PHYS 630: Homework I

due date: Thursday, September 11th, 2008 at class meeting.

You are welcome to work together. If you partially use work from other (e.g. something you might have found in a book or a journal paper), you should properly credit the author by citing the material used.

1. **Paraboloidal Beams Solution of the Helmholtz Equation (10 pts):** Verify that the complex amplitude associated to a Paraboloidal beam

$$U(\overrightarrow{r}) = \frac{A}{z} e^{-ikz} e^{-ik\frac{x^2+y^2}{2z}}.$$

   where A is a complex constant, satisfies the paraxial Helmholtz equation.

2. **Validity of the Paraxial Equation for a Gaussian Beam (10 pts):** The complex envelop $A(\overrightarrow{r})$ of a Gaussian beam is an exact solution of the paraxial Helmholtz equation but its corresponding complex amplitude $U(\overrightarrow{r}) = A(\overrightarrow{r})\exp(-ikz)$ is only an approximate solution of the Helmholtz equation. This is because the paraxial Helmholtz equation is itself approximate (slowly varying envelope approximate: $\partial_z A \ll kA$). Show that if the divergence angle θ_0 of a Gaussian beam is small ($\theta_0 \ll 1$) the condition $\partial_z A \ll kA$ is satisfied.

3. **Elliptic Gaussian Beams (15 pts):** The paraxial Helmholtz equation admits a Gaussian beam solution with intensity $I(x, y, 0) = |A_0|^2 \exp[-2(x^2/W_{0x}^2 + y^2/W_{0y}^2)]$ in the $z = 0$ plane, with beam waist radii W_{0x} and W_{0y} in the x and y-direction respectively. The contour of constant intensity in (x,y) plane are therefore ellipses instead of circles. Write expressions for the beam depth of focus, angular divergence, and radii of curvature in the x and y directions, as function of W_{0x}, W_{0y} and the wavelength λ. If $W_{0x} = 2W_{0y}$ sketch the shape of the beam spot in the $z = 0$ plane and in the far field (z much greater than the depth of focus).

4. **Determination of a Beam with Given Width and Curvature (15 pts):** Assuming that the beam width W and curvature R are known at some point on the beam axis. Find the location of the beam waist z and its width W_0.

5. **Beam Relaying (15 pts):** A Gaussian beam of width W_0 and wavelength λ is repeatedly focused by a sequence of identical lenses, each with focal length f and separated by a distance d. Show that a necessary condition to have the focused beam waist radius equal to the incident waist radius is $d \leq 4f$.

6. **Passage of a Gaussian Beam through a Lens (15 pts):** A Gaussian beam is transmitted through a thin lens of focal length f. Show that the locations of the waists of the incident and transmitted beams, z and z' are related via
$$\frac{z'}{f} - 1 = \frac{z/f - 1}{(z/f - 1)^2 + (z_0/f)^2}.$$
where z_0 is the Rayleigh length of the incident beam.

7. **Transmission of a Gaussian Beam through a Transparent Plate (10 pts):** Use the ABCD law to examine the transmission of a Gaussian beam from air, through a transparent plate of reflective index n and thickness d, and again into air. Assume the beam axis is normal to the plate.

8. **"Donut" Beams (10 pts):** We consider a wave which is a superimposition of two Hermite-Gaussian beams of orders (1,0) and (0,1) of equal intensities. The two beams have independent and random phases so their intensities add with no interference. Show that the total intensity is a donut-shaped circularly symmetric function. Assuming that $W_0 = 1$ mm, find the radius of the circle of peak intensity and the radii of the two circles corresponding to the $1/e^2$ times the peak intensity at the beam waist.
