PHYS 630: Homework III

due date: Tuesday, October 21st, 2008 at class meeting.

You are welcome to work together. If you partially use work from other (e.g. something you might have found in a book or a journal paper), you should properly credit the author by citing the material used.

1. **Optics in a doubly-negative (left handed) meta-material (50 pts) [Problem from Candidacy exam of September 2008]:** In man-made dielectric and magnetic material unusual situations arise when the electric permittivity and magnetic permeability are both real and negative. We consider a plane wave with electric field $E = E_0 \exp[-i(k.r - \omega t)]$ and magnetic field $H = H_0 \exp[-i(k.r - \omega t)]$.

   (a) Let first consider a standard material with permittivity ϵ and magnetic permeability μ both real and positive. Find the Maxwell equations satisfied by E_0 and H_0 in such a medium, the velocity of light in such a medium, and what combination of E_0, k and H_0 form a right-handed set. (10 pts)

   (b) From now on we consider a doubly-negative meta-material and write $\epsilon = -|\epsilon|$ and $\mu = -|\mu|$. Write the Maxwell equations satisfied by E0 and H0 in the meta-material and show that the role of E0 and H0 are interchanged compared to the standard case explored in (1). (10 pts)

   (c) What can you state about the set (E_0, H_0, k)? (5 pts)

   (d) We now specialize to a plane wave propagating along the z axis and take $E = E_0 \exp[-i(kz-\omega t)]\hat{x}$ and $H = H_0 \exp[-i(kz-\omega t)]\hat{y}$ (where $\hat{u}$ stands for the unit vector along the u direction). Find the Poynting vector $\overrightarrow{S}$ in the meta-material and compare its direction to k. (10 pts)

   (e) Discuss the implication of (d) to the sign of the index of refraction. Consider an incoming optical ray propagating at the interface of a standard material (with index of refraction $n_1 > 0$) and a meta-material (with index of refraction $n_2 = -|n_2| < 0$). Take the incident and refracted angles to be respectively θ_1 and θ_2. Write Snells refraction law at the interface and draw a schematic stressing the difference(s) with the usual situation of refraction at the interface between two standard materials. (15 pts)

2. **Angular momentum beams as secure information carriers (20 pts):** Read the attached paper from *New Scientist* and explain why light beam carrying angular momentum can be used in secure communications.

3. **Stokes parameters to characterize the polarization of a wave (30 pts):**

   (a) Using the x and y linearly polarized vectors

   $$\left\{ \hat{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ \hat{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$$

   as an expansion basis, the expansion coefficient for the Jones vector

   $$\overrightarrow{J} = \begin{bmatrix} A_x \\ A_y \end{bmatrix}$$

   are by definition $\alpha_1 = A_x$ and $\alpha_2 = A_y$. Find the expansion coefficient in the basis

   $$\left\{ \hat{e}'_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \ \hat{e}'_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$$

   which will be called α_{45} and α_{135}, i.e., $\overrightarrow{J} = \alpha_{45}\hat{e}'_1 + \alpha_{135}\hat{e}'_2$.

   (b) Similarly, find the expansion coefficient in the basis

   $$\left\{ \hat{e}''_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}, \ \hat{e}''_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} \right\}$$

   which will be called α_R and α_L, i.e., $\overrightarrow{J} = \alpha_R\hat{e}''_1 + \alpha_L\hat{e}''_2$.

   (c) Show that the S_1, S_2, and S_3 Stokes parameters (see Lesson 12) associated to the vector $\overrightarrow{J}$ can be written

   $$S_1 = |A_x|^2 - |A_y|^2,$$

   $$S_2 = |\alpha_{45}|^2 - |\alpha_{135}|^2, \text{ and } S_3 = |\alpha_R|^2 - |\alpha_L|^2$$

   note that the "0" Stokes parameters is $S_0 = |A_x|^2 + |A_y|^2$
