Introduction

- Course's webpage:
  http://nicadd.niu.edu/~piot/phys_571/
- Grading:
  - **Homework** **50 %** **(ok to work/discuss together)**
  - **MidTerm** **20 %** **(class exam?)**
  - **Final** **30 %** **(class exam?)**
- Instructor:
  - Philippe Piot (NIU/FNAL & ANL) [philippe.piot@gmail.com]
    - **Generally at NIU on Tues, Wed, and Th.**
    - At FNAL on Mondays.
    - At ANL on Fridays.
- Handouts:
  - Current version distributed (watch out for typos & mistakes!)
  - Updated regularly and available on web
  - Slides (only available on the web), papers will be distributed when needed and made available on the web (in protected way due to copyright…)

Notes on textbooks

- **<u>Required:</u> (the reference in E&M)**
  - J. D. Jackson *classical electrodynamics,* J. Wiley and Sons 3rd Ed
- **Other very good books you may want to check**
  - C. Brau, *Modern problems in classical electrodynamics*, Oxford Univ. Press: same level as JDJ but more applied with many references to contemporary problems (e.g. lasers nonlinear optics)
  - Landau & Lifchitz, Classical Field Theory, a reference
  - S. Parrott, *Relativistic Electrodynamics and Differential Geometry,* Springer-Verlag (1986): very good for the mathematical formalism
  - A. O. Barut, *Electrodynamics and Classical Theory of Fields and Particles*, The McMillan Company (1964): very good – hard to find definitely a reference!
  - F. Rohrlich, *Classical Charged Particle*, Addison-Wesley (Reading 1965): simply excellent but hard to find… BUT new 3rd edition just out (I did not check it).

Maxwell's equation I

- Maxwell's equations in a medium (ε,μ) and charge/current density (ρ,**J**)

$$
\begin{aligned}
\vec{\nabla}.\vec{D} &= \rho, \text{ Coulomb law}\\
\vec{\nabla}.\vec{B} &= 0, \text{ no magnetic charges}\\
\vec{\nabla}\times\vec{E} &= -\partial_t \vec{B}, \text{ Faraday law}\\
\vec{\nabla}\times\vec{H} &= \vec{J} + \partial_t \vec{D}, \text{ Ampère- Maxwell law}
\end{aligned}
$$

- Where

electric field

Electric displacement $\vec{D} \equiv \epsilon \vec{E} + \vec{P}$ polarization

induction

Magnetic displacement $\vec{H} \equiv \vec{B}/\mu - \vec{M}$ magnetization

Maxwell's equation II

- Generally

Linear susceptibility

$$\vec{D} = \underbrace{\varepsilon\vec{E} + \varepsilon\overleftrightarrow{\chi}\vec{E}}_{\text{Linear optics}} + \underbrace{\ldots}_{\text{NL optics}} \equiv \boxed{\overleftrightarrow{\varepsilon}\vec{E}}$$

Tensor (or matrix)

$$\vec{B} = \overleftrightarrow{\mu}\vec{H}$$

- Consider "simple case" of homogenous, non conducting, non dissipating isotropic medium then:

$$\vec{D} = \varepsilon\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\vec{E} = \varepsilon\vec{E} \qquad \vec{H} = \frac{\vec{B}}{\mu}$$

Maxwell's equation III

- If no source terms are present (assume no charge in the medium) then Maxwell's equations reduce to

$$\boxed{\begin{aligned} \vec{\nabla}.\vec{D} = 0 \Rightarrow \vec{\nabla}.\vec{E} = 0; \vec{\nabla}.\vec{B} = 0; \\ \vec{\nabla} \times \vec{E} + \partial_t \vec{B} = 0; \\ \vec{\nabla} \times \vec{H} - \partial_t \vec{D} = 0 \Rightarrow \nabla \times \vec{B} - \mu\epsilon\partial_t \vec{E} = 0. \end{aligned}}$$

- Note if **medium is vacuum** then:

$$\varepsilon \rightarrow \varepsilon_0 \qquad \epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C}^2/\mathrm{N\ m}^2$$

$$\mu \rightarrow \mu_0 \qquad \mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A}^2$$

Boundaries conditions

- You already saw this (PHYS 570) [check JDJ p. 154 & p. 194]

Surface charge density

$$\begin{cases}\left(\vec{D}_2-\vec{D}_1\right).\hat{n}_{21}=\sigma \\ \left(\vec{E}_2-\vec{E}_1\right)\times\hat{n}_{21}=0\end{cases} \qquad \begin{cases}\left(\vec{B}_2-\vec{B}_1\right).\hat{n}_{21}=0 \\ \hat{n}_{21}\times\left(\vec{H}_2-\vec{H}_1\right)=\vec{K}\end{cases}$$

Surface's normal pointing from (1) to (2)

Surface current density

- For at the boundary of a **perfect** conductor we have: (2) conductor (1) material with (ε,μ) :

$$\varepsilon_2\to\infty \quad \mu_2\to\infty$$

$$\boxed{\hat{n}.\vec{B}=\hat{n}\times\vec{E}=0}$$

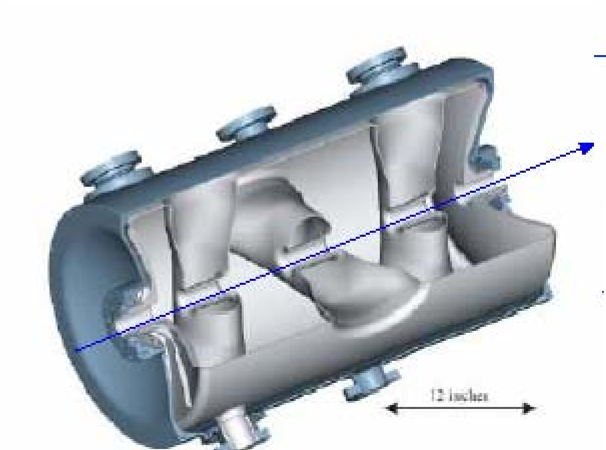
Resonant cavities -- introductory remarks I

- Why do we care?
  - The e.m. “hamornic oscillator”
  - Acceleration & manipulation of charged-particle beams
  - Solid state physics e.g. measurement of dielectric permittivity of materials
  - Lasers (amplification occurs as a medium is placed in an optical cavity) e.g. a Fabry-Perot resonators
  - Non conducting wall cavities/waveguides include dielectric slabs and fibers that can support eigenmodes

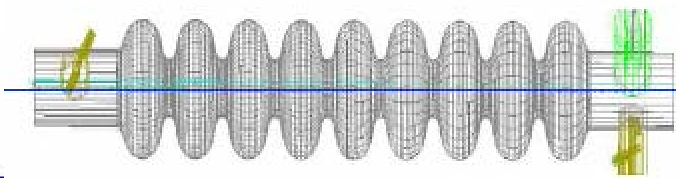
- You studied (?) the simple case of rectangular waveguides and cavities (the shoebox type); eigenmodes are easy (sin/cos functions).
- We are going to concentrate on the case of cylindrical-symmetric cavities more exciting Bessel function
- We will not deal with spherically symmetric cavities (the eigenmodes are linear combinations of Spherical Harmonic functions)

Resonant cavities -- introductory remarks II

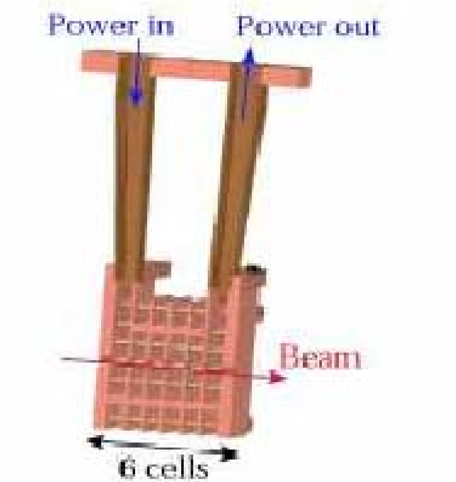
- Example of cavities



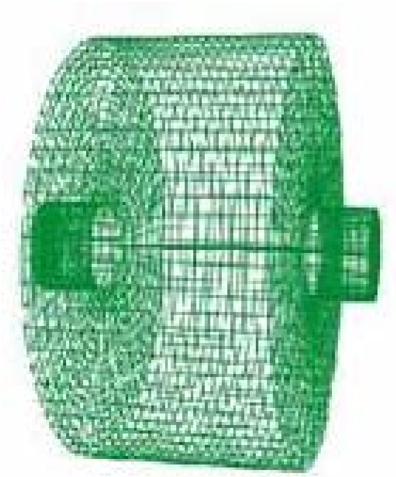
Heavy ions accelerating cavities



accelerating cavities for the International Linear Collider



Photonic band-gap type accelerating cavity (new concept)



Pillbox w apertures

- The academic model we are going to consider is the case of a “pillbox” cavity – note it has no practical application since we will assumed the cavity is closed (no aperture)…

e.m. field in a cylindrical-symmetric cavity

- Consider the e.m. fields of the form:

$$\left[\begin{array}{c} \overrightarrow{E}(r,\phi,z,t) \\ \overrightarrow{B}(r,\phi,z,t) \end{array}\right] = \left[\begin{array}{c} \overrightarrow{E}(r,\phi) \\ \overrightarrow{B}(r,\phi) \end{array}\right] e^{\pm ikz - i\omega t}.$$

- Note that

$$\left[\begin{array}{c} \overrightarrow{E} \\ \overrightarrow{B} \end{array}\right] = \left[\begin{array}{c} \overrightarrow{E}_t \\ \overrightarrow{B}_t \end{array}\right] + \left[\begin{array}{c} E_z \\ B_z \end{array}\right] \hat{z};$$

$$\left[\begin{array}{c} E_z \\ B_z \end{array}\right] = \hat{z}. \left[\begin{array}{c} \overrightarrow{E} \\ \overrightarrow{B} \end{array}\right]; \text{ and} \left[\begin{array}{c} \overrightarrow{E}_t \\ \overrightarrow{B}_t \end{array}\right] = \left(\hat{z} \times \left[\begin{array}{c} \overrightarrow{E} \\ \overrightarrow{B} \end{array}\right]\right) \times \hat{z}.$$

Wave equation

- Take $\overrightarrow{\nabla}\times$ of Faraday's law:

$$\begin{aligned}\overrightarrow{\nabla}\times(\overrightarrow{\nabla}\times\overrightarrow{E})+\partial_t\overrightarrow{\nabla}\times\overrightarrow{B} &= \overrightarrow{\nabla}(\overrightarrow{\nabla}.\overrightarrow{E})-\nabla^2\overrightarrow{E}+\mu\epsilon\partial_t^2\overrightarrow{E}\\ &= -\nabla_t^2\overrightarrow{E}+k^2\overrightarrow{E}-\mu\epsilon\omega^2\overrightarrow{E}=0\end{aligned}$$

- Take $\overrightarrow{\nabla}\times$ of Ampere-Maxwell's equation:

$$\begin{aligned}\overrightarrow{\nabla}\times(\overrightarrow{\nabla}\times\overrightarrow{B})-\mu\epsilon\partial_t\overrightarrow{\nabla}\times\overrightarrow{E} &= \overrightarrow{\nabla}(\overrightarrow{\nabla}.\overrightarrow{B})-\nabla^2\overrightarrow{B}+\mu\epsilon\partial_t^2\overrightarrow{B}\\ &= -\nabla_t^2\overrightarrow{B}+k^2\overrightarrow{B}-\mu\epsilon\omega^2\overrightarrow{B}=0\end{aligned}$$

- Summary:

$$\boxed{[\nabla_t^2+(\mu\epsilon\omega^2-k^2)]\left[\begin{array}{c}\overrightarrow{E}\\ \overrightarrow{B}\end{array}\right]=0.}$$

Relation between transverse and axial e.m. fields I

- From Maxwell's equations

$$\left[\hat{z}\times(\overrightarrow{\nabla}\times\overrightarrow{E})\right]\times\hat{z}-i\omega\overrightarrow{B}_t=0,$$
$$\left[\hat{z}\times(\overrightarrow{\nabla}\times\overrightarrow{B})\right]\times\hat{z}+i\mu\epsilon\omega\overrightarrow{E}_t=0.$$

- Generally $\hat{z}\times(\overrightarrow{\nabla}\times\overrightarrow{V})=\overrightarrow{\nabla}(\hat{z}.\overrightarrow{V})-(\hat{z}.\overrightarrow{\nabla})\overrightarrow{V}=\overrightarrow{\nabla}V_z-\partial_z\overrightarrow{V}$

$$\begin{aligned}\left[\hat{z}\times(\overrightarrow{\nabla}\times\overrightarrow{V})\right]\times\hat{z} &= \overrightarrow{\nabla}V_z\times\hat{z}-\partial_z(\overrightarrow{V}\times\hat{z})\\ &= -\hat{z}\times\overrightarrow{\nabla}_tV_z+\hat{z}\times\partial_z\overrightarrow{V}_t.\end{aligned}$$

- So

$$\boxed{\begin{aligned}i\omega\overrightarrow{B}_t &= \hat{z}\times\partial_z\overrightarrow{E}_t-\hat{z}\times\overrightarrow{\nabla}_tE_z, \quad \text{(B)}\\ -i\omega\mu\epsilon\overrightarrow{E}_t &= \hat{z}\times\partial_z\overrightarrow{B}_t-\hat{z}\times\overrightarrow{\nabla}_tB_z. \quad \text{(E)}\end{aligned}}$$

Relation between transverse and axial e.m. fields II

- Apply "$\hat{z} \times \partial_z$" to the "B"-equation:

$$
\begin{aligned}
i\omega \hat{z} \times \partial_z \overrightarrow{B}_t &= \hat{z} \times (\hat{z} \times \partial_z^2 \overrightarrow{E}_t) - \quad \hat{z} \times (\hat{z} \times \overrightarrow{\nabla}_t \partial_z E_z) \\
&= -\partial_z^2 \overrightarrow{E}_t - \quad (-)\overrightarrow{\nabla}_t \partial_z E_z \\
&= +k^2 \overrightarrow{E}_t \pm ik \overrightarrow{\nabla}_t E_z .
\end{aligned}
$$

$$
\Rightarrow \hat{z} \times \partial_z \overrightarrow{B}_t = \frac{-i}{\omega} (k^2 \overrightarrow{E}_t \pm ik \overrightarrow{\nabla}_t E_z). \quad \textbf{(B')}
$$

- Similarly for the "E"-equation

$$
\hat{z} \times \partial_z \overrightarrow{E}_t = \frac{+i}{\mu\epsilon\omega} (k^2 \overrightarrow{B}_t \pm ik \overrightarrow{\nabla}_t B_z). \quad \textbf{(E')}
$$

Relation between transverse and axial e.m. fields III

- Insert (E') into (B)

$$i\omega B_t = \frac{i}{\mu\epsilon\omega}(k^2 \overrightarrow{B}_t \pm ik\overrightarrow{\nabla}_t B_z) - \hat{z} \times \overrightarrow{\nabla}_t E_z$$

$$i(\mu\epsilon\omega^2 - k^2)\overrightarrow{B}_t = \mp k\overrightarrow{\nabla}_t B_z + \mu\epsilon\omega\hat{z} \times \overrightarrow{\nabla}_t E_z$$

$$\boxed{\Rightarrow \overrightarrow{B}_t = \frac{i}{\mu\epsilon\omega^2 - k^2}(\pm k\overrightarrow{\nabla}_t B_z + \mu\epsilon\omega\hat{z} \times \overrightarrow{\nabla}_t E_z).}$$

- Insert (B') into (E)

$$-i\mu\epsilon\omega E_t = \frac{-i}{\omega}(k^2 \overrightarrow{E}_t \pm ik\overrightarrow{\nabla}_t E_z) - \hat{z} \times \overrightarrow{\nabla}_t B_z$$

$$-i(\mu\epsilon\omega^2 - k^2)\overrightarrow{E}_t = \pm k\overrightarrow{\nabla}_t E_z - \omega\hat{z} \times \overrightarrow{\nabla}_t B_z$$

$$\boxed{\Rightarrow \overrightarrow{E}_t = \frac{i}{\mu\epsilon\omega^2 - k^2}(\pm k\overrightarrow{\nabla}_t E_z - \omega\hat{z} \times \overrightarrow{\nabla}_t B_z).}$$

Prescription for e.m. field calculations

1. Find $E_z(r,\phi)$ and $B_z(r,\phi)$ from $[\nabla_t^2 + (\mu\epsilon\omega^2 - k^2)] \left[\begin{array}{c} \overrightarrow{E} \\ \overrightarrow{B} \end{array} \right] = 0.$

2. Find $\overrightarrow{E}_t(r,\phi)$ and $\overrightarrow{B}_t(r,\phi)$ from

$$\left[\begin{array}{c} \overrightarrow{E}_t(r,\phi) \\ \overrightarrow{B}_t(r,\phi) \end{array} \right] = \frac{i}{\mu\epsilon\omega^2 - k^2} \left\{ \pm k \overrightarrow{\nabla}_t \left[\begin{array}{c} E_z(r,\phi) \\ B_z(r,\phi) \end{array} \right] - \omega \hat{z} \times \overrightarrow{\nabla}_t \left[\begin{array}{c} B_z(r,\phi) \\ -\mu\epsilon E_z(r,\phi) \end{array} \right] \right\}.$$

3. The total field is:

$$\left[\begin{array}{c} \overrightarrow{E} \\ \overrightarrow{B} \end{array} \right] = \left[\begin{array}{c} \overrightarrow{E}_t \\ \overrightarrow{B}_t \end{array} \right] + \left[\begin{array}{c} E_z \\ B_z \end{array} \right] \hat{z}.$$

4. Incorporate the boundary conditions ($\mathcal{S}$: cavity side surface):

$$\hat{n} \times \overrightarrow{E} = 0 \Rightarrow E_z\big|_{\mathcal{S}} = 0 \text{ and, } \overrightarrow{E}_t = 0 \text{ at end plates}$$
$$\hat{n}.\overrightarrow{B} = 0 \Rightarrow \hat{n}.\overrightarrow{\nabla}_t B_z = 0 \Rightarrow \partial_n B_z\big|_{\mathcal{S}} = 0$$