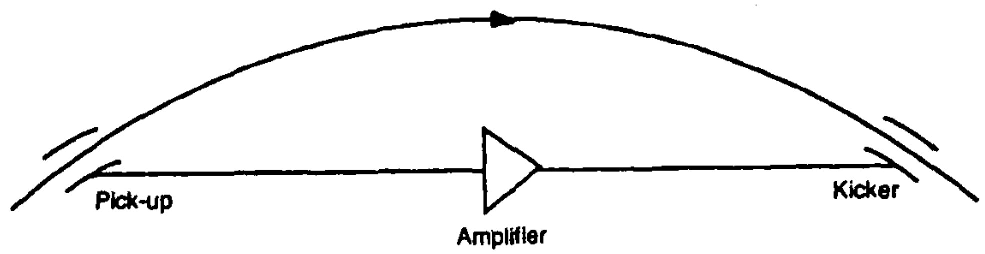


Figure 7.12. Stochastic cooling system consisting of pickup electrodes, amplifier, and beam deflector.

$$f_{max} = \frac{v}{\lambda_{min}} = \frac{kv}{2C} = \frac{k}{2T}, \qquad (7.136)$$

where T is the revolution period. For a system with a flat frequency response from $f = 0$ to $f = W$, W determines f_{max}. So the number of particles in a sample, in terms of the bandwidth W, is given by

$$N_s = \frac{N}{k} = \frac{N}{2TW}. \qquad (7.137)$$

where in the second step we have assumed that the various x_i are uncorrelated. Thus, keeping terms up to first order in $1/N_s$, we have for the rate of change of $\langle x^2 \rangle$

$$\frac{d\langle x^2\rangle}{dn} = -\frac{2g}{N_s}\langle x^2\rangle + \frac{g^2}{N_s}\langle x^2\rangle. \qquad (7.143)$$

The cooling rate is then

$$\frac{1}{\epsilon}\frac{d\epsilon}{dn} = -\left(\frac{2g - g^2}{N_s}\right), \qquad (7.144)$$

or, in terms of time,

$$\frac{1}{\tau} \equiv -\frac{1}{\epsilon}\frac{d\epsilon}{dt} = -\frac{1}{\epsilon}\frac{d\epsilon}{dn}\frac{1}{T} = \frac{2g - g^2}{N_s T} = \frac{2W}{N}(2g - g^2). \qquad (7.145)$$

Averaging over many samples, $\langle x_n \rangle = 0$ and so

$$\frac{1}{\langle x^2\rangle}\frac{d\langle x^2\rangle}{dn} = \left[-2g + g^2(1 + U)\right]\frac{1}{N_s}, \qquad (7.149)$$

$$M = \frac{T_s}{\Delta T}, \qquad (7.150)$$

where $T_s = (N_s/N)T = 1/(2W)$ is the sample time, and ΔT is the change in the revolution period due to the momentum deviation $\Delta p/p$. Then

$$M = \frac{1}{2WT|\eta|(\Delta p/p)}. \qquad (7.151)$$

For ideal mixing, $M = 1$. Intuitively, one would expect the cooling rate to degrade by a factor of M as we depart from perfect mixing. Actually, this factor of M appears only in the incoherent term, and so the emittance decreases according to

$$\epsilon = \epsilon_0 e^{-t/\tau}, \qquad (7.152)$$

where we have for the cooling rate

$$\frac{1}{\tau} = \frac{2W}{N}\left[2g - g^2(M + U)\right].$$

*D. Mohl, G. Petrucci, L. Thorndahl, and S. van der Meer, "Physics and Technique of Stochastic Cooling," Physics Reports **58, No.**2 (1980).

Longitudinal Stochastic Cooling

Northern Illinois University

- Cooling

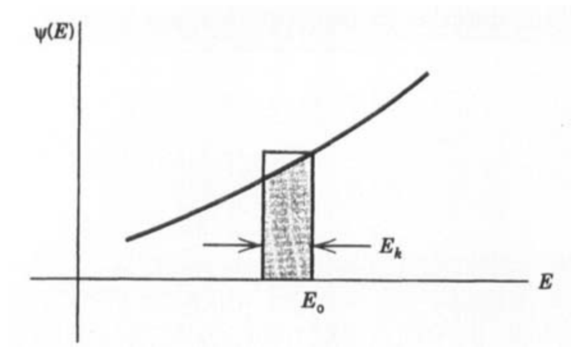


Figure 7.13. Particle density function $\psi(E)$.

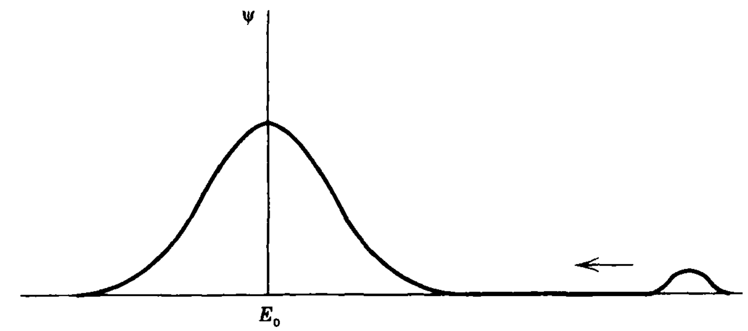


Figure 7.14. Momentum stacking.

As in the discussion of beam-gas scattering, the flux arising from diffusion can be written in the form

$$\vec{J} = -D\nabla\psi, \tag{7.154}$$

where $\vec{J}$ is the particle flux. In the case under consideration here, since energy is the only degree of freedom,

$$J = -D(E)\frac{\partial\psi}{\partial E}, \tag{7.155}$$

where the diffusion "constant" may be a function of energy. To this, we must add coherent forces. If the rate of energy gain is $C(E)$, then we must add $\psi C(E)$ to the flux, obtaining

$$J = C(E)\psi - D(E)\frac{\partial\psi}{\partial E}. \tag{7.156}$$

in which the flux is zero. Suppose there is a coherent force driving particles toward some central energy E_0, where the force is proportional to the energy deviation $E - E_0$, and suppose that the diffusion force is a constant. Then $C(E) = -\alpha(E - E_0)$ and $D(E) = D_0$. So this static situation is described by

$$J = -\alpha(E - E_0)\psi - D_0\frac{\partial\psi}{\partial E} = 0. \tag{7.163}$$

The solution to the above equation is the Gaussian

$$\psi = \psi_0 e^{-\alpha(E-E_0)2/2D_0}. \tag{7.164}$$

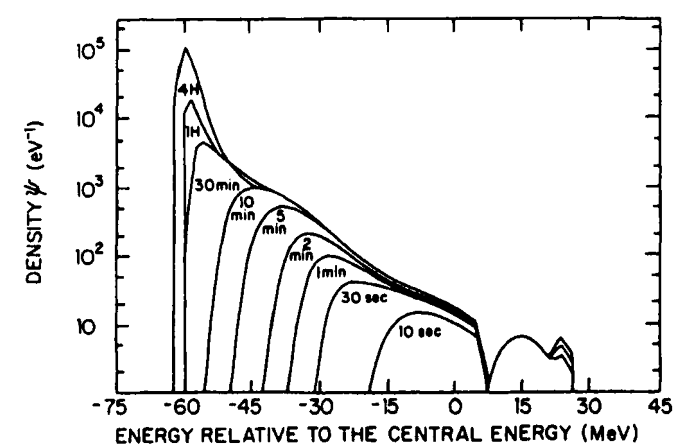


Figure 7.15. Design curves for antiproton energy density at FNAL Accumulator Ring, showing development of the "$\bar{p}$ stack" over time. From Tollestrup and Dugan, with permission.

So we have

$$\frac{\partial\psi}{\partial E} = -\frac{e^2\psi}{4J_0T^2A} + \frac{e^2\psi}{2J_0T^2A} = \frac{e^2\psi}{4AT^2J_0} \equiv \frac{\psi}{E_d}, \tag{7.171}$$

or

$$\psi = \psi_0 e^{(E-E_i)/E_d}. \tag{7.172}$$

Ionization Cooling*

Applications of Ionization Cooling

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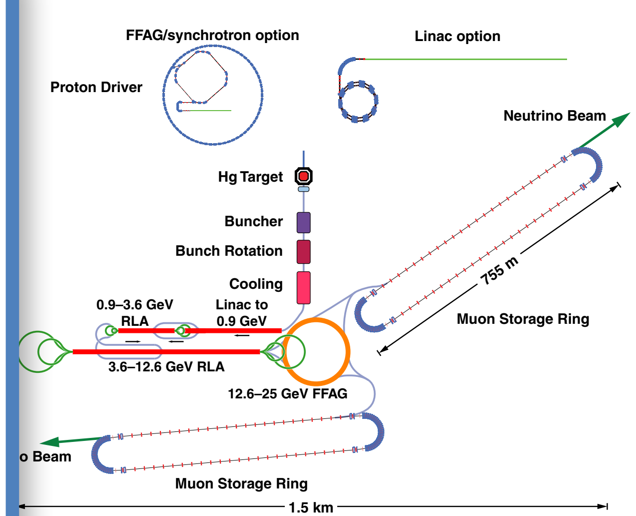
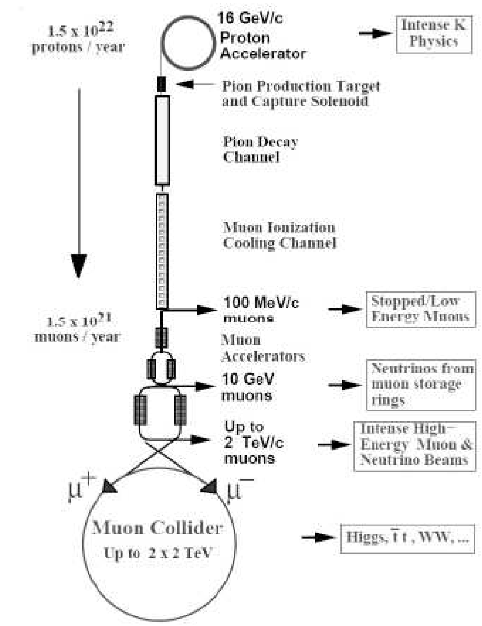


Muon Collider



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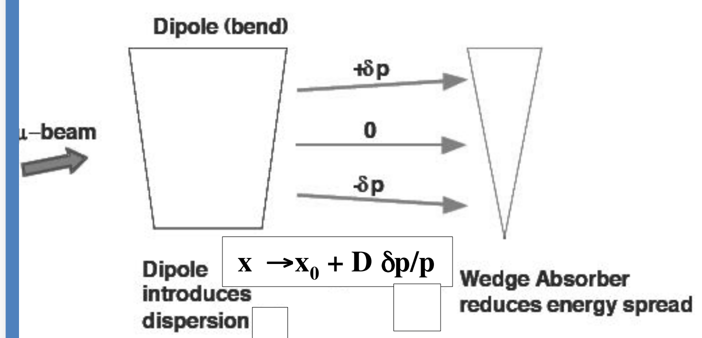
increased longitudinal cooling
by longitudinal-transverse emittance exchange



$$\frac{d\sigma_E^2}{ds} = -2\frac{\partial(dE/ds)}{\partial E}\sigma_E^2 + \frac{d\langle \Delta E_{rms}^2\rangle}{ds}$$

cooling term   heating term

cooling, if $\frac{\partial(dE/ds)}{\partial E} > 0$

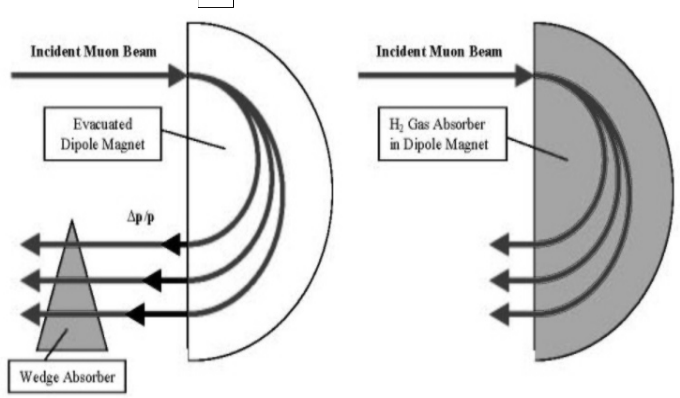


Figure 1. Use of a Wedge Absorber for Emittance Exchange

Figure 2. Use of Continuous Gaseous Absorber for Emittance Exchange

emittance exchange

increased longitudinal cooling

$$\frac{\partial \frac{dE}{ds}}{\partial E} \Rightarrow \frac{\partial \frac{dE}{ds}}{\partial E}\Big|_0 + \frac{dE}{ds}\frac{D\rho'}{\beta c p \rho_0}$$

reduced transverse cooling

$$\frac{d\epsilon_N}{ds} = -\frac{1}{\beta^2 E}\frac{dE}{ds}\left(1 - \frac{D\rho'}{\rho_0}\right)\epsilon_N$$

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Muon Ionization Cooling Experiment at RAL

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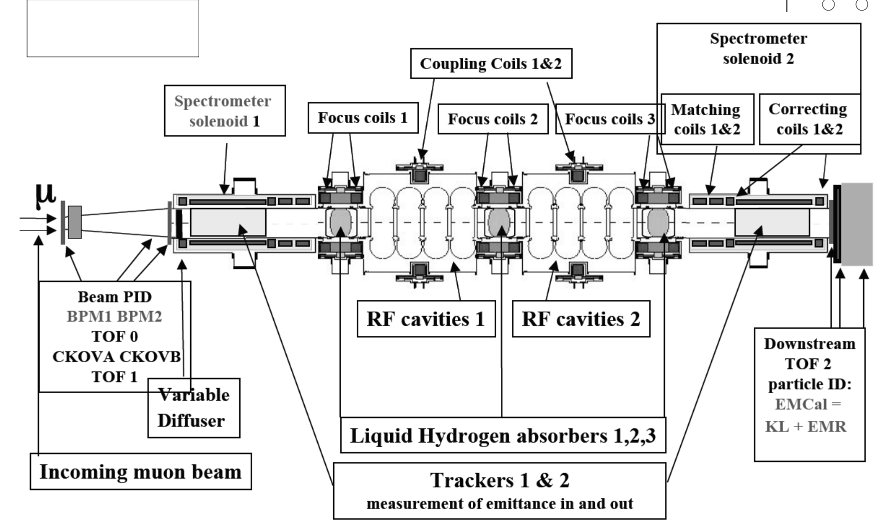

* G.I. Budker, in: Proceedings of 15th International Conference on High Energy Physics, Kiev, 1970