

Acceleration

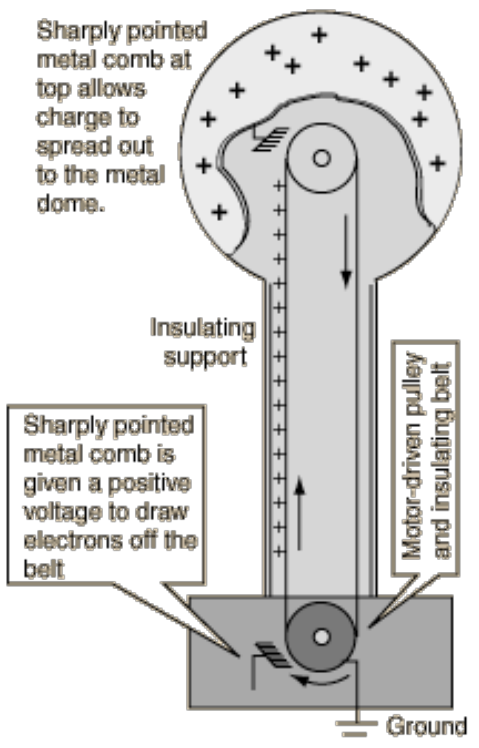
- DC High Voltage Sources
 - ▶ Van de Graaff (and Tandem)
 - ▶ Pelletron
 - ▶ Cockcroft-Walton
- AC Acceleration
 - ▶ Linac
 - ▶ Cyclotron
 - ▶ Betatron
 - ▶ Synchrotrons

- DC Acceleration

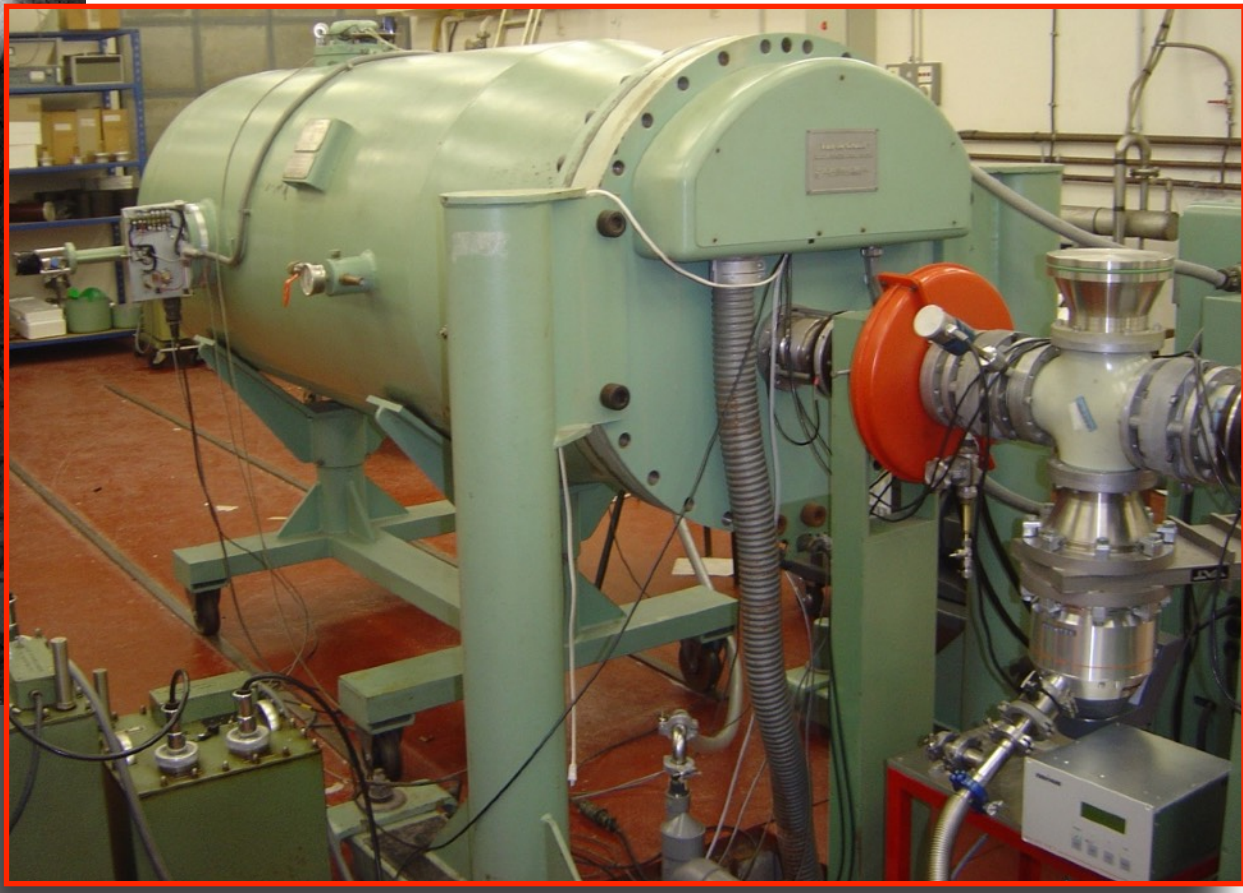
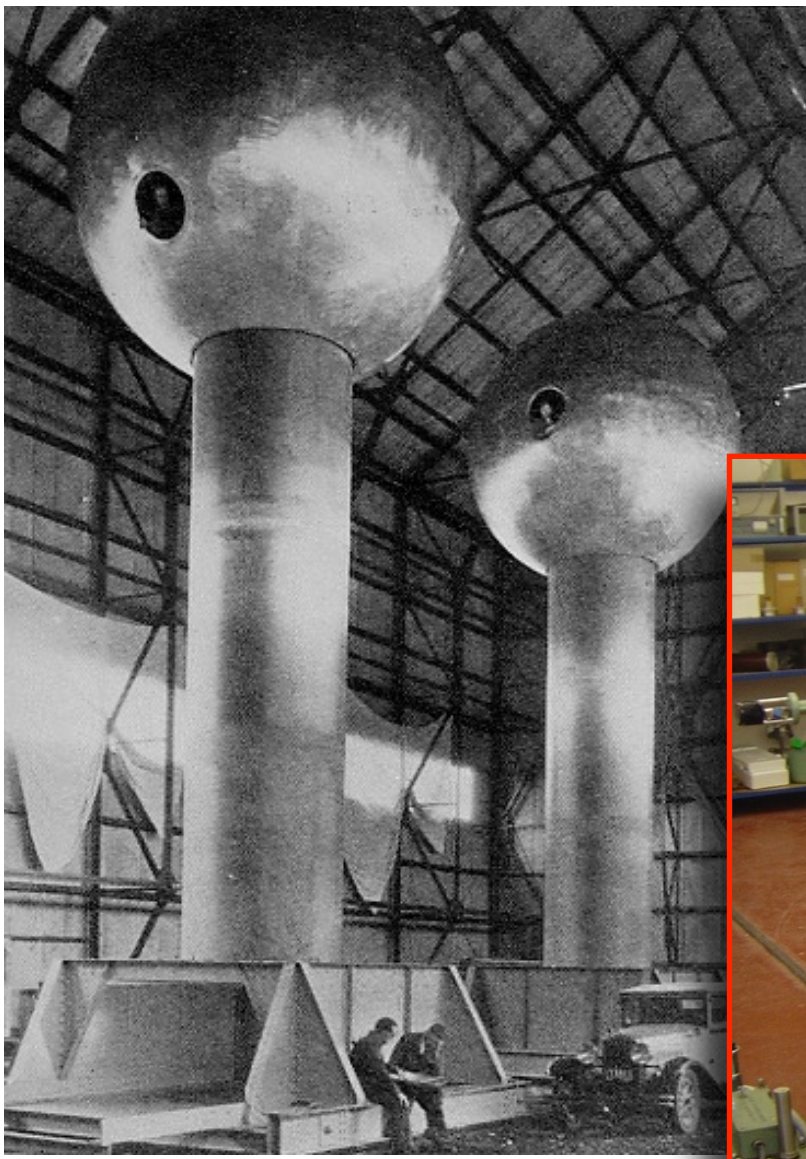
The Van de Graaff

Van de Graaff

(1929)



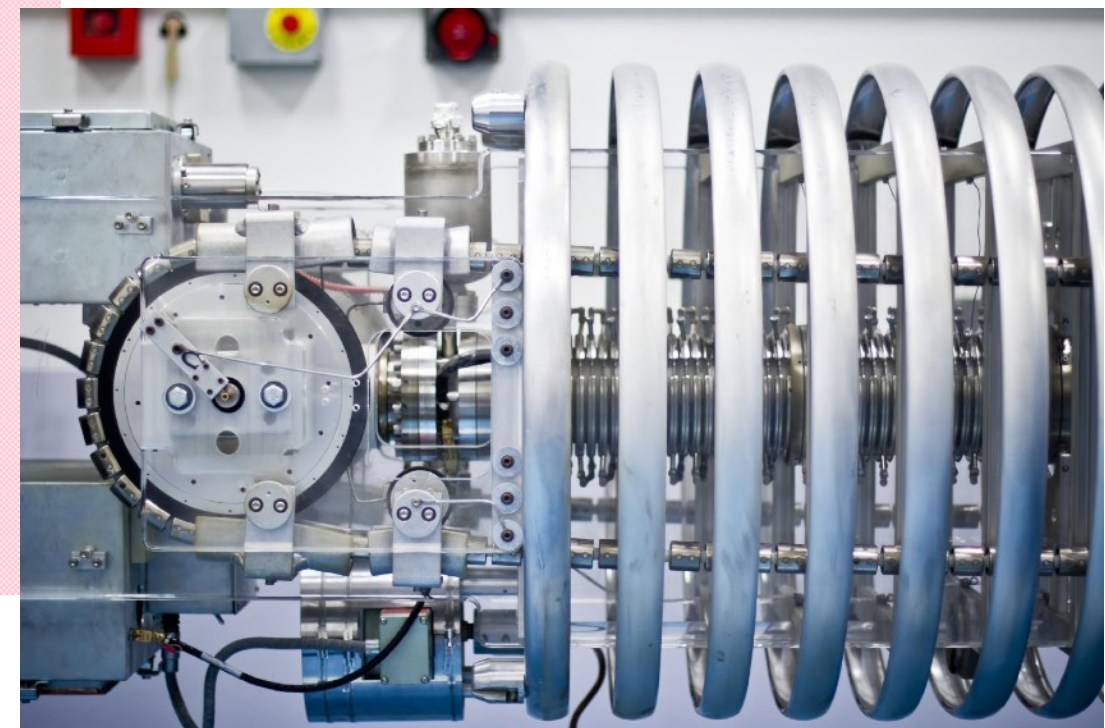
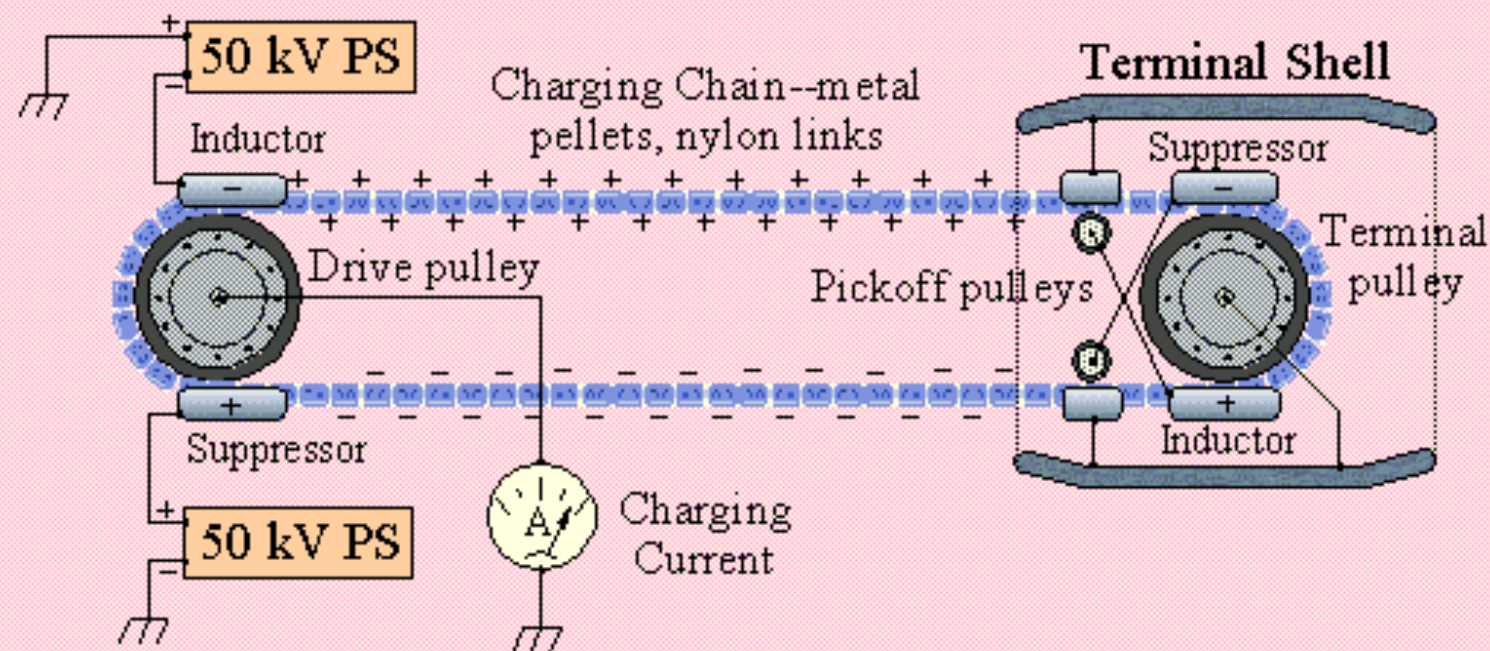
MIT, c.1940s



The Pelletron

- Use mechanical chain of pellets (short conductive tubes connected by links made of insulating material) to build up high voltages on the Pelletron terminal.
- The pellet chain can operate at higher velocity than rubber belt, attaining both higher voltage and currents. Create more stable voltage on terminal.

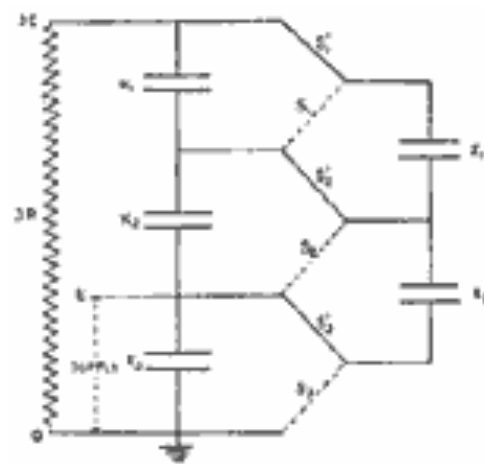
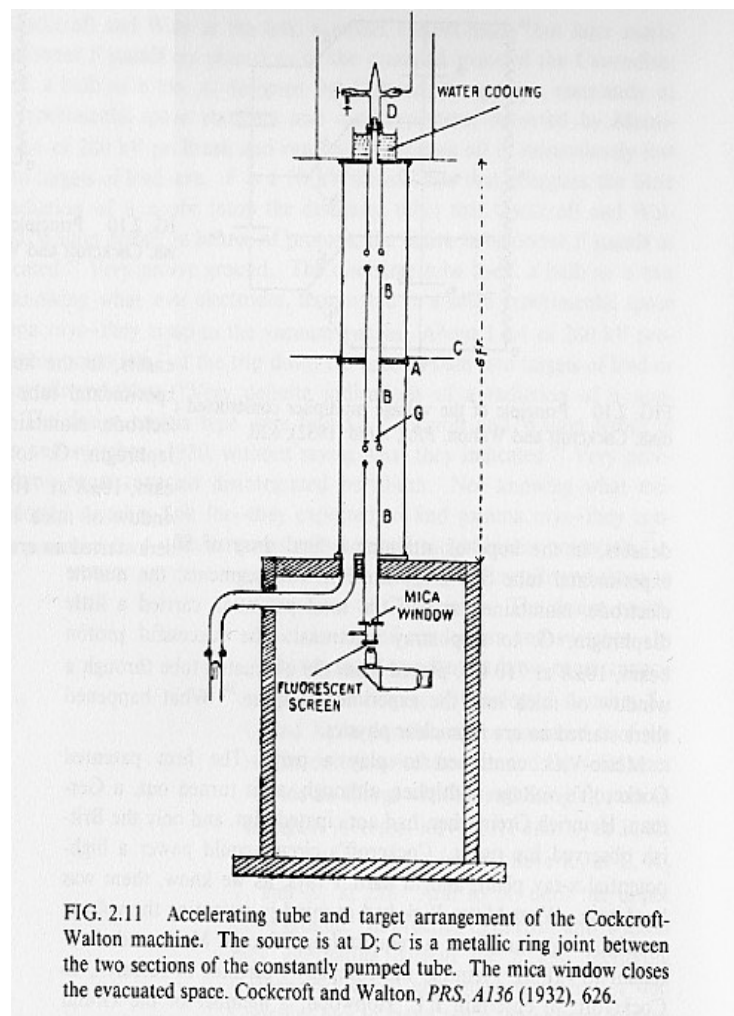
Pelletron Charging System
(Positive configuration shown)



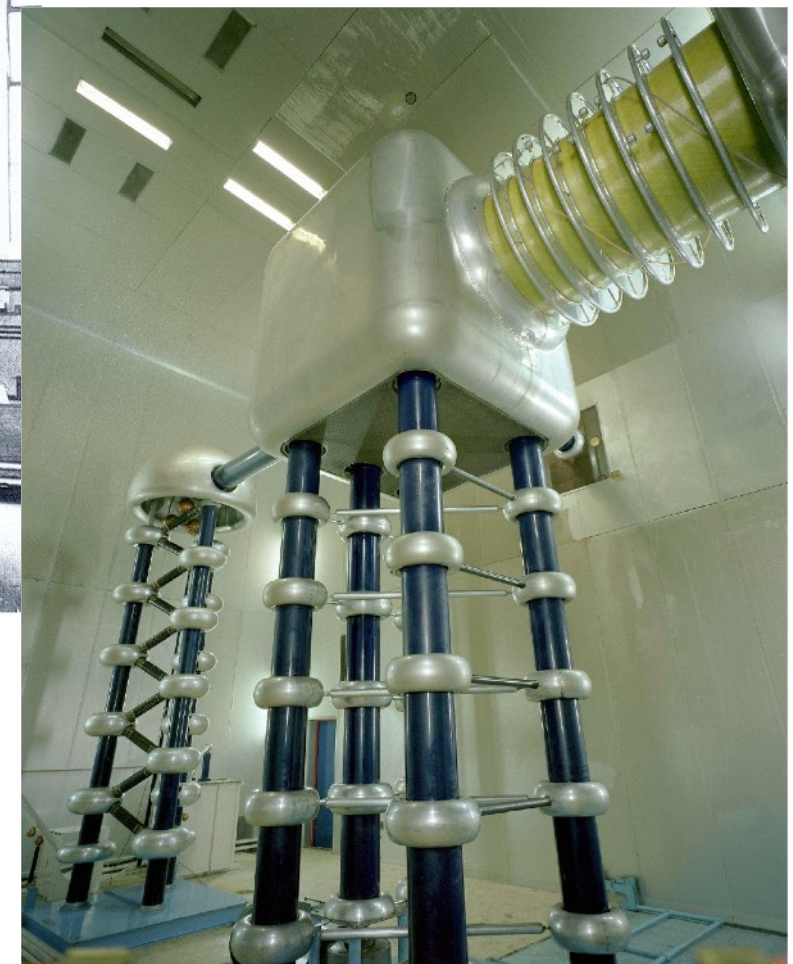
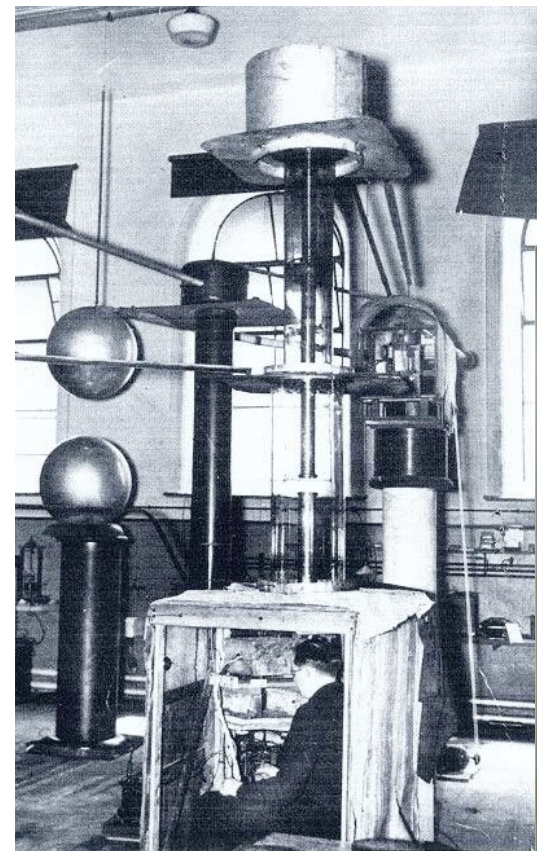
<http://www.pelletron.com/> (National Electronics Corp.)

Cockcroft and Walton

- Voltage Multiplier



Converts AC

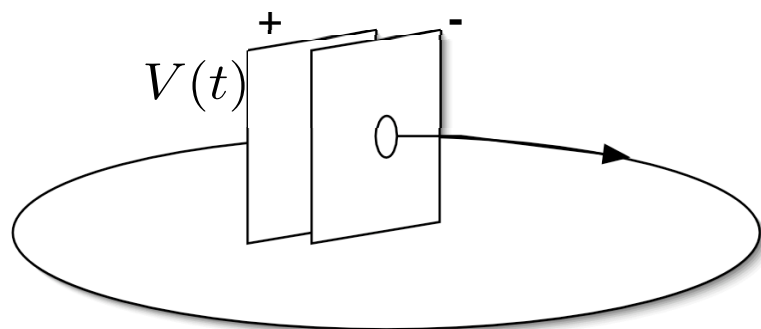


Fermilab (recently decommissioned)

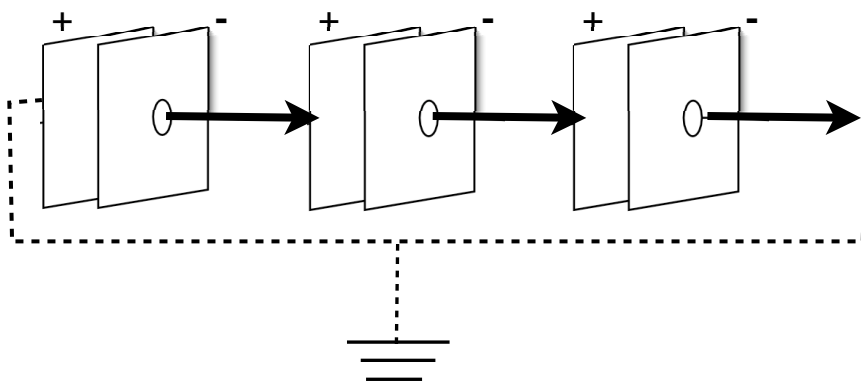
The Route to Even Higher Energies

■ The Need for AC Systems...

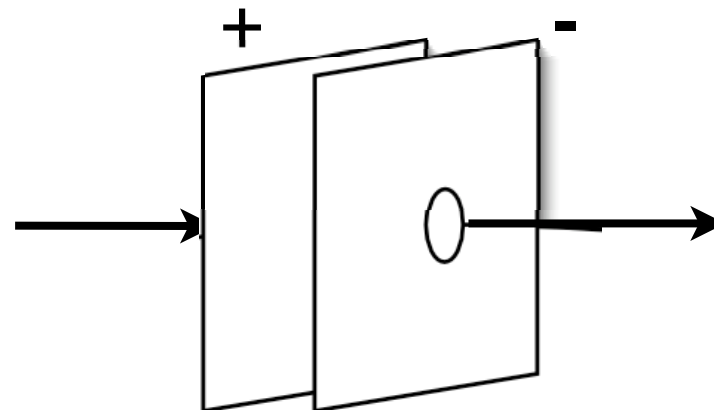
Circular Accelerator



Linear Accelerator



$$\text{energy gain} = q \cdot V$$



DC systems limited to a few MV

$$\oint (q\vec{E}) \cdot d\vec{s} = \text{work} = \Delta(\text{energy})$$

To gain energy, a time-varying field is required:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{\partial}{\partial t} \oint \vec{B} \cdot d\vec{A}$$

The Cyclotron

- A charged particle in a magnetic field, B , moves in a circular path of radius r at a speed v . The time to orbit once is

$$T = 2\pi r/v.$$

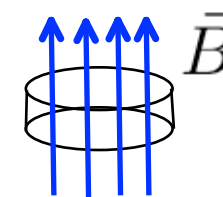
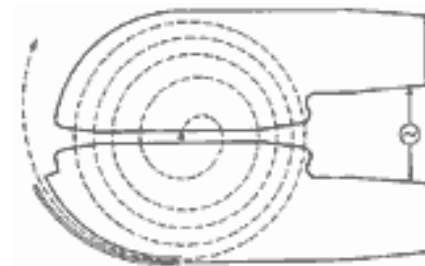
- The force due to the magnetic field is

$$F = qvB = mv^2/r$$

- thus, $r/v = m/eB$

- and so, $f = 1/T = (q/m)(B/2\pi)$

- Oscillate the voltage with a frequency f_{RF} and adjust the magnetic field until $1/f_{RF} = 1/f$ above.
- As the particle passes the gap, it gets accelerated, circulates on a slightly larger orbit, but the time to go around remains fixed.
- Eventually, the orbit gets big enough that the particle leaves the device.



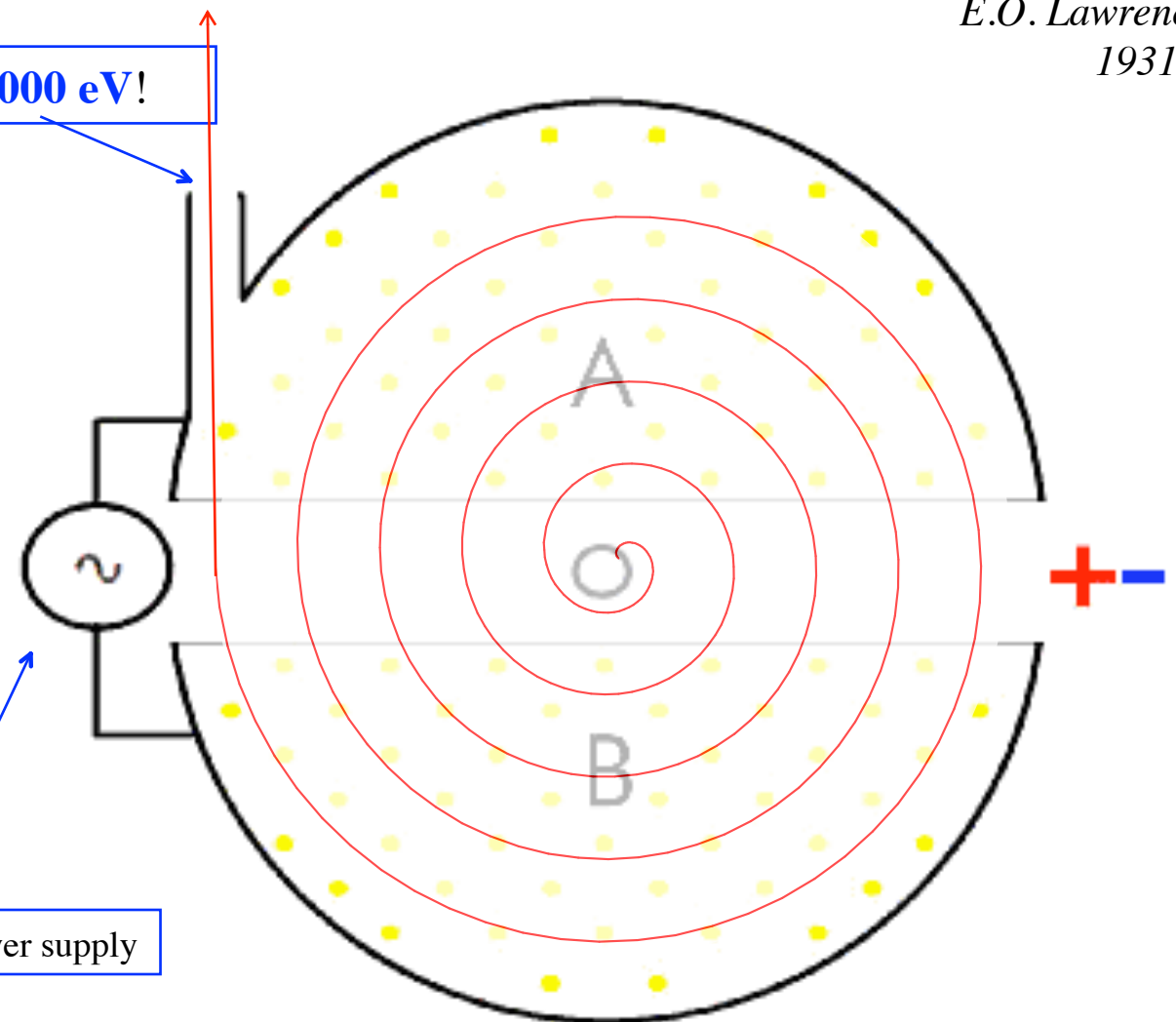
4.5 in.
↔



*E.O. Lawrence,
1931*

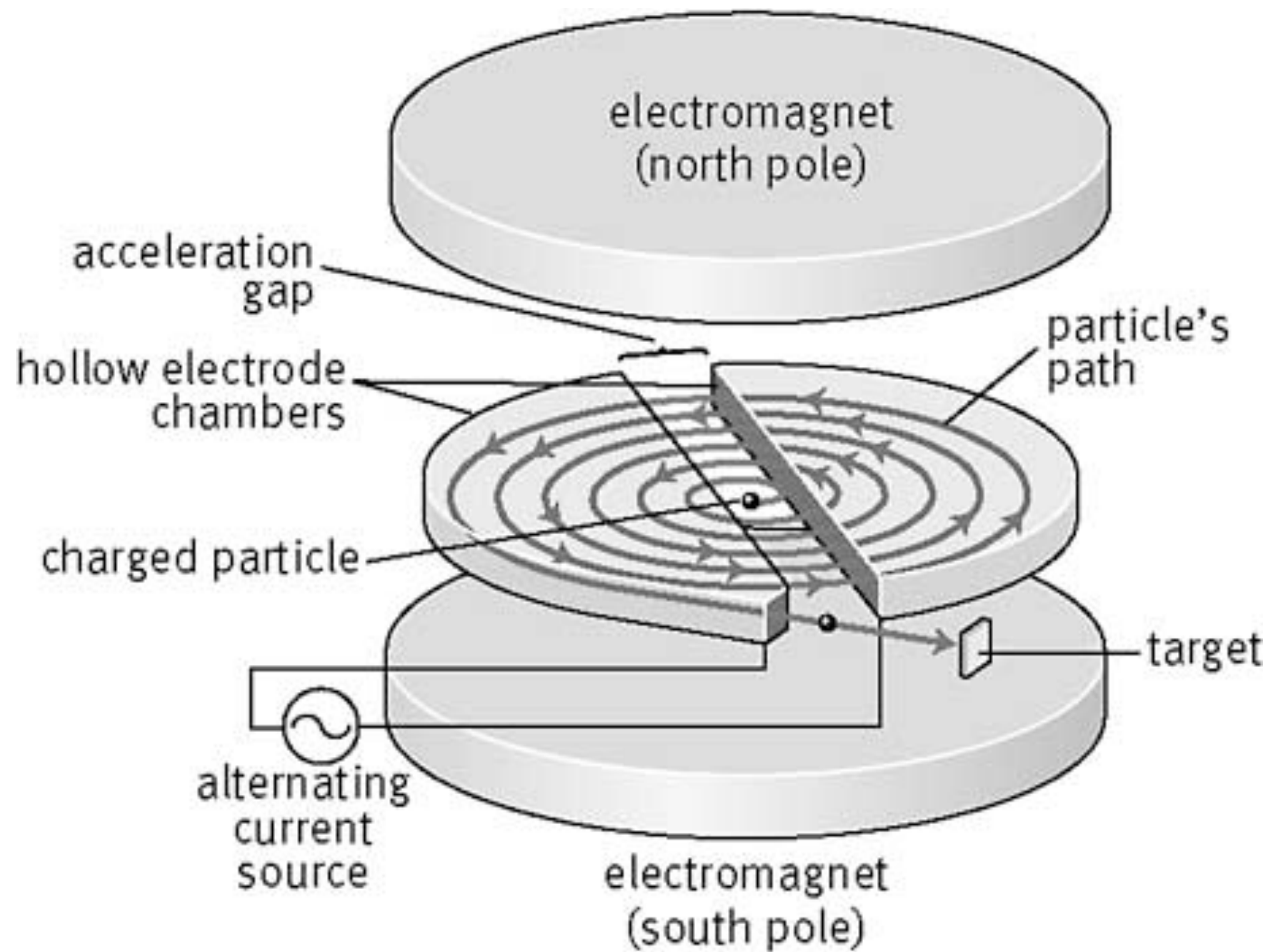
$K.E. = 80,000 \text{ eV!}$

1,800 V power supply



Cyclotrons

- Relatively easy to operate and tune (only a few parts).
- Tend to be used for isotope production and places where reliable and reproducible operation are important
- Intensity is moderately high, acceleration efficiency is high, cost low
- Relativity is an issue, so energy is limited to a few hundred MeV/u.
- RIKEN Superconducting Ring Cyclotron 350 MeV/u

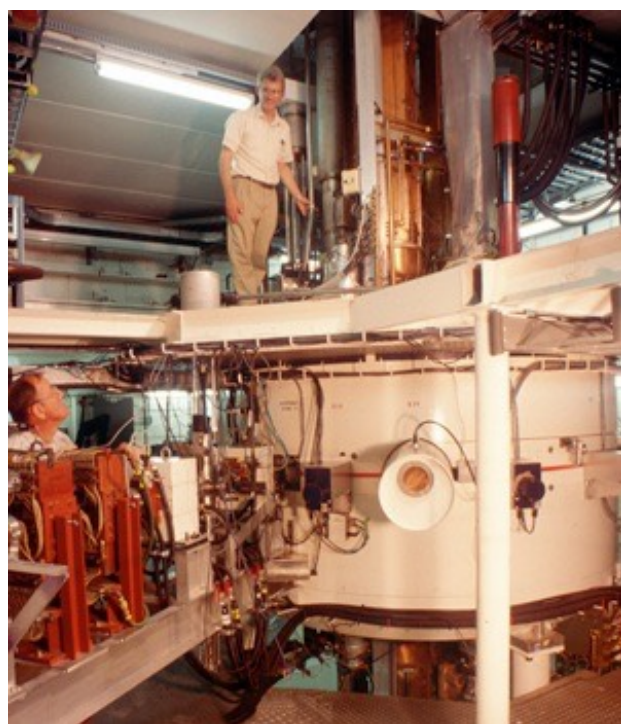


<http://images.yourdictionary.com/cyclotron>

National Superconducting Cyclotron Laboratory

(MSU)

- First use of superconducting magnet technology in a major particle accelerator — K500; next was K1200



The K500 superconducting cyclotron, early-1980's;
Note the compact size compared to earlier picture of 184" cyclotron(!), which was of comparable energy

Accelerating "dees" of the K1200 cyclotron



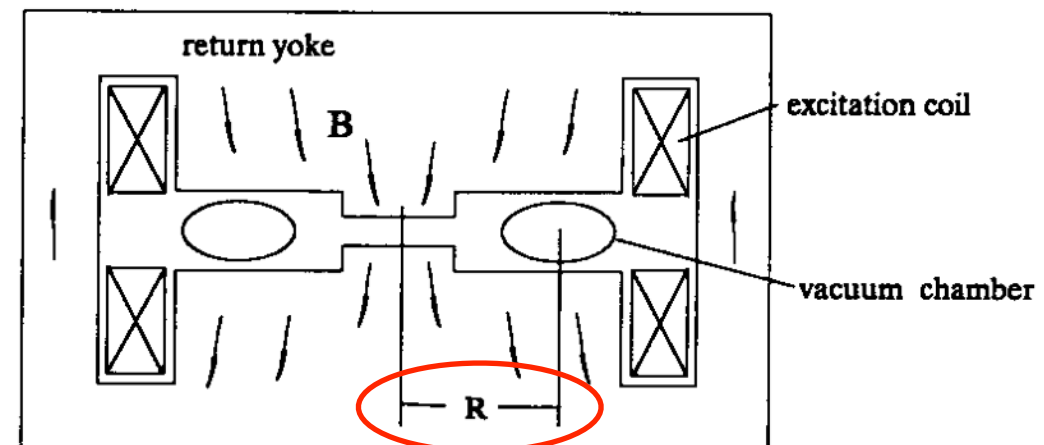
The Betatron

- 1940, Kerst (U. Illinois)

- induction accelerator

- $\oint \vec{E} \cdot d\vec{s} = -\frac{\partial}{\partial t} \oint \vec{B} \cdot d\vec{A}$

- used for electrons
 - beam dynamics heavily studied
 - » “betatron oscillations”



$$BR = p/q \quad \rightarrow \quad \dot{p} = qR\dot{B}$$

$$\dot{\Phi} = -\oint \vec{E} \cdot d\vec{r} = 2\pi RE$$

$$\dot{p} = F = qE$$

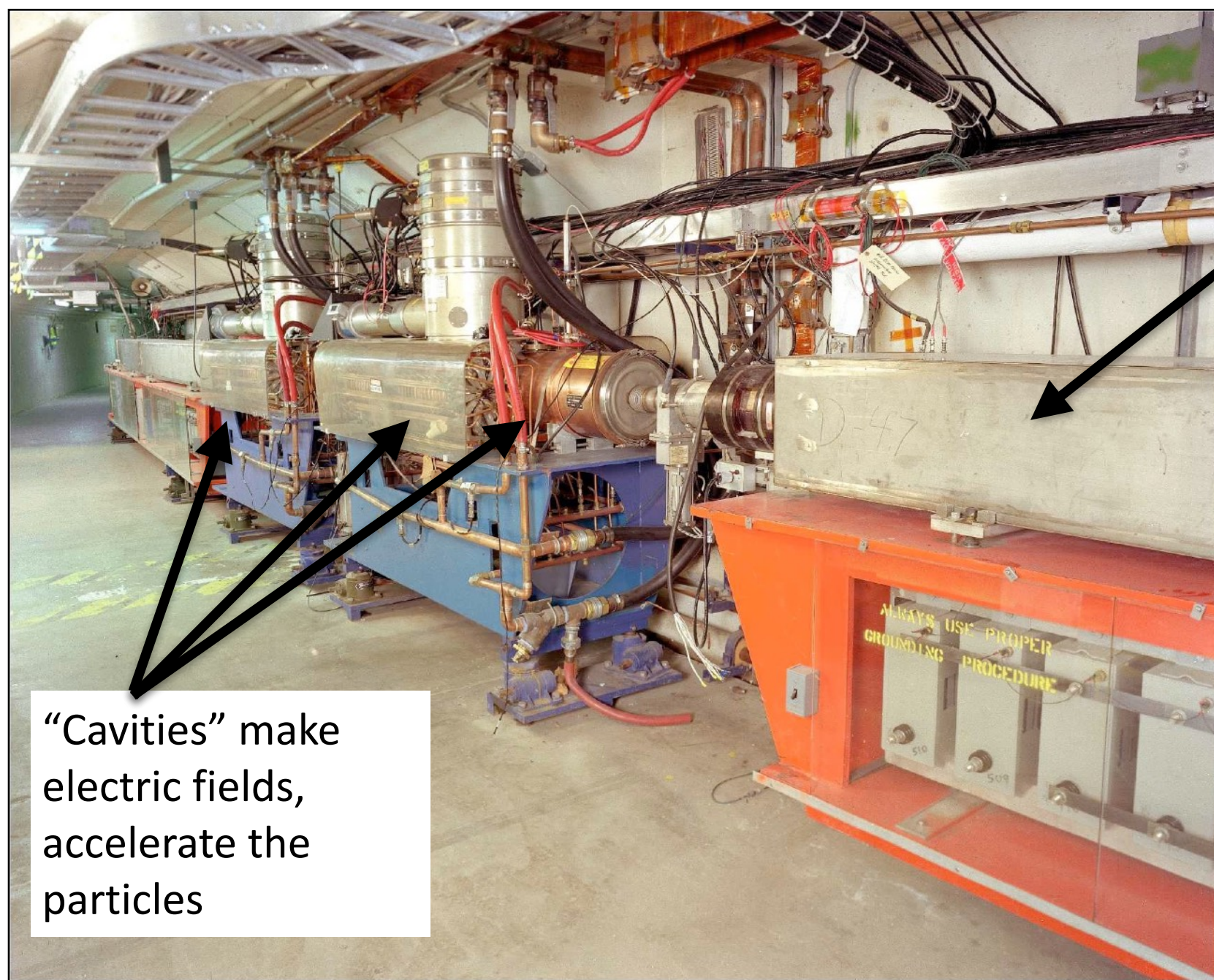
flux “through” the orbit of radius R

$$\dot{\Phi} = 2(\pi R^2)\dot{B}$$

shape poles so that field is stronger near the center,
adjusting the flux, until the *Betatron condition* is met

field on orbit of radius R

A Synchrotron



“Cavities” make electric fields, accelerate the particles

Magnets steer the particles in a circle

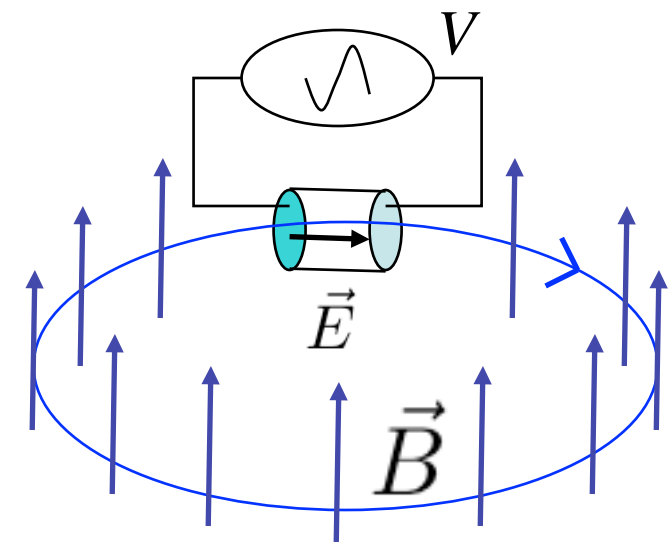
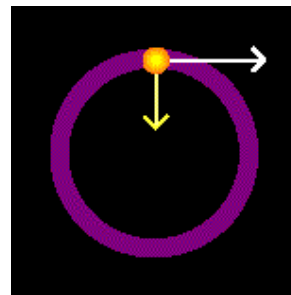
Booster Synchrotron, Fermilab (Batavia, IL)

The Synchrotron

- As the electromagnet fields are slowly increased, the particle will be accelerated by the cavity enough to keep its momentum in step with the magnetic field and keep the orbit radius constant:

$$mv^2/R = evB \quad ==> R = mv / eB$$

$$= p / eB$$



What frequencies do we need?

Let's say $v \sim c$,
and say $R = 1 \text{ m}$

then,

$$\begin{aligned} f &= v / 2\pi R \\ &= (3 \times 10^8 \text{ m/s}) / (2\pi \text{ 1m}) \\ &= 5 \times 10^7 / \text{s} = 50 \text{ MHz} \end{aligned}$$

FM Radio Stations: 88 - 108 MHz!
thus, we use RF cavities and power sources

If increase circumference, can still use same frequency.
Will generate h groups, or “bunches”, of particles that
will be separated in time by $1/f_{RF}$.

harmonic number:

$$h = (\text{RF frequency}) / (\text{revolution frequency})$$



Resonant Cavities



MJS

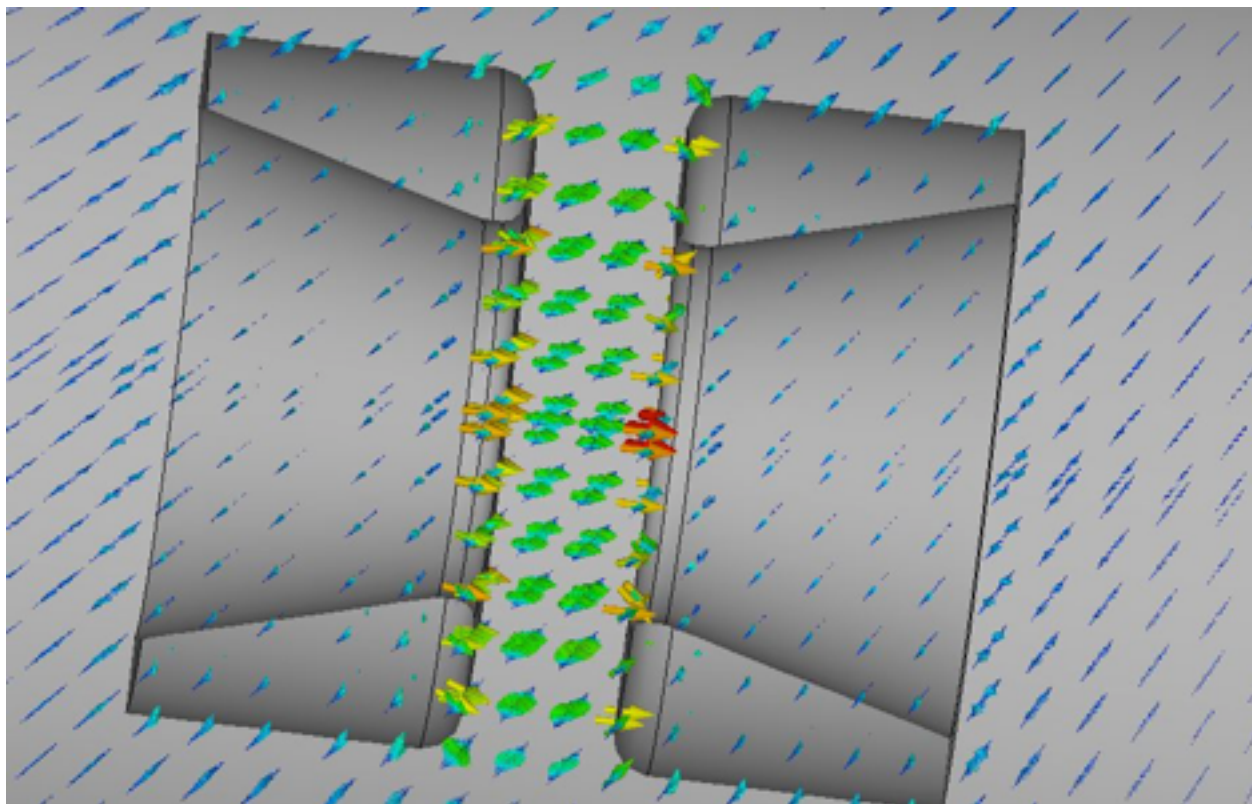
Acceleration using AC Fields

- Pass through a gap with an oscillating field...
- $\Delta W = q E d = qV$ $\Delta W/A \text{ [eV/u]} = (Q/A) eV$
- But here, V is an “average” or “effective” potential; depends upon the frequency of the field in the gap, the incoming speed of the particle (due to the field varying with time), and the phase of the oscillation relative to the particle arrival time

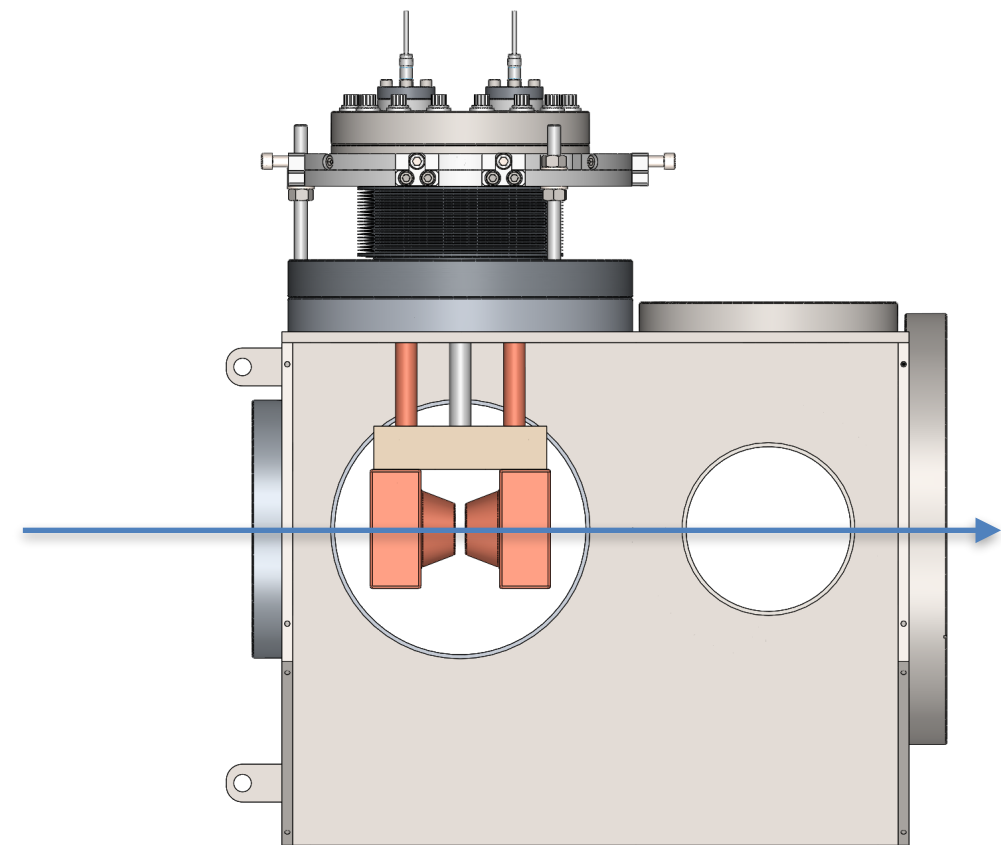
$$\Delta W \text{ [per nucleon]} = (Q/A) T(\beta) eV_0 \cos(\phi)$$

Accelerating Gap

- Create electric field / potential across two “plates”



- Vary the field with frequency, f



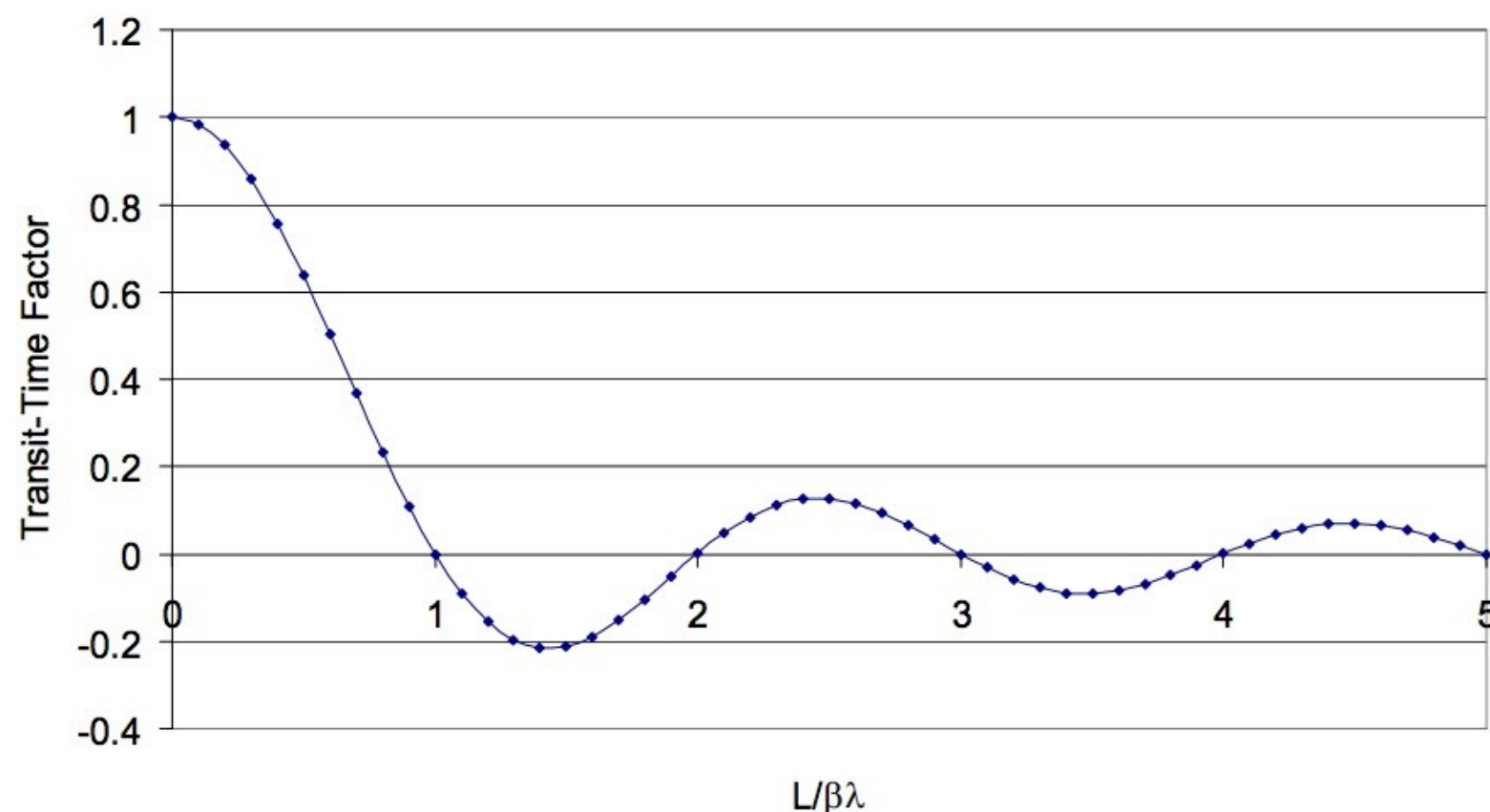
D. Alt, et al., MSU

Transit Time Factor

•

$$T(\beta) = \frac{1}{V_0} \int_{-g/2}^{g/2} E(0, s) \cos(ks) ds$$

$$V_0 = \int_{-g/2}^{g/2} E(0, s) ds \quad k \equiv \frac{2\pi}{\beta\lambda}$$



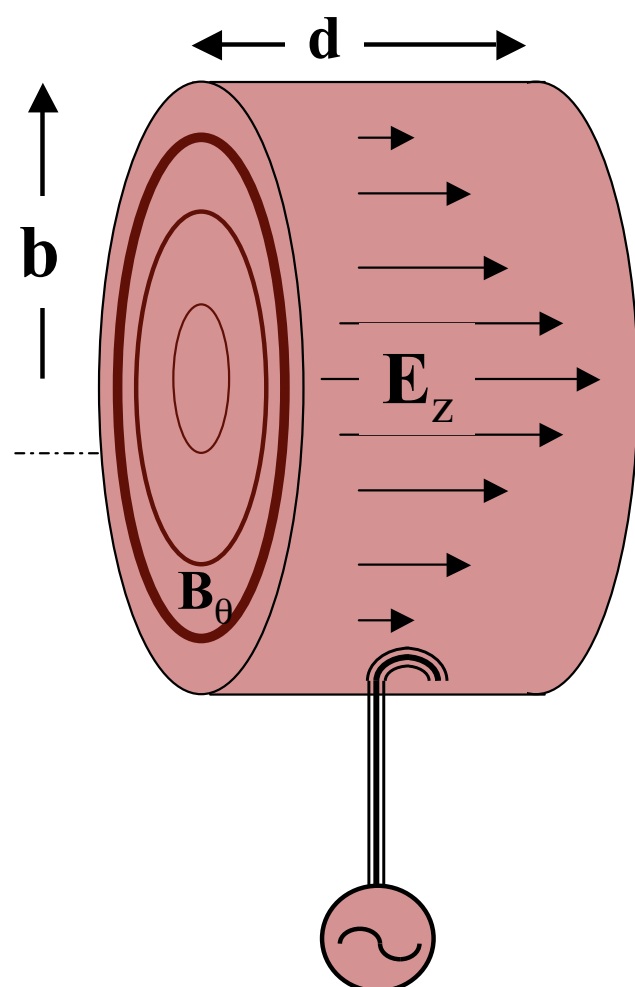
- For $v = c$, and for a gap = $\lambda/2$, the TTF will be

$$\frac{1}{\lambda/2} \int_{-\lambda/4}^{\lambda/4} \cos(2\pi z/\lambda) dz = \frac{2}{\pi}$$

- Once get up to higher velocities ($v \sim c$), then can consider multiple-cell cavities since the velocity is no longer changing.



Properties of the RF pillbox cavity



$$\sigma_{walls} = \infty$$

- * We want lowest mode: with only E_z & B_θ
- * Maxwell's equations are:

$$\frac{1}{r} \frac{\partial}{\partial r} (r B_\theta) = \frac{1}{c^2} \frac{\partial}{\partial t} E_z \quad \text{and} \quad \frac{\partial}{\partial r} E_z = \frac{\partial}{\partial t} B_\theta$$

- * Take derivatives

$$\frac{\partial}{\partial t} \left[\frac{1}{r} \frac{\partial}{\partial r} (r B_\theta) \right] = \frac{\partial}{\partial t} \left[\frac{\partial B_\theta}{\partial r} + \frac{B_\theta}{r} \right] = \frac{1}{c^2} \frac{\partial^2 E_z}{\partial t^2}$$

$$\frac{\partial}{\partial r} \frac{\partial E_z}{\partial r} = \frac{\partial}{\partial r} \frac{\partial B_\theta}{\partial t}$$

\Rightarrow

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r} \frac{\partial E_z}{\partial r} = \frac{1}{c^2} \frac{\partial^2 E_z}{\partial t^2}$$

slides courtesy
W. Barletta



For a mode with frequency ω



✱
$$E_z(r, t) = E_z(r) e^{i\omega t}$$

✱ Therefore,
$$E_z'' + \frac{E_z'}{r} + \left(\frac{\omega}{c}\right)^2 E_z = 0$$

→ (Bessel's equation, 0 order)

✱ Hence,

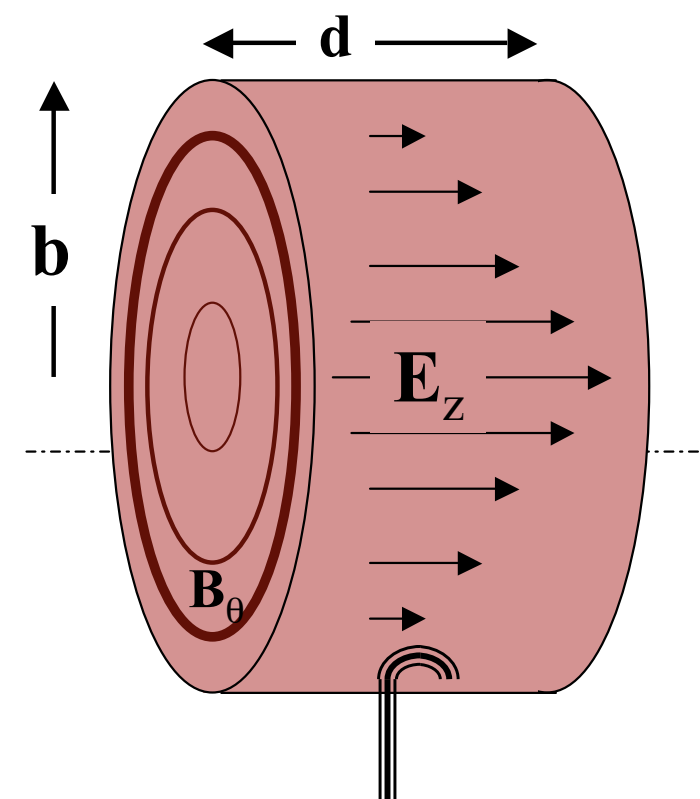
$$E_z(r) = E_o J_0\left(\frac{\omega}{c} r\right)$$

✱ For conducting walls, $E_z(R) = 0$, therefore

$$\frac{2\pi f}{c} b = 2.405$$

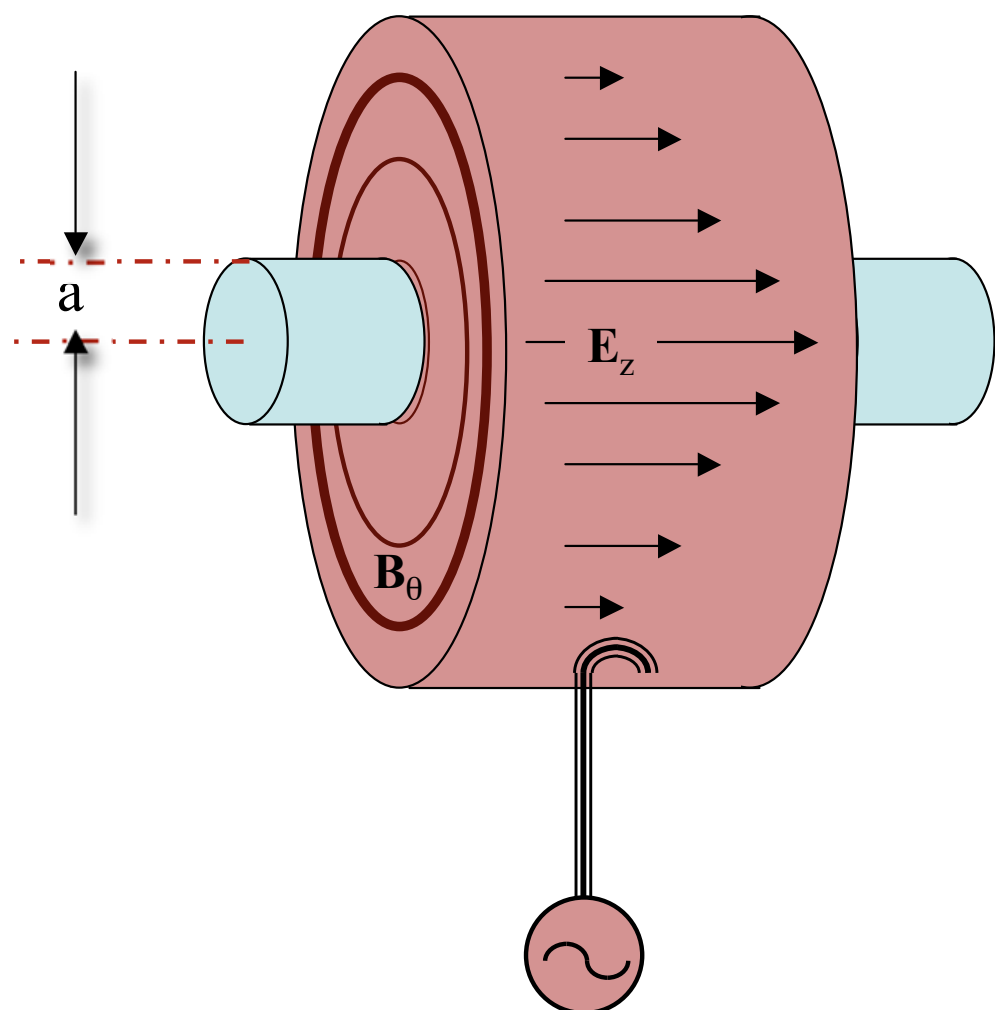
Size Matters

<i>Frequency (MHz)</i>	<i>Diameter (2b) (m)</i>
1	240
10	24
50	5
100	2.5
1000	0.25
3000	0.08





The beam tube makes the field modes (& cell design) more complicated



- * Peak E no longer on axis
 - $E_{pk} \sim 2 - 3 \times E_{acc}$
 - $FOM = E_{pk}/E_{acc}$
- * ω_0 more sensitive to cavity dimensions
 - Mechanical tuning & detuning

The Pill Box Cavity



Basic components of an RF cavity



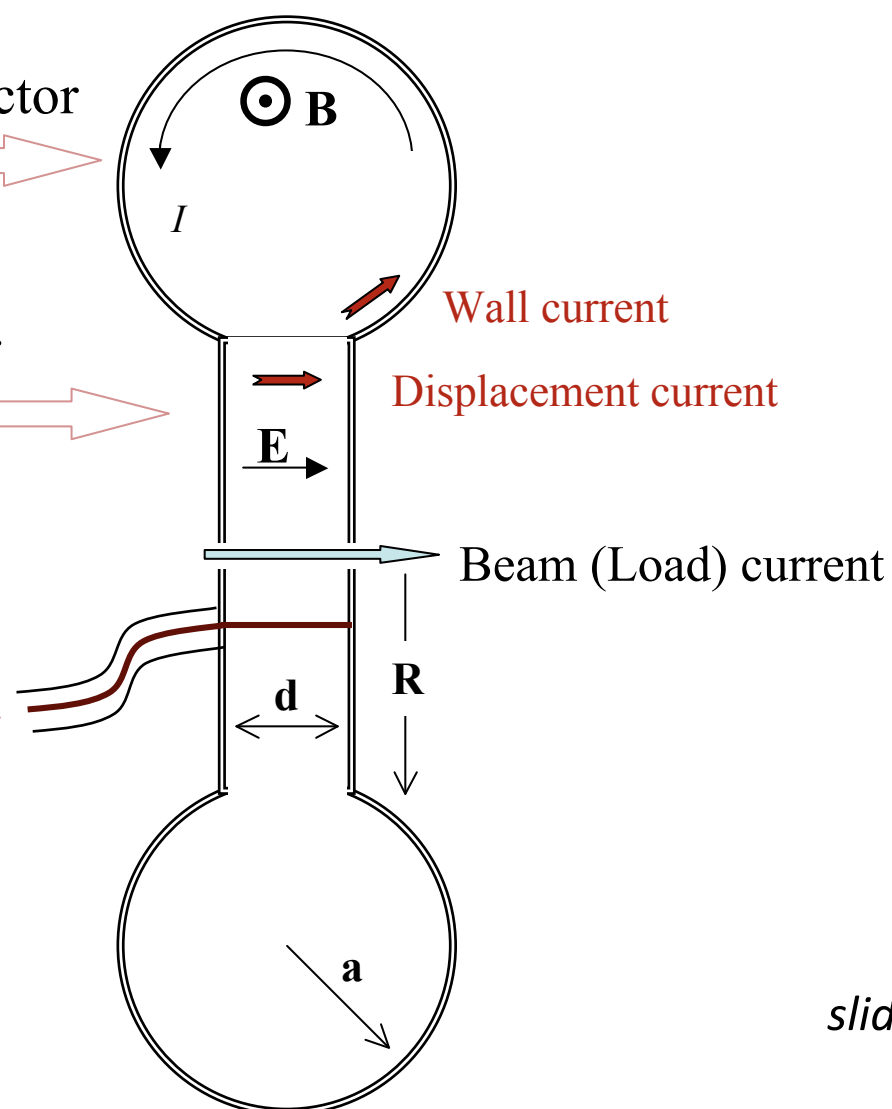
Outer region: Large, single turn Inductor



Central region: Large plate Capacitor



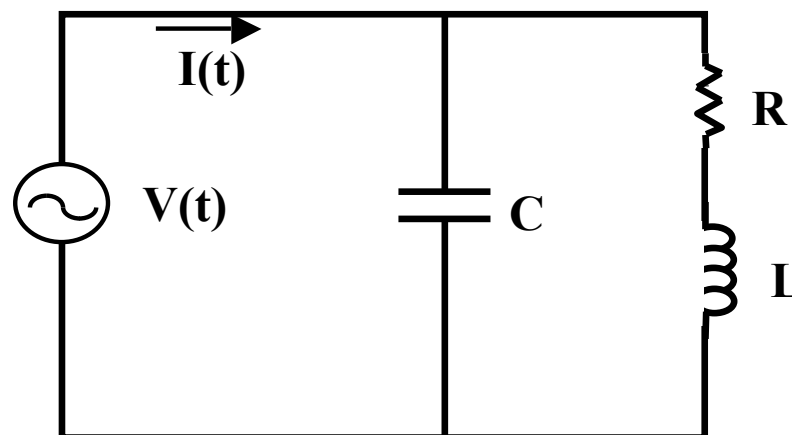
Power feed from rf - generator



slides courtesy W. Barletta



Lumped circuit analogy of resonant cavity



$$Z(\omega) = \left[j\omega C + (j\omega L + R)^{-1} \right]^{-1}$$

$$Z(\omega) = \frac{1}{j\omega C + (j\omega L + R)^{-1}} = \frac{(j\omega L + R)}{(j\omega L + R)j\omega C + 1} = \frac{(j\omega L + R)}{(1 - \omega^2 LC) + j\omega RC}$$

The resonant frequency is $\omega_o = \frac{1}{\sqrt{LC}}$



More basics from circuits - Q



$$Q = \frac{\omega_o \circ \text{Energy stored}}{\text{Time average power loss}} = \frac{2\pi \circ \text{Energy stored}}{\text{Energy per cycle}}$$

$$\mathcal{E} = \frac{1}{2} L I_o I_o^* \quad \text{and} \quad \langle \mathcal{P} \rangle = \langle i^2(t) \rangle R = \frac{1}{2} I_o I_o^* R_{\text{surface}}$$

$$\therefore Q = \frac{\sqrt{L/C}}{R} = \left(\frac{\Delta\omega}{\omega_o} \right)^{-1}$$

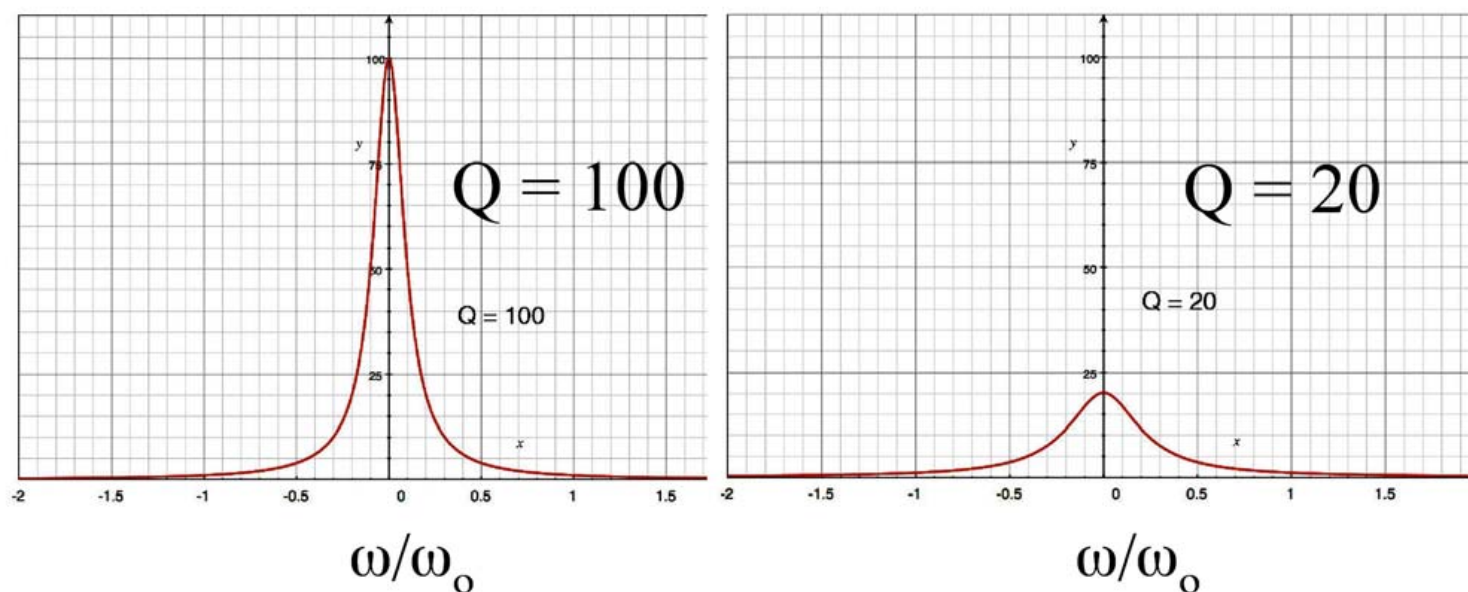


Q of the lumped circuit analogy



Converting the denominator of Z to a real number we see that

$$|Z(\omega)| \sim \left[\left(1 - \frac{\omega^2}{\omega_o^2} \right)^2 + (\omega RC)^2 \right]^{-1}$$



The width is $\frac{\Delta\omega}{\omega_0} = \frac{R}{\sqrt{L/C}}$



Translate circuit model to a cavity model: Directly driven, re-entrant RF cavity



Outer region: Large, single turn Inductor

$$L = \frac{\mu_o \pi a^2}{2\pi(R + a)}$$



Central region: Large plate Capacitor

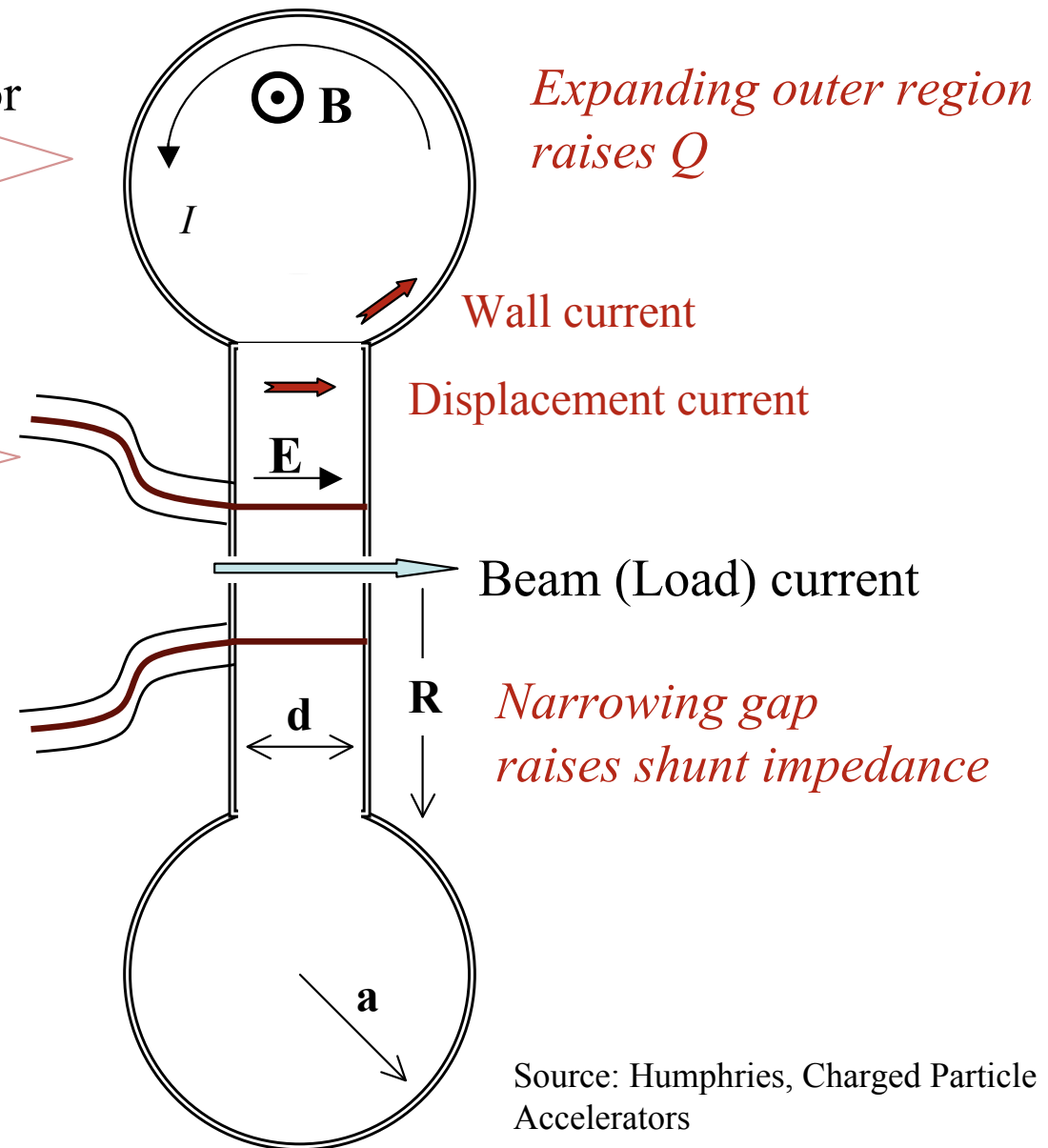
$$C = \epsilon_o \frac{\pi R^2}{d}$$



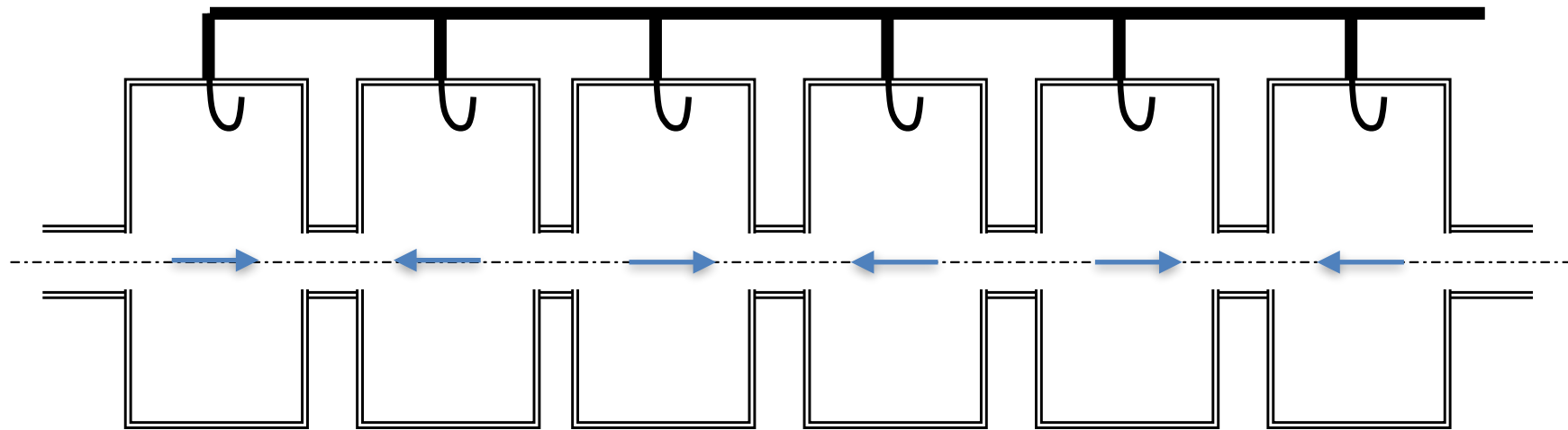
$$\omega_o = \frac{1}{\sqrt{LC}} = c \left[\frac{2((R + a)d)}{\pi R^2 a^2} \right]^{1/2}$$

Q – set by resistance in outer region

$$Q = \sqrt{L/C} / R$$



At high velocities, make a linac out of a series of pillboxes ...



make the cells $\lambda/2$ apart

Power the cavities so that $E_z(z, t) = E_z(z) \sin(2\pi f t)$

- Once get up to higher velocities ($v \sim c$), then can consider multiple-cell cavities since the velocity is no longer changing.
- For $v = c$, the TTF will be

$$\frac{1}{\lambda/2} \int_{-\lambda/4}^{\lambda/4} \cos(2\pi z/\lambda) dz = \frac{2}{\pi}$$

Other Choices of Cavity Type

- Other variations might be required
 - ▶ particles may NOT already be at the speed of light, so transit time effects become significant
 - ▶ in a synchrotron, the speed of the particles may vary significantly during acceleration, so will require a changing RF frequency
 - ▶ slowly moving particles (like heavy ions, for instance: $v \sim 0.01c$, etc.) will need special attention
- Many variants are available; likely more to come

low- β cavities: **Just cavities that accelerate efficiently particles with $\beta < 1$...**

low- β cavities are often further subdivided in low-, medium-, high- β

$\beta=1$ SC resonators:
“elliptical” shapes



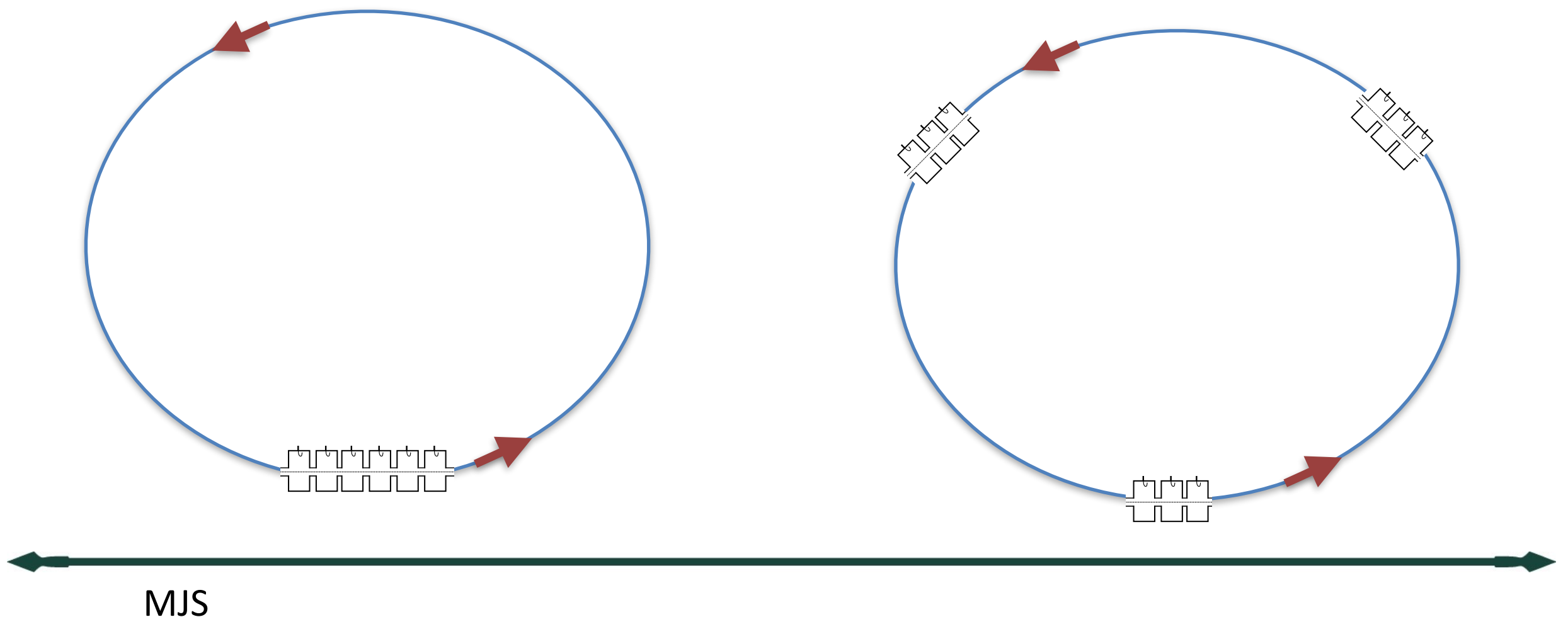
Courtesy A. Facco

$\beta < 1$ resonators, from very low ($\beta \sim 0.03$) to intermediate ($\beta \sim 0.5$):
many different shapes and sizes



Synchrotron RF Distribution

- Might have all cavities in one location in a synchrotron, or may be distributed around the circumference..



Superconducting Cavities

- Using superconducting materials, cavities can have Q very high ($>10^{10}$) with much lower power costs, and bunches of particles can be continuously traversing the system over time
 - ▶ “CW” -- continuous wave
- Can achieve higher acceleration per meter if use a system wherein the particle traverses several gaps, perhaps being shielded from the field while its direction changes within the cavity
- If particles were near light speed, then the cavity structure can be formed so that the gaps are separated by $\lambda/2$; elliptical cavities are common for high gradient acceleration
 - ▶ for lower speeds -- e.g., heavy ions -- must be spaced by $\sim \beta\lambda/2$. This might also limit the number of gaps per cavity to ~ 2

Normal vs. Superconducting Cavities

DTL tank - Fermilab



*Normal conducting
Cu cavity @ 300K
 $R_s \sim 10^{-3} \Omega$
 $Q \sim 10^4$*

*Superconducting
Nb Cavity @ 4.2K
 $R_s \sim 10^{-8} \Omega$
 $Q \sim 10^9$*



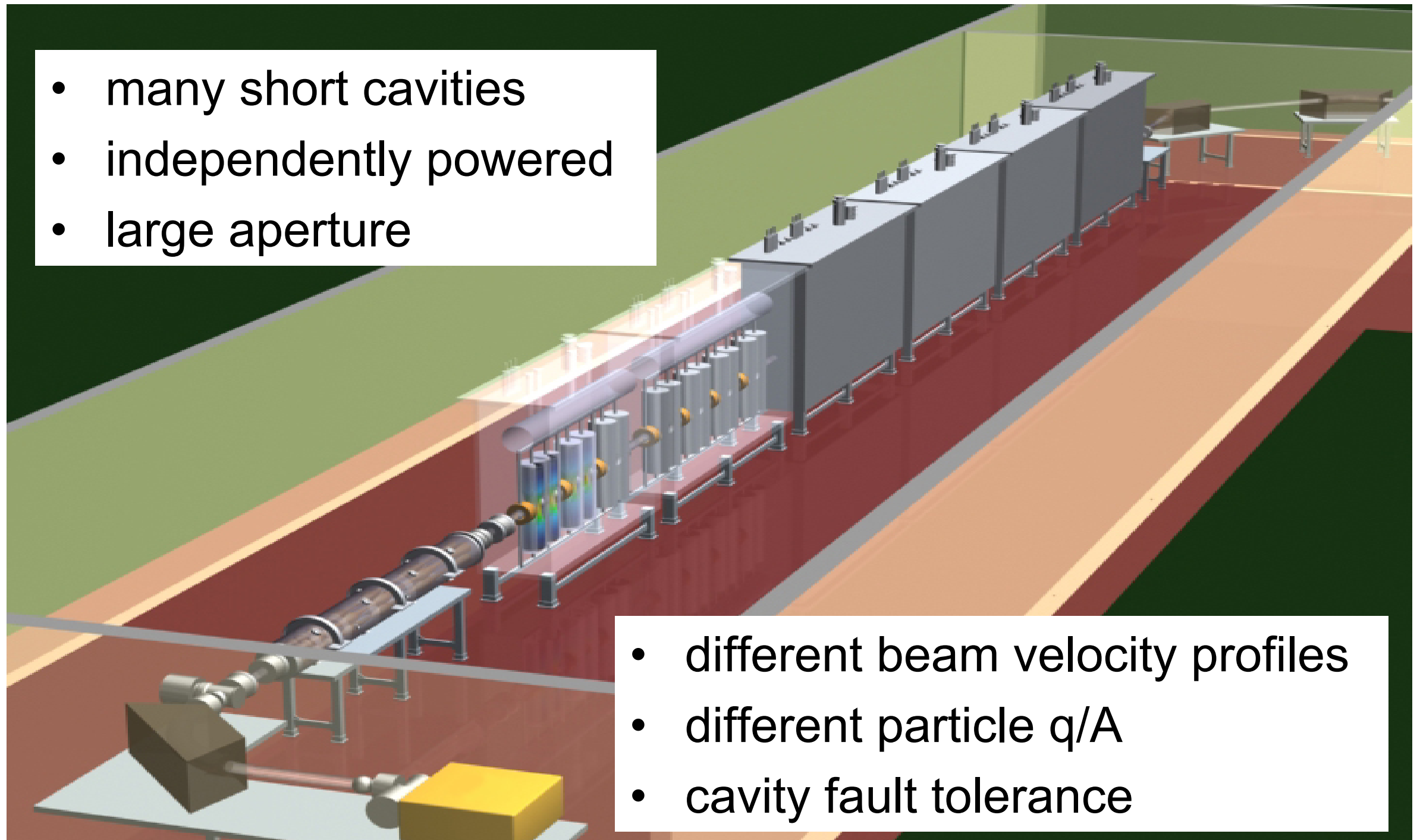
LNL PIAVE 80 MHz, $\beta = 0.047$ QWR

Superconductivity allows

- great reduction of rf power consumption even considering cryogenics (1W at 4.2K ~ 300W at 300K)*
- the use of short cavities with wide velocity acceptance*

Low- β Superconducting Linacs

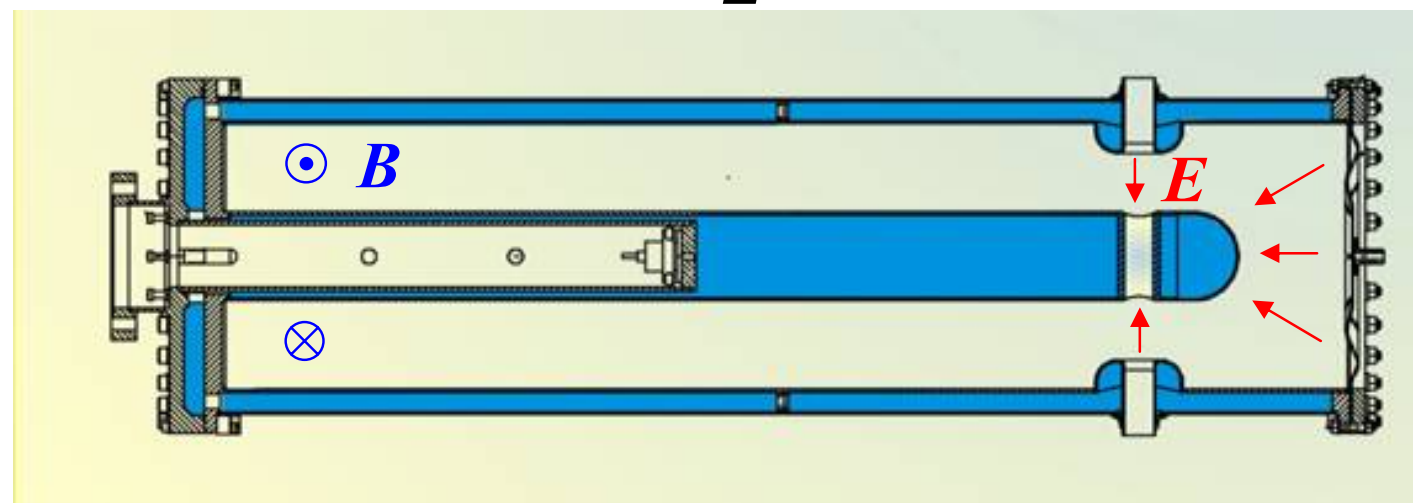
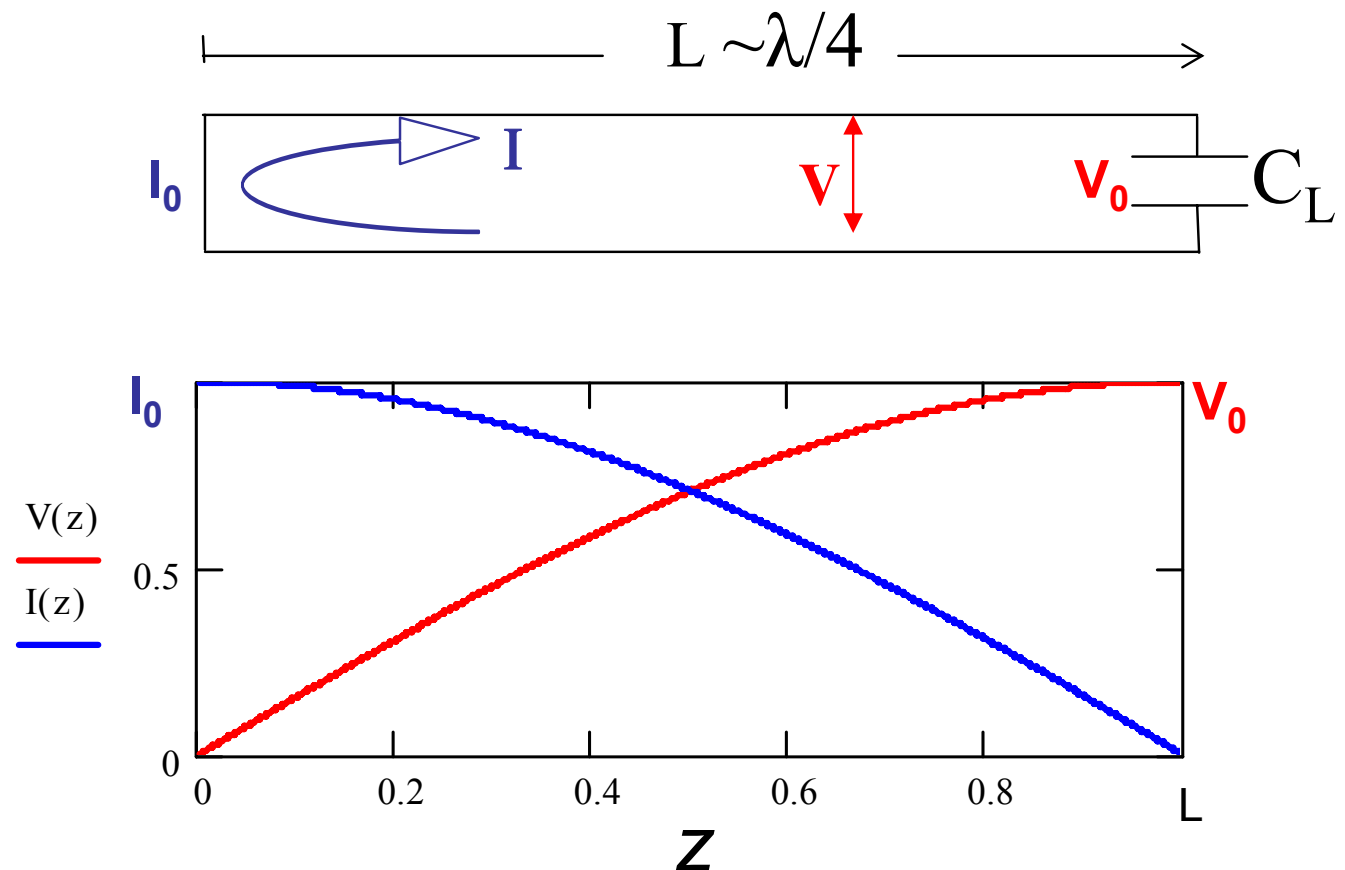
- many short cavities
- independently powered
- large aperture



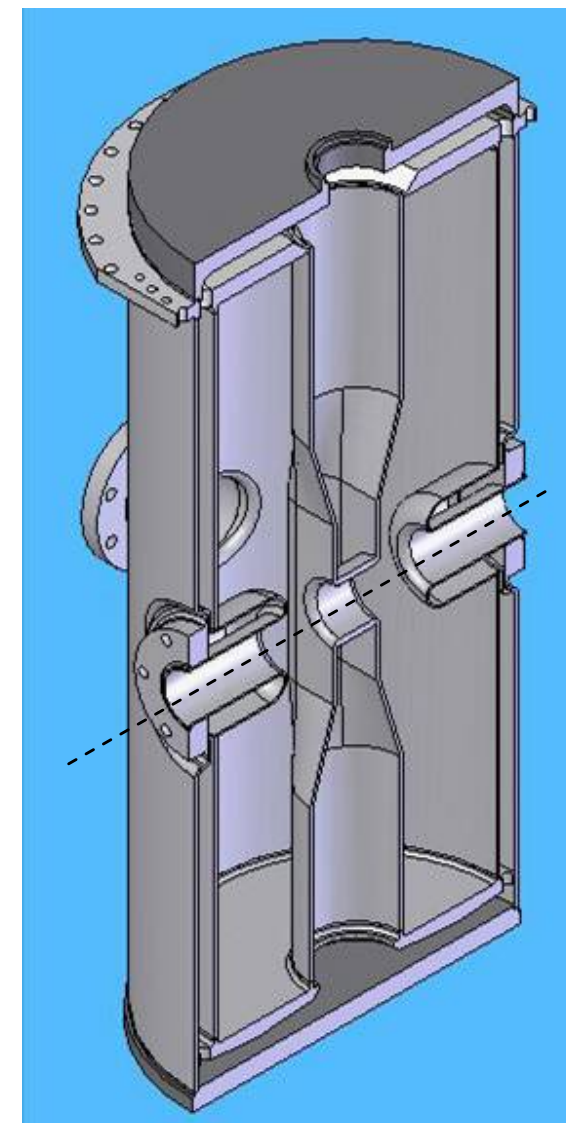
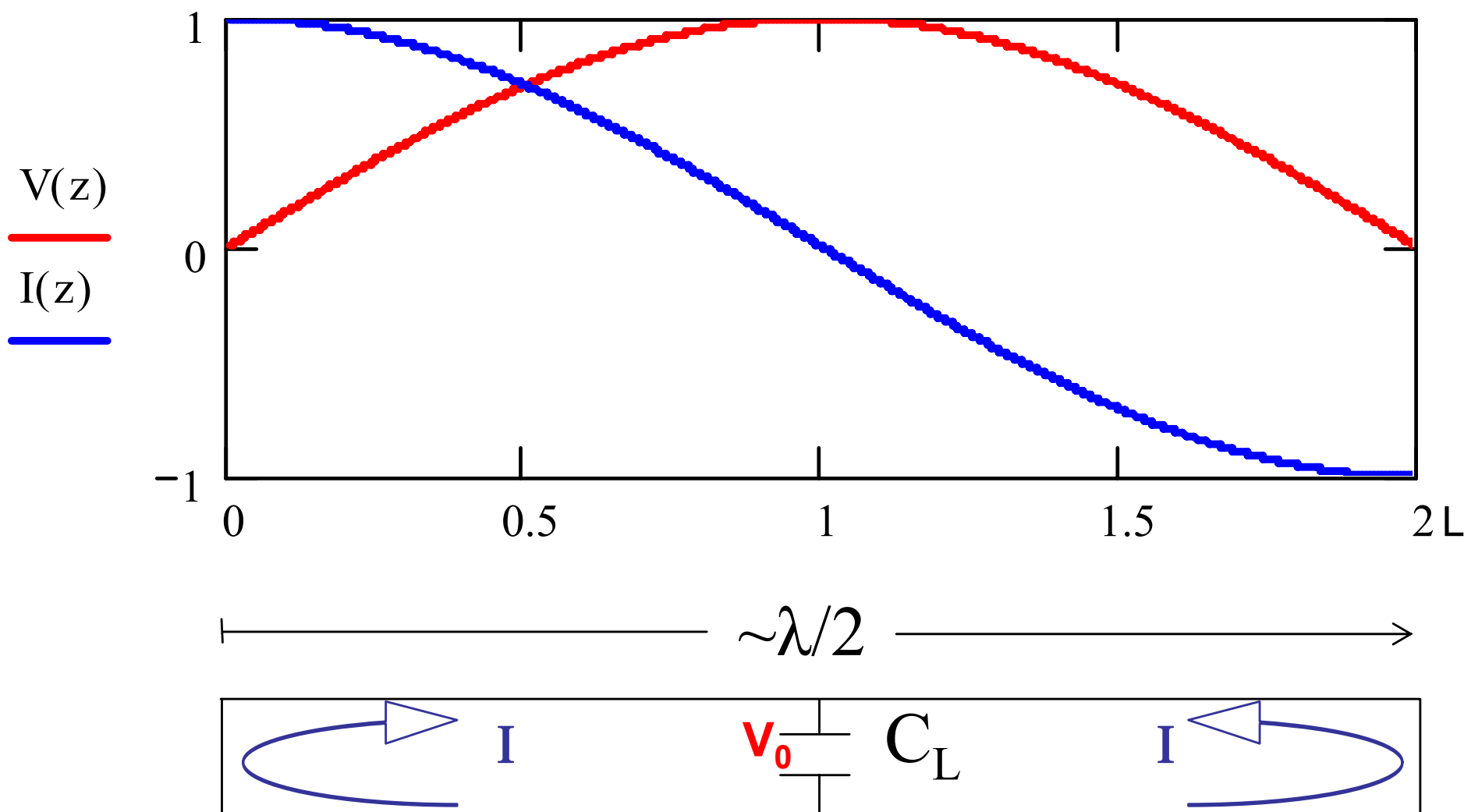
- different beam velocity profiles
- different particle q/A
- cavity fault tolerance

Quarter-wave Structures

- Imagine a coaxial waveguide set to transmit an EM wave of frequency f , and wavelength $\lambda = c/f$
- Make the waveguide of length $L = \lambda/4$, and “close” one end
 - create a “standing wave” within the structure
- At the end where the E field is strong, allow the beam to pass through two gaps
 - separate gaps by distance $\beta\lambda/2$



Half-wave Structures -- More Symmetry

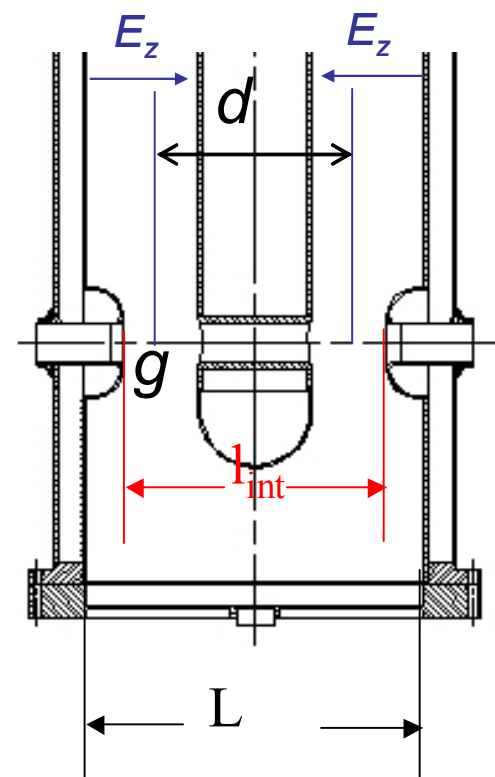
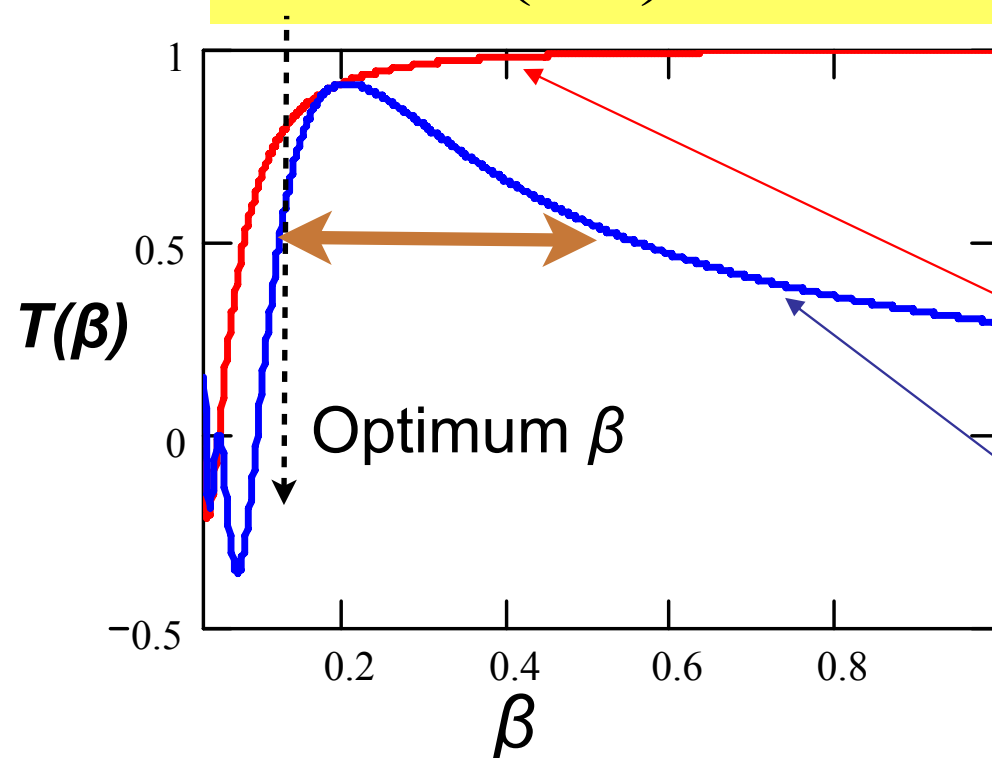


- A half-wave resonator is equivalent to 2 QWRs facing each other and connected

Transit Time Factor for 2-gap π -mode Cavity

(constant E_z approximation)

$$T(\beta) \cong \frac{\sin\left(\frac{\pi g}{\beta\lambda}\right)}{\left(\frac{\pi g}{\beta\lambda}\right)} \sin\left(\frac{\pi d}{\beta\lambda}\right)$$



1° term: 1-gap effect $\rightarrow g < \beta\lambda/2$

2° term: 2 gap effect $\rightarrow d \sim \beta\lambda/2$

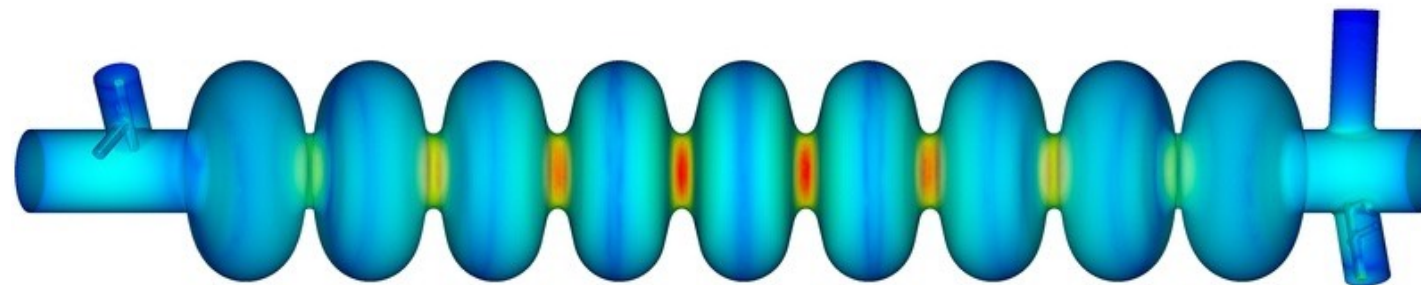
1° + 2° term TTF curve

(For more than 2 equal gaps in π mode, the formulas change only in the 2° term)

Multi-cell Cavities

- Here: “ILC” 9-cell style cavity

as $v \rightarrow c$, can use multiple cells in succession



have achieved > 35 MV/m average accelerating gradient with superconducting cavities

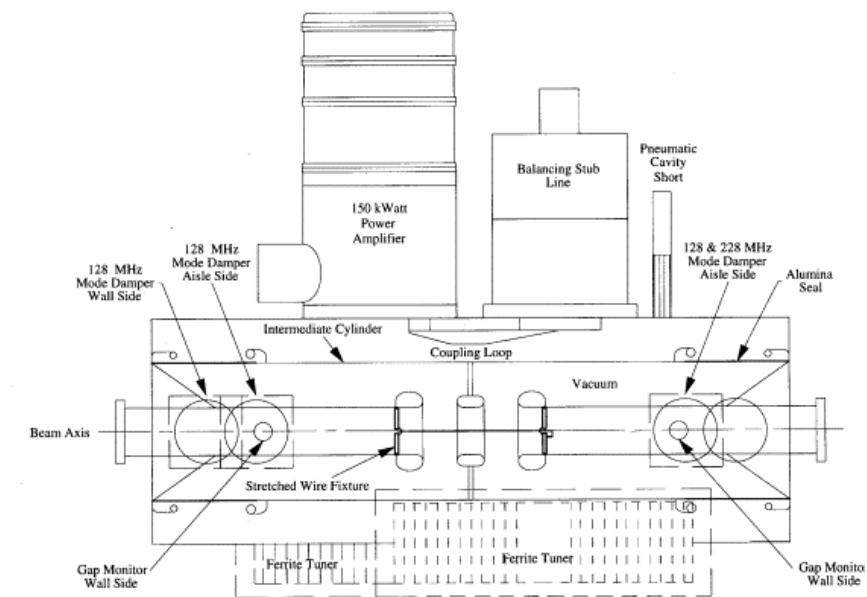
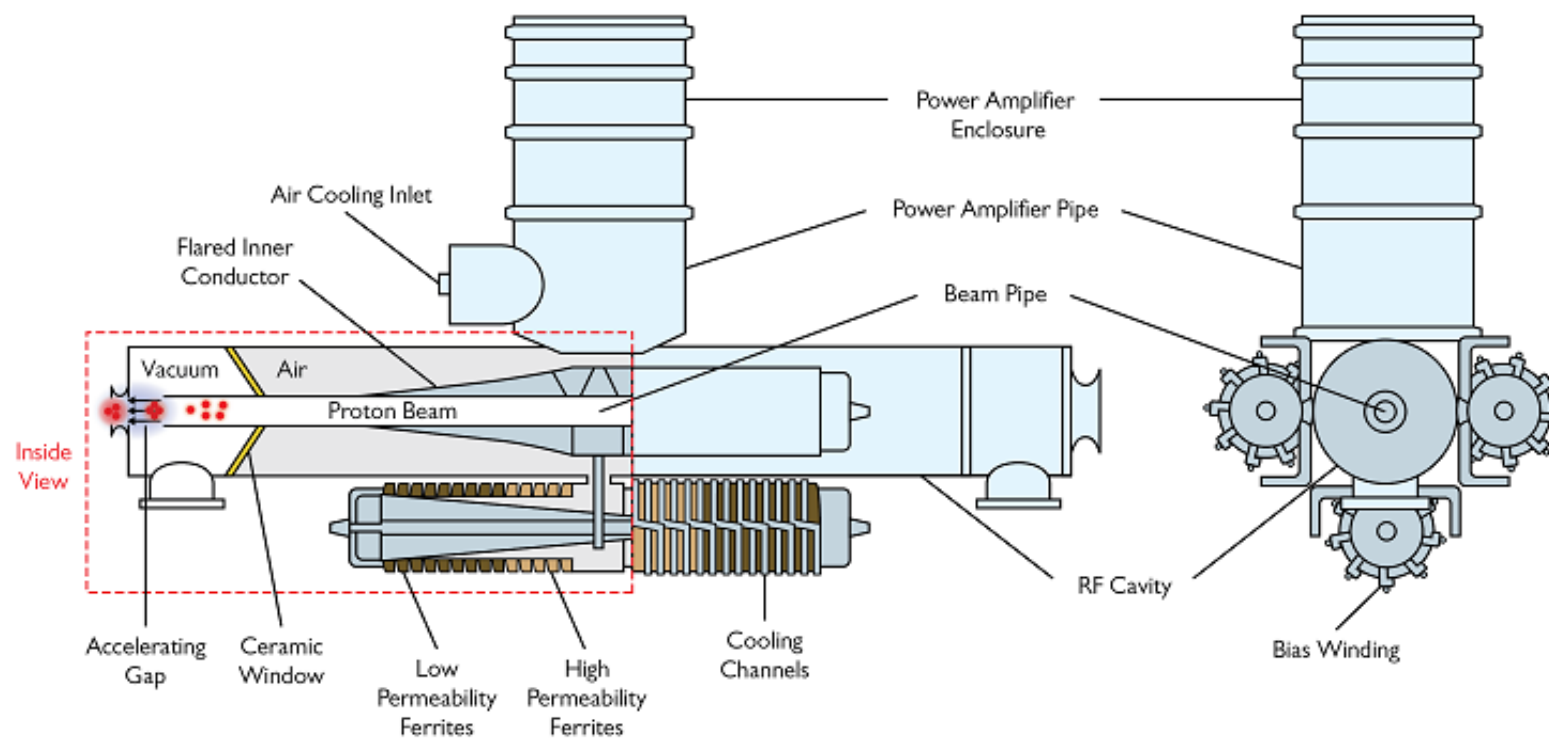
(Note: even larger gradients achieved with non-SC, but very power intensive)

Ex: 37-53 MHz, Ferrite-tuned Cavities

- essentially “half-wave” structures with variable frequency



changing the bias voltage on the ferrite changes its μ , which changes the inductance, L , changing the resonant f





Longitudinal Focusing

and Related Transverse Effects



Longitudinal Focusing

and Related Transverse Effects

- sometimes referred to as “phase focusing” or “time focusing”
- particles of different energy (momentum) move at different speeds, so tend to “spread out” relative to the “ideal” particle which is assumed to exist traveling with perfect synchronism with respect to the oscillating fields
- wish to study the (longitudinal) motion of particles relative to this “synchronous particle”

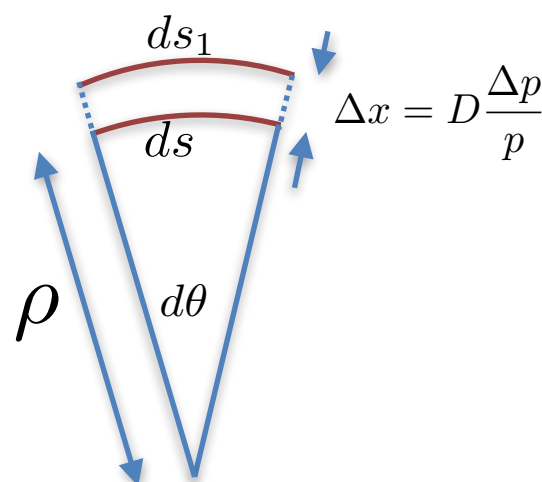
Longitudinal Focusing

- time of flight — the “slip factor”
- Evolution due to dp/p or dW/W
- Longitudinal focusing, time of arrival:
 - ▶ bunchers, rebunchers, debunchers

Momentum Compaction Factor

- How does path length along the beam line depend upon momentum?
 - ▶ in straight sections, no difference; in bending regions, *can* be different

Look closely at an infinitesimal section along the ideal trajectory...



$$d\theta = \frac{ds}{\rho} = \frac{ds_1}{\rho + \Delta x}$$

$$\begin{aligned} ds_1 - ds &= \left(\frac{\rho + \Delta x}{\rho} - 1 \right) ds \\ &= \frac{\Delta x}{\rho} ds = \frac{D}{\rho} \frac{\Delta p}{p} ds \end{aligned}$$

if L = path length along ideal trajectory
between 2 points, then

$$\frac{\Delta L}{L} = \frac{\int \frac{D(s)}{\rho(s)} ds}{\int ds} \cdot \frac{\Delta p}{p}$$

The relative change in path length, per
relative change in momentum, is called
the *momentum compaction factor*,

$\alpha_p = \langle D/\rho \rangle$ along the ideal path

The Slip Factor

$$t = \frac{L}{v}$$

$$\frac{dt}{t} = \frac{dL}{L} - \frac{dv}{v}$$

$$\frac{dv}{v} = \frac{1}{\gamma^2} \frac{dp}{p}$$

$$\frac{dt}{t} = \left(\alpha_p - \frac{1}{\gamma^2} \right) \frac{dp}{p}$$

$$\frac{dt}{t} = \eta \frac{dp}{p}$$

Momentum Compaction Factor:

$$\alpha_p \equiv \left(\frac{dL/L}{dp/p} \right)$$

$$\alpha_p = \frac{\int [D(s)/\rho(s)] ds}{\int ds}$$

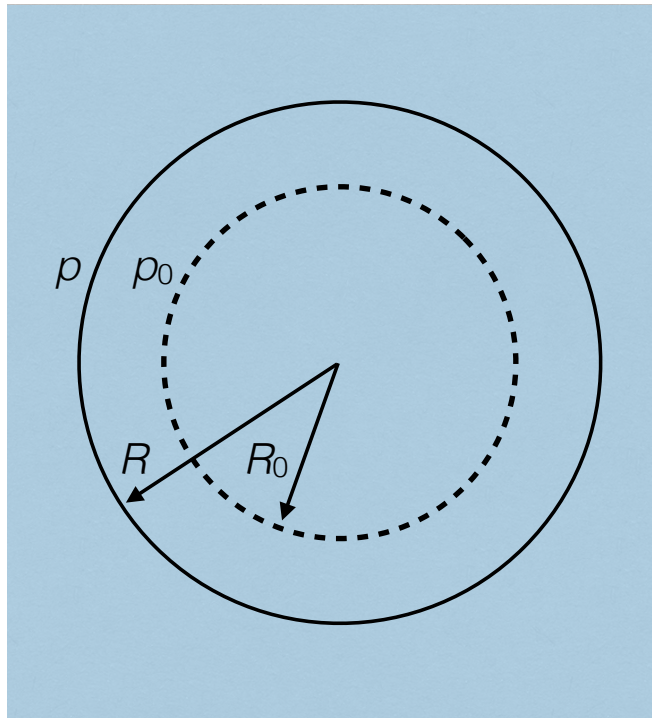
$$= \langle D/\rho \rangle$$

$D = \text{dispersion}$

The Slip Factor:

$$\eta \equiv \alpha_p - \frac{1}{\gamma^2}$$

A Simple Example...



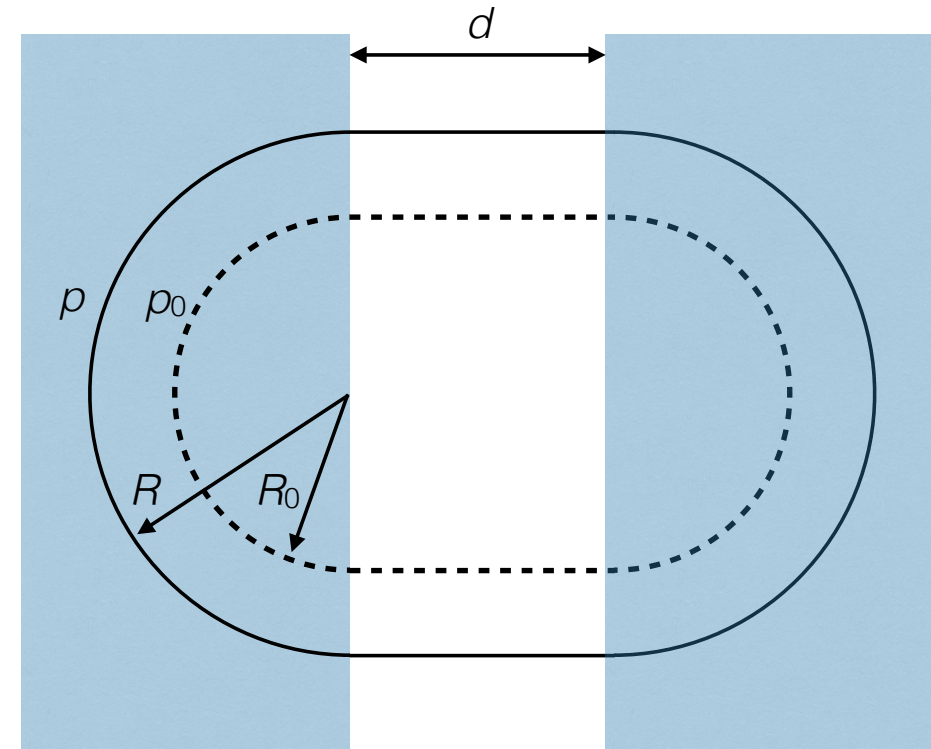
$$R/R_0 = p/p_0$$

$$\frac{\Delta v}{v} = \frac{1}{\gamma^2} \frac{\Delta p}{p}$$

$$\tau = 2\pi R/v$$

$$\frac{\Delta L}{L_0} = \frac{\Delta R}{R_0} = \frac{\Delta p}{p_0}$$

$$\frac{\Delta \tau}{\tau_0} = \left(1 - \frac{1}{\gamma_0^2}\right) \frac{\Delta p}{p_0}$$



$$\tau = (2\pi R + 2d)/v$$

$$\frac{\Delta L}{L_0} = \frac{2\pi(R - R_0)}{2\pi R_0 + 2d} = \frac{1}{1 + d/\pi R_0} \frac{\Delta p}{p_0}$$

$$\frac{\Delta \tau}{\tau_0} = \left(\frac{1}{1 + d/\pi R_0} - \frac{1}{\gamma_0^2} \right) \frac{\Delta p}{p_0}$$

Implications of the Slip Factor

$$\eta = \alpha_p - \frac{1}{\gamma^2} = \left\langle \frac{D}{\rho} \right\rangle - \frac{1}{\gamma^2}$$

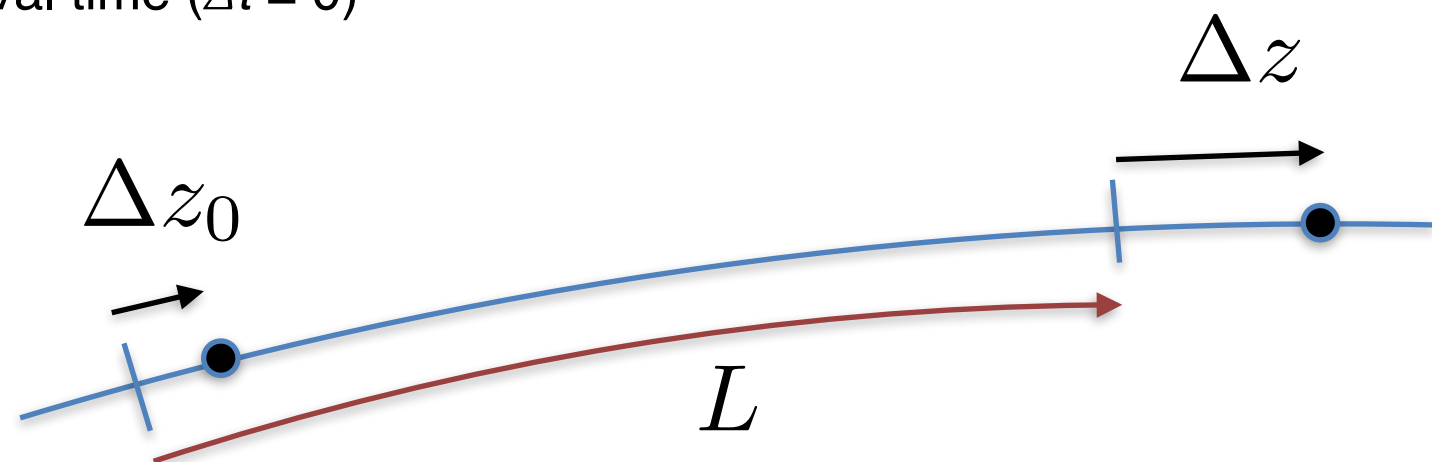
- Suppose no bending in the line (e.g., linac), or, perhaps have bending yet $\gamma^2 < 1/\alpha_p$
 - ▶ then, the slip factor is negative, and particles of higher momentum take less time to traverse the same distance as the ideal particle
- If the energy of the particles is high enough in the presence of bending, then can have $\gamma^2 > 1/\alpha_p$
 - ▶ in this case, the slip factor is positive — the changes in path length outweigh the changes in speed when determining the time of flight difference
 - ▶ here, a higher-momentum particle will actually take **longer** to traverse the same distance as the ideal particle, even though it's moving faster

Linear Motion Very Near the Ideal Particle

Particles moving along the ideal trajectory move toward or away from the ideal particle according to their speed (momentum/energy) and path length differences

Δt = arrival time relative to the ideal arrival time ($\Delta t = 0$)

$$\Delta z = -\beta c \Delta t$$



$$\tau = L/v = L/(\beta c)$$

Note : $\frac{\Delta p}{p} = \frac{1}{\beta^2} \frac{\Delta E}{E} = \frac{1}{\beta^2} \frac{\gamma - 1}{\gamma} \frac{\Delta W}{W}$

$$\Delta z = \Delta z_0 - \eta L \frac{\Delta p}{p}$$

$$\Delta t = \Delta t_0 + \eta \frac{L}{\beta c} \frac{\Delta p}{p}$$

Linear Motion Very Near the Ideal Particle [2]

- Imagine a particle on the ideal trajectory and that has the ideal energy, W_s . A second particle on the ideal trajectory, but with a different energy, W , may be ahead of or lagging behind the ideal particle.
- We will use **radio frequency (RF) cavities** to provide an accelerating voltage to the particles as they pass by.
- The ideal particle will arrive at the cavity at the “ideal” time or, equivalently, at an ideal phase, ϕ_s , to receive an appropriate increase in its energy (which might be an increase of “0”).
- We will keep track of the “difference” in energy between our test particle and the ideal particle:

$$W_s = \text{“ideal” energy}$$

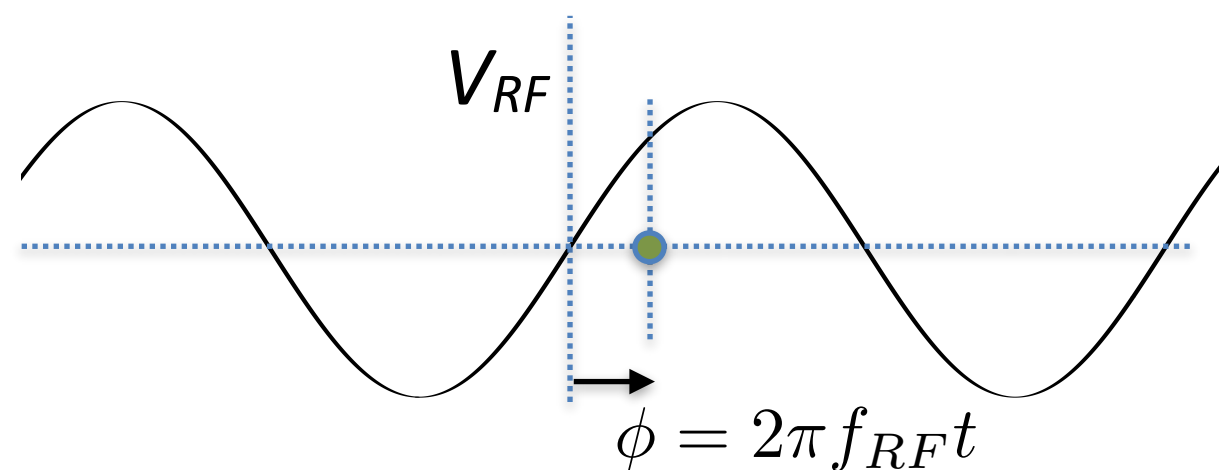
$$\Delta W \equiv W - W_s$$

Linear Motion Very Near the Ideal Particle [3]

- If a group of particles passes through a cavity such that the ideal (synchronous) particle receives no net energy gain, can give particles that are ahead/behind a decrease/increase in energy

W_s = “ideal” energy

$$\Delta W \equiv W - W_s$$



$$\Delta W = \Delta W_0 + qV(\sin \phi - \sin \phi_s)$$

$$= \Delta W_0 + qV[\sin(\phi_s + \Delta\phi) - \sin \phi_s]$$

$$\Delta W \approx \Delta W_0 + qV \cos \phi_s \Delta\phi = \Delta W_0 + qV \cos \phi_s (2\pi f_{RF}) \Delta t_0$$

- Can use matrix techniques to propagate the longitudinal motion

Linear Motion through Cavities and Drifts

- Keep track of time differences and energy differences...

drift:

$$\begin{pmatrix} \Delta t \\ \Delta W \end{pmatrix} = \begin{pmatrix} 1 & \eta \frac{L}{c} \frac{1}{\beta^3 \gamma} \frac{1}{mc^2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta t \\ \Delta W \end{pmatrix}_0$$

through cavity:
longitudinal focusing

$$\begin{pmatrix} \Delta t \\ \Delta W \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ (2\pi f_{RF})qV \cos \phi_s & 1 \end{pmatrix} \begin{pmatrix} \Delta t \\ \Delta W \end{pmatrix}_0$$

remember —

$$\eta \equiv \alpha_p - \frac{1}{\gamma^2}$$

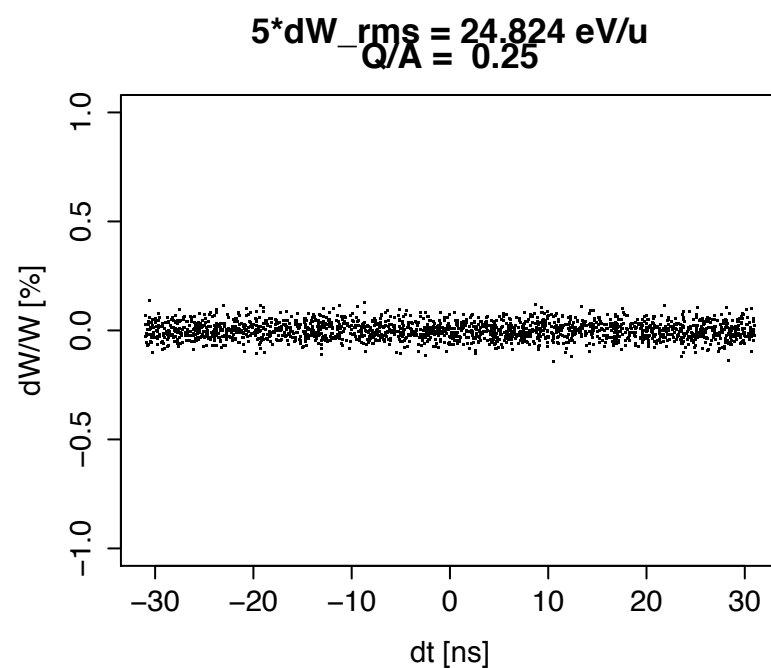
$$\alpha_p = \frac{\int [D(s)/\rho(s)] ds}{\int ds}$$

Bunchers, Re-bunchers, Debunchers

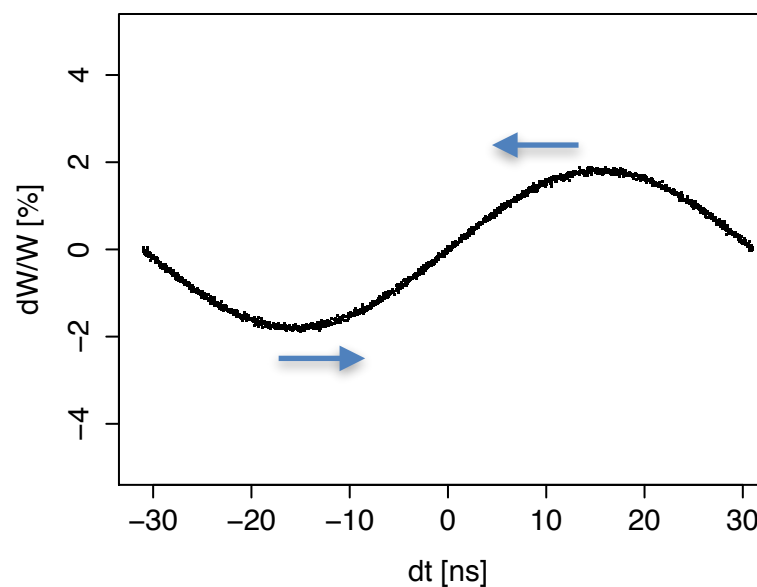
- If start with continuous stream of particles (DC current, with no strong “AC” component), can create bunches (AC) beam using a single cavity (buncher)
- If have bunched beam that is allowed to travel a certain distance, the particles within the bunch will begin to spread out due to the inherent spread in momentum
 - ▶ re-buncher: mitigate this effect
 - ▶ debuncher: enhance this effect

Beam Buncher

incoming DC beam

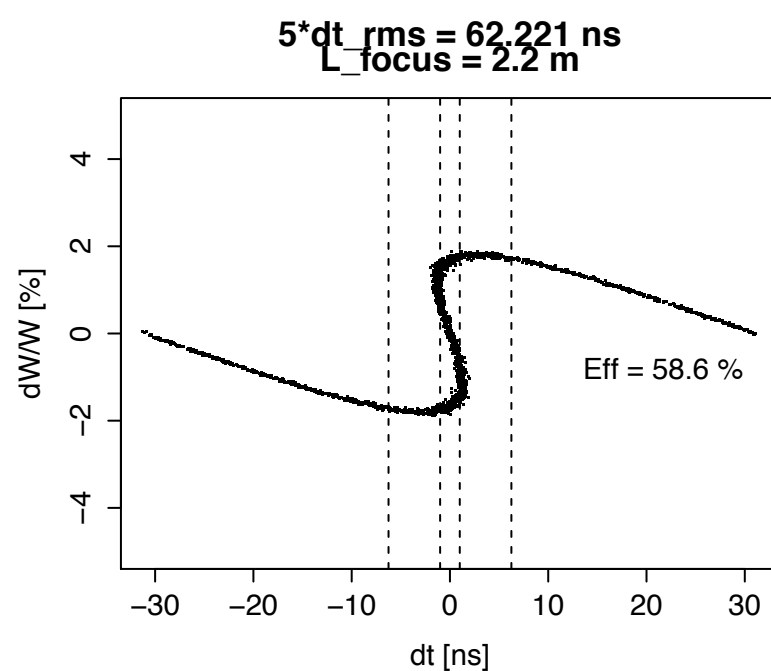


$W_{width} 469.8 \text{ eV/u}$

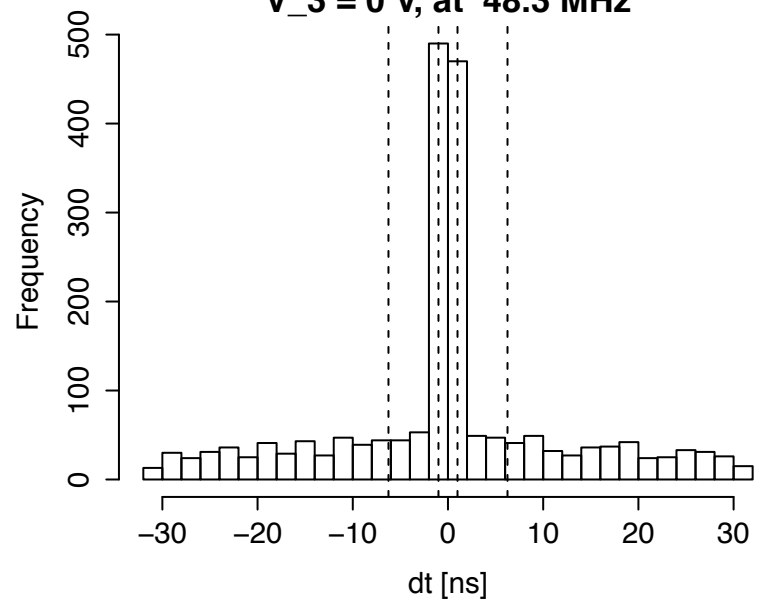


after the cavity

downstream of cavity



$V_1 = 900 \text{ V, at } 16.1 \text{ MHz}$
 $V_2 = 0 \text{ V, at } 32.2 \text{ MHz}$
 $V_3 = 0 \text{ V, at } 48.3 \text{ MHz}$



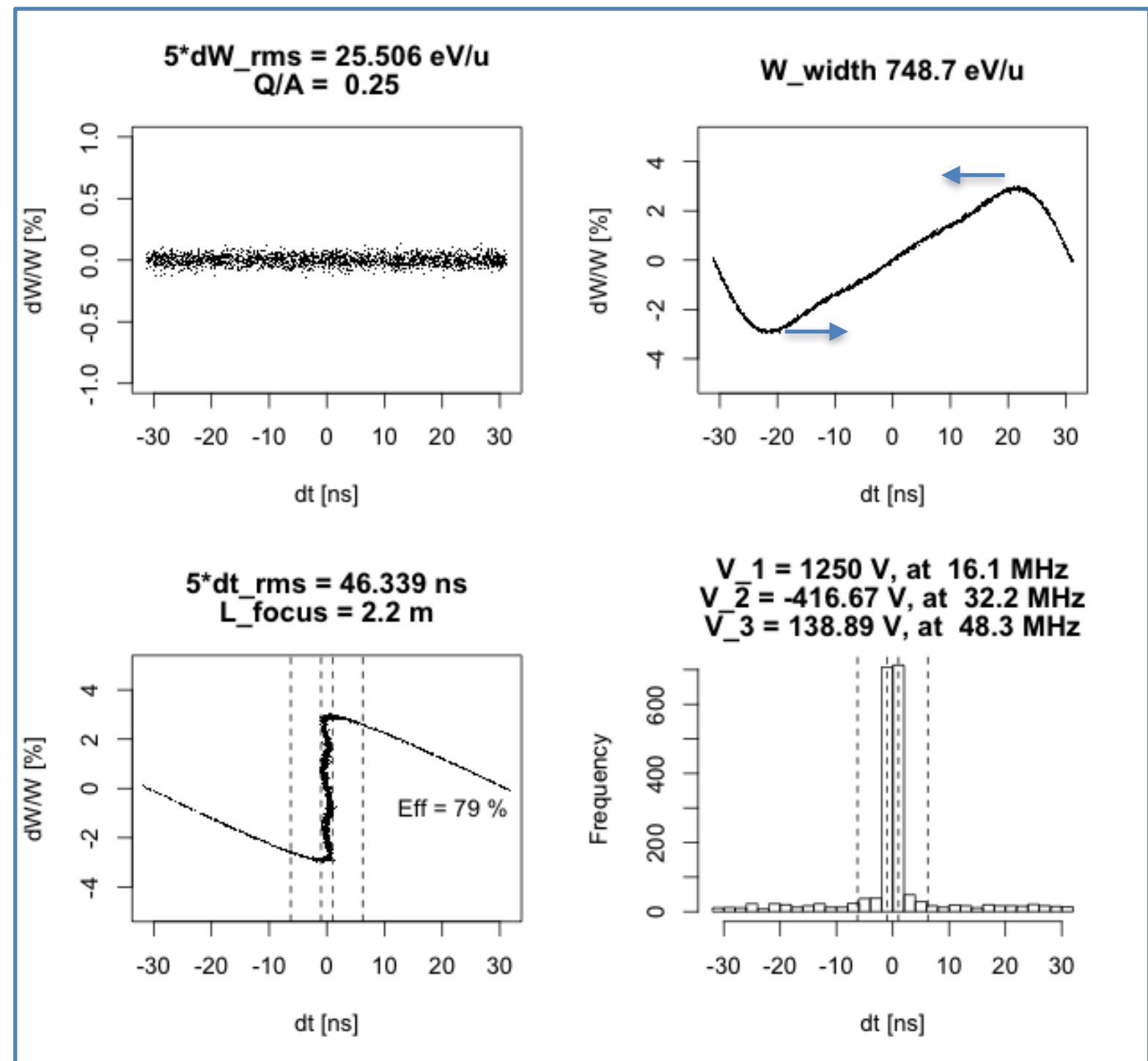
resulting time profile

Multi-harmonic Buncher

- Use 2, or 3 (or 4?) harmonics of the fundamental frequency to smooth out the sine wave into a more linear waveform

$$V(t) = V_1 \sin(2\pi ft) + V_2 \sin(4\pi ft) + V_3 \sin(6\pi ft) + V_4 \sin(8\pi ft) + \dots$$

ReA pre-buncher; Alt, *et al.* (MSU)



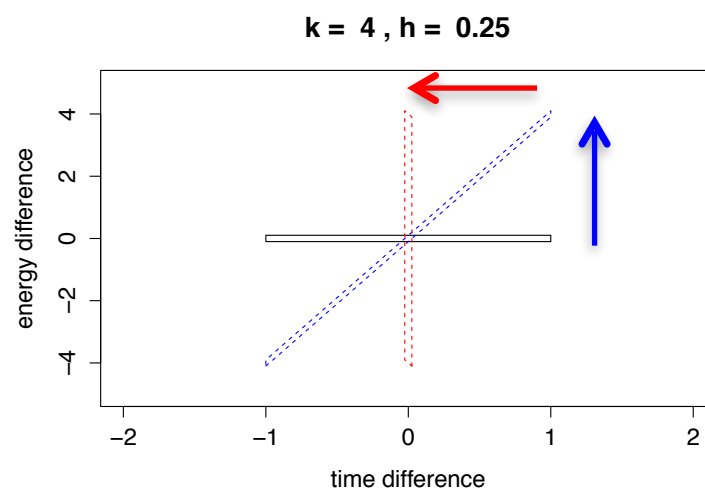
Dependence upon initial energy spread

$k \sim$ voltage

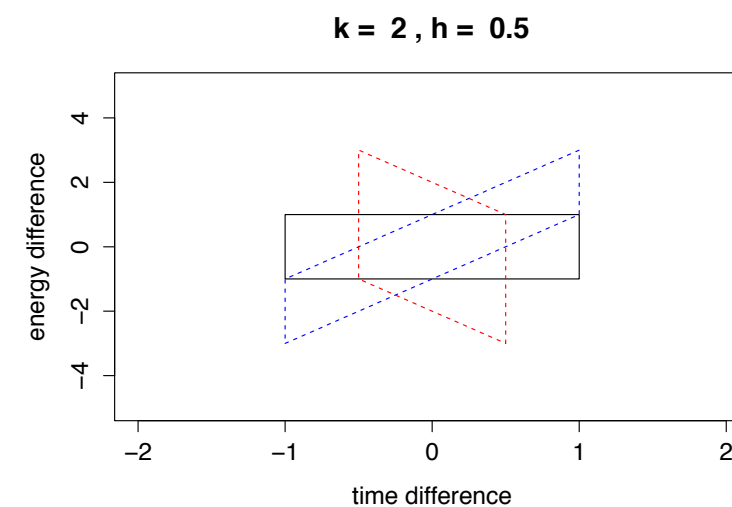
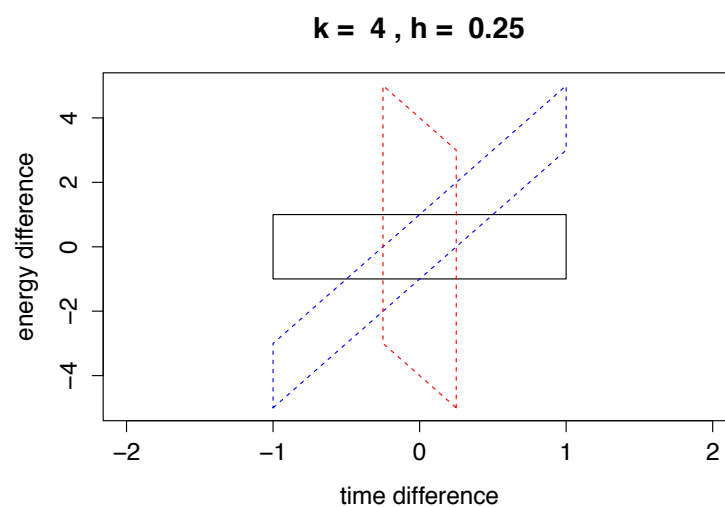
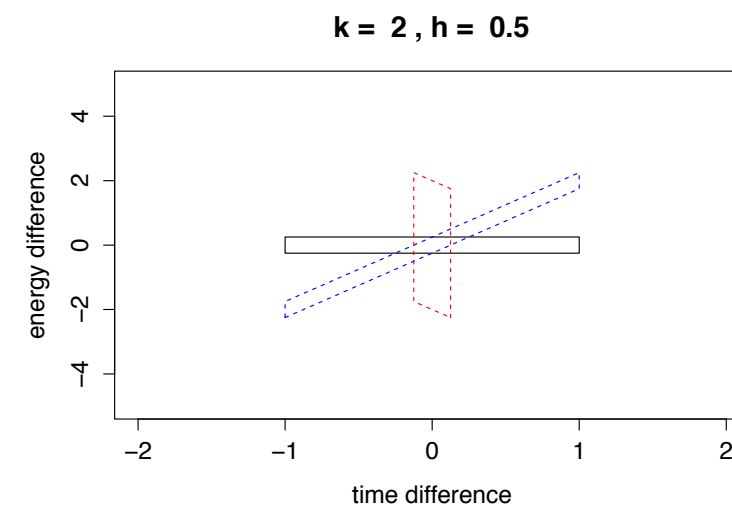
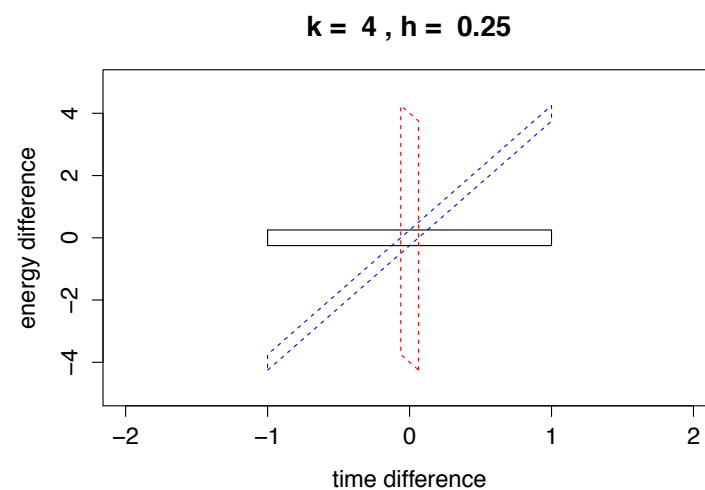
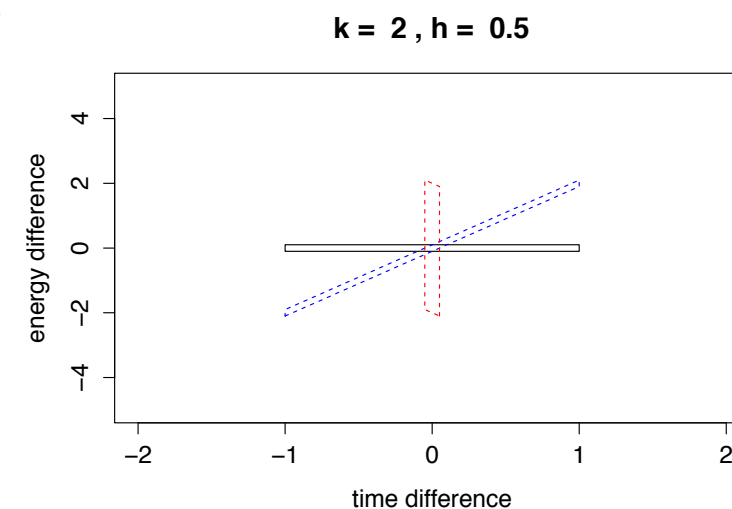
$h \sim$ travel distance

Increasing initial energy spread

lower voltage, longer distance



$\uparrow y \leftarrow y + kx$
 $\leftarrow x \leftarrow x - hy$

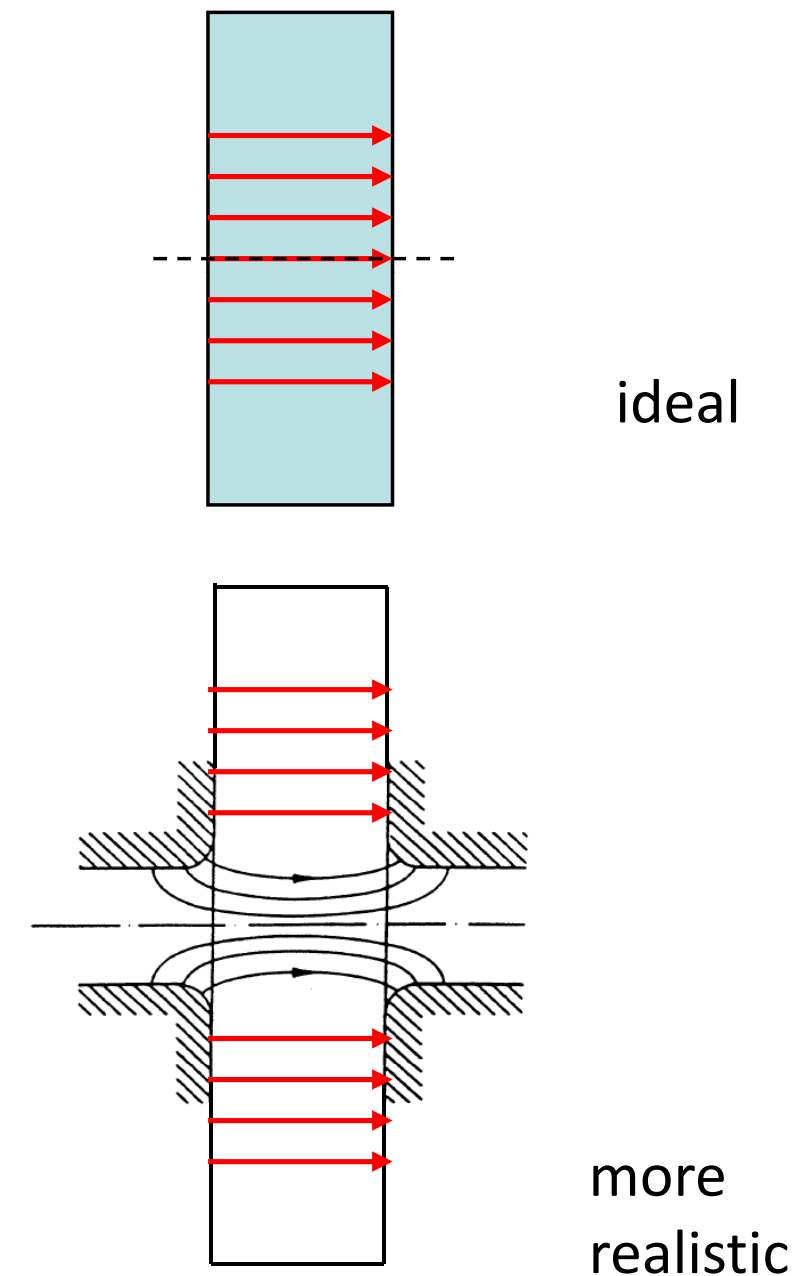


areas
(emittance)
preserved

RF Defocusing

- When a particle enters a cavity off center, the field lines will have an inward component; and they will have an outward component upon exit from the cavity.
- However, the strength of the field is changing — typically, increasing — during transit.
- Thus, the outward “kick” due to the field will be greater than the inward kick — defocusing effect
- This “RF defocusing” is more important at lower energies

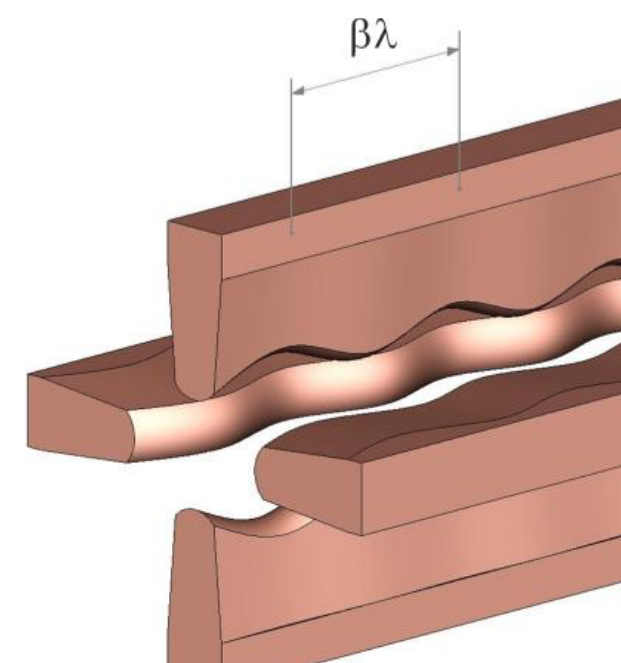
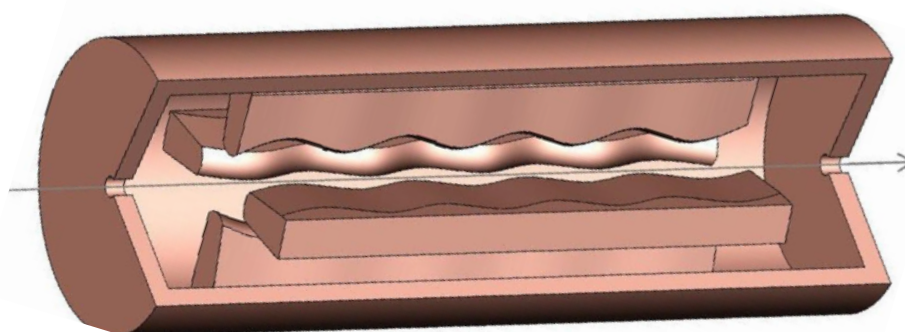
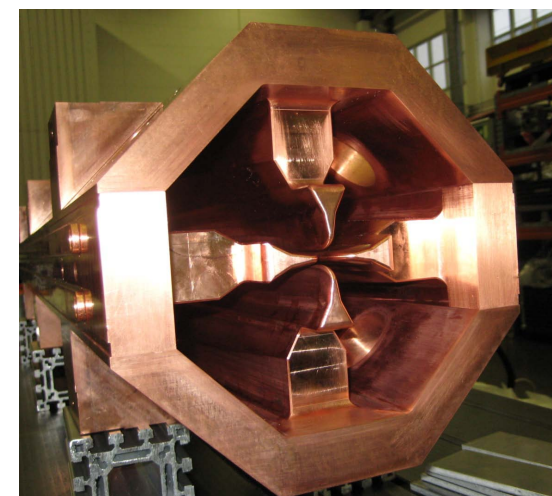
$$\frac{1}{f} = \frac{\Delta x'}{x} \approx \pi \frac{eV_{\text{eff}}}{mc^2} \frac{T \cos \phi_s}{\lambda(\beta\gamma)^2}$$



see T. Wangler, *RF Linear Accelerators*

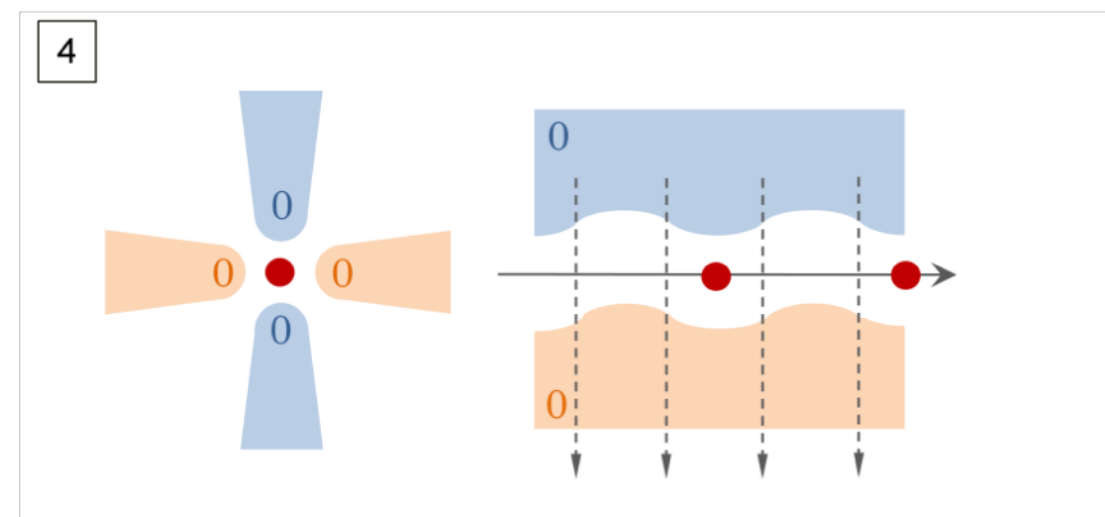
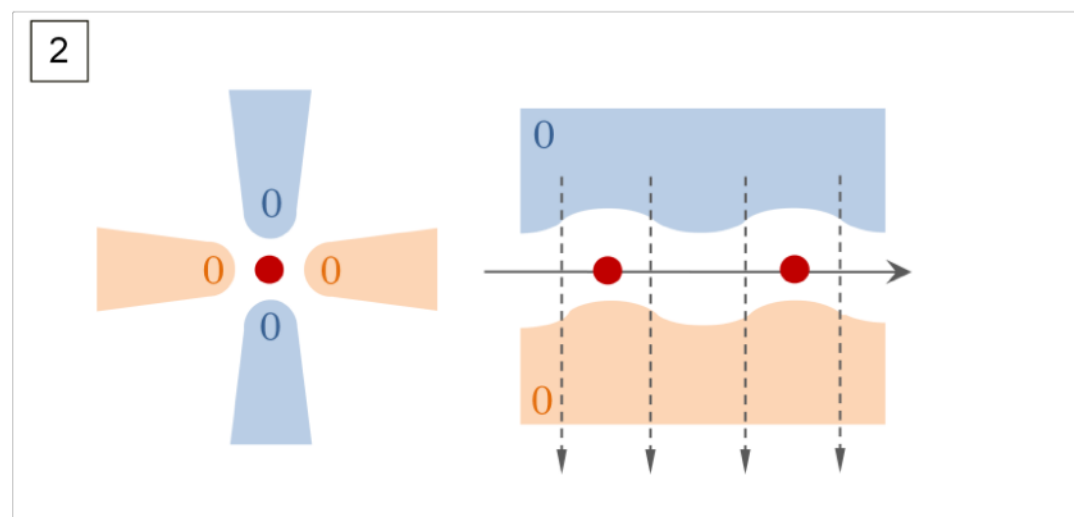
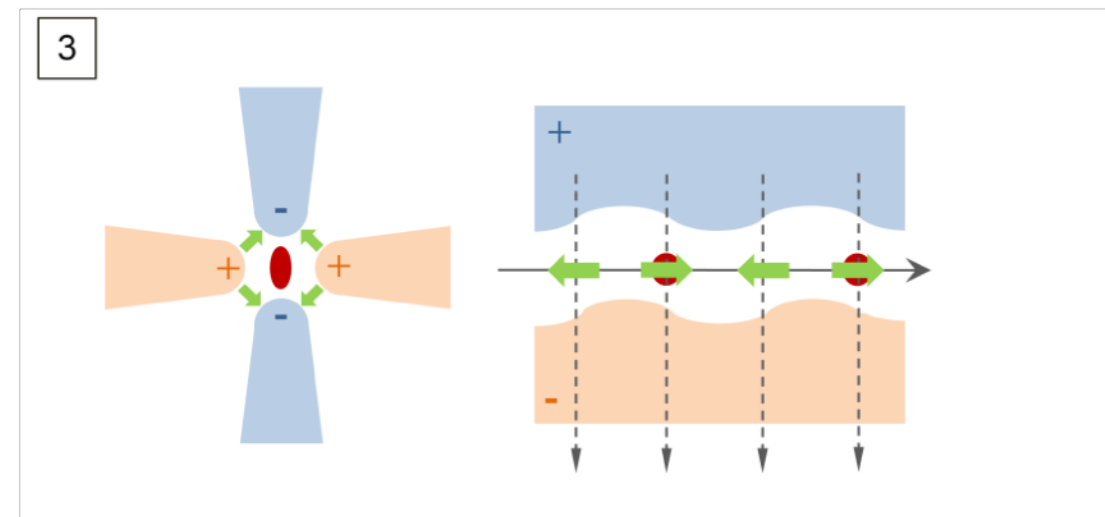
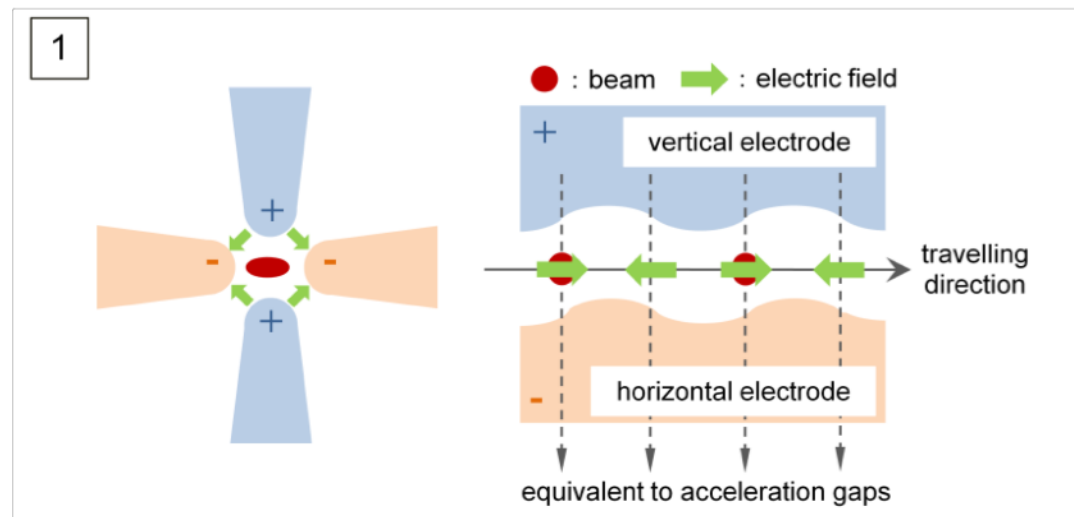
The Radio Frequency Quadrupole (RFQ)

- Create quadrupole electric field for x/y focusing
- Modulate the distance to the quad pole tips along the length of the cavity
 - ▶ modulation period will vary along the length, according to desired velocity profile of ideal particle
 - Period = $\beta\lambda = \beta c/f_{RF}$; *varies as $\beta=v/c$ varies*



- The varying vane structure creates a longitudinal electric field, accelerating the particle; thus, simultaneously accelerate & focus

The Radio Frequency Quadrupole (RFQ)



Many variants: 4-vanes, 4-rods, Al/Cu, large/small

Typical energy range — up to few MeV (protons, ions typically; also electrons)



Repetitive Systems of Acceleration



Repetitive Systems of Acceleration

- ▶ We will assume that particles are propagating through a system of accelerating cavities. Each cavity has oscillating fields with frequency f_{RF} , and maximum “applied” voltage V (i.e., this takes into account TTF’s, etc.). The ideal particle would arrive at the cavity at phase ϕ_s .
- ▶ We will choose ϕ_s to be relative to the “positive zero-crossing” of the RF wave, such that the ideal particle acquires an energy gain of

$$\Delta E = \Delta W = qV \sin \phi_s = QeV \sin \phi_s$$

- this definition used for synchrotrons; linacs more often define ϕ_s relative to the “crest” of the RF wave
 - apologies for this possible *further* confusion...

Acceleration of Ideal Particle

Wish to accelerate the ideal particle. As the particle exits the $(n+1)$ -th RF cavity/station we would have

$$E_s^{(n+1)} = E_s^{(n)} + QeV \sin \phi_s$$

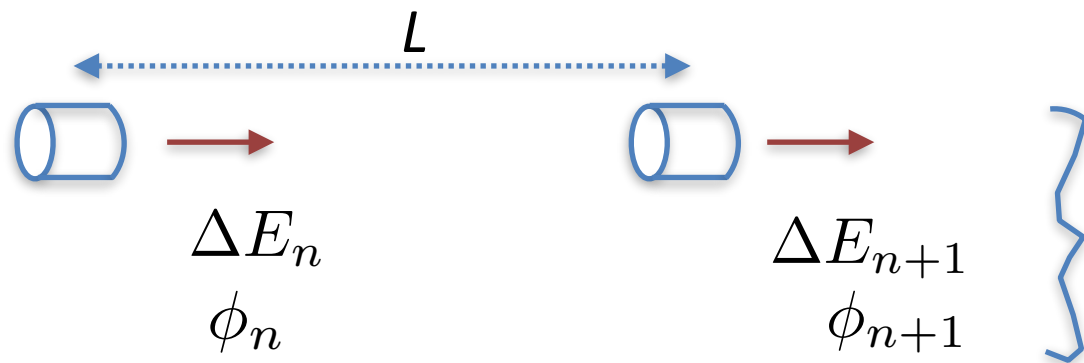
If we are considering a synchrotron, we can consider the above as the total energy gain on the $(n+1)$ -th revolution. The ideal energy gain per second would be:

$$dE_s/dt = f_0 QeV \sin \phi_s$$

Next, look at (longitudinal) motion of particles near the ideal particle: ϕ = phase w.r.t. RF system

$$\Delta E \equiv E - E_s = \text{energy difference from the ideal}$$

- Assume that our accelerating system of cavities is set up so that the ideal particle always arrives at the next cavity when the accelerating voltage V is at the same phase (called the “synchronous phase”)



$$\phi_{n+1} = \phi_n + \frac{2\pi h \eta}{\beta^2 E} \Delta E_n$$

$$\Delta E_{n+1} = \Delta E_n + QeV(\sin \phi_{n+1} - \sin \phi_s)$$

Notes:

$$h = L/\beta\lambda, \quad \lambda = c/f_{\text{rf}}$$

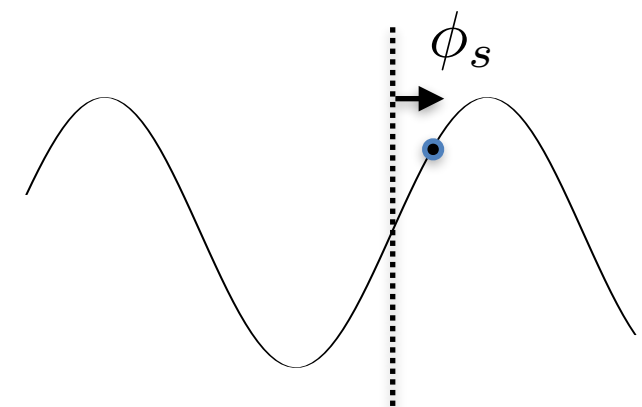
(difference equations)

If L is circumference of a synchrotron then: $h = f_{\text{rf}}/f_0$

where f_0 is the revolution frequency,

In this case, h is called the “harmonic number”

$$E = mc^2 + W; \quad \Delta E \Leftrightarrow \Delta W$$



Applying the Difference Equations

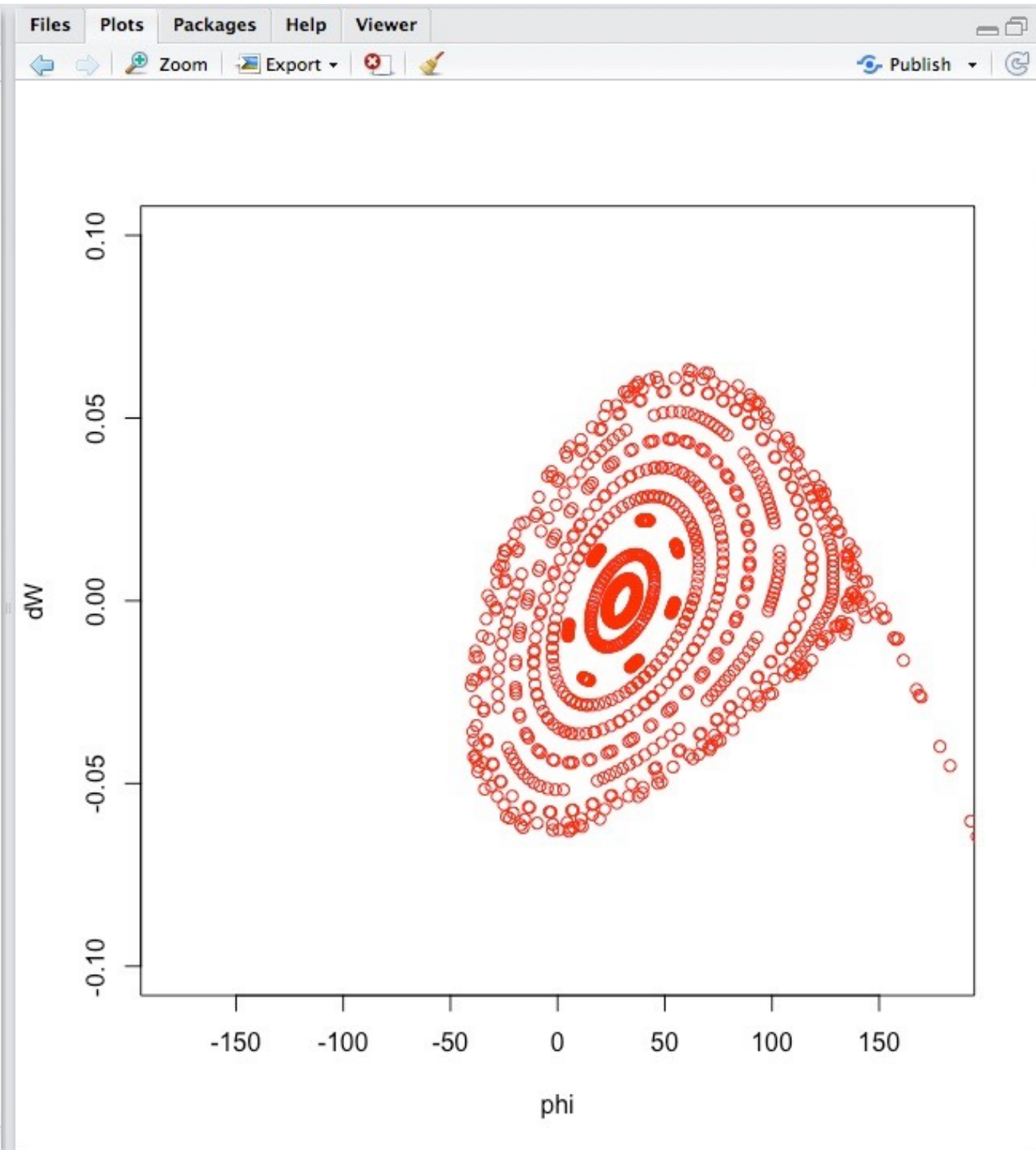
```
while (i < Nturns+1) {  
    phi = phi + k*dW  
    dW = dW + QonA*eV*(sin(phi)-sin(phis))  
    points(phi*360/2/pi, dW, pch=21,col="red")  
    i = i + 1  
}
```

Let's run a code...

```

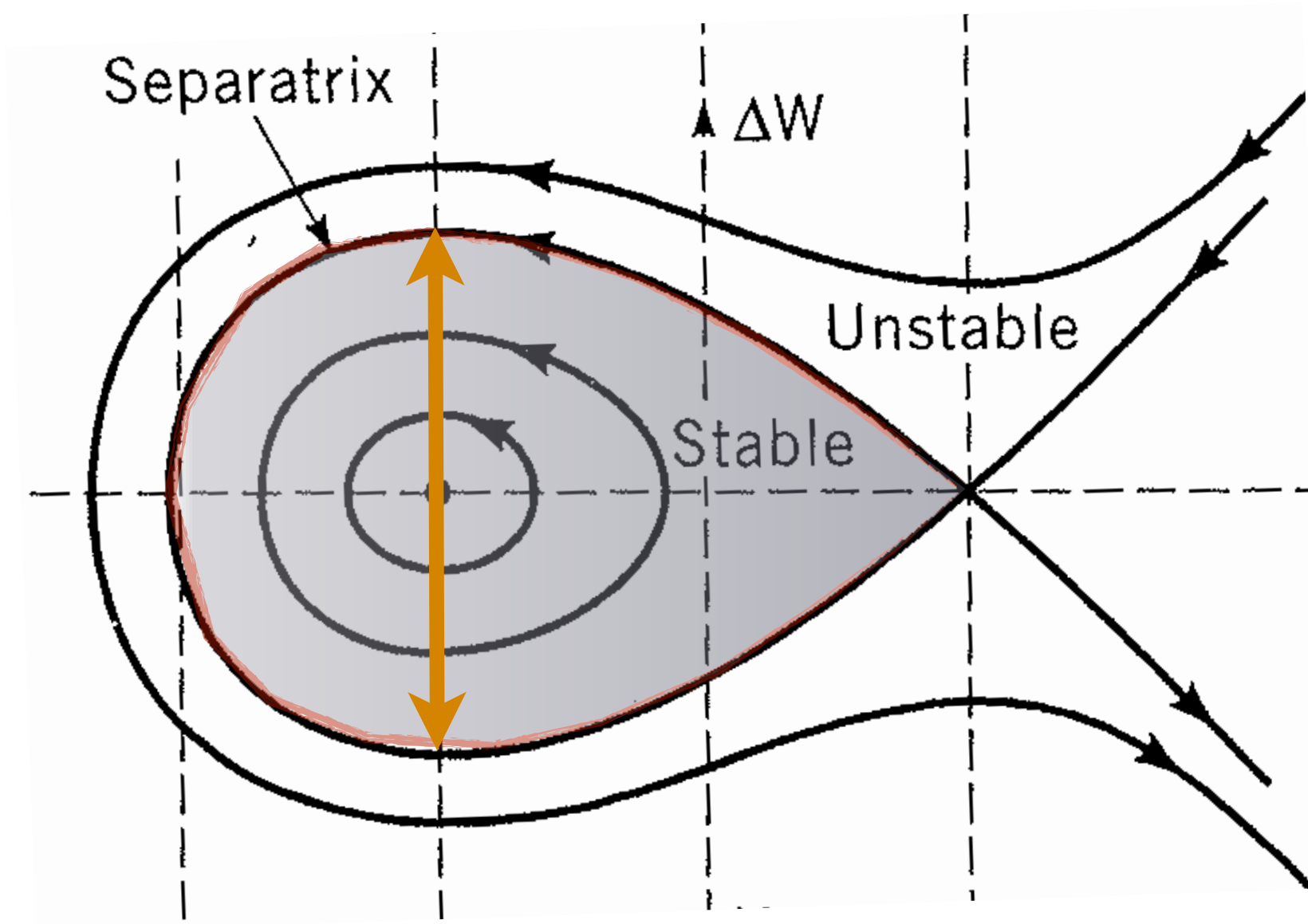
v0_RFtrack.R
1 # Program to plot longitudinal phase space motion
2 # through a system of cavities (just an example...)
3
4 Nturns = 100
5
6 # Some Parameters
7 Ws = 1.0 # MeV/u
8 phis = 30*pi/180 # synchronous phase angle
9 eV = 0.2 # MeV/u
10 QonA = 0.25
11 gamma = (931+Ws)/931
12 beta = sqrt(1-1/gamma^2)
13 eta = -1/gamma^2
14 h = 1/(beta*3e8/80.5e6)
15 k = 2*pi*h*eta/beta^2*(gamma-1)/gamma/Ws
16
17 # initialize the phase space plot
18 phi = 0
19 dW = 0
20 plot(phi, dW, xlim=c(-180,180), ylim=c(-0.1,0.1), typ="n")
21
22 trk = 1
23 while (trk < 16) {
24 # initialize particle positions in phase space
25 u0 <- locator(1)
26 phi <- u0$x/180*pi
27 dW <- u0$y
28 # track the particle...
29 i = 1
30 while (i < Nturns+1) {
31 phi = phi + k*dW
32 dW = dW + QonA*eV*(sin(phi)-sin(phis))
33 points(phi*360/2/pi, dW, pch=21,col="red")
34 i = i + 1
35 }
36 trk = trk + 1
37 }
38
38:1 (Top Level) R Script

```



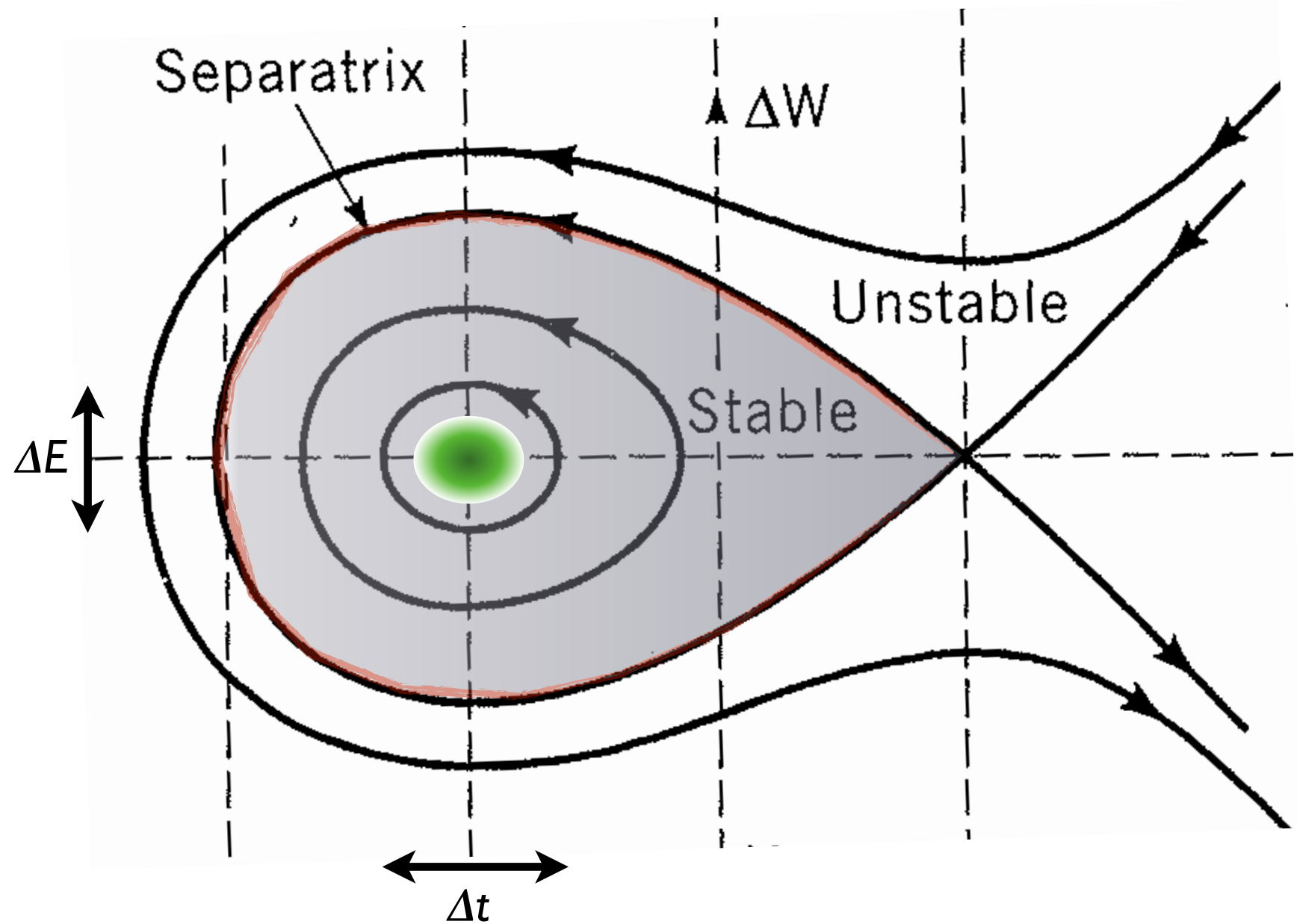
Acceptance and Emittance

- Stable region often called an RF “bucket”
 - “contains” the particles
- Maximum vertical extent is the maximum spread in energy that can be accelerated through the system



Acceptance and Emittance

- Stable region often called an RF “bucket”
 - “contains” the particles
- Maximum vertical extent is the maximum spread in energy that can be accelerated through the system
- Desire the beam particles to occupy much smaller area in the phase space



area: “eV-sec”
Note: E, t canonical

$$\phi_{n+1} = \phi_n + \frac{2\pi h\eta}{\beta^2 E} \Delta E_n$$

$$\Delta E_{n+1} = \Delta E_n + QeV(\sin \phi_{n+1} - \sin \phi_s)$$

differential approach...

start with above difference eqs $\rightarrow \frac{d\phi}{dn} = \frac{2\pi h\eta}{\beta^2 E} \Delta E, \quad \frac{d\Delta E}{dn} = QeV(\sin \phi - \sin \phi_s)$

$$\rightarrow \frac{d^2\phi}{dn^2} = \frac{2\pi h\eta}{\beta^2 E} \frac{d\Delta E}{dn} = \frac{2\pi h\eta}{\beta^2 E} QeV(\sin \phi - \sin \phi_s) \quad (1)$$

$$\Rightarrow \frac{d^2\phi}{dn^2} - \frac{2\pi h\eta}{\beta^2 E} QeV(\sin \phi - \sin \phi_s) = 0$$

find 1st integral:

$$\int \left(\frac{d^2\phi}{dn^2} \right) \frac{d\phi}{dn} dn - \frac{2\pi h\eta}{\beta^2 E} QeV \int (\sin \phi - \sin \phi_s) \frac{d\phi}{dn} dn = 0$$

$$\frac{1}{2} \left(\frac{d\phi}{dn} \right)^2 + \frac{2\pi h\eta}{\beta^2 E} QeV (\cos \phi + \phi \sin \phi_s) = \text{constant}$$

or,

$$\Delta E^2 + 2 \frac{\beta^2 E}{2\pi h\eta} QeV (\cos \phi + \phi \sin \phi_s) = \text{constant} \quad (2)$$

The equation of the **trajectories** in phase space!

Synchrotron Oscillations

- Particles near the synchronous phase and ideal energy will oscillate about the synchronous particle with the “synchrotron frequency” (this is call synchrotron motion)
 - ▶ “synchrotron tune” == # of synch. osc.’s per revolution

compute small oscillation frequency:

$$\text{in (1), let } \phi = \phi_s + \Delta\phi \rightarrow \sin \phi - \sin \phi_s = \sin \phi_s \cos \Delta\phi + \cos \phi_s \sin \Delta\phi - \sin \phi_s \\ \approx \Delta\phi \cos \phi_s$$

$$\Rightarrow \frac{d^2 \Delta\phi}{dn^2} - \left(\frac{2\pi h\eta}{\beta^2 E} QeV \cos \phi_s \right) \Delta\phi = 0$$

\swarrow $(2\pi\nu_s)^2$ \searrow

\Rightarrow

$$\nu_s = \sqrt{-\frac{h\eta QeV}{2\pi\beta^2 E} \cos \phi_s}$$

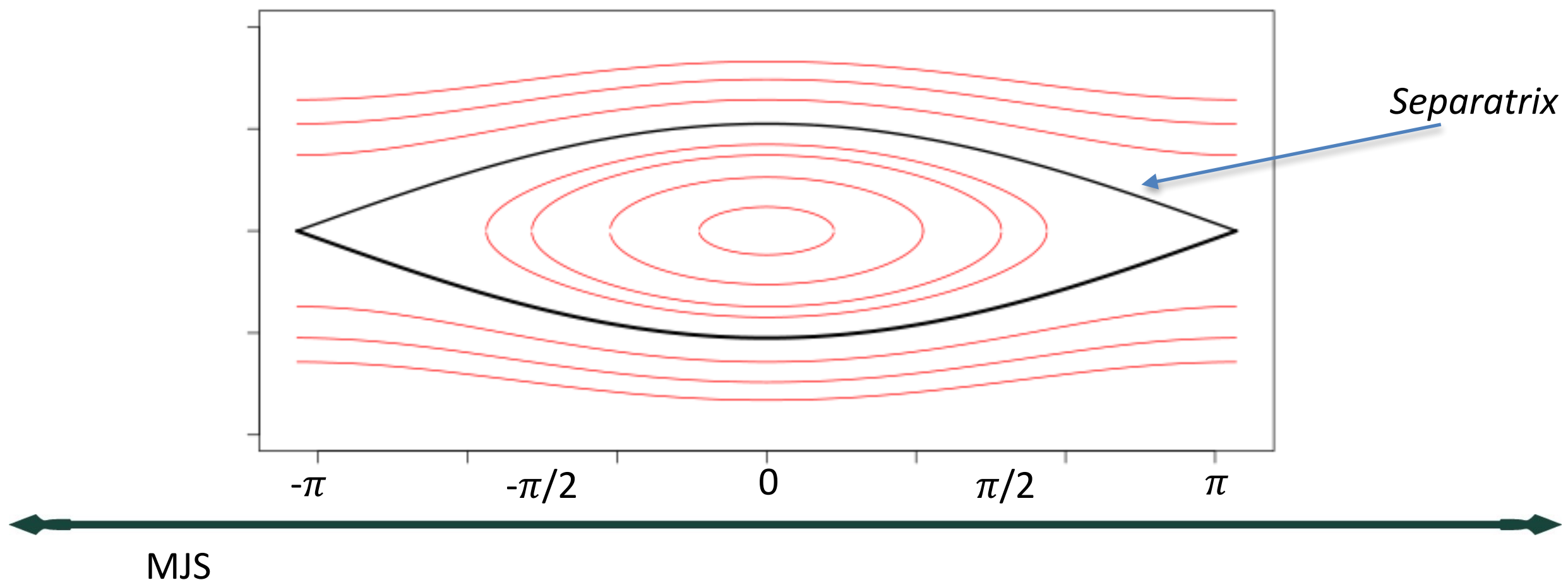
if $\eta > 0$, choose $\cos \phi_s < 0$

Comment on Frequencies of the Motion

- From what we've just seen, the synchrotron motion in a circular accelerator takes many (perhaps hundreds of) revolutions to complete one synchrotron period
- On the other hand, in the transverse plane, a particle will typically undergo many betatron oscillations during one revolution
- Thus, transverse/longitudinal dynamics typically occur on very different time scales — this actually justifies us studying them independently

The Stationary Bucket

- Suppose do not wish to accelerate the ideal particle...
 - ▶ for lower energies, where the slip factor is negative, then need to choose $\phi_s = 0^\circ$



“stationary” bucket: $\phi_s = 0, 2\pi$ ($\sin \phi_s = 0$)

anticipate stability: \rightarrow choose $\phi_s = 0, \eta < 0$

then,
$$\Delta E^2 + 2 \frac{\beta^2 E}{2\pi h \eta} QeV \cos \phi = \text{constant}$$

on the separatrix: $\Delta E = 0$ at $\phi = \pm\pi$

$$0 - 2 \frac{\beta^2 E}{2\pi h \eta} QeV = \text{constant}$$

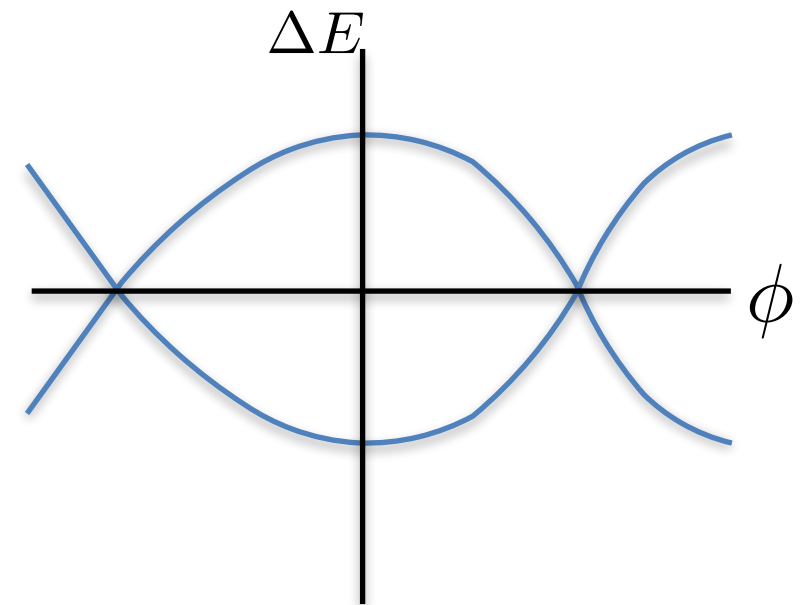
thus, the Eq. of separatrix:
$$\Delta E^2 + (1 + \cos \phi) \frac{\beta^2 E}{\pi h \eta} QeV = 0$$

$$\Delta E^2 + \frac{2\beta^2 E}{\pi h \eta} QeV \cos^2(\phi/2) = 0$$

separatrix:

$$\Delta E = \pm \sqrt{-\frac{2\beta^2 E}{\pi h \eta} QeV \cos^2(\phi/2)}$$

(for “stationary bucket”)



thus, “bucket height”: $a = \sqrt{\frac{2\beta^2 E}{\pi h |\eta|}} QeV$

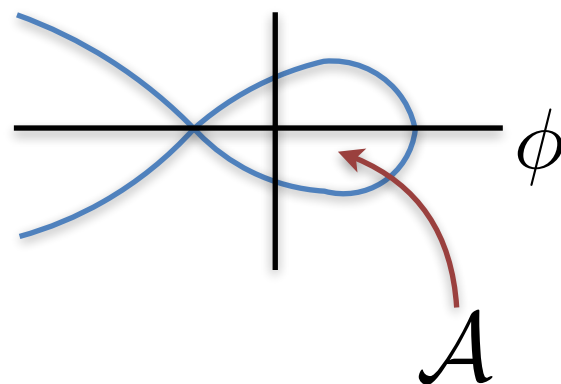
Phase space area of a stationary bucket: $4 \int_0^\pi a \cos(\phi/2) d\phi = 8a$

and, if use ΔE - Δt coordinates rather than ΔE - ϕ , then area of a *stationary* bucket is...

$$\mathcal{A}_0 \equiv \frac{8}{\pi f_{\text{rf}}} \sqrt{\frac{\beta^2 E QeV}{2\pi h |\eta|}} \quad \boxed{\text{(here, units of eV-sec)}}$$

since $\phi = 2\pi f_{\text{rf}} t$

Note: for $\sin \phi_s \neq 0$

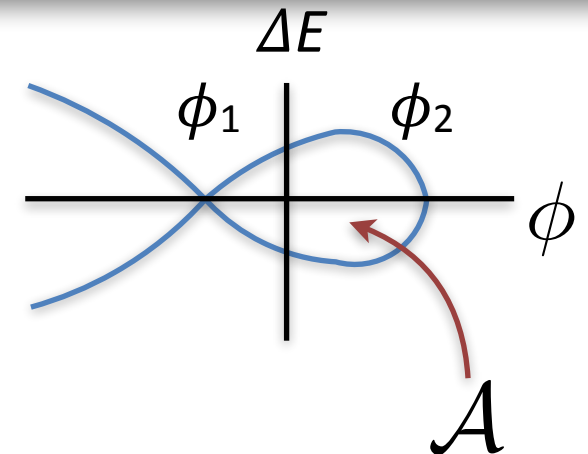


$$\mathcal{A} = \mathcal{A}_0 \cdot \mathcal{F}(\phi_s)$$

where $0 < \mathcal{F} < 1$

(determined numerically)

Finding the Bucket Area

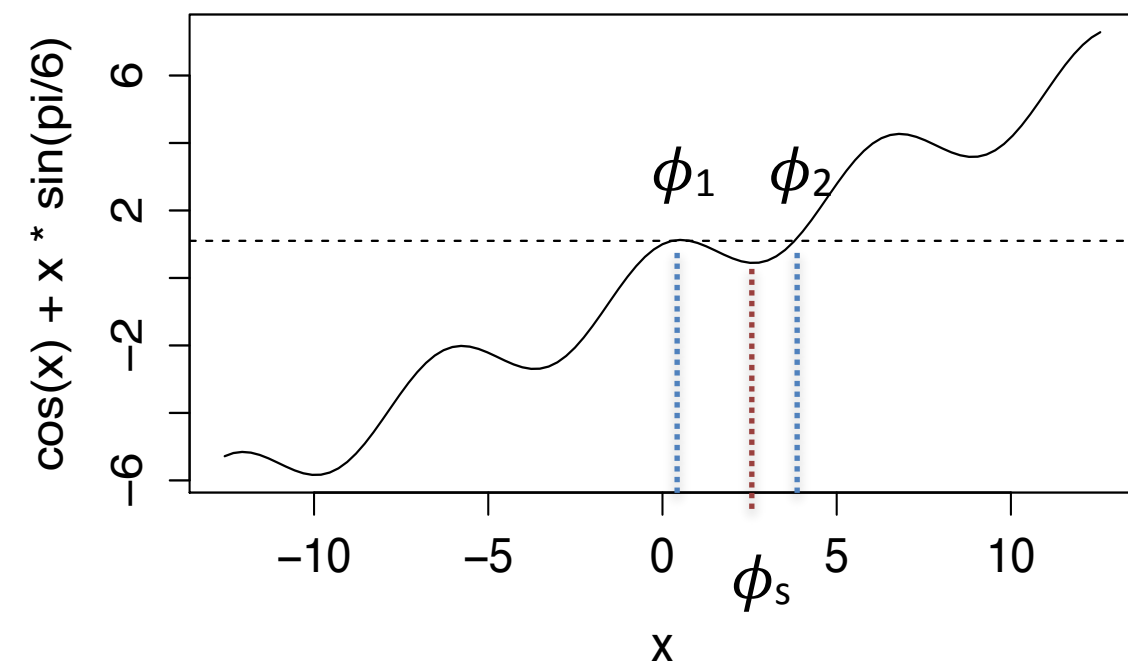


curve: $\Delta E^2 + 2 \frac{\beta^2 E}{2\pi h \eta} QeV (\cos \phi + \phi \sin \phi_s) = \text{constant}$

“kinetic”-like

“potential”-like

“total Energy”-like



ϕ_1 is where

“potential like”

has derivative = 0: $\phi_1 = \pi - \phi_s$

Given $\phi_1 = \pi - \phi_s$, can now determine the “constant”: $\Delta E = 0$ at ϕ_1 , and so...

$$(0)^2 + 2 \frac{\beta^2 E}{2\pi h \eta} QeV (\cos \phi_1 + \phi_1 \sin \phi_s) = \text{constant}$$

Then, find that ϕ_2 must satisfy: $\cos \phi_2 + \phi_2 \sin \phi_s + \cos \phi_s + (\pi - \phi_s) \sin \phi_s = 0$

Numerical Solution for Bucket Area

```
# Solve for bucket area; phis = 0 is "stationary"

Xout <- array(0,dim=c(91,4))
phisDeg <- -1

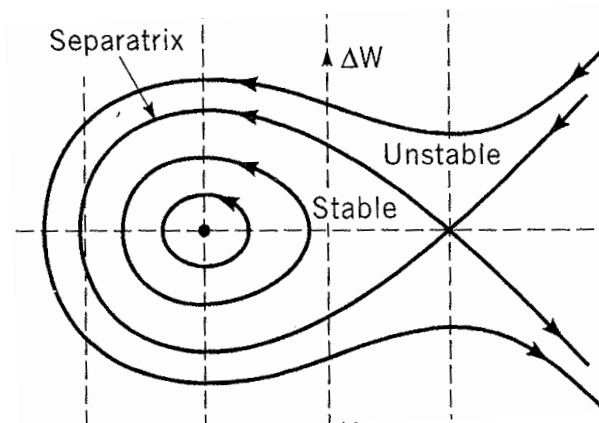
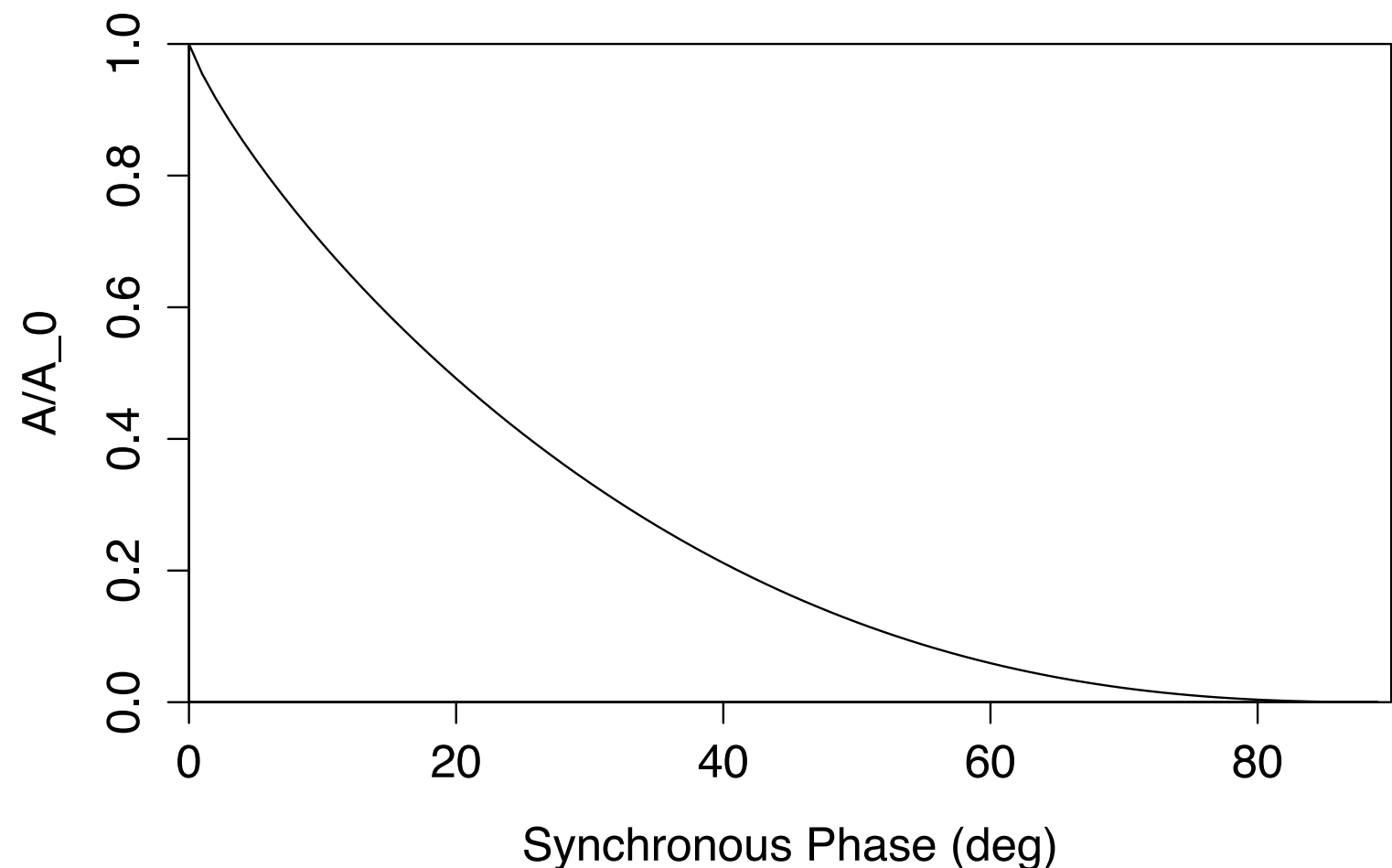
for(i in (1:90)){
  phisDeg <- phisDeg + 1
  phis <- phisDeg*pi/180

  f <- function(x){
    cos(x)+x*sin(phis)+cos(phis)-(pi-phis)*sin(phis) }
  dE <- function(x){
    sqrt(cos(phis)-(pi-phis)*sin(phis)+cos(x)+x*sin(phis)) }

  phi1 <- pi-phis
  phi2 <- uniroot( f, c(-pi, 2*pi))$root
  A <- -1/4/sqrt(2)*integrate(dE, phi1, phi2)$value

  Xout[i,] = c(phis*180/pi, phi1*180/pi, phi2*180/pi, A) }

plot(Xout[,1],Xout[,4],typ="l",
     xlab="Synchronous Phase (deg)", ylab="A/A_0",
     xaxs="i", yaxs="i",xlim=c(0,90))
```



Back to Small Oscillations...

from (2),
$$\Delta E^2 + 2 \frac{\beta^2 E}{2\pi h \eta} Q e V (\cos \phi + \phi \sin \phi_s) = \text{constant}$$

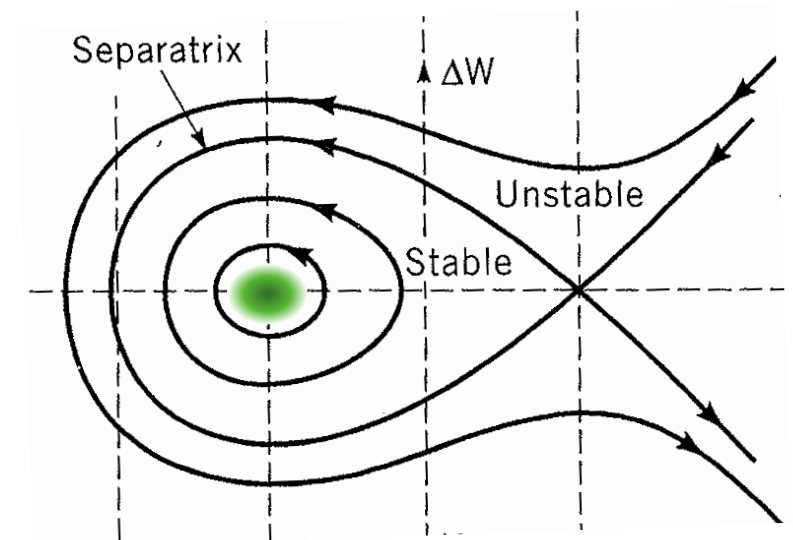
if $\phi = \phi_s + \Delta\phi$, then ...
(small)

$$\Delta E^2 + 2 \frac{\beta^2 E}{2\pi h \eta} Q e V (\cos \phi_s \cos \Delta\phi - \sin \phi_s \sin \Delta\phi + (\phi_s + \Delta\phi) \sin \phi_s) = \text{constant}$$

$$\Delta E^2 + 2 \frac{\beta^2 E}{2\pi h \eta} Q e V (\cos \phi_s (1 - \frac{1}{2} \Delta\phi^2) - \sin \phi_s \Delta\phi + \phi_s \sin \phi_s + \Delta\phi \sin \phi_s) = \text{constant}$$

$$\Delta E^2 + \left(-\frac{\beta^2 E}{2\pi h \eta} Q e V \cos \phi_s \right) \Delta\phi^2 = \text{constant} \quad (3)$$

This Eqn. represents trajectories in longitudinal phase space of particles **near** the ideal particle.



Beam Longitudinal Emittance

Suppose beam is well contained within an ellipse given by (3), and suppose we know either $\Delta\hat{E}$ or $\Delta\hat{\phi}$ (or, $\Delta\hat{t}$) of the distribution (i.e., maximum extent). Then, the *constant* is easily seen to be:

$$\text{constant} = \Delta\hat{E}^2 = -\frac{\beta^2 E}{2\pi h\eta} Q e V \cos \phi_s \Delta\hat{\phi}^2$$

So, area of ellipse (the *longitudinal emittance*) is: $\pi \Delta\hat{E} \Delta\hat{\phi}$

or, in E - t coordinates,
$$S \equiv \pi \Delta\hat{E} \Delta\hat{t} = \pi \Delta\hat{E} \frac{\Delta\hat{\phi}}{2\pi f_{\text{rf}}}$$

➡
$$S = \frac{1}{2f_{\text{rf}}} \sqrt{-\frac{\beta^2 E e V}{2\pi h\eta} Q \cos \phi_s} \Delta\hat{\phi}^2$$

or,

$$S = 2\pi^2 f_{\text{rf}} \sqrt{-\frac{\beta^2 E e V}{2\pi h\eta} Q \cos \phi_s} \Delta\hat{t}^2$$

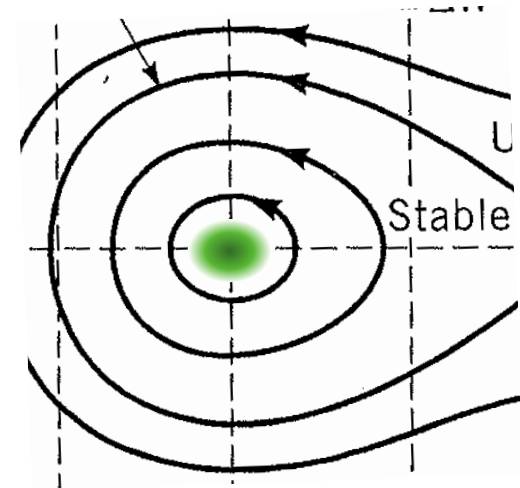
units: "eV-sec"

Motion Near the Ideal Particle

Linearize the motion, and write in matrix form...

$$\phi_{n+1} = \phi_n + \frac{2\pi h\eta}{\beta^2 E} \Delta E_n$$

$$\begin{aligned} \Delta E_{n+1} &= \Delta E_n + QeV(\sin \phi_{n+1} - \sin \phi_s) \\ &= \Delta E_n + QeV(\sin \phi_s \cos \Delta \phi_{n+1} + \sin \Delta \phi_{n+1} \cos \phi_s) - \sin \phi_s \\ &= \Delta E_n + QeV \cos \phi_s \Delta \phi_{n+1} \\ &= \Delta E_n + QeV \cos \phi_s \left[\Delta \phi_n + \frac{2\pi h\eta}{\beta^2 E} \Delta E_n \right] \end{aligned}$$



Thus,

$$\Delta \phi_{n+1} = \Delta \phi_n + \frac{2\pi h\eta}{\beta^2 E} \Delta E_n$$

$$\Delta E_{n+1} = QeV \cos \phi_s \Delta \phi_n + \left(1 + \frac{2\pi h\eta}{\beta^2 E} QeV \cos \phi_s \right) \Delta E_n$$

or,

$$\begin{pmatrix} \Delta\phi \\ \Delta E \end{pmatrix}_{n+1} = \begin{pmatrix} 1 & \frac{2\pi h\eta}{\beta^2 E} \\ QeV \cos \phi_s & \left(1 + \frac{2\pi h\eta}{\beta^2 E} QeV \cos \phi_s\right) \end{pmatrix} \begin{pmatrix} \Delta\phi \\ \Delta E \end{pmatrix}_n$$

$$= \begin{pmatrix} 1 & 0 \\ QeV \cos \phi_s & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{2\pi h\eta}{\beta^2 E} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta\phi \\ \Delta E \end{pmatrix}_n$$

$$M = M_c \cdot M_d$$

“thin” cavity *drift*
(acts as longitudinal focusing element)

Note: for $\eta < 0$, M_d is a “backwards” drift; i.e., $\Delta\phi$ decreases for $\Delta E > 0$
 (when no bending)

$\eta = -1/\gamma^2$ in straight region (linac)



Remember from transverse motion, $x \propto \sqrt{\beta} \sin \Delta\psi$
and when M was periodic,

$$M = \begin{pmatrix} \cos \Delta\psi + \alpha \sin \Delta\psi & \beta \sin \Delta\psi \\ -\gamma \sin \Delta\psi & \cos \Delta\psi - \alpha \sin \Delta\psi \end{pmatrix} \quad \text{and} \quad \text{tr} M = 2 \cos \Delta\psi$$

$\Delta\psi$ = phase advance through periodic section

Can imagine “longitudinal” $\beta, \alpha, \gamma, \Delta\psi$ parameters as well

Note: from M of previous page, if represents periodic structure (linac), then

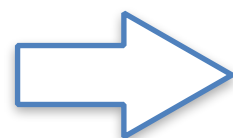
$$\text{tr} M = 2 + \frac{2\pi h\eta}{\beta^2 E} QeV \cos \phi_s = 2 \cos \Delta\psi_s$$

longitudinal phase advance

$$\Delta\psi_s = 2\pi\nu_s$$

oscillation frequency
w.r.t. cavity number, “ n ”
(e.g., synchrotron *tune*)

$$\cos \Delta\psi_s \approx 1 - \frac{1}{2}(\Delta\psi_s)^2 = 1 + \frac{2\pi h\eta}{\beta^2 E} QeV \cos \phi_s \left[= \frac{1}{2} \text{tr} M \right]$$



$$\nu_s = \sqrt{-\frac{h\eta}{2\pi\beta^2 E} QeV \cos \phi_s}$$

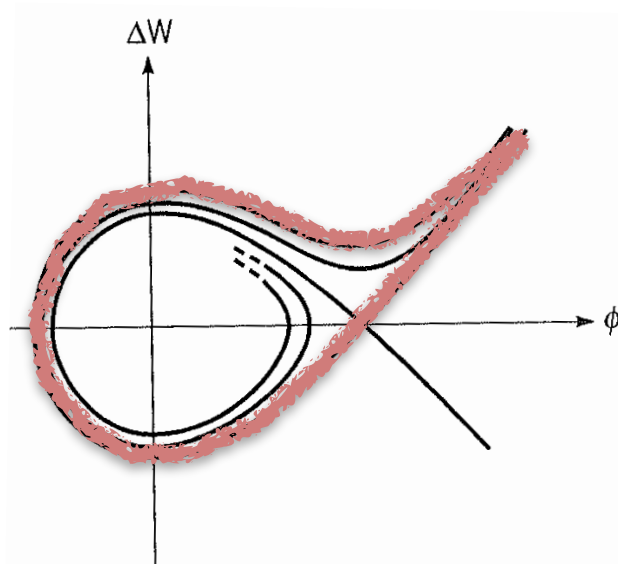
as found previously!

Golf Clubs vs. Fish

- Our analysis “assumes” slowly changing variables (including the energy gain!). Quite reasonable in many Alvarez-style linacs and in synchrotrons
- In linacs, fractional energy change can be large, and so this will distort the phase space
- Plots from Wangler’s book:

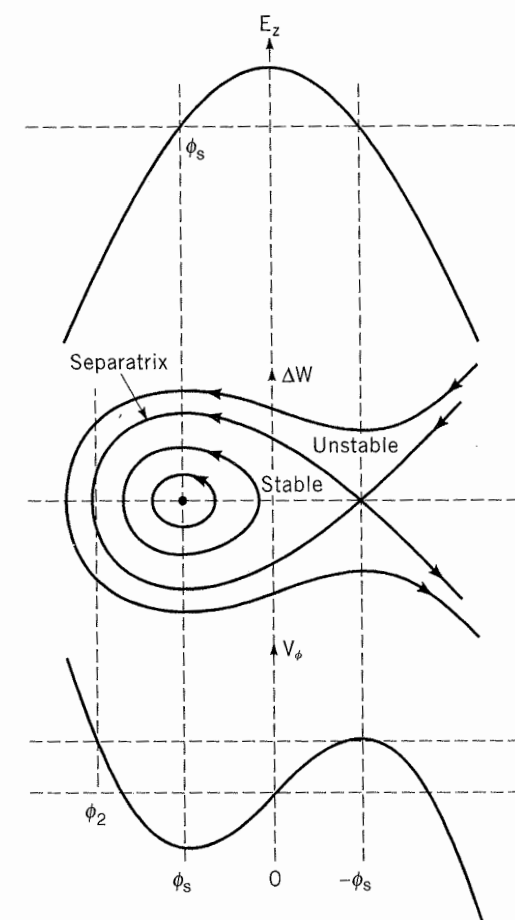
Here, a more rapid acceleration is included

(linac)



Here, assume that energy is “constant” or varying very slowly

(synchrotron)



Transition Energy

- In a synchrotron, there can be an energy at which the slip factor changes sign — this is called the “transition energy”

$$\eta = \alpha_p - \frac{1}{\gamma^2} = \left\langle \frac{D}{\rho} \right\rangle - \frac{1}{\gamma^2}$$

$$\eta = 0 = \alpha_p - \frac{1}{\gamma^2}$$

$$\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$$

$$\gamma_t \equiv \frac{1}{\sqrt{\alpha_p}}$$

- In a typical FODO-style synchrotron, the transition gamma is roughly equal to the betatron tune

Transition

We had... $\Rightarrow \frac{d^2 \Delta \phi}{dn^2} - \left(\frac{2\pi h \eta}{\beta^2 E} QeV \cos \phi_s \right) \Delta \phi = 0$

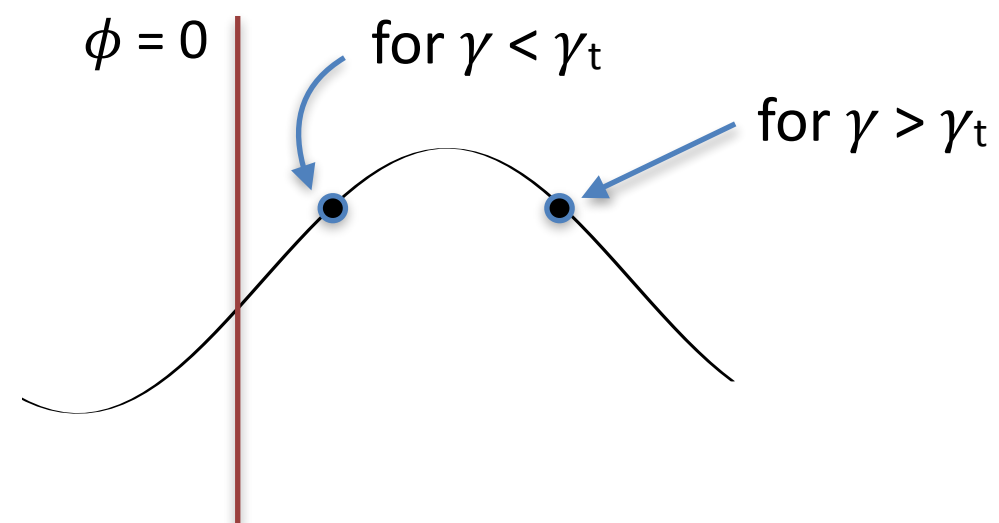
$$\nu_s = \sqrt{-\frac{h\eta}{2\pi\beta^2 E} QeV \cos \phi_s}$$

if $\eta > 0$, choose $\cos \phi_s < 0$

So,

when $\eta < 0$, we want $\cos \phi_s > 0$

when $\eta > 0$, we want $\cos \phi_s < 0$

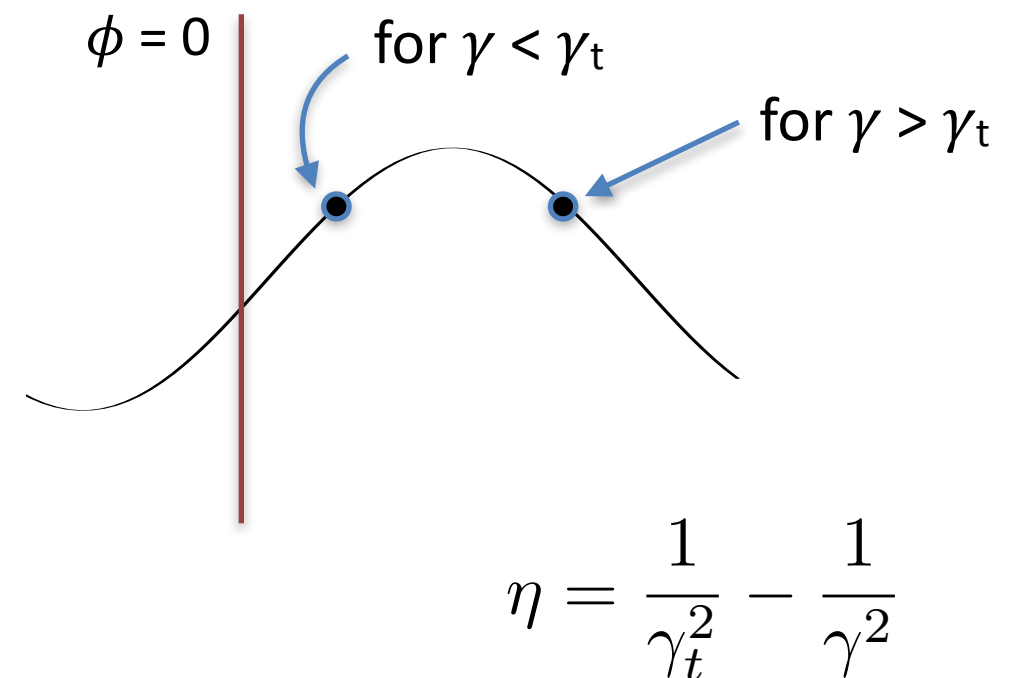


\therefore if γ_t exists, need “phase jump” to occur at transition crossing

$$\gamma_t mc^2 = \text{transition energy}$$

Transition Crossing

- If the synchrotron accelerates through its transition energy, then the phase of the RF system has to be shifted at the time of transition crossing
- The synchrotron motion slows down as approach transition — it would stop if the slip factor were exactly zero!
 - loss of phase stability!
 - momentum spread also gets larger near transition
- So, best to accelerate quickly through this energy region!



$$\nu_s = \sqrt{-\frac{h\eta}{2\pi\beta^2 E} QeV \cos \phi_s}$$

Some Movies...

- Bucket Transformation
- Snap Capture
- Adiabatic Capture
- Parabolic acceleration
- Parabolic acceleration — full bucket
- Transition Crossing





Buckets, Bunches, Batches, ...



Buckets, Bunches, Batches, ...

- Have seen definition of “buckets” — stable phase space area
- Buckets can be occupied by “bunches” of particles
 - ▶ note: need not be — can have “empty buckets”
 - ▶ thus, can (in principle) adjust bunch spacing, bunch arrangements, etc.
- A set of bunches that are created in an accelerator (pulsed) is often called a Batch (especially if from a synchrotron)
 - ▶ can also be called a Bunch Train as well (especially if from a linac)

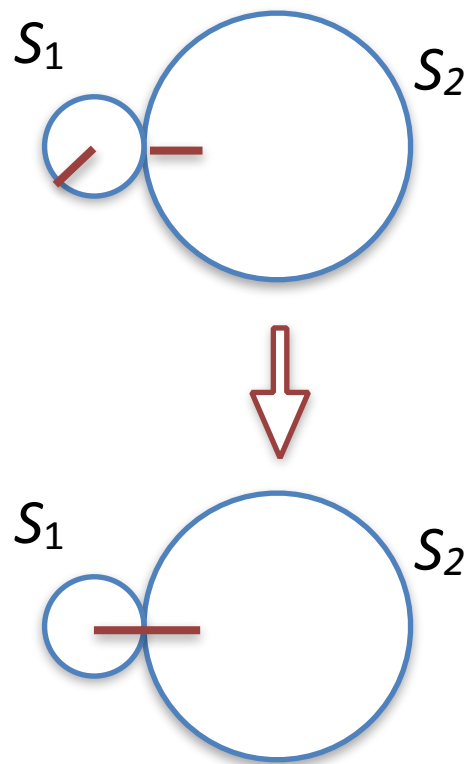
Bunch Manipulations in Synchrotrons

- Cogging
- Slip Stacking
- Bunch Rotation
- Bunch Coalescing
- Barrier Buckets



Cogging

- Essentially, phase slippage by changing the relative momentum
- Ex: beam transfers between two synchrotrons



Suppose $C_2 = 2C_1$; want to inject bunch in synchrotron S_1 into a particular “bucket” location in synchrotron S_2

need to adjust the revolution frequency of one ring (pick S_1 , say) until the two revolving “markers” line up

if $C_2 = 2C_1 \iff f_1 = 2f_2$, and may *never* line up!

So, make $\Delta\tau_1/\tau_1 = \eta \Delta p/p$ such that, after N turns,

$$N |\Delta\tau_1| = \Delta C_1/v$$

Cogging [2]

$$\longrightarrow \quad \frac{\Delta\tau}{\tau} = -\frac{\Delta f}{f} = -\frac{\Delta f_{\text{rf}}}{f_{\text{rf}}} = \eta \frac{\Delta p}{p}$$

Suppose want to “cog” beam by one RF bucket in $S_1 \dots$ then $\Delta C_1 = C_1/h$

adjust Δf_{RF} which yields $\Delta\tau_1$ each turn; leave on for N turns; $N = (\text{time between buckets})/\Delta\tau_1$

to cog by one bucket, $N |\Delta\tau_1| = 1/f_{\text{rf}} \Rightarrow N (\tau_1 \eta \Delta p/p) = 1/f_{\text{rf}} \Rightarrow N \Delta p/p = 1/(\tau_1 \eta h f_1)$

$$\text{or, } N \Delta p/p = 1/(\eta h)$$

Note: when generate an average $\Delta p/p$, the average horizontal displacement in the synchrotron at a particular position where there is **dispersion** will be $\Delta x = D \Delta p/p$.

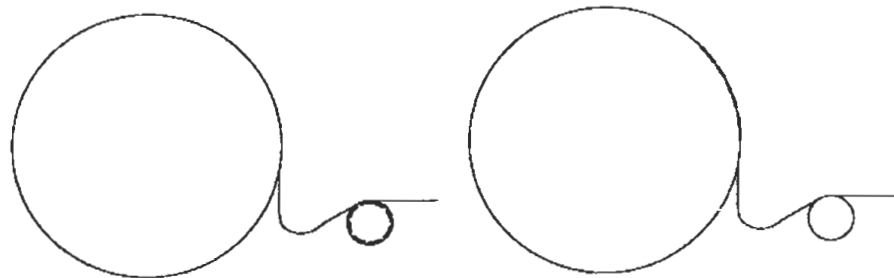
$$\text{Thus, } N \Delta x = D/(\eta h)$$

Ex: Suppose we can accommodate radial motion on the scale of 10 mm where the dispersion function has value 2.5 m in a synchrotron with $\eta = 0.05$ and $h=100$. Then, to cog by one RF bucket would take $N = (2.5 \text{ m} / 0.01 \text{ m}) / (0.05 * 100) = 50$ revolutions.

Ex: Slip Stacking (ex: FNAL Main Injector)

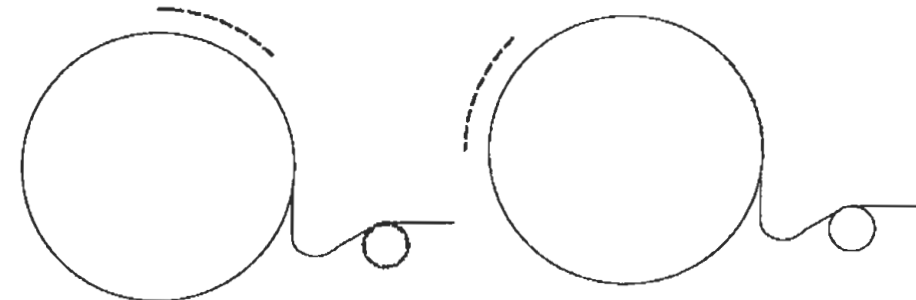
- Essentially “cogging” during injection
 - ▶ inject $\sim 1/2$ -circumference-worth of beam
 - accelerate slightly \rightarrow moves orbit outward
 - » (use RF system “A”, say)
 - ▶ inject 2nd batch into the ring, behind the first batch
 - decelerate slightly \rightarrow moves orbit inward
 - » (using RF system “B”, say)
 - ▶ Δp between these 2 orbits implies they will “slip” in time until they line up
 - ▶ re-capture with a higher voltage RF in order to match the bucket shape to the beam emittance

Slip Stacking cartoon (1)



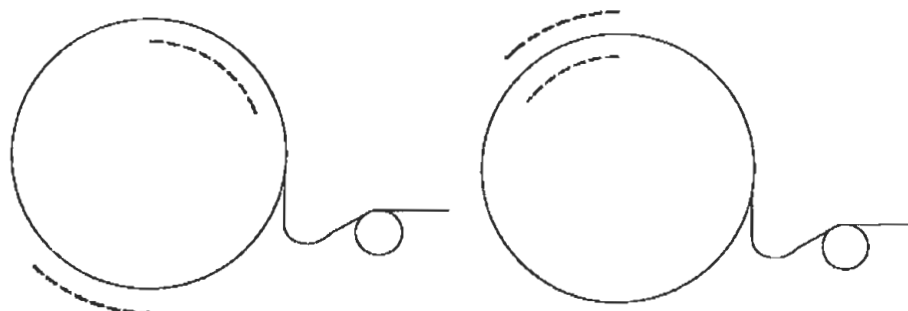
- First Booster Batch accelerated in Booster
- First Booster Batch injected onto MI central orbit with RF system A

Slip Stacking Cartoon (2)



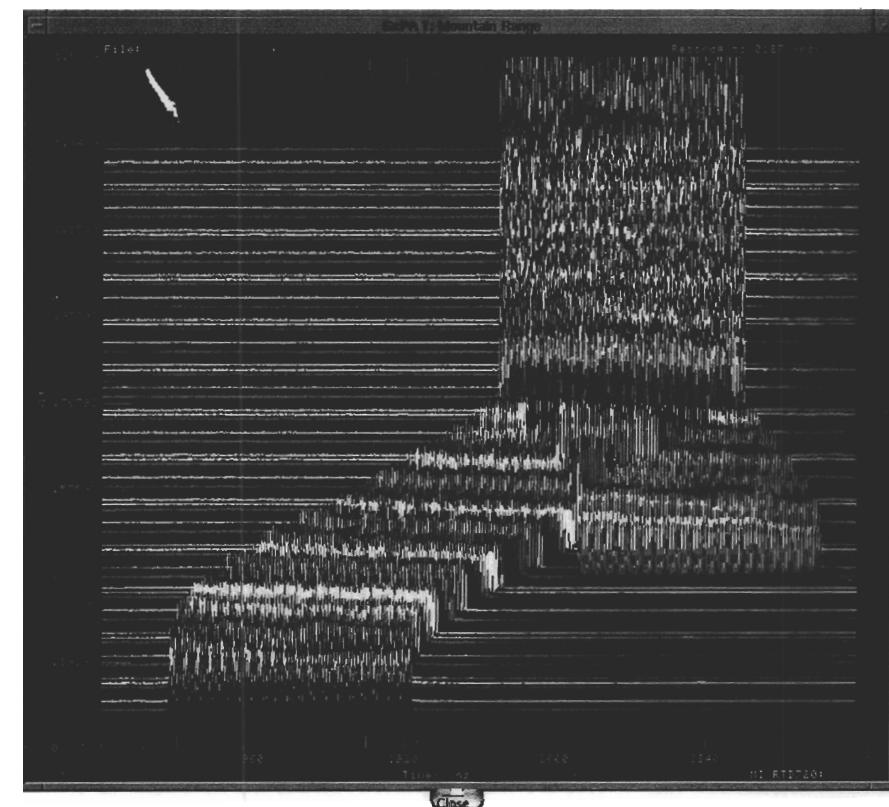
- First Booster Batch slightly accelerated in MI with RF System A
- Second Booster Batch injected onto MI central orbit with RF system B
- Second Booster Batch accelerated in Booster

Slip Stacking Cartoon (3)



- Second Booster Batch slightly decelerated in MI with RF System B
- Wait till batches line up and snap on RF system C while turning of RF systems A & B

Protons on Target, I. Kourbanis

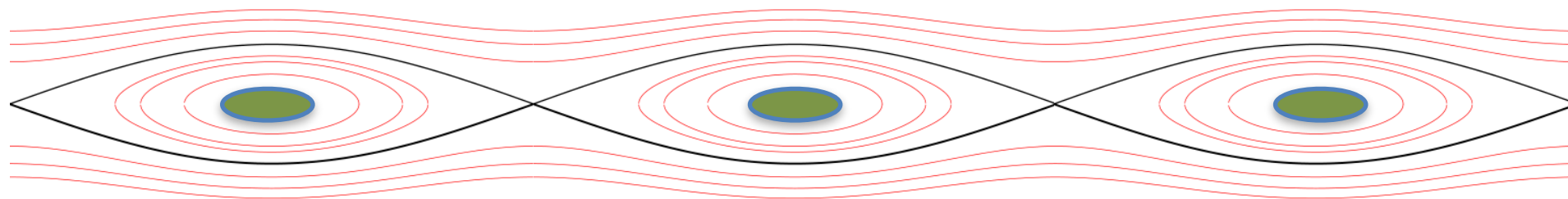


data

Protons on Target, I. Kourbanis

Bunch Rotation

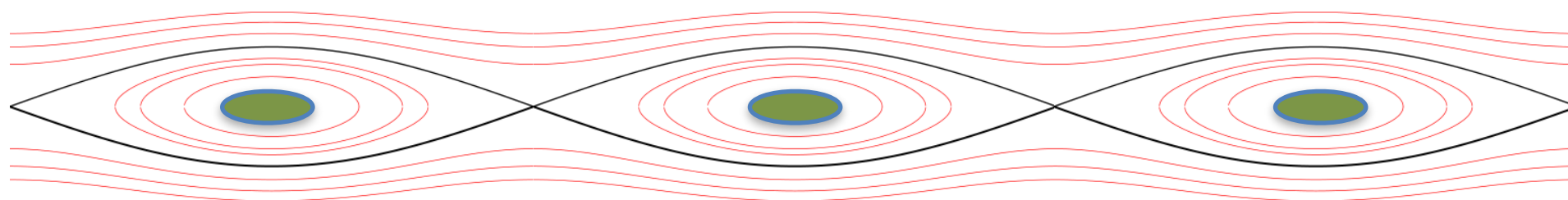
start:



MJS

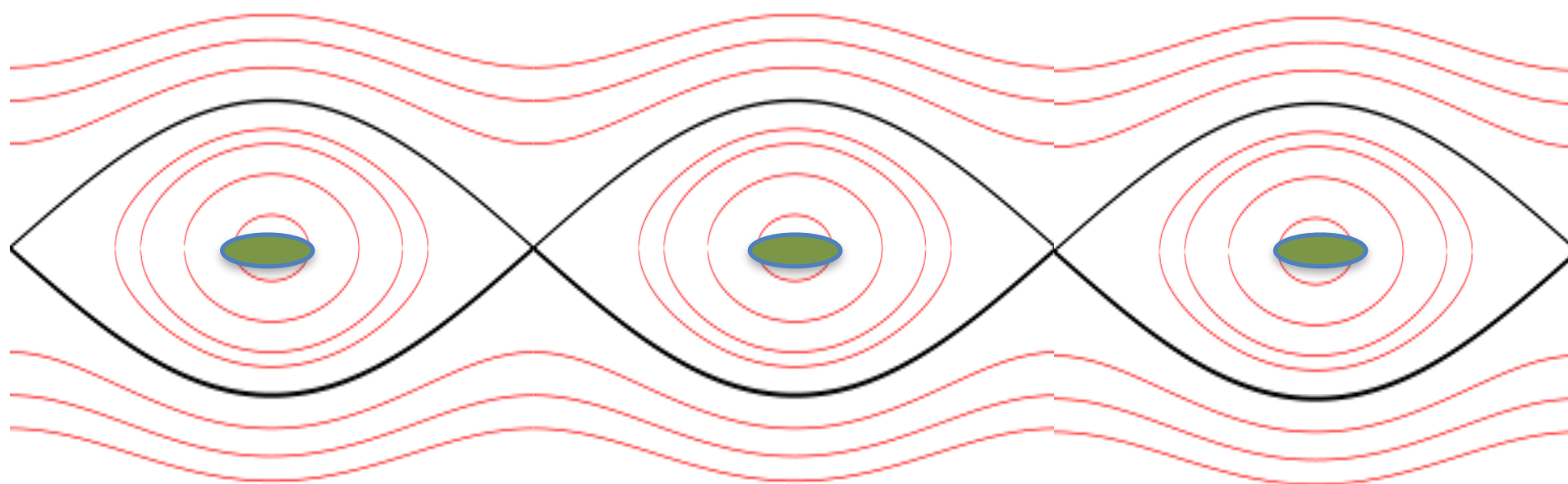
Bunch Rotation

start:



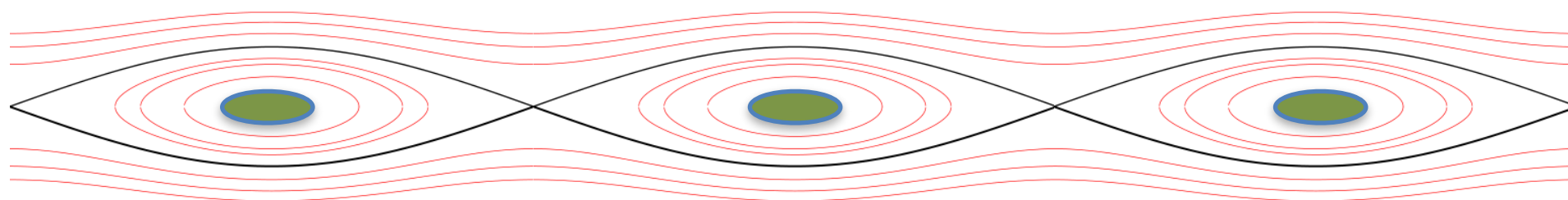
instantly raise RF voltage...

bunches will begin to rotate in phase space:



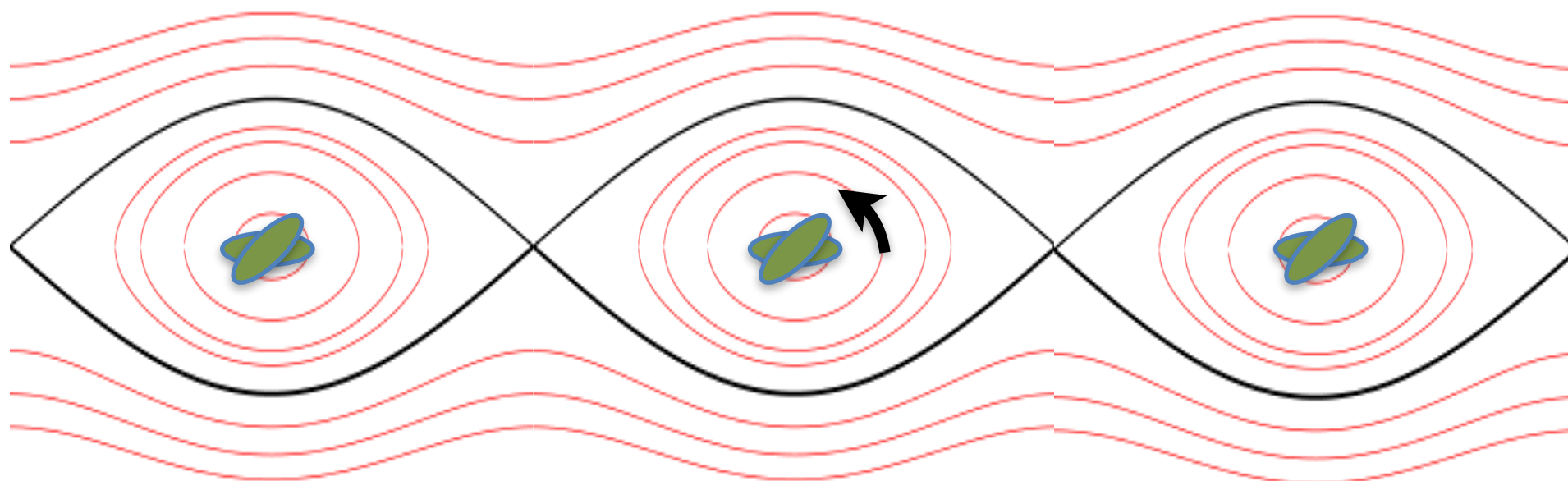
Bunch Rotation

start:



instantly raise RF voltage...

bunches will begin to rotate in phase space:

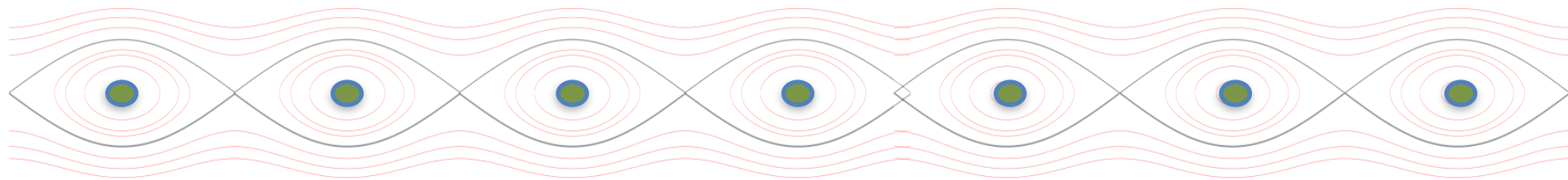


when rotated by 90° can rapidly switch to a higher-harmonic RF system in order to maintain the shorter bunch length; *or*, for example, extract the beam and send to a target!



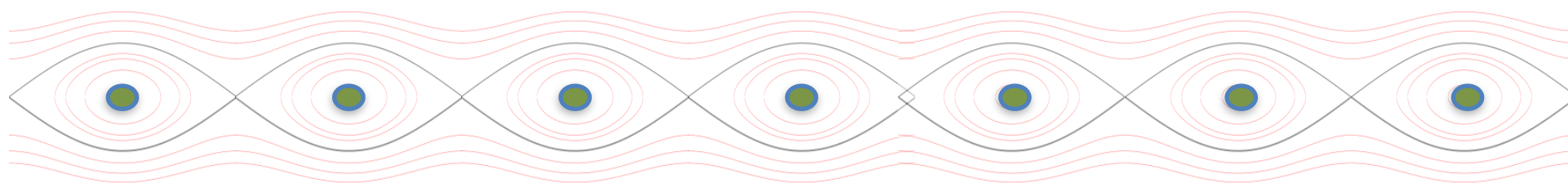
Bunch Coalescing

similar to bunch rotation, but also involves a change in RF frequency (harmonic)

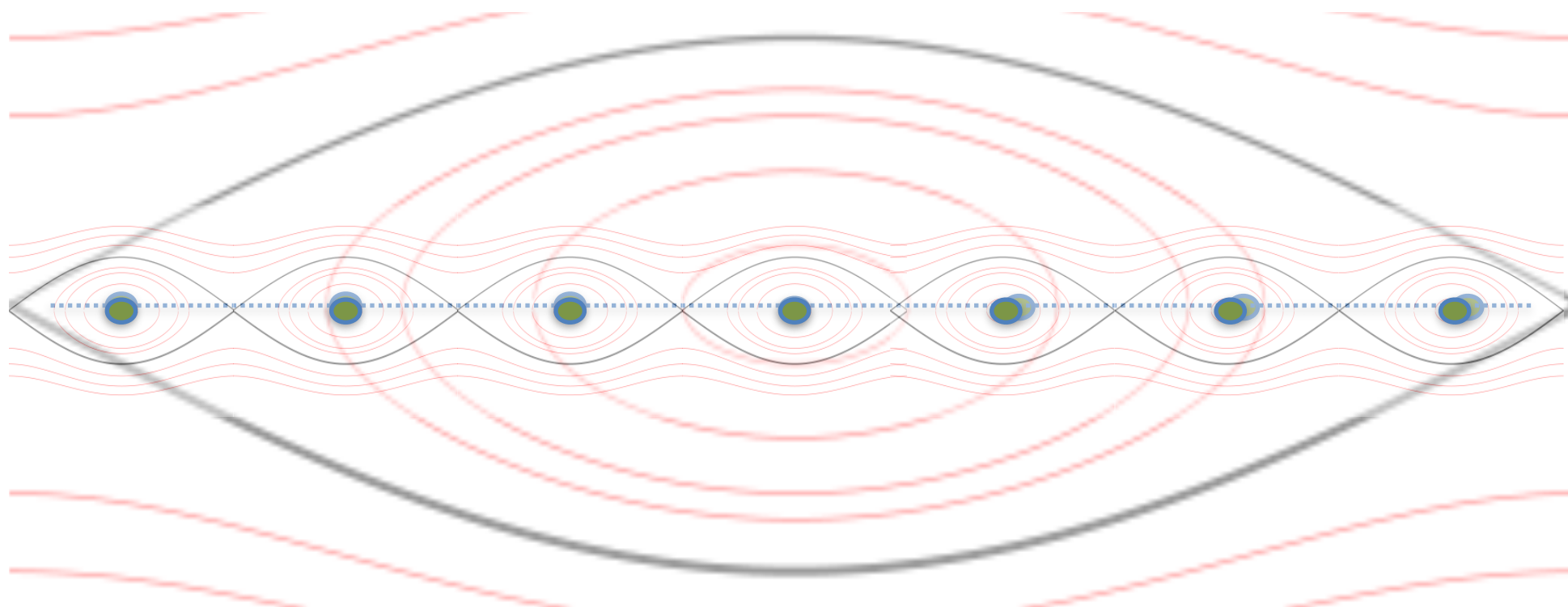


Bunch Coalescing

similar to bunch rotation, but also involves a change in RF frequency (harmonic)

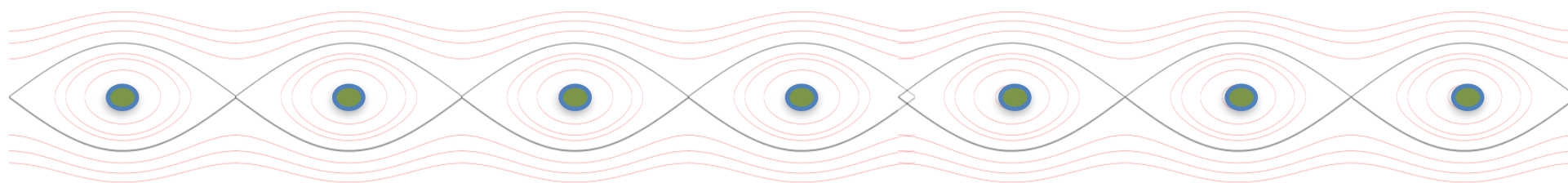


switch off high frequency, low voltage system,
switch on low frequency, high voltage system...

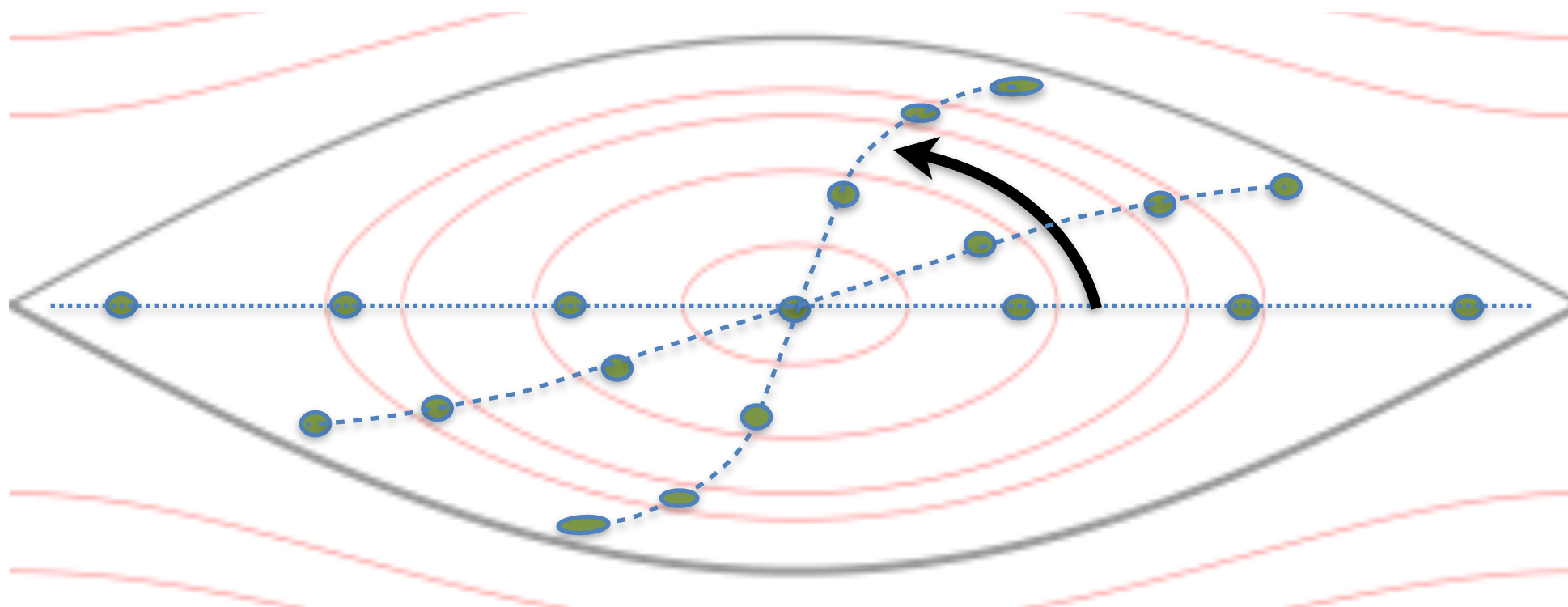


Bunch Coalescing

similar to bunch rotation, but also involves a change in RF frequency (harmonic)

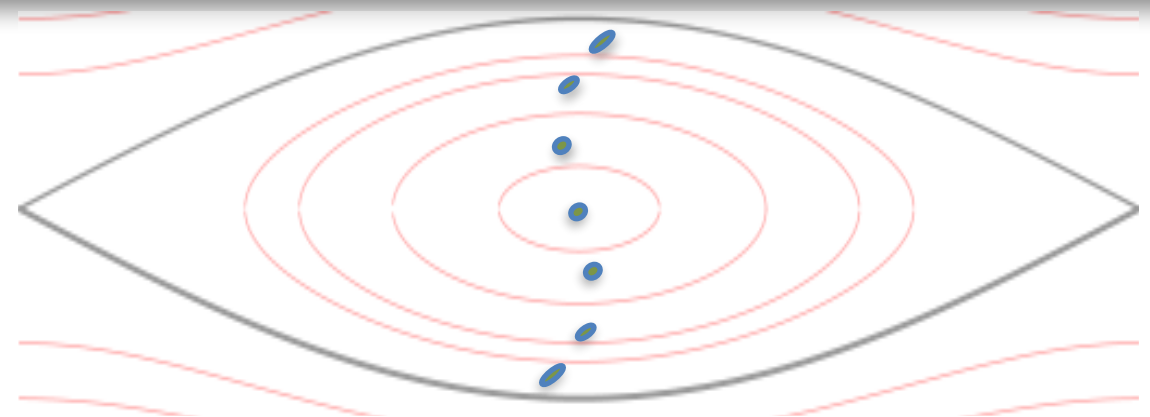


switch off high frequency, low voltage system,
switch on low frequency, high voltage system...

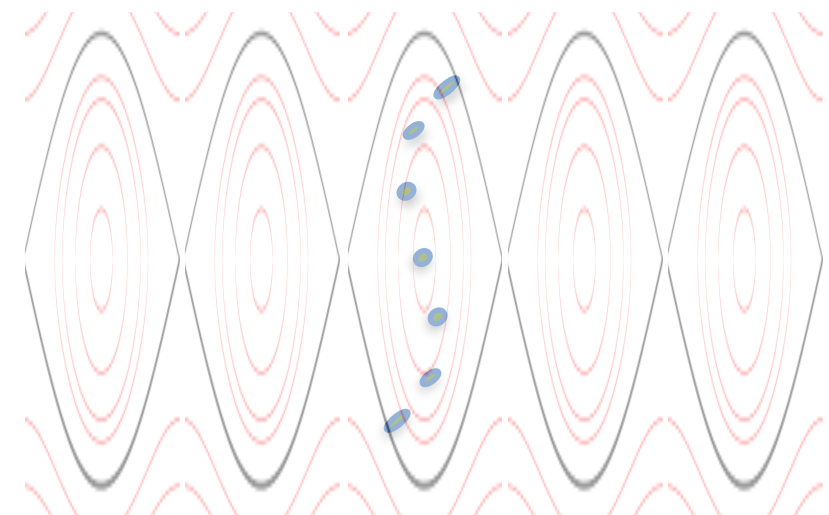


Bunch Coalescing [2]

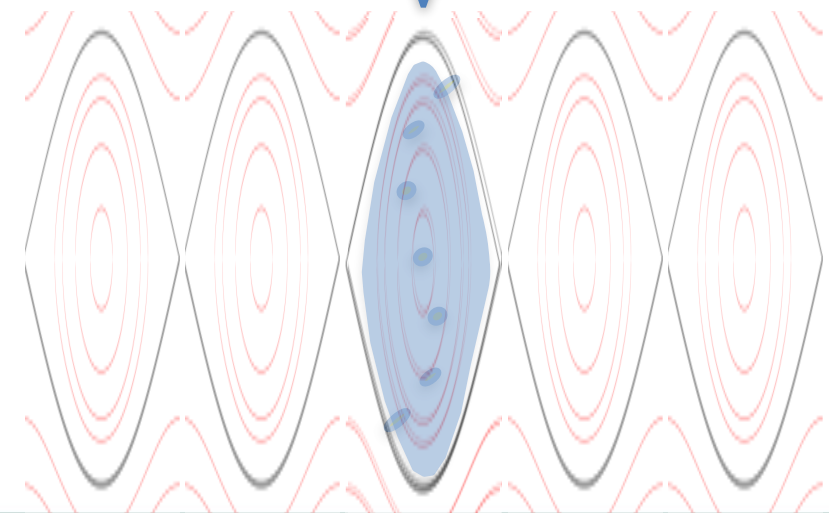
- Can use coalescing techniques to take bunched beam from one accelerator, make intense bunches, and inject into downstream accelerators to increase throughput of particles
 - downside: increased longitudinal emittance



then, recapture with the original harmonic system @ higher voltage



after dilution...



Barrier Bucket(s)

- Use a **pulsed** RF waveform to produce a “barrier” potential to contain (or exclude) beam in certain lengths of the circulating beam
- provides essentially DC beam, with a “gap” or gaps, which might be useful to provide time for kicker magnets to energize (injection, extraction) or for performing bunch compression to make room for incoming pulses of particles
- can adjust pulse separation, voltages adiabatically in order to control beam density, etc.

