Longitudinal Focusing

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- sometimes referred to as "phase focusing" or "time focusing"
- particles of different energy (momentum) move at different speeds, so tend to "spread out" relative to the "ideal" particle which is assumed to exist traveling with perfect synchronism with respect to the oscillating fields
- wish to study the (longitudinal) motion of particles relative to this "synchronous particle"

Longitudinal Focusing

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- time of flight — the "slip factor"
- Evolution due to *dp/p* or *dW/W*
- Longitudinal focusing, time of arrival:
  - bunchers, rebunchers, debunchers

The Slip Factor

$$t = \frac{L}{v}$$

$$\frac{dt}{t} = \frac{dL}{L} - \frac{dv}{v}$$

$$\frac{dv}{v} = \frac{1}{\gamma^2}\frac{dp}{p}$$

$$\frac{dt}{t} = \left(\alpha_p - \frac{1}{\gamma^2}\right)\frac{dp}{p}$$

$$\frac{dt}{t} = \eta\,\frac{dp}{p}$$

Momentum Compaction Factor:

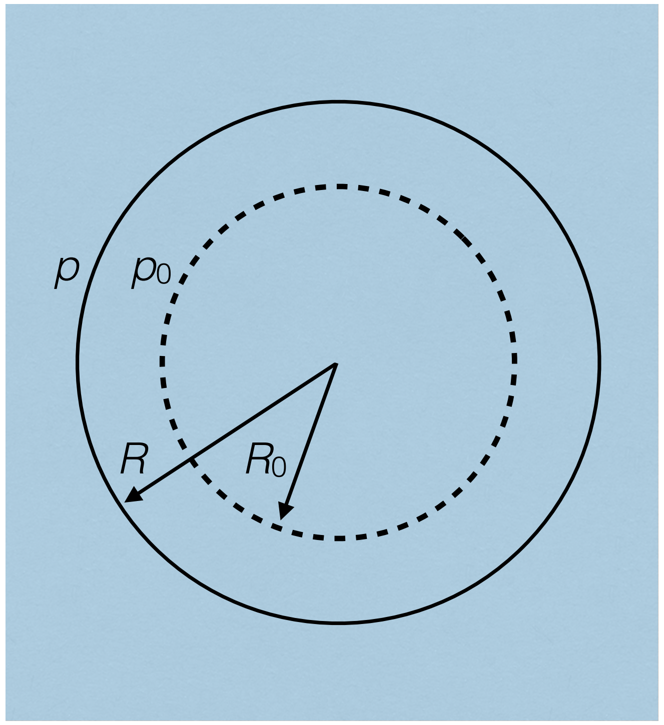
$$\alpha_p \equiv \left(\frac{dL/L}{dp/p}\right)$$

The Slip Factor: $$\eta \equiv \alpha_p - \frac{1}{\gamma^2}$$

For a straight section, or a linac, $\eta = -1/\gamma^2 < 0$

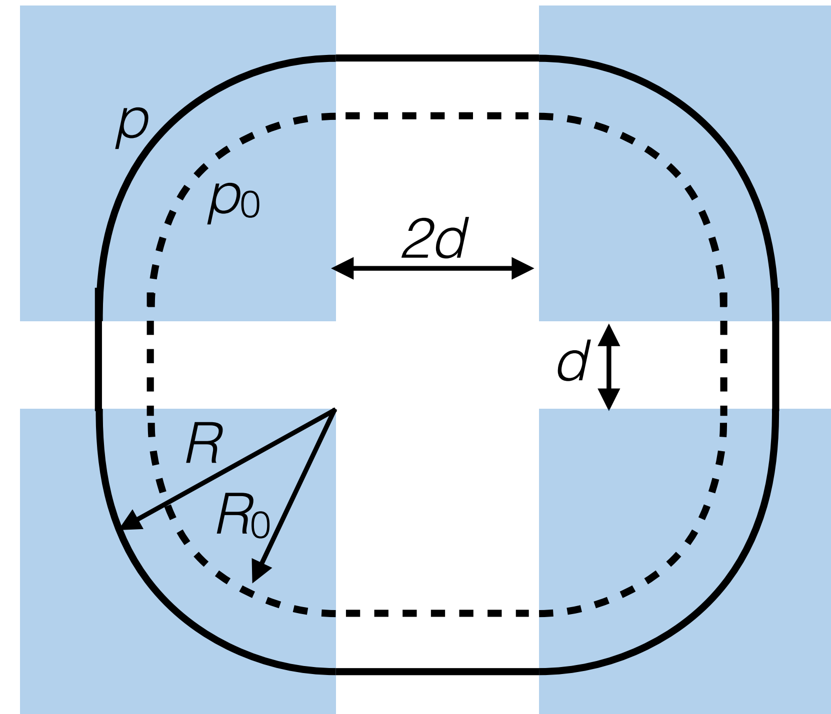
For a region with bending, α_p might not be zero

A Simple Example



$$R/R_0 = p/p_0$$

$$\frac{\Delta v}{v} = \frac{1}{\gamma^2}\frac{\Delta p}{p}$$



$$\tau = 2\pi R/v$$

$$\tau = (2\pi R + 6d)/v$$

$$\frac{\Delta L}{L_0} = \frac{\Delta R}{R_0} = \frac{\Delta p}{p_0}$$

$$\frac{\Delta L}{L} = \frac{2\pi(R - R_0)}{2\pi R_0 + 6d} = \frac{1}{1 + 3d/\pi R_0}\frac{\Delta p}{p_0}$$

$$\frac{\Delta \tau}{\tau_0} = (1 - \frac{1}{\gamma_0^2})\frac{\Delta p}{p_0}$$

$$\frac{\Delta \tau}{\tau_0} = \left(\frac{1}{1 + 3d/\pi R_0} - \frac{1}{\gamma_0^2}\right)\frac{\Delta p}{p_0}$$

Implications of the Slip Factor

- Suppose no bending in the line (e.g., linac), or, perhaps have bending yet $\gamma^2 < 1/\alpha_p$
  - then, the slip factor is negative, and particles of higher momentum take less time to traverse the same distance as the ideal particle

$$\eta = \alpha_p - \frac{1}{\gamma^2}$$

- If the energy of the particles is high enough in the presence of bending, then can have $\gamma^2 > 1/\alpha_p$
  - in this case, the slip factor is positive — the changes in path length outweigh the changes in speed when determining the time of flight difference
  - here, a higher-momentum particle will actually take ***longer*** to traverse the same distance as the ideal particle, even though it's moving faster

Achromatic / Isochronous Sections

- Isochronous Section:

$$\frac{\Delta T}{T} = 0$$ all particles take same time

essentially: $$\eta = \underbrace{\frac{\Delta L/L}{\Delta p/p}}_{\frac{1}{L}\int_0^L \frac{D(s)}{\rho(s)}\,ds \;\approx\; \frac{1}{L}\cdot \sum D_i\theta_i} - \frac{1}{\gamma^2} = 0 \qquad \left(\frac{\Delta T}{T} = \eta \frac{\Delta p}{p}\right)$$

$\therefore$ make $$\sum D_i\theta_i \approx \frac{L}{\gamma^2}$$

ex: 180° bend w/ 4 magnets: $$4\bar{D}\cdot\frac{\pi}{4} \approx \frac{L}{\gamma^2} \;\rightarrow\; \bar{D} \approx \frac{L}{\pi\gamma^2}$$

Isochronous Bending Section in FRIB

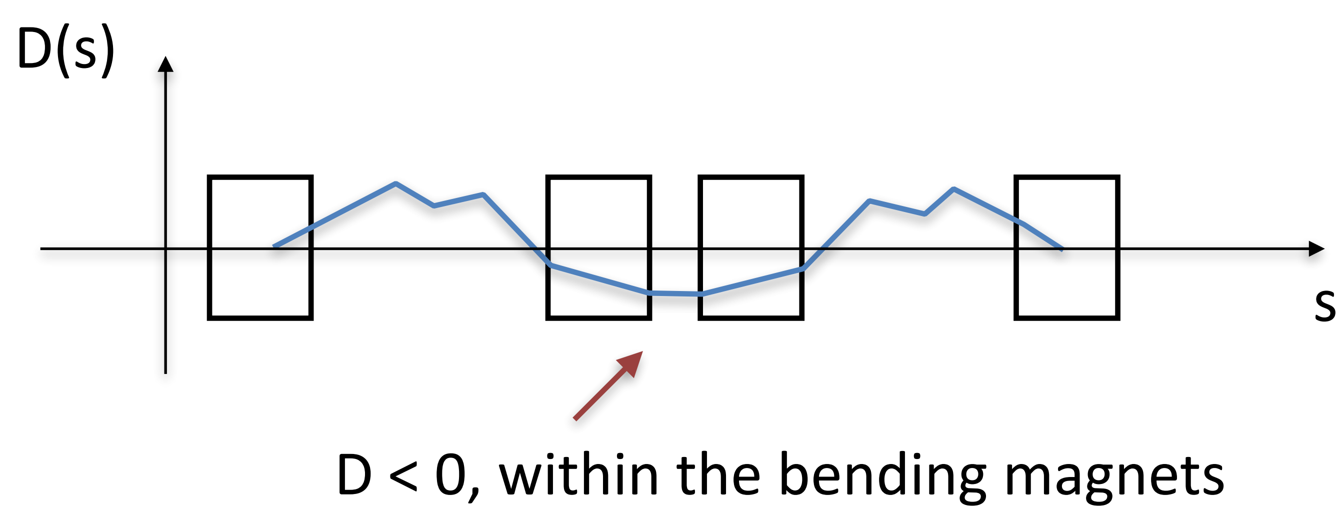
- FRIB beam — *dp/p* ≲ 0.1%; but will accelerate several charge states simultaneously: $dQ/Q = \pm 2/78 = \pm 2.5\%$!!

$$\frac{d(B\rho)}{B\rho} = \frac{dp}{p} - \frac{dQ}{Q} \approx -\frac{dQ}{Q} \qquad \frac{\Delta L}{L} = \alpha_p \frac{\Delta p}{p} \longrightarrow \alpha_p \left(\frac{\Delta p}{p} - \frac{\Delta Q}{Q} \right)$$

$$\frac{\Delta t}{t} = \left(\alpha_p - \frac{1}{\gamma^2} \right) \frac{\Delta p}{p} - \alpha_p \frac{\Delta Q}{Q}$$

~1   *small* *(10^{-3})*   *larger* *($25\ 10^{-3}$)*

$$\alpha_p = \langle D/\rho \rangle \approx \frac{1}{L} \sum_i D_i \theta_i$$



adjust α_p to make total $\Delta t/t$ ~ 0

Magnetic Chicane

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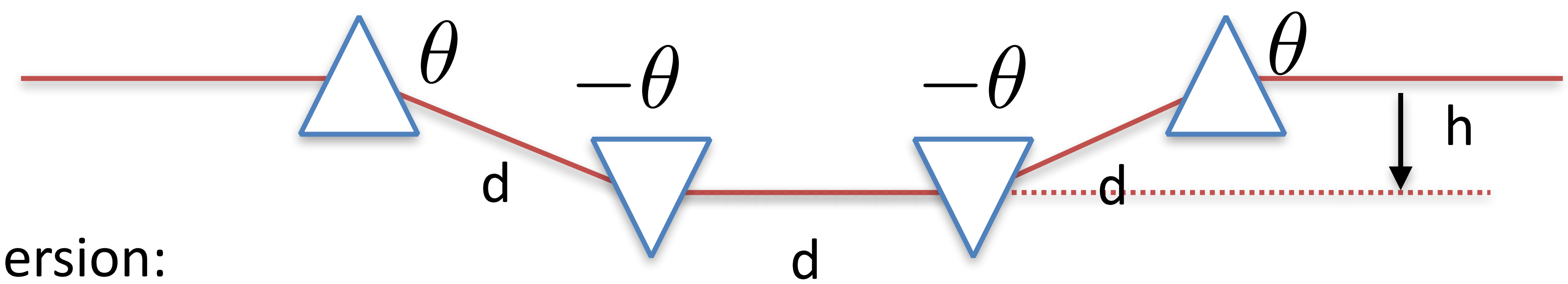
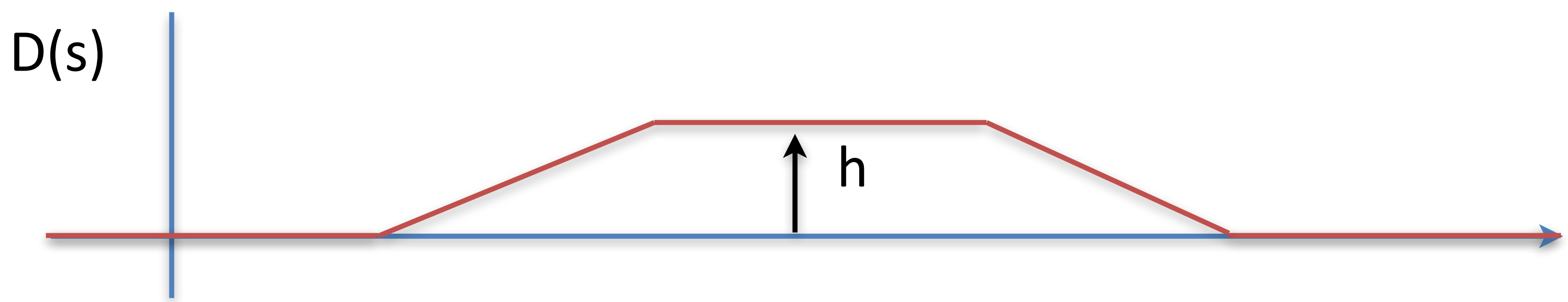


dispersion:



Higher rigidity will take shorter path; thus often used for bunch length manipulations (especially in e- linacs/beam lines)

$$\eta = \langle D/\rho \rangle - \frac{1}{\gamma^2} \approx \frac{1}{L_{tot}} \sum D_i \cdot \theta_i - \frac{1}{\gamma^2}$$

$$= \frac{1}{3d} \cdot 2 \cdot h \cdot (-\theta) - \frac{1}{\gamma^2} = -\frac{2}{3}\frac{h}{d}\theta - \frac{1}{\gamma^2} = -\frac{2}{3}\theta^2 - \frac{1}{\gamma^2}$$

$$\frac{\Delta t}{t} = \eta \frac{\Delta p}{p} < 0$$

control *dt*/*dp* by controlling θ

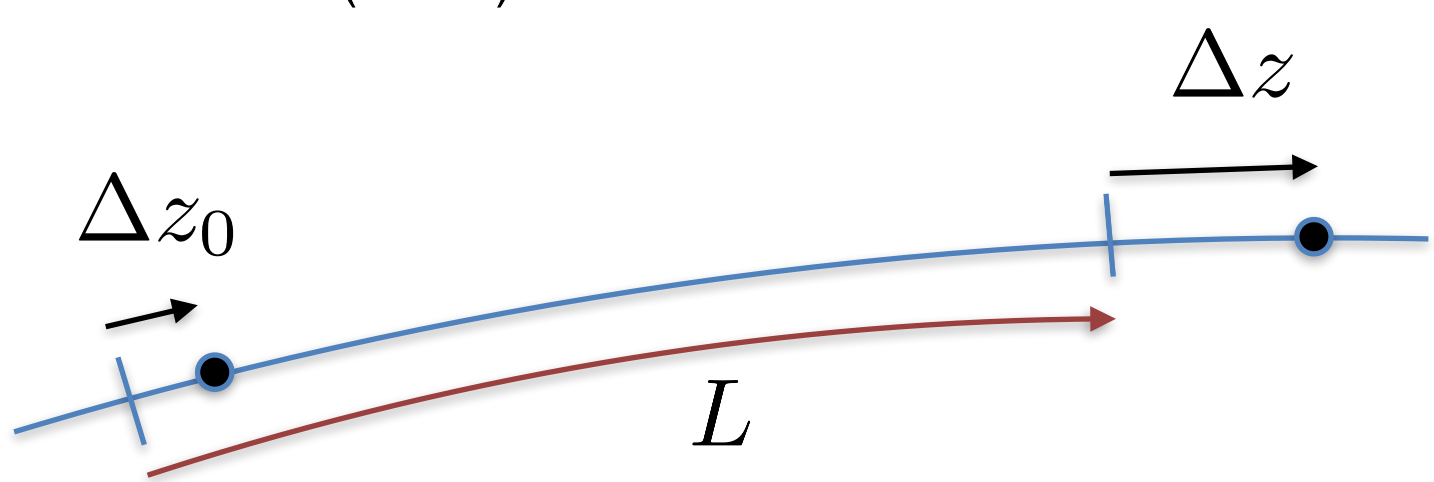
can be very small, for e- beams!

Linear Motion Very Near the Ideal Particle

- Particles moving along the ideal trajectory move toward or away from the ideal particle according to their speed (momentum/energy) and path length differences

Δt = arrival time relative to the ideal arrival time ($\Delta t = 0$)

$$\Delta z = -\beta c \, \Delta t$$



$$\tau = L/v = L/(\beta c)$$

$$\Delta z = \Delta z_0 - \eta L \frac{\Delta p}{p}$$

$$\Delta t = \Delta t_0 + \eta \frac{L}{\beta c} \frac{\Delta p}{p}$$

$$\text{Note}: \quad \frac{\Delta p}{p} = \frac{1}{\beta^2}\frac{\Delta E}{E} = \frac{1}{\beta^2}\frac{\gamma - 1}{\gamma}\frac{\Delta W}{W}$$

Linear Motion Very Near the Ideal Particle [2]

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- Imagine a particle on the ideal trajectory and that has the ideal energy, W_s. A second particle on the ideal trajectory, but with a different energy, W, may be ahead of or lagging behind the ideal particle.
- We will use **radio frequency (RF) cavities** to provide an accelerating voltage to the particles as they pass by.
- The ideal particle will arrive at the cavity at the "ideal" time or, equivalently, at an ideal phase, ϕ_s, to receive an appropriate increase in its energy (which might be an increase of "0").
- We will keep track of the "difference" in energy between our test particle and the ideal particle:

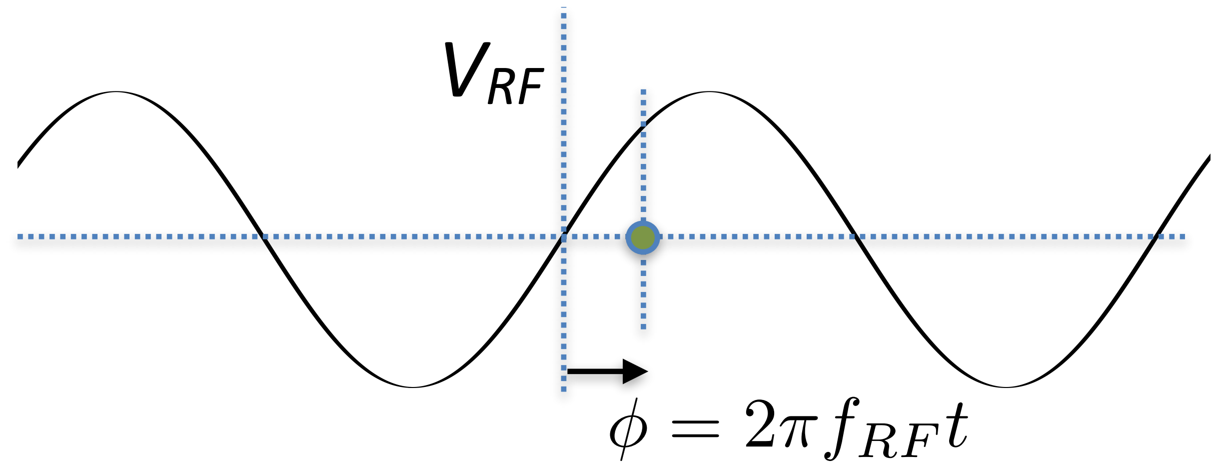
$$W_s = \text{``ideal'' energy}$$

$$\Delta W \equiv W - W_s$$

Acceleration using AC Fields

- Pass through a gap with an oscillating field, particle gains energy …

$$W = W_0 + q\ E\ g\ \sin(\phi)$$
$$= W_0 + qV\sin(\phi)$$



- Here, ϕ is the "phase" of the oscillating field at the time of arrival
- But here, V is an "average" or "effective" potential; depends upon the frequency of the field in the gap, the incoming speed of the particle (due to the field varying with time), and the phase of the oscillation relative to the particle arrival time:

$$W = W_0 + q\ T(\beta)\ V_0 \sin(\phi)$$

- For our purposes today, we will lump the transit time factor, *T*, and the peak voltage, V_0, into a single "effective voltage", *V*

Linear Motion Very Near the Ideal Particle

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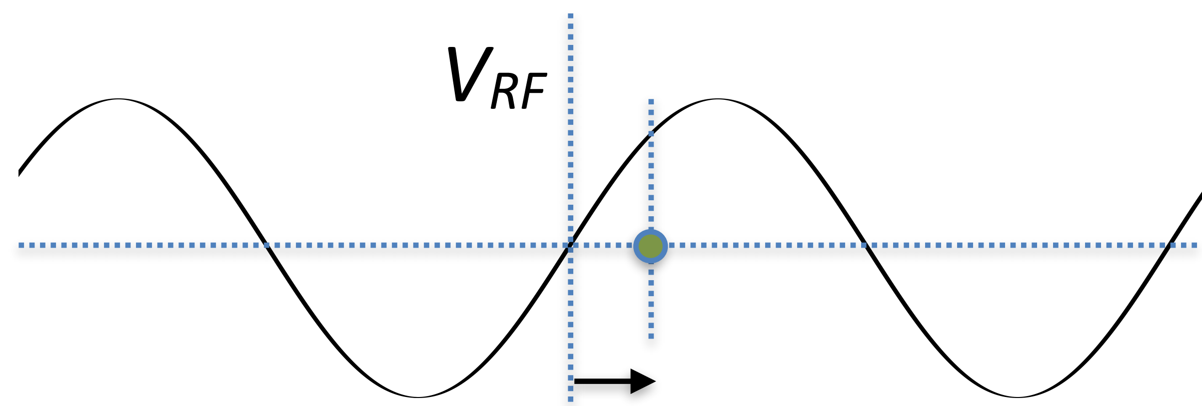
- Let the ideal particle receive energy gain according to:

$$W_s = W_{s,0} + qV \sin(\phi_S) \qquad W_s = \text{“ideal” energy}$$

- As nearby particles passes through the cavity, will give particles that are ahead/behind a decrease/increase in energy



a particle's energy
difference from the ideal: $\Delta W \equiv W - W_s$

$$\Delta W = \Delta W_0 + qV(\sin\phi - \sin\phi_s)$$
$$= \Delta W_0 + qV[\sin(\phi_s + \Delta\phi) - \sin\phi_s]$$
$$\Delta W \approx \Delta W_0 + qV\cos\phi_s \Delta\phi = \Delta W_0 + qV\cos\phi_s(2\pi f_{RF})\Delta t_0$$

- Can use matrix techniques to propagate the longitudinal motion

Linear Motion through Cavities and Drifts

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- Keep track of time differences and energy differences…

drift:

$$\begin{pmatrix} \Delta t \\ \Delta W \end{pmatrix} = \begin{pmatrix} 1 & \eta \frac{L}{c} \frac{1}{\beta^3 \gamma} \frac{1}{mc^2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta t \\ \Delta W \end{pmatrix}_0$$

through cavity:
longitudinal focusing

$$\begin{pmatrix} \Delta t \\ \Delta W \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ (2\pi f_{RF}) qV \cos\phi_s & 1 \end{pmatrix} \begin{pmatrix} \Delta t \\ \Delta W \end{pmatrix}_0$$

remember —

$$\eta \equiv \alpha_p - \frac{1}{\gamma^2}$$

Note: this "linearization" valid when $\sin(2\pi f_{RF}\,\Delta t) \approx 2\pi f_{RF}\,\Delta t$

Linear Motion through Cavities and Drifts

- So, with this in mind, can create a system to transport beams with large momentum spread that keeps the particles "together" in time along the path

$$M = \ldots \; M_{drift} M_{cavity} M_{drift} M_{cavity} M_{drift}$$

focus

Most important for low-energy beams, such as high-charge-state ion beams

Bunchers, Re-bunchers, Debunchers

- If start with continuous stream of particles (DC current, with no strong “AC” component), can create bunches (AC beam) using a single cavity (buncher)

- If already have bunched beam that is allowed to travel a certain distance, the particles within the bunch will begin to spread out due to the inherent spread in momentum
  - re-buncher: mitigate this effect (last slide)
  - debuncher: enhance this effect
    - for example, to spread beam out when injected into a storage ring or synchrotron

Beam Buncher

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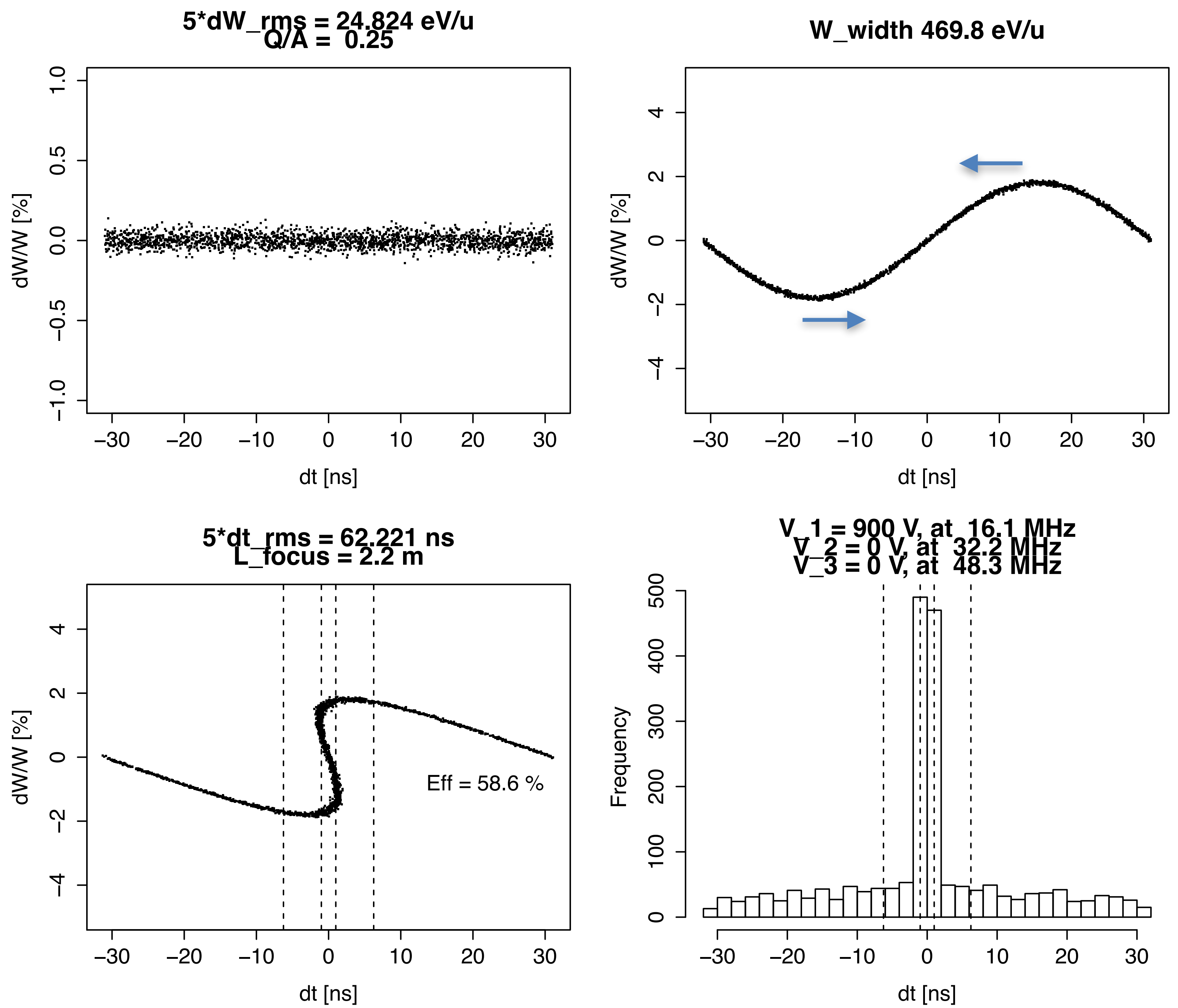
incoming DC beam

after the cavity



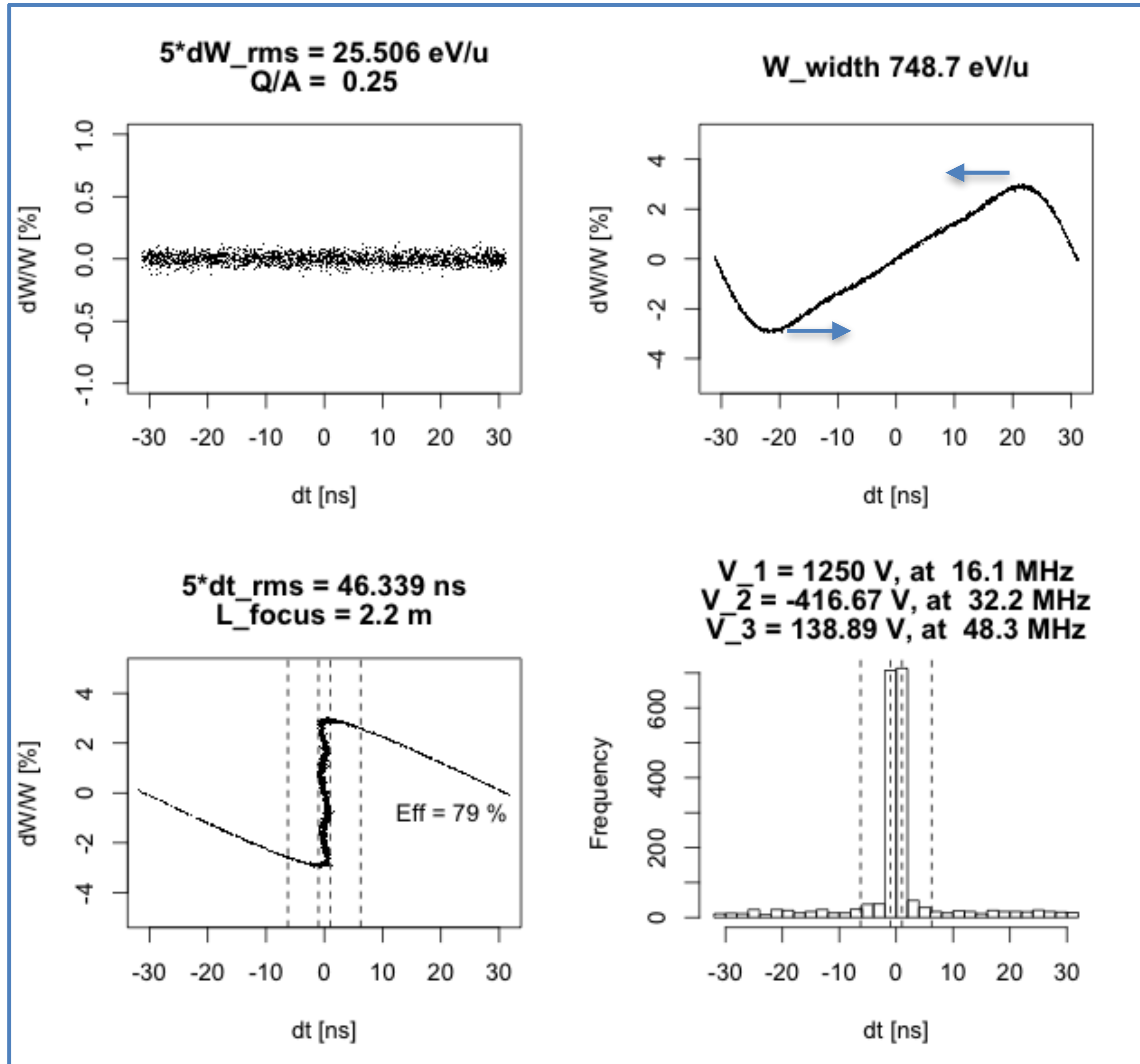
downstream of cavity

resulting time profile

Multi-harmonic Buncher

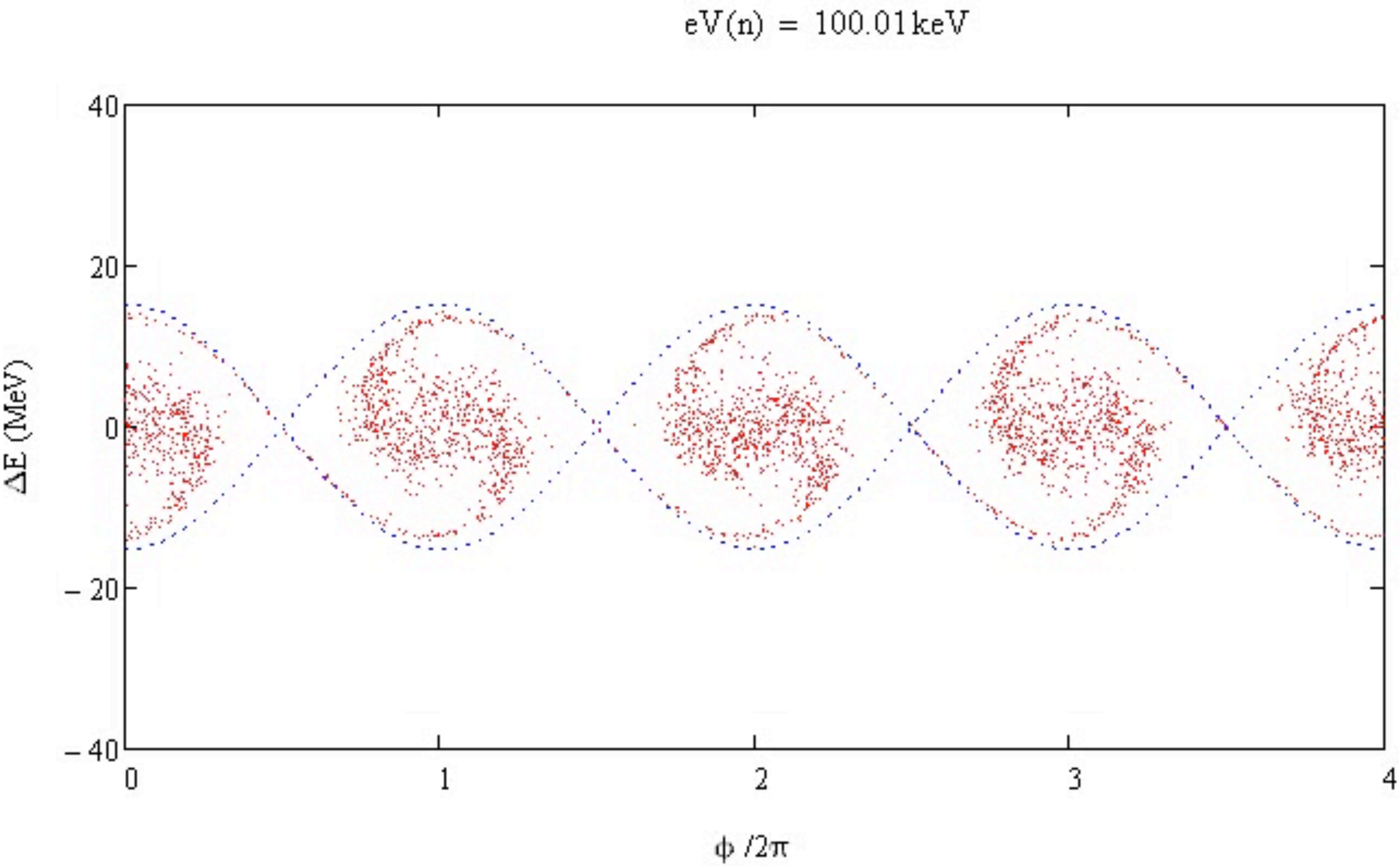
- Use 2, or 3 (or 4?) harmonics of the fundamental frequency to smooth out the sine wave into a more linear waveform

$$V(t) = V_1 \sin(2\pi ft) + V_2 \sin(4\pi ft) + V_3 \sin(6\pi ft) + V_4 \sin(8\pi ft) + \ldots$$



ReA pre-buncher; Alt, *et al.* (MSU)

Adiabatic Capture in a Storage Ring



Here, a single cavity — operating at 4x the revolution frequency (sine wave)— has its voltage gradually increased to turn DC beam into bunched beam