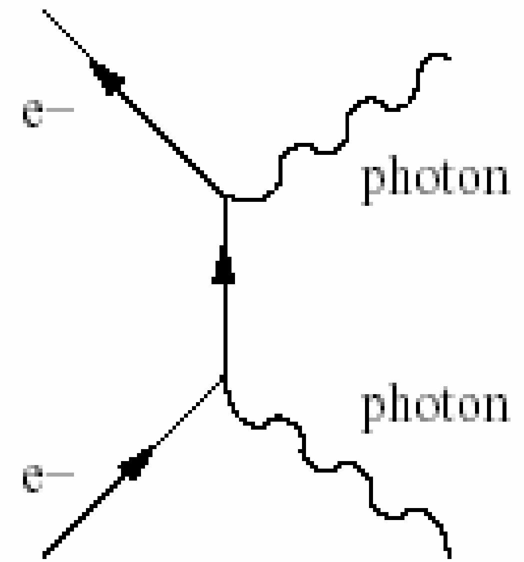
"Scattering" of light on charged particle

- A charged particle has no surface,
  so "scattering" of light is a metaphor.

- Quantum view: collision photon/electron

$$P^{\alpha}_{\gamma} + P^{\alpha}_{\gamma} = P^{\alpha}_{\gamma'} + P^{\alpha}_{e_-'}$$
$$P^{\alpha}_{e_-'} = P^{\alpha}_{e_-} + P^{\alpha}_{\gamma} - P^{\alpha}_{\gamma'}$$



- Then

$$m^2c^2 = (P^{\alpha}_{e_-} + P^{\alpha}_{\gamma} - P^{\alpha}_{\gamma'})(P_{e_-,\alpha} + P_{\gamma,\alpha} - P_{\gamma',\alpha})$$

- and $P^{\alpha}_{\gamma'} P_{e_-,\alpha} - P^{\alpha}_{\gamma'} P_{e_-,\alpha} - P^{\alpha}_{\gamma'} P_{e_-,\alpha} = 0$

"Scattering" of light on charged particle

- We have

$$\begin{aligned} P_{e-,\alpha} &= mc(E_\gamma/2) - \overrightarrow{p}_{e-}.\overrightarrow{p}_\gamma = mE_\gamma \\ P^\alpha_{\gamma'} P_{\gamma,\alpha} &= \frac{E_\gamma}{c}\frac{E_{\gamma'}}{c} - \overrightarrow{p}_\gamma.\overrightarrow{p}_{\gamma'} = -p_\gamma p_{\gamma'}\cos\theta + \frac{E_\gamma E_{\gamma'}}{c^2} \\ P_{e-,\alpha} P^\alpha_{\gamma'} &= mE_\gamma. \end{aligned}$$

- Taking $E_\gamma \equiv \frac{hc}{\lambda}$ and similarly for γ', we have

$$\boxed{\lambda - \lambda' = \frac{h}{mc}(1-\cos\theta).}$$

this is the usual Compton scattering formula. The non-relativistic limit yields $\lambda=\lambda'$, which is the regime of Thomson scattering

Linear Thomson Scattering: cross section I

- Cross section in a figure-of-merit.

$$\sigma \equiv \frac{\text{E radiated/time/solid angle}}{\text{incident flux/unit area/time}}.$$

- Since the electron is at rest:

$$\frac{dP(t')}{d\Omega} = \frac{e^2}{4\pi c}\frac{\left|\hat{n}\times[(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}]\right|^2}{\kappa^5}$$

$$\xrightarrow{\beta\to 0} \frac{e^2}{4\pi c}\frac{\left|\hat{n}\times[\hat{n}\times\dot{\vec{\beta}}]\right|}{\kappa^5} = \frac{e^2}{4\pi c}\dot{\beta}^2\sin^2\Theta$$

where $\Theta\angle(\hat{n},\dot{\vec{\beta}})$

acceleration

- So finally

$$\boxed{\frac{dP(t')}{d\Omega} = \frac{e^2}{4\pi c^3}a^2\sin^2\Theta.}$$

Linear Thomson Scattering: cross section II

- Note that in the non-relativistic limit

$$\frac{dP(t)}{d\Omega} = \frac{dP(t')}{d\Omega}$$

- Let's now specialize our problem and consider a plane wave:

$$\vec{E}(\vec{x},t) = \hat{\epsilon} E_0 e^{i(\vec{k}.\vec{x}-\omega t)}$$

- The acceleration is therefore given by:

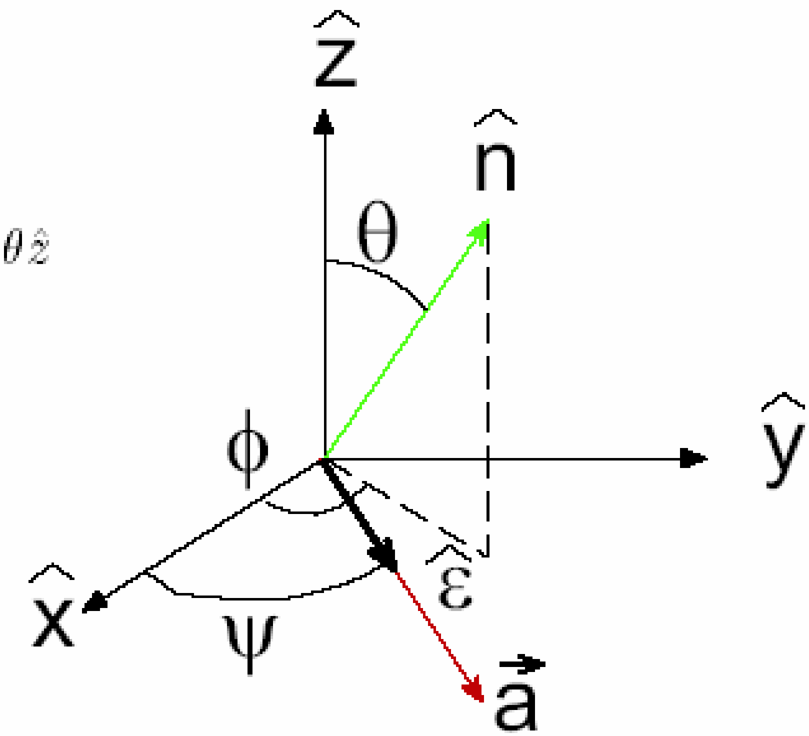
$$\vec{a}(t) = \frac{e}{m}\hat{\epsilon} E_0 e^{i(\vec{k}.\vec{x}-\omega t)}$$

- We ignore the B-field associated to the plane wave because we assume β=0

Linear Thomson Scattering: cross section III

- Given the geometry of the problem we have

$$\begin{aligned}\hat{\epsilon} &= \cos\psi\hat{x} + \sin\psi\hat{y} \\ \hat{n} &= \sin\theta\cos\phi\hat{x} + \sin\theta\sin\phi\hat{y} + \cos\theta\hat{z} \\ \hat{n}.\overrightarrow{a} &= as_\theta(c_\psi c_\phi + s_\psi s_\phi) = as_\theta c_{\psi-\phi} \\ &= a\sin\theta\cos(\psi-\phi) = a\cos\Theta.\end{aligned}$$



- Thus

$$\sin^2\Theta = 1 - \sin^2\theta\cos^2(\psi-\phi).$$

- So the time averaging gives

$$\langle a^2\sin^2\Theta\rangle_t = \frac{1}{2}\left(\frac{eE_0}{m}\right)^2[1 - \sin^2\theta\cos^2(\psi-\phi)].$$

Linear Thomson Scattering: cross section IV

- Assume the incoming wave is unpolarized then

$$\langle \cos^2(\psi - \phi) \sin^2\theta \rangle_\psi = \frac{1}{2}\sin^2\theta.$$

- So

$$\langle a^2 \sin^2\Theta \rangle_{t,\psi} = \frac{1}{2}\left(\frac{eE_0}{m}\right)^2 [1 - \frac{1}{2}\sin^2\theta]$$

- So finally

$$\langle \frac{dP}{d\Omega} \rangle_{t,\psi} = \frac{cE_0^2}{16\pi}\left(\frac{e^2}{mc^2}\right)^2 [1 + \cos^2\theta] = \frac{c}{16\pi} r_e (1 + \cos^2\theta).$$

- The Poynting vector is given by

$$S = \frac{c}{8\pi} \vec{E} \times \vec{H}^*$$

Linear Thomson Scattering: cross section V

- The time averaged power per unit of area is

$$\frac{dP}{d\sigma} = S = \frac{c}{8\pi} E_0^2.$$

- And so the cross-section is

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \frac{d\sigma}{dP}\frac{dP}{d\Omega} = \frac{\frac{cr_e^2}{16\pi}E_0^2}{\frac{c}{8\pi}E_0^2}[1+\cos^2\theta] \\ &= \frac{1}{2}r_e^2(1+\cos^2\theta)\end{aligned}$$

this is the scattering Thomson cross section. The integrated cross section is:

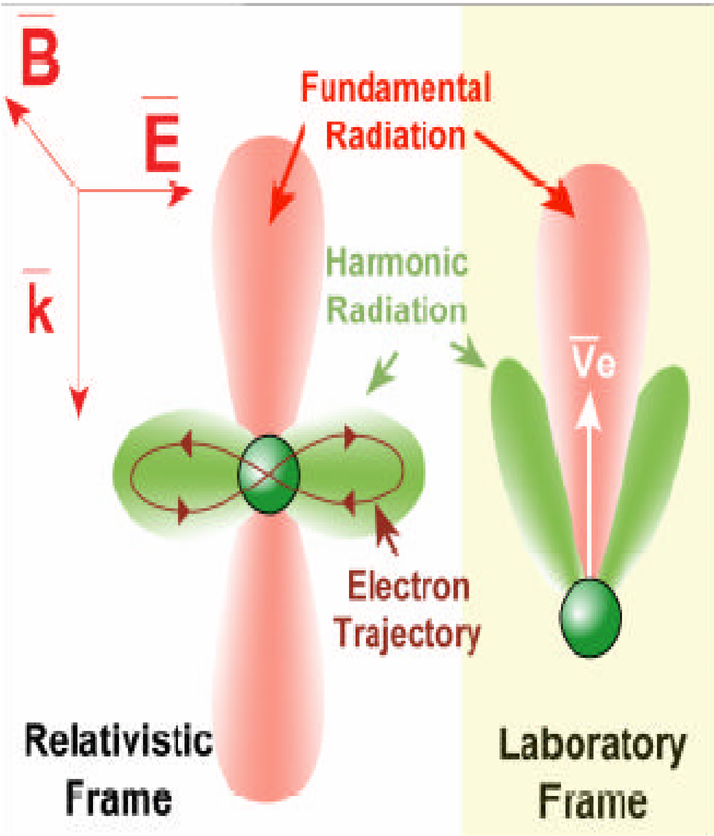
$$\begin{aligned}\sigma &= \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \frac{d\sigma}{d\Omega} = \frac{16\pi}{3}\frac{1}{2}r_e^2 \\ &= \frac{8\pi}{3}r_e^2.\end{aligned}$$

Notes on Nonlinear Thomson Scattering I

- Classical Thomson scattering, the scattering of low-intensity light by e-, is a linear process: it does not change the frequency of the radiation;
- The magnetic-field component of light is not involved.

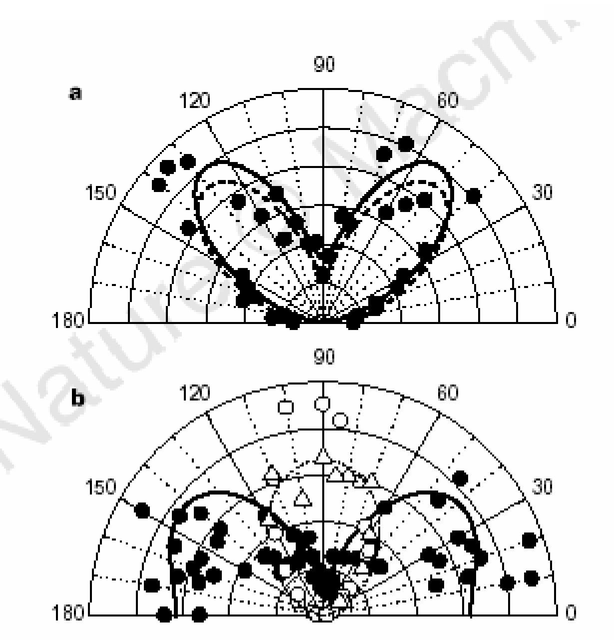
- But if the light intensity is extremely high (~10^{18}W.cm^{-2}), the electrons oscillate during the scattering process with velocities approaching *c*.

- In this relativistic regime, the effect of the magnetic and electric fields on the electron motion should become comparable

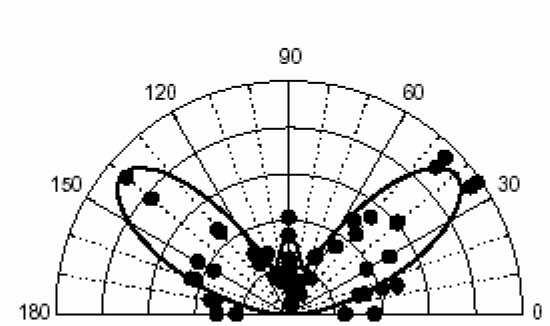


Notes on Nonlinear Thomson Scattering II

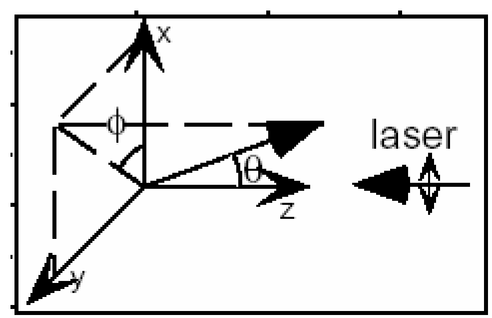
- First experimentally observed in 1998
  Nature **396** issue of Dec. 17th, 1998



2nd harmonic patterns



3rd harmonic patterns



Experimental observation of relativistic nonlinear Thomson scattering

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Case of a bounded electron I

- Compton and Thomson scatterings apply to free electrons
- What happen if an electron is bounded (i.e. to an atom)?

- We assume the equation of motion of the bounded electron to be described by:

$$\vec{a} + \Gamma \vec{v} + \omega_0^2 \vec{x} = \frac{q}{m}\vec{E}.$$

external force

acceleration

restoring force

friction term

- As before we take $\vec{E} = \hat{\epsilon} E_0 e^{i(\vec{k}.\vec{x} - \omega t)}$ and $\vec{x} = \vec{x}_0 e^{-i\omega t}$, then

$$(-\omega^2 - i\omega\Gamma + \omega_0^2)\vec{x}_0 = \hat{\epsilon}\frac{q}{m}E_0 e^{i\vec{k}.\vec{x}}$$

Case of a bounded electron II

- We further assume that $\vec{k}.\vec{x} = 0$ i.e. |x|<<λ .
- then
$$\vec{x}_0 \simeq \frac{\frac{e}{m}E_0}{\omega_0^2 - \omega^2 - i\omega\Gamma}\hat{\epsilon},$$
- so
$$\vec{a} = -\omega^2\vec{x} \Rightarrow |a^2| = \left(\frac{e}{m}E_0\right)^2 \frac{\omega^4}{(\omega_0^2 - \omega^2)^2 + \omega^2\Gamma^2}.$$
- And finally
$$|a^2| = \frac{\left(\frac{e}{m}E_0\right)^2}{\left(\frac{\omega_0}{\omega}\right)^2 + \left[\left(\frac{\Gamma}{\omega}\right)^2 - 1\right]^2}.$$
- Same as before but the denominator is different

Case of a bounded electron III

- The radiated power is therefore

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{16\pi} r_e^2 E_0^2 \frac{1+\cos^2\theta}{\left[\left(\frac{\omega_0}{\omega}\right)^2 - 1\right]^2 + \left(\frac{\Gamma}{\omega}\right)^2},$$

- and the associated cross section is

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_e^2 \frac{1+\cos^2\theta}{\left[\left(\frac{\omega_0}{\omega}\right)^2 - 1\right]^2 + \left(\frac{\Gamma}{\omega}\right)^2},$$

- $\omega \ll \omega_0$ and $\omega \ll \Gamma$ corresponds to Thomson scattering

- $\omega \ll \omega_0$ and $\omega \gg \Gamma$ gives the Raleigh scattering cross section

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_e^2 \left(\frac{\omega}{\omega_0}\right)^4 [1+\cos^2\theta] \propto \omega^4$$

The reason why the sky is blue…