PHYS 630: Homework II

due date: Tuesday, September 27th, 2008 at class meeting.

You are welcome to work together. If you partially use work from other (e.g. something you might have found in a book or a journal paper), you should properly credit the author by citing the material used.

1. **Fresnel zone plate (20 pts):** Consider the two dimensional "binary plate" with complex amplitude transmittance given by

$$f(x, y) = \Theta\left[\cos\left(\pi\frac{x^2 + y^2}{\lambda f}\right)\right],$$

where Θ is the unit step function [$\Theta(x) = 0$ for $x < 0$ and $\Theta(x) = 1$ for $x \geq 0$]. Such a component is known as Fresnel zone plate. Show that this system acts as a lens with multiple focal lengths equal to ∞, $\pm f$, $\pm f/2$,...

2. **Condition of Validity of Fresnel versus Fraunhofer Approximations: a comparison (10 pts):** Demonstrate that the Fraunhofer approximation is more restrictive than the Fresnel approximation by taking $\lambda = 0.5$ μm and assuming that the object points lie within a circular aperture of radius $b = 1$ cm and the observation point lies within a circular aperture $a = 2$ cm. Determine the range of distances d between the object and observation planes for which each of these approximations is applicable.

3. **Fresnel Diffraction from a Gaussian Aperture (20 pts):** We consider a Gaussian aperture with function

$$p(x, y) = \exp[-(x^2 + y^2)/W_0^2]$$

Considering an incident plane wave with amplitude $\sqrt{I_i}$, show that under the Fresnel approximation, the intensity in an observation plane located at a distance d downstream is

$$I(x, y) = I_i\left[\frac{W_0}{W(d)}\right]^2 \exp\left[-2\frac{x^2 + y^2}{W^2(d)}\right]$$

where $W^2(d)$ should be given. Sketch the intensity distribution along one direction, e.g. $I(x, 0)$, for $N_f = 10, 1, 0.5, 0.1$ and compare to the aperture size $2W_0$.

4. **Diffraction by Circular Apertures (20 pts):** We consider a mask which consists of two circular holes of radius a with a center-to-center separation D. Under the Fraunhofer approximation give and sketch the intensity distribution associated to the diffraction pattern at an observation plane located a distance d downstream of the mask. Discuss what happens as d increases. Sketch the diffraction pattern for $D < 2a$.

5. **Passage through a Square Array of Circular Apertures (10 pts):** We consider the circular aperture of the previous problem but now assume the mask is a two-dimensional array of apertures (N apertures along x and y directions) with center-to-center separation D in both directions. Derive and sketch the Fraunhofer diffraction pattern at a location d downstream of the mask.

6. **Impulse-response Function of a Single-lens System (20 pts):** An optical setup consisting of an aperture (with aperture function $p(x, y)$ immediately followed by a thin lens of focal length f is used to image a point in the object plane onto the image plane; see Figure 1. We assume the aperture and lens to be in the same plane. Consider a point source in the object plane which produces in the lens-aperture plane a spherical wave

$$U(x, y) = h_1 exp\left[-ik\frac{x^2 + y^2}{2d_1}\right].$$

Show that the impulse-response function of the system is of the form

$$h(x, y) = h_0 P_1\left(\frac{x}{\lambda d_2}, \frac{y}{\lambda d_2}\right),$$

where P_1, which needs to be explicited, reduces to P [the Fourier transform of $p(x, y)$] when the system is focused i.e. when $1/d_1 + 1/d_2 = 1/f$. Using the class Notes deduce the impulse-response function of a focussed imaging system with a circular aperture of diameter D. Show that a point like source in the object plane gives an Airy pattern and show that the radius of its central maximum is $1.22\lambda d_2/D$.

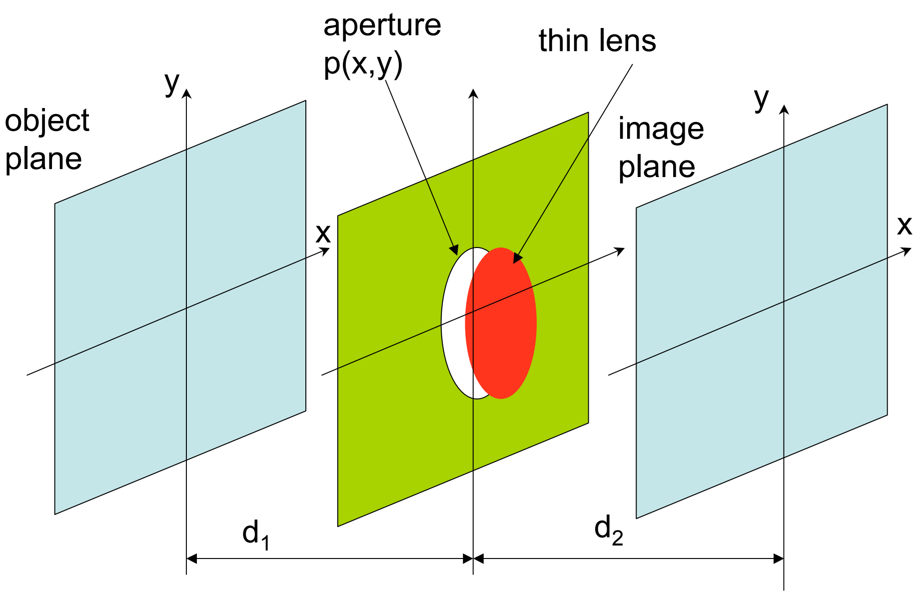

Figure 1: Problem 7 geometry.