# PHYS 630: Homework V

due date: Monday, December 8th, 2008 in my mailbox.

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You are welcome to work together. If you partially use work from other (e.g. something you might have found in a book or a journal paper), you should properly credit the author by citing the material used.

1. **Threshold of a Ruby Laser (20 pts)**

   (a) At the line center of $\lambda_0 = 694.3$ nm transition, the absorption coefficient of ruby in thermal equilibrium (i.e. without pumping) at 300 K is $\alpha(\nu_0) = -\gamma(\nu_0) \simeq 0.2\ \text{cm}^{-1}$. If the concentration of $Cr^{3+}$ ions responsible for the transition is $N_a = 1.58 \times 10^{19}\ \text{cm}^{-3}$, determine the transition cross section $\sigma_0 \equiv \sigma(\nu_0)$. (7 pts)

   (b) A ruby laser makes use of a 10 cm long rod (with refractive index $n = 1.76$) with a 1 $\text{cm}^2$ cross section and operates at the transition $\lambda_0 = 694.3$ nm. Both ends of the rod are polished and coated so that each has a reflectance of 80%. Assuming that these are the only sources of losses, determine the resonator loss coefficient $\alpha_r$ and the corresponding photon lifetime $\tau_p$. (7 pts)

   (c) As the laser is pumped $\gamma(\nu_0)$ increases from is initial thermal equilibrium value $-0.2\ \text{cm}^{-1}$ to positive values thereby providing gain. Determine the population difference threshold $N_t$ for laser oscillation. (6 pts)

2. **Spectral Broadening of a Saturated Amplifier (20 pts)** Consider a homogeneously broadened amplifier with a Lorentzian lineshape function with width $\Delta\nu$:

$$g(\nu) = \frac{\Delta\nu/(2\pi)}{(\nu-\nu_0)^2 + (\Delta\nu/2)^2}.$$

   Show that for a photon-flux density $\varphi$, the amplifier gain coefficient $\gamma(\nu)$ assumed a Lorentzian lineshape with width

$$\Delta\nu_s = \Delta\nu\sqrt{1 + \frac{\varphi}{\varphi_s(\nu_0)}}$$

   where $\varphi_s$ is the "saturated flux" defined in the Lecture Notes. (20 pts)

3. **Frequency pulling effect (30 pts):** In the Lecture Notes, we show that the second condition for laser oscillation (besides the gain coefficient larger than the loss coefficient) was

$$2kd + 2\varphi(\nu)d = 2\pi q.$$

If the first term in the right-hand side is much larger than the second term then the modes of the laser are given by the resonator modes. We also briefly discussed the "frequency pulling" effect occurring when the second term is non-negligible. The purpose of this exercise is to quantify this effect.

   (a) Assume $\varphi(\nu)$ is given by the phase shift associated to a Lorentzian lineshape function and write down an expression for $\nu_q \equiv qc/(2d)$ as a function of $\nu$. (10 pts)

   (b) Make a sketch for $\nu_q$ as a function of $\nu$ and explain how one can graphically determine the frequency of laser oscillation from the plot. The laser modes have frequencies $\nu_{q'}$. (15 pts).

   (c) Assuming the lasing frequency is very close to the cold resonator model, derive a simple and approximative analytical equation for $\nu_{q'}$. (5 pts)

4. **Numerical investigation of pulsing via Mode-Locking (30 pts)**: Write a simple computer program to plot the intensity $I(t) = |A(t)|^2$ (in arbitrary units) of a wave whose amplitude $A(t)$ is given by the sum

$$A(t) = \sum_{q=1}^{M} A_q e^{\frac{iq2\pi ct}{2d}},$$

where $d$ is the length of the resonator. For each the following three cases provide a plot of $|I(t)|$ for a total number of mode $M = 11$, 21, and 51. You will consider the three following cases for the complex amplitude $A_q$:

   (a) Equal magnitudes and same phases. (10 pts)

   (b) Magnitudes that obey a Gaussian spectral profile $|A_q| = \exp\left[-\frac{1}{2}(q/5)^2\right]$ and the same phases. (10 pts)

   (c) Equal magnitudes but random phases uniformly distributed in $[0, 2\pi]$. (10 pts)

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