Modulation and Demodulation

• Principle borrow from radio telecommunication
  – Take a wide band signal [few Hz to kHz]
  – Modulate it with a single frequency “the carrier signal” which is the frequency associated to a given station
  – Emit the signal
  – Given the station frequency, a receiver follows a demodulation procedure that reconstructs the audio signal

• Type of modulation
  – Amplitude Modulation -- analog to holography
  – Frequency Modulation -- analog to spectral holography
  – Phase Modulation
  – Digital methods
Amplitude Modulation I

- Consider $f(x)$ the “baseband” function
- The modulated function is going to be $f(x)\cos(2\pi f x)$

\[ f(x)\cos(2\pi f x) \]

\[ \text{carrier frequency} \]

\[ f(x) \]

\[ \cos(2\pi f x) \]

\[ \text{multiplication} \]

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Amplitude Modulation II

- Consider $f(x)$ the “baseband” function
- The modulated function is going to be

$$f(x)\cos(2\pi f x)$$

carrier frequency
Demodulation

- To demodulate:
  - Multiply modulated signal by $\cos(2\pi f x)$
  - Use a low pass filter to cut high frequencies
Optical analogy

• Usually a wave is described by a complex amplitude, i.e. an amplitude and phase information

• Usually a detector, e.g. a CCD camera, cannot detect the phase (time scale associated to optical phenomena is ~10 fs too fast!!). Only intensity can be measured

• So amplitude information can be recovered but not phase informations

• Can we record the phase information of a wave?
  – Yes: this is holography

• Holography enables the reconstruction of a wave…
Intensity recording

- Recording the intensity of a wave on a photographic emulsion allows the reconstruction of the intensity ONLY.
Holography I

- Mix the original wave (object wave) with a reference wave
- Record the sum intensity

\[ t \propto |U_0 + U_r|^2 = |U_r|^2 + |U_0|^2 + U_0 U_r^* + U_0^* U_r \]
\[ \propto I_r + I_0 + 2 \sqrt{I_r I_0} \cos[\arg(U_r) - \arg(U_0)]. \]

Hologram: a record of the sum intensity of \( U_0 \) and \( U_r \)
Holography II

• To decode the information stored on the hologram: use the same reference wave and illuminate the hologram
• The wave generated in the hologram plane is:

\[ U = tU_r = U_r(I_r + I_0) + I_rU_0 + U_r^2U_0^* \]

• Need to separate the relevant information…

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Example of a plane object wave

• Specialize to the case of a uniform plane wave for the reference wave the “normalized” amplitude downstream of the hologram is

$$\bar{U} \equiv \frac{U}{I_r^{1/2}} = I_r + I_0(x, y) + I_r^{1/2}U_0(x, y) + I_r^{1/2}U_0^*(x, y)$$

• Take the special case of a plane object wave

$$U_0(x, y) = I_0^{1/2} \exp[-ik \sin \theta x]$$

$$\bar{U} = (I_r + I_0) + \sqrt{I_rI_0} \exp[-ik \sin \theta x] + \sqrt{I_rI_0} \exp[+ik \sin \theta x]$$
Example of a plane spherical wave

• Still consider a uniform plane wave for the reference wave

• Take the special case of a spherical object wave

\[ U_0(x, y) = I_0^{1/2} \exp[-ik(r - r_0)]/|r - r_0| \]

where \( r = (x, y, 0), r_0 = (0, 0, -d) \)
Off-axis Holography

- One mean of separating the three term in the “reconstructed” wave is to use an off-axis reference wave

\[ U_0(x, y) = f(x, y) \exp[-ik \sin \theta x] \]

The reconstructed wave is

\[ U(x, y) = I_r + I_r |f(x, y)|^2 + I_r^{1/2} f(x, y) \exp[-ik \sin \theta x] + I_r^{1/2} f^*(x, y) \exp[+ik \sin \theta x] \]

We must have \( \theta_s \equiv \arcsin(\lambda \nu_s) \ll \theta_x \)

maximum frequency associated to \( f(x,y) \)
Fourier Holography

- One can compute optically the Fourier transform of an object and encode it on an hologram
- Then do an inverse Fourier transform of the reconstructed wave
- Can use this technique to do spatial filtering (Vander Lugt holography)
Practical implementation

- Hologram generation

- Note: One can also create “virtual” objects by digitally generating the hologram

- Object restoration

(Images from Wikipedia)
Analogy between AM & holography

AM Radio
Modulation

\[ f(x) \quad \times \quad \cos(2\pi f_c x) \quad \rightarrow \quad \text{modulated } f(x) \]

Demodulation

\[ \text{modulated } f(x) \quad \times \quad \cos(2\pi f_c x) \quad \rightarrow \quad f(x) \]

Holography
Recording

\[ S \]

\[ |R + S|^2 \]

\[ R = e^{i2\pi f_0 x} \]

Reconstruction

\[ |R + S|^2 \]

\[ R = e^{i2\pi f_0 x} \]

\[ S \]

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