Transition Radiation

- Transition radiation is emitted whenever a charged particle crosses the boundary between two media with different electrical properties.

- If we consider the case of an electron crossing a vacuum/perfect conductor interface.

- Then the problem can be treated as the collapsing of the electron with its image.

- Both particles are decelerated...
TR fluence I

• Start with the spectral fluence:

\[
\frac{d^2 I(\hat{n}, \omega)}{d\Omega d\omega} = \frac{q^2 \omega^2}{4\pi^2 c} \left| \int_{-\infty}^{+\infty} dt [\hat{n} \times (\hat{n} \times \vec{\beta})] e^{i\omega[t - \frac{\hat{n} \cdot \vec{r}(t)}{c}]} \right|^2
\]

• Let \( t=0 \) be the time corresponding to the charge hitting the boundary, so at \( t=0 \) the charge suddenly disappear.

\[
\frac{d^2 I}{d\Omega d\omega} = \frac{q^2 \omega^2}{4\pi^2 c} \left| \int_{-\infty}^{0} dt [\hat{n} \times (\hat{n} \times \vec{\beta})] e^{i\omega t'[1 - \hat{n} \cdot \vec{\beta}]} \right|^2
\]

\[
= \frac{q^2}{4\pi^2 c} \left| \begin{array}{c}
\hat{n} \times (\hat{n} \times \vec{\beta}) \\
1 - \hat{n} \cdot \vec{\beta}
\end{array} \right|^2
\]

\[
= \frac{q^2}{4\pi^2 c} \frac{\sin^2 \theta}{(1 - \beta \cos \theta)^2}.
\]

Angle between \( \beta \) and \( n \)
TR fluence II

• The spectral fluence seems independent of frequency!

• Physically impossible integrating over the frequency spectrum should be a \textit{finite} energy value

• Simple argument…
  – Another way of explaining transition radiation is to consider the e.m. fields associated to the moving charge
  – When the charge passes through the foil these field are “reflected”
  – Reflection impose the interface to be a good mirror, and this generally introduce a frequency dependence
  – For instance the X-ray components of the e.m. field will not be reflected. The typical cut-off frequency is the plasma frequency
  – A similar argument hold for the low frequency (diffraction!)
TR fluence III

• In the relativistic and small angle approximation

\[
\sin \theta \simeq \theta, \cos \theta \simeq 1 - \theta^2/2 \text{ and } \beta \simeq 1 - 1/(2\gamma^2)
\]

• the fluence simplifies to

\[
\frac{d^2 I}{d\Omega d\omega} = \frac{q^2 \gamma^2}{\pi^2 c} \frac{(\gamma \theta)^2}{[1 + (\gamma \theta)^2]^2}.
\]

• So the angular distribution is peaked at \(\theta = \pm 1/\gamma\)
Angular distribution of TR fluence
Forward and backward TR

- We consider the case of the particle which suddenly disappears this gives the forward transition radiation.

- Considering the particle which suddenly appears give the backward transition radiation.
Angular distribution of TR (polar plots)

Figure 5.9: Polar plot of the normalized radiation pattern for an aluminum foil with an electron under normal incidence (i.e. what we derived in the text) (A) and with a 45 deg incidence (B) (not derived here). For these plots the Lorentz factor was chosen to be \( \gamma = 10 \) for clarity of the figure.
Angle integrated TR

• To compute the total energy radiated per unit of frequency, we just need to evaluate the integral over the solid angle

\[
\frac{dI}{d\omega} = \frac{q^2}{4\pi^2 c} \int_0^\pi \frac{\sin^3 \theta}{(1 - \beta \cos \theta)^2} d\theta,
\]

\[
= \frac{q^2}{2\pi c} \int_0^\pi \frac{\sin^3 \theta}{(1 - \beta \cos \theta)^2} d\theta
\]

\[
= \frac{q^2}{2\pi c \beta^3} \left[ \frac{\beta^2 - 1}{\beta \cos x - 1} + \beta - 2 \ln[\beta \cos x - 1] \right]_0^\pi
\]

\[
= \frac{q^2}{\pi c \beta^3} \left[ \ln \left( \frac{1 + \beta}{1 - \beta} \right) - 2\beta \right]
\]

• In the relativistic limit

\[
\frac{dI}{d\omega} = \frac{q^2}{2\pi c} \left( \ln 2 - 1 + \ln \gamma \right)
\]
Angle integrated TR

(A) Fraction of TR energy within $\theta$

(B) Fraction of TR within $1/\gamma$ cone

$E=10$ MeV, $E=42$ MeV, $E=800$ MeV, $E=4000$ MeV

Energy (MeV)
Example of use of Optical TR

• Angular distribution of transition radiation can be used to infer some of a charged particle beam properties:
  – Energy
  – Divergence

Figure 3: Experimental picture of the OTR polarisation. The beam energy is 3.8 MeV

Give the energy
Example of use of Optical TR II

Figure 3: Image of the beam on the OTR screen (a) and its projection (b).

Figure 4: OTR angular distribution: a) image with background, b) measured and simulated projection.
Example of use of Coherent TR

- CTR can be used to measure the time distribution of charged particle beam


FIG. 1. (Color) Michelson interferometer. Detector 1 is used to record the autocorrelation function. Detector 2 is a reference detector used to normalize the autocorrelation function.