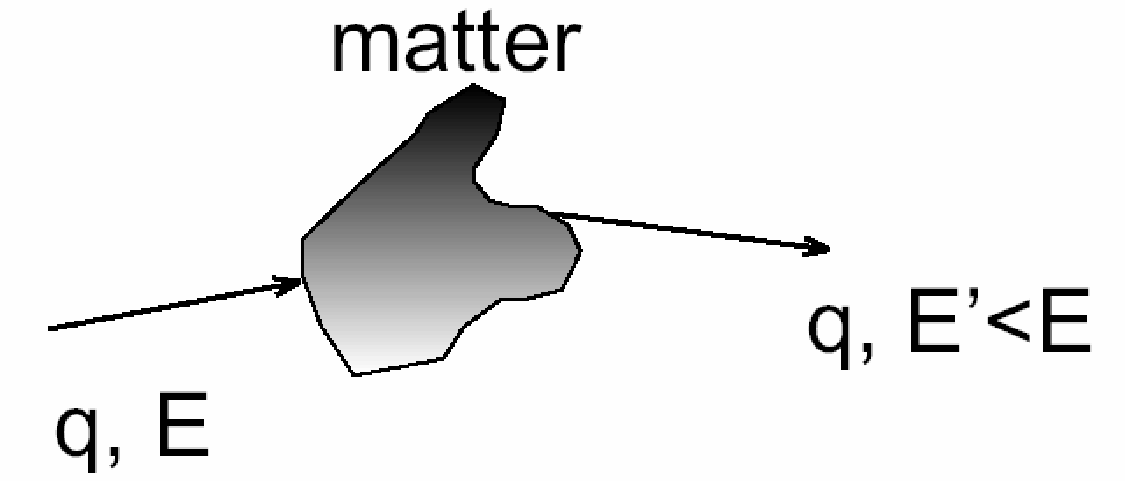
Scattering - Introduction

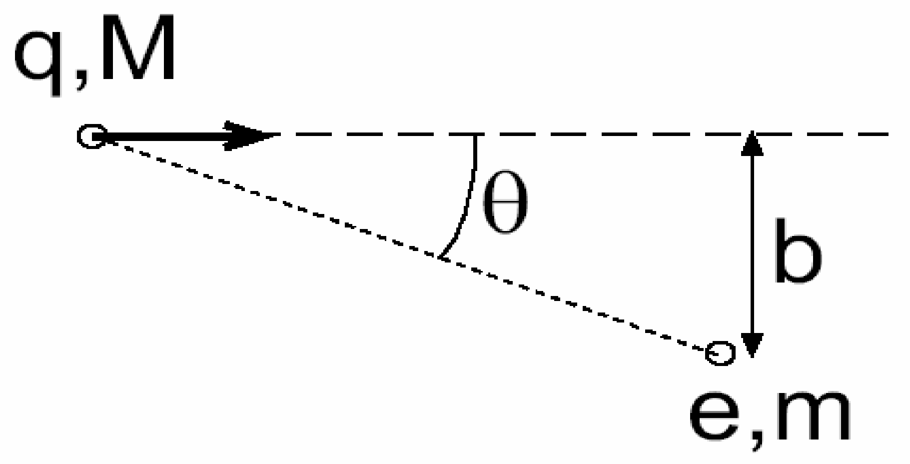
- We will consider two types of scattering
  - Scattering on electron (q=-e, me=9.110-31 kg) which results in high energy losses but small deflections
  - Scattering on nuclei (q=Ze, mn>>me) which are associated to low energy losses but large deflections

- Naively, since matter is composed of much more electron that nuclei (a factor Z), we may conjecture that electron scattering is the dominant type of scatter



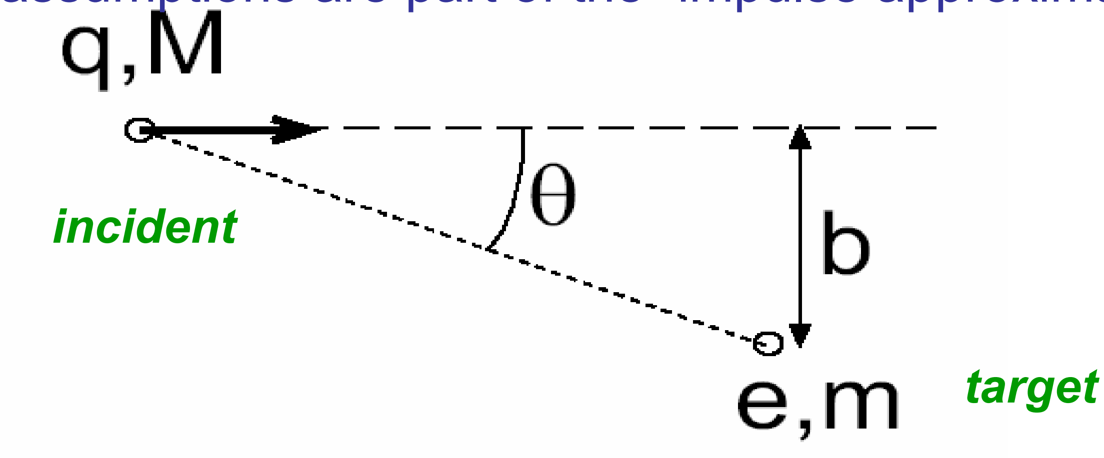
Energy transfer

- We now are going to compute the energy transfer between two particle during scattering.
- The technique is imply to consider the “matter” particle at rest and the particle to be scattered moving and penetrating the matter block.
- Scattering is not a point like collision, it occur via long-range electromagnetic interaction (considering the e.m. field of the moving particle)



Energy transfer: the impulse approximation

- Calculation of energy transfer in the most general case can be tedious,
- So we make some simplifying assumptions:
  - Incident particle is NOT locally deflected by collision (rather a momentum kick is imparted and as the particle drifts away might be deflected)
  - Target particle is stationary during collision
- These two assumptions are part of the "impulse approximation" (IA)



Energy transfer: the impulse approximation

- The E-field generated by the incident particle at the location of the target particle is

$$\overrightarrow{E}(x=-b, y=z=0,t) = -\gamma q \frac{b\hat{x}+vt\hat{z}}{(b^2+\gamma^2v^2t^2)^{3/2}}$$

- The momentum transfer from *q* to *e* is

$$\begin{aligned}
\Delta\overrightarrow{p} &= \int_{-\infty}^{+\infty} dt e\overrightarrow{E} = -qe\gamma\int_{-\infty}^{+\infty} dt\frac{b\hat{x}+vt\hat{z}}{(b^2+\gamma^2v^2t^2)^{3/2}}, \\
&= -qe\gamma\int_{-\infty}^{+\infty} dt\frac{b\hat{x}}{(b^2+\gamma^2v^2t^2)^{3/2}} = \frac{-qe}{vb}\hat{x}\int_{-\infty}^{+\infty}\frac{du}{(1+u^2)^{3/2}}, \\
&= \frac{-qe}{vb}\hat{x}\left[\frac{u}{\sqrt{1+u^2}}\right]_{-\infty}^{+\infty} = -\frac{2qe}{vb}\hat{x}.
\end{aligned}$$

Energy transfer: the impulse approximation

- The associated kinetic energy change

NR

$$\Delta T = \sqrt{(\Delta p_e c)^2 + (mc^2)^2} - mc^2 \simeq \frac{2}{m}\left(\frac{qe}{vb}\right)^2$$

- The electrons $\Delta T_e \equiv \Delta T \propto \frac{e^2}{m}.$

- For nuclei $\Delta T_n \propto \frac{q_n^2}{m_n} \propto \frac{Z^2 e^2}{m_n}.$

- So we have

$$\frac{\Delta T_n}{\Delta T_e} = \left(\frac{q_n}{e}\right)^2 \frac{m_e}{m_n} = Z^2 \frac{m_e}{m_n} = \frac{Z}{1836} \ll 1.$$

Energy transfer: the impulse approximation

- Let's what are the implication of the IA
  - Deflection angle is given by

$$\theta = \frac{\Delta p_e}{\gamma M v} = \frac{2qe}{\gamma M v^2 b} = \frac{2}{\gamma}\frac{qe/b}{M v^2} = 2\frac{\text{E electrostat.}}{\text{incident Energy}} = 2\frac{\mathcal{V}}{E}.$$

so $\theta \ll 1 \Rightarrow \mathcal{V} \ll E.$

  - Target is stationary means that recoil of the target during collision is much smaller than impact parameter: *d<<b*
    - Interaction time during collision given by $\tau \sim \frac{b}{\gamma v}$,
    - The corresponding recoil is $d \sim \frac{\Delta p_e}{m}\tau$

$$d \ll b \Rightarrow \frac{\Delta p_e}{m}\frac{b}{\gamma v} \ll 1 \Rightarrow \frac{2qe/b}{\gamma m v^2} \ll 1.$$

Energy transfer: the impulse approximation

- This is a stronger condition than the small deflection angle condition [by a factor M/m], so if the latter condition is fulfilled then the 1st condition is fulfilled and IA is legitimate

- So for IA to be valid we need

$$\frac{2}{\gamma}\frac{q}{e}\frac{e^2/(mc^2)}{b}\frac{c^2}{v^2} \ll 1$$

- Which can also be written (β<1)

$$\boxed{\frac{2}{\beta\gamma}\frac{q}{e}\frac{r_e}{b} \ll 1}$$

NR approximation

- NR approximation implies

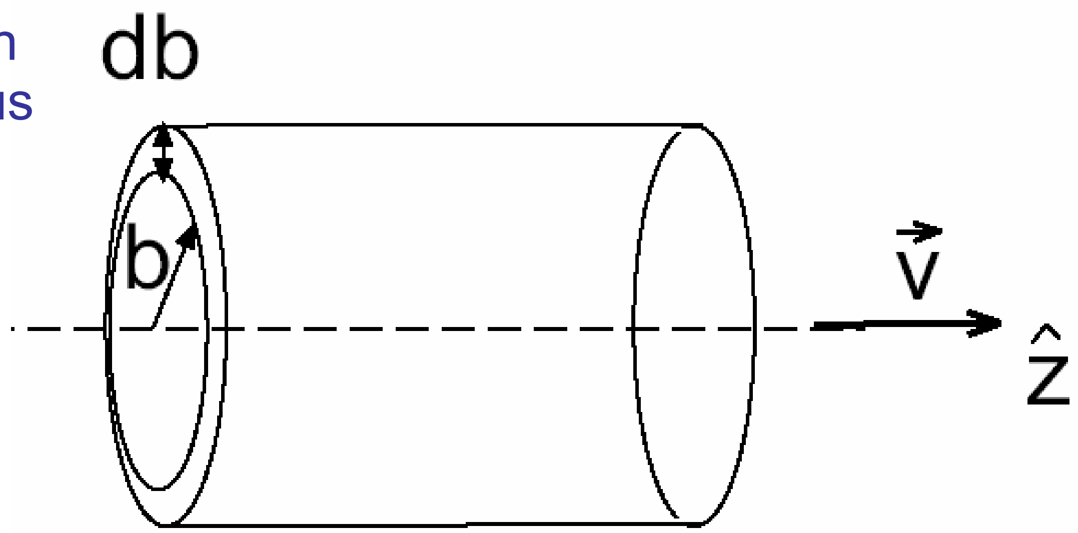
$$\frac{\Delta p_e}{mc} \ll 1 \Rightarrow \frac{2qe}{mvcb} \ll 1 \Rightarrow 2\frac{q}{e}\frac{e^2/(mc^2)}{\beta b} \ll 1$$
$$\Rightarrow \boxed{\frac{2}{\beta}\frac{q}{e}\frac{r_e}{b} \ll 1.}$$

- This is the SAME condition as for IA to be valid but just with $\gamma \rightarrow 1$

Passage through a bulk of matter

- Now we generalize our treatment to the case of a particle passing through a bulk of matter (many electrons).
- We associate a electronic density n_e to this block of matter
- The total number of electron in a cylindrical shell or radius *b* is

$$N_e = n_e(vdt)(2\pi b db)$$



Passage through a bulk of matter

- The differential energy loss by the charge *q* is

$$\frac{d^2T_q}{dtdb} = -2\pi n_e v b \left[\frac{2}{m}\left(\frac{qe}{bv}\right)^2\right]$$

- Integrate over *b*

$$\begin{aligned}\frac{dT_q}{dt} &= -4\pi n_e \frac{(qe)^2}{mv}\int_{b_{min}}^{b_{max}} \frac{db}{b} \\ &= -4\pi n_e \frac{(qe)^2}{mv} \ln \frac{b_{max}}{b_{max}}.\end{aligned}$$

Passage through a bulk of matter

- Limit of the integral
  - When *b* goes to zero IA is no more valid so we must limit our integral to values such that

$$\frac{2}{\beta^2\gamma}\frac{q}{e}\frac{r_e}{b} \ll 1$$

that is $b_{min} \equiv \frac{2}{\beta^2\gamma}\frac{q}{e}r_e = \frac{qe}{\gamma m v^2}.$

  - When *b* goes to infinity the stationary condition breaks: electron orbit with an angular frequency $\omega_e = E_e/h.$ so we must make sure $\tau \ll \omega_0^{-1}$

$$\tau \sim \frac{b}{\gamma v};\ \frac{b_{max}}{\gamma v} = \frac{1}{\omega_0}$$
$$\Rightarrow b_{max} = \frac{\gamma v}{\omega_0}.$$

Passage through a bulk of matter

- So finally

$$\frac{dT_q}{dt} = -4\pi n_e \frac{(qe)^2}{mv} \ln \frac{\frac{\gamma v}{\omega_0}}{\frac{qe}{\gamma m v^2}} = -4\pi n_e \frac{(qe)^2}{mv} \ln \frac{\gamma v}{\omega_0 b_{min}},$$
$$= -4\pi n_e \frac{(qe)^2}{mv} \ln \frac{\gamma^2 m v^3}{qe\omega_0}$$

- Since $d\Delta T_q/dt = dE/dt$ (E is total energy of q), and $(1/v)d/dt = d/dz$ we have

$$\boxed{\frac{dE}{dz} = -4\pi n_e \frac{(qe)^2}{mv^2} \ln \frac{\gamma^2 m v^3}{qe\omega_0} \text{ [JDJ Eq (13.9)].}}$$

- Compare to Beth (1915)

$$\boxed{\frac{dE}{dz} = -4\pi n_e \frac{(qe)^2}{mv^2} \left[\ln \frac{1.123\gamma^2 m v^2}{qe\langle\omega\rangle} - \frac{1}{2}\frac{v^2}{c^2}\right]}$$