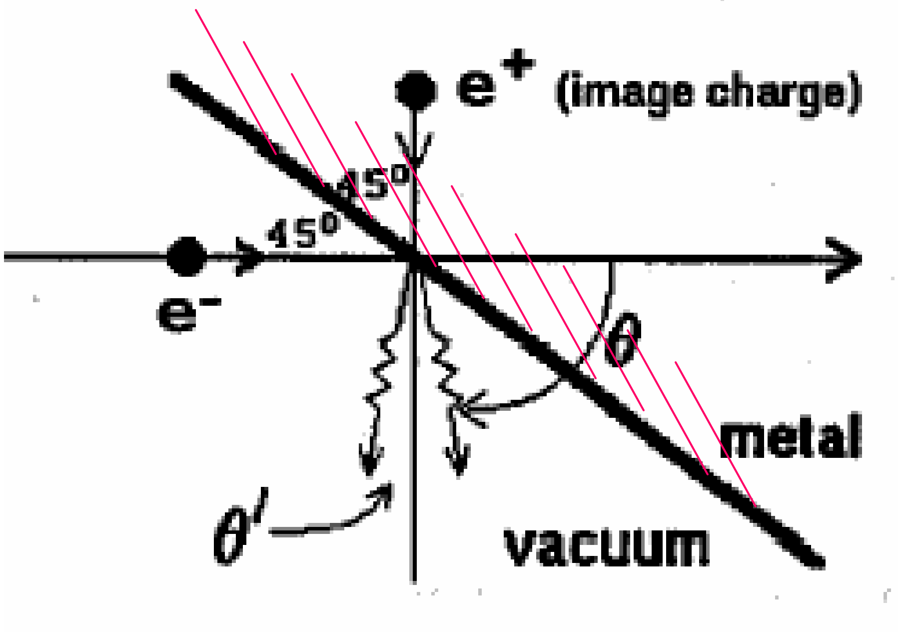
Transition Radiation

- Transition radiation is emitted whenever a charged particle cross the boundary between two media with different electrical properties

- If we consider the case of an electron crossing a vacuum/ perfect conductor interface

- Then the problem can be treated as the collapsing of the electron with its image

- Both particle are decelerated…



TR fluence I

- Start with the spectral fluence:

$$\frac{d^2I(\hat{n},\omega)}{d\Omega d\omega} = \frac{q^2\omega^2}{4\pi^2 c}\left| \int_{-\infty}^{+\infty} dt[\hat{n} \times (\hat{n} \times \overrightarrow{\beta})]e^{i\omega[t' - \frac{\hat{n}.\overrightarrow{r}(t)}{c}]} \right|^2$$

- Let t=0 be the time corresponding to the charge hitting the boundary, so at t=0 the charge suddenly disappear.

$$\begin{aligned}
\frac{d^2I}{d\Omega d\omega} &= \frac{q^2\omega^2}{4\pi^2 c}\left| \int_{-\infty}^{0} dt[\hat{n} \times (\hat{n} \times \overrightarrow{\beta})]e^{i\omega t'[1-\hat{n}.\overrightarrow{\beta}]} \right|^2 \\
&= \frac{q^2}{4\pi^2 c}\left| \frac{\hat{n} \times (\hat{n} \times \overrightarrow{\beta})}{1-\hat{n}.\overrightarrow{\beta}} \right|^2 \\
&= \frac{q^2}{4\pi^2 c}\frac{\sin^2\theta}{(1-\beta\cos\theta)^2}.
\end{aligned}$$

*Angle between β and **n***

TR fluence II

- The spectral fluence seems independent of frequency!
- Physically impossible integrating over the frequency spectrum should be a **finite** energy value
- Simple argument…
  - Another way of explaining transition radiation is to consider the e.m. fields associated to the moving charge
  - When the charge passes through the foil these field are "reflected"
  - Reflection impose the interface to be a good mirror, and this generally introduce a frequency dependence
  - For instance the X-ray components of the e.m. field will not be reflected. The typical cut-off frequency is the plasma frequency
  - A similar argument hold for the low frequency (diffraction!)

TR fluence III

- In the relativistic and small angle approximation

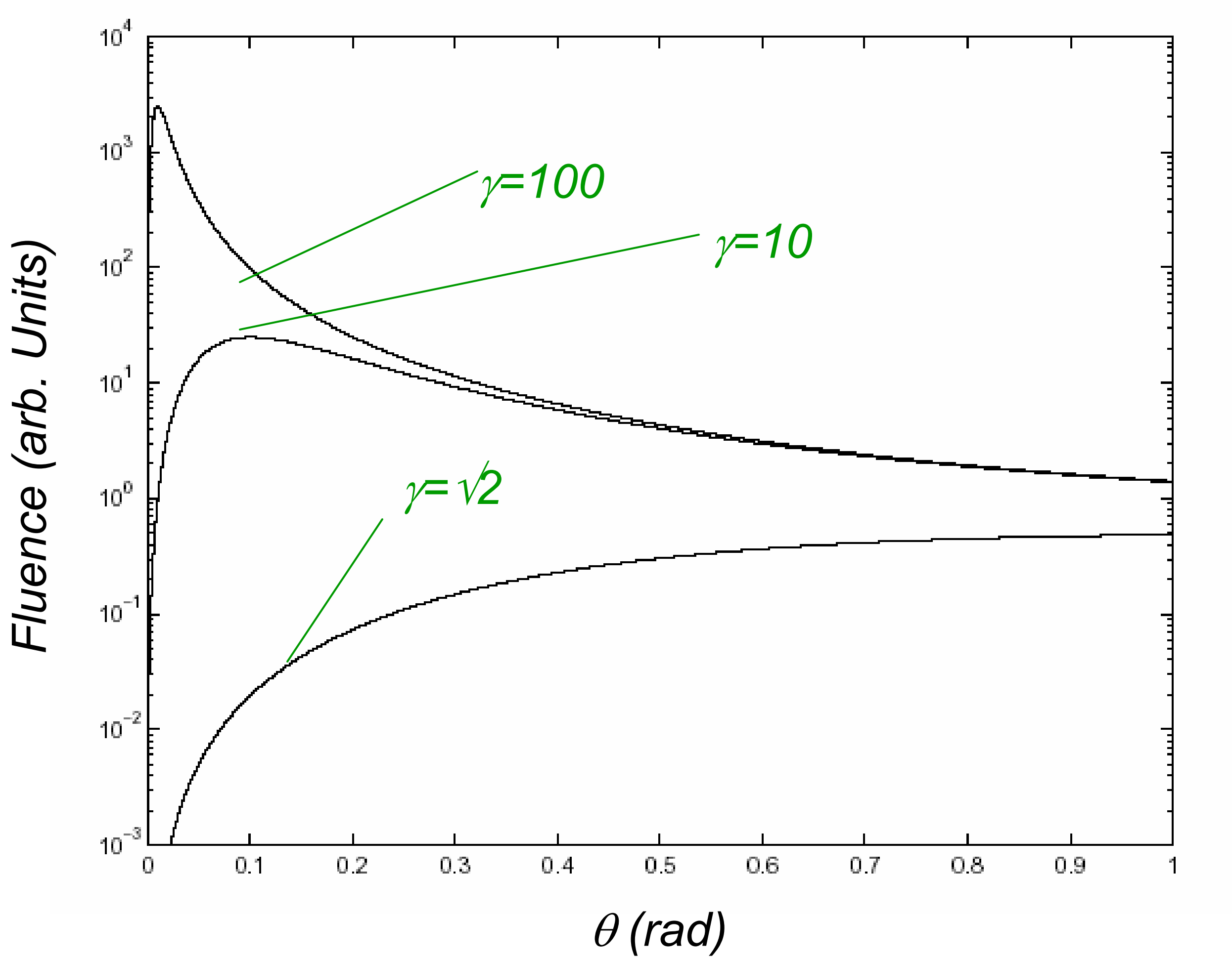
$$\sin\theta \simeq \theta, \cos\theta \simeq 1-\theta^2/2 \text{ and } \beta \simeq 1-1/(2\gamma^2)$$

- the fluence simplifies to

$$\frac{d^2 I}{d\Omega d\omega} = \frac{q^2\gamma^2}{\pi^2 c}\frac{(\gamma\theta)^2}{[1+(\gamma\theta)^2]^2}.$$

- So the angular distribution is peaked at $\theta=\pm 1/\gamma$

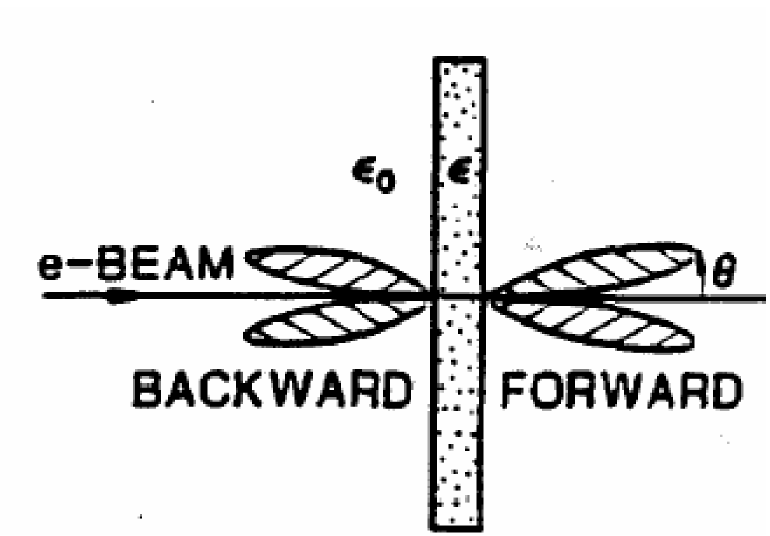
Angular distribution of TR fluence



Forward and backward TR

- We consider the case of the particle which suddenly disappears this gives the forward transition radiation

- Considering the particle which suddenly appears give the backward transition radiation



Angular distribution of TR (polar plots)

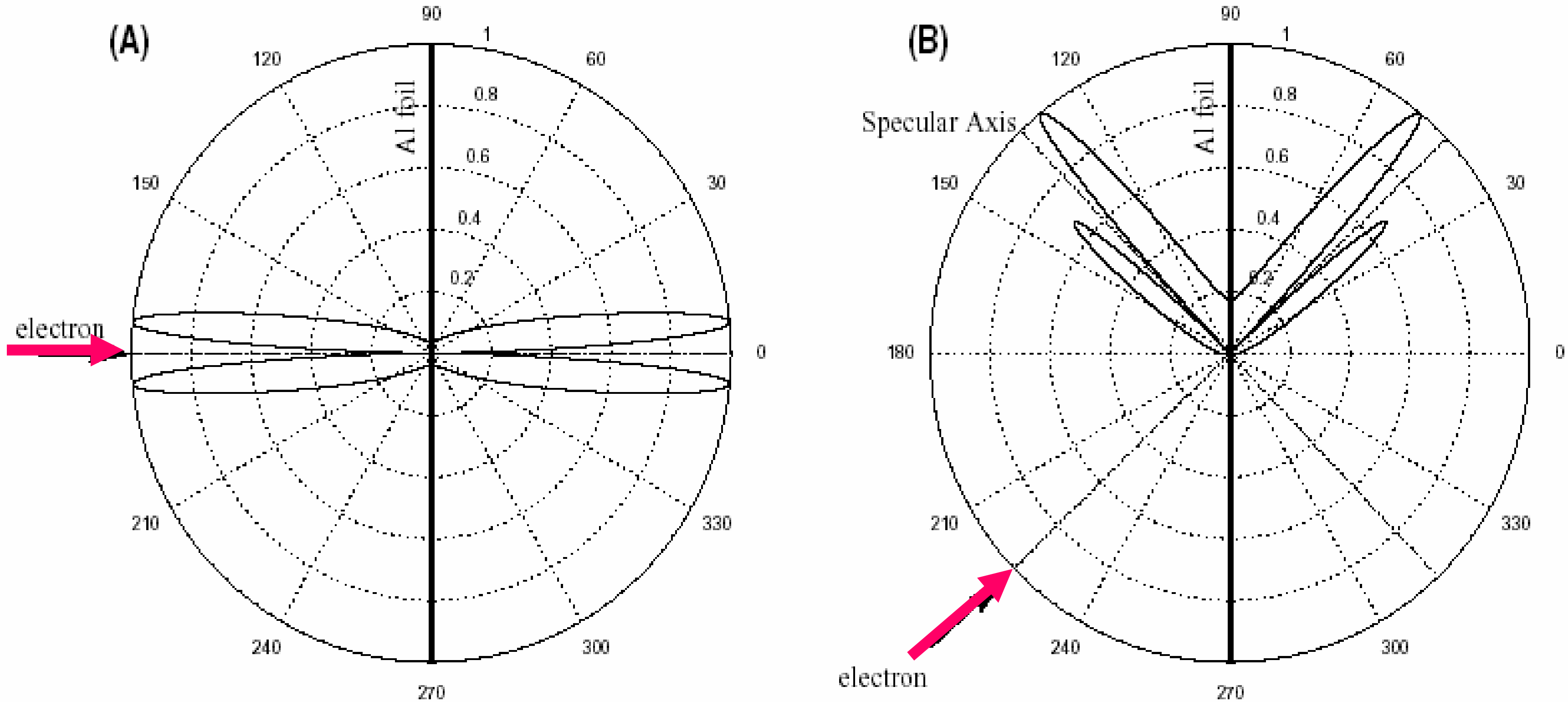


Figure 5.9: Polar plot of the normalized radiation pattern for an aluminum foil with an electron under normal incidence (i.e. what we derived in the text) (**A**) and with a 45 deg incidence (**B**) (not derived here). For these plots the Lorentz factor was chosen to be $\gamma = 10$ for clarity of the figure.

Angle integrated TR

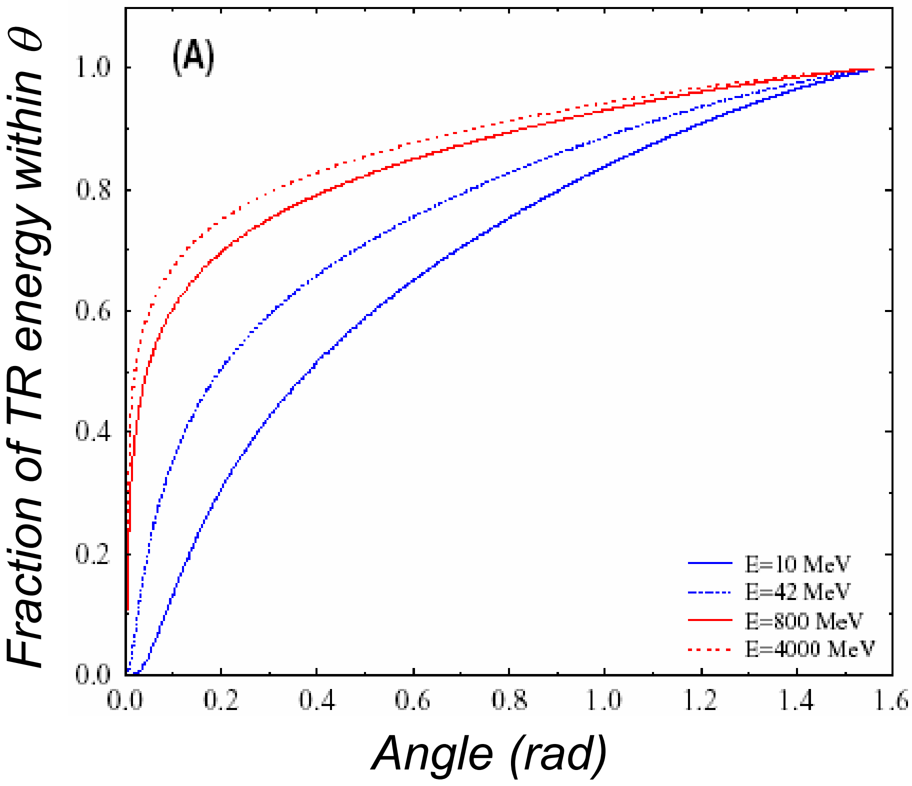
- To compute the total energy radiated per unit of frequency, we just need to evaluate the integral over the solid angle

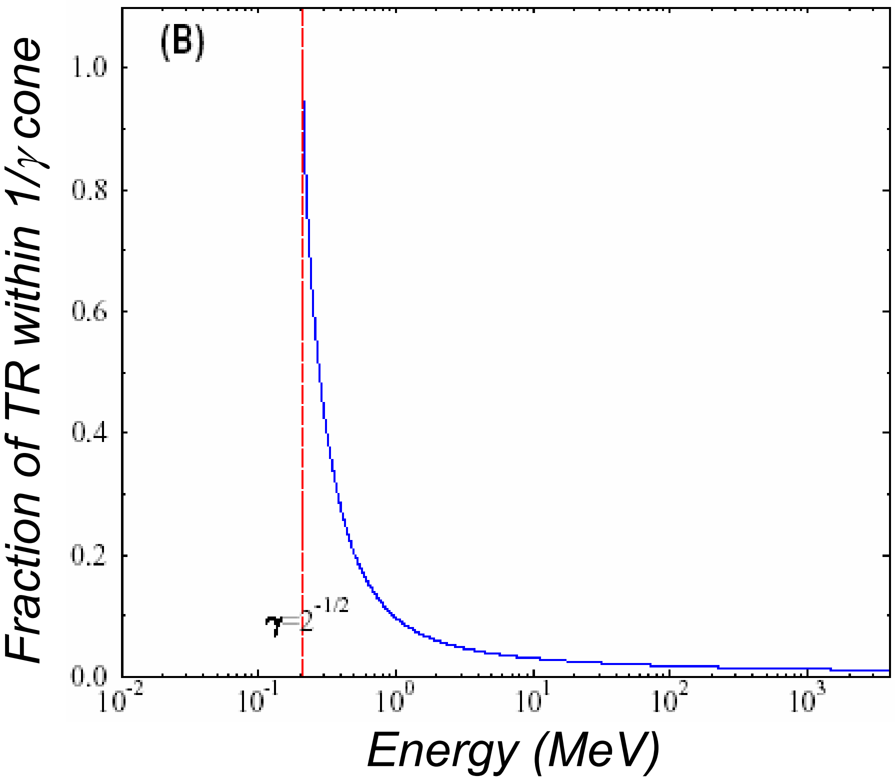
$$
\begin{aligned}
\frac{dI}{d\omega} &= \frac{q^2}{4\pi^2 c} 2\pi \int_0^{\pi} \frac{\sin^3\theta}{(1-\beta\cos\theta)^2} d\theta, \\
&= \frac{q^2}{2\pi c} \int_0^{\pi} \frac{\sin^3\theta}{(1-\beta\cos\theta)^2} d\theta \\
&= \frac{q^2}{2\pi c \beta^3} \left[\frac{\beta^2 - 1}{\beta\cos x - 1} + \beta - 2\ln[\beta\cos x - 1] \right]_0^{\pi} \\
&= \frac{q^2}{\pi c \beta^3} \left[\ln\left(\frac{1+\beta}{1-\beta}\right) - 2\beta \right]
\end{aligned}
$$

- In the relativistic limit

$$
\frac{dI}{d\omega} = \frac{q^2}{2\pi c} (\ln 2 - 1 + \ln\gamma)
$$

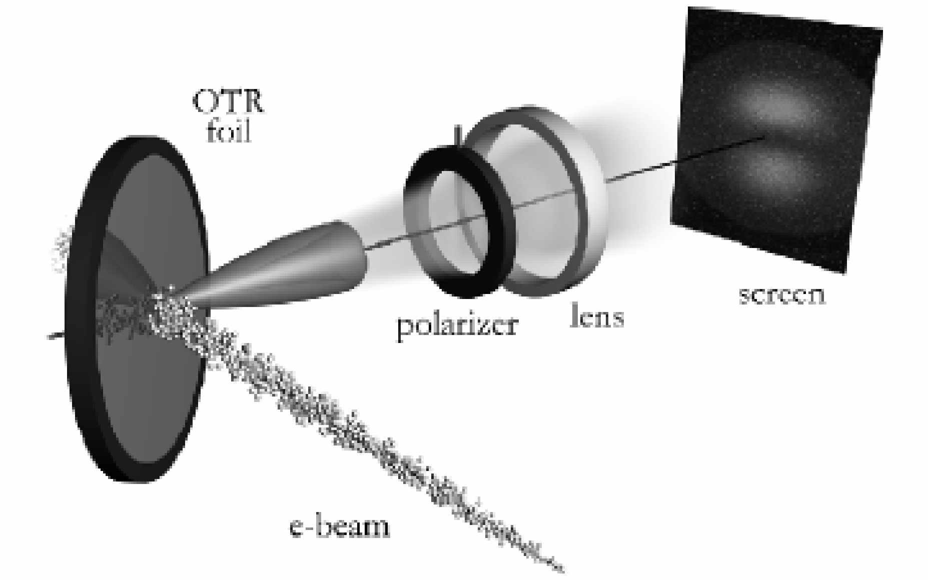
Angle integrated TR

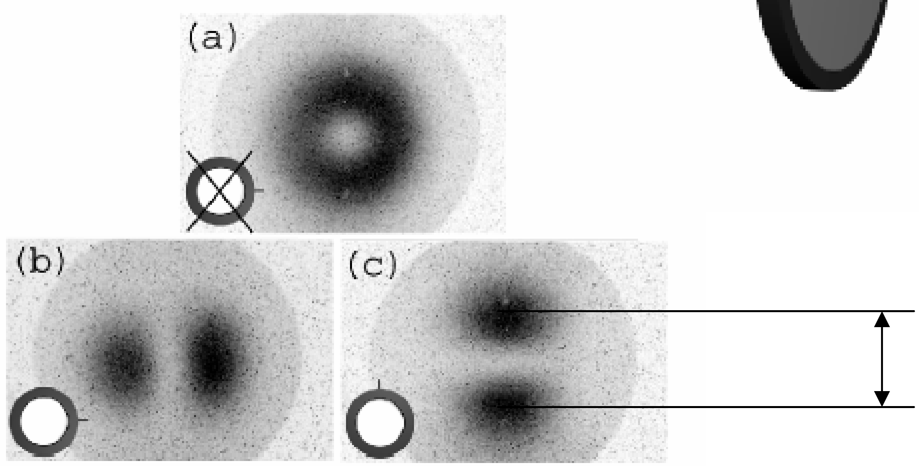




Example of use of Optical TR I

- Angular distribution of transition radiation can be used to infer some of a charged particle beam properties:
  - Energy
  - divergence

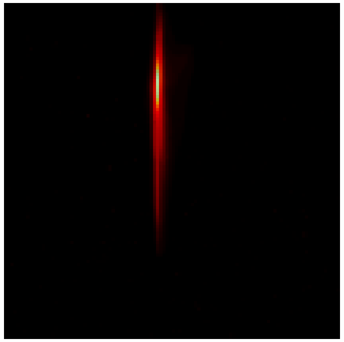




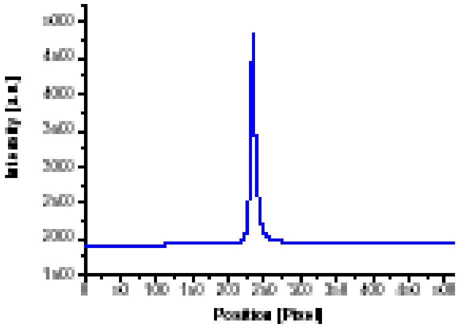
Give the energy

Figure 3: Experimental picture of the OTR polarisation. The beam energy is 3.8 MeV

Example of use of Optical TR II



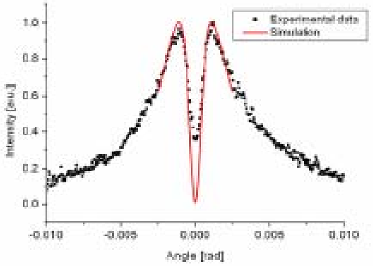
(a) Beam image



(b) Beam projection

Figure 3: Image of the beam on the OTR screen (a) and its projection (b).

(a) OTR angular distribution and background



(b) OTR projection

Figure 4: OTR angular distribution: a) image with background, b) measured and simulated projection.

Example of use of Coherent TR

- CTR can be used to measure the time distribution of charged particle beam

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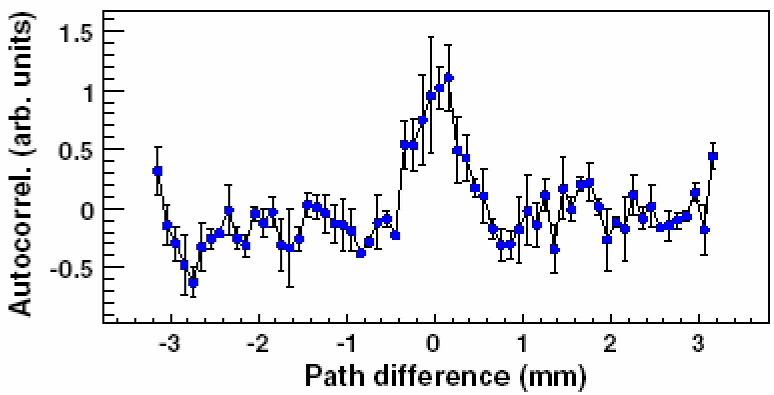

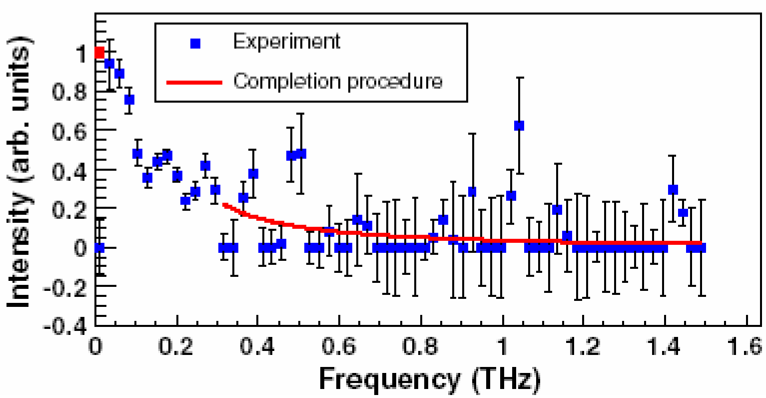

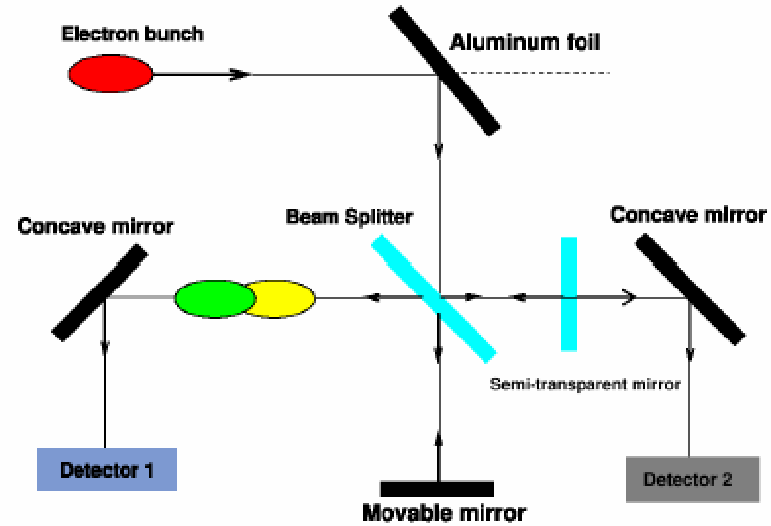

FIG. 1. (Color) Michelson interferometer. Detector 1 is used to record the autocorrelation function. Detector 2 is a reference detector used to normalize the autocorrelation function.