Laser oscillator

• How does one build a laser
  – Laser medium
    • Will dictate wavelength, pulse duration, achievable power
  – Pumping: Amplified Stimulated Emission (ASE)
    • Multimode in time and space
  – Resonator: laser oscillator
    • Mode selection

From R. Trebino

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Laser amplification I

• Amplification occurs within a frequency bandwidth given by the linewidth of the considered atomic transition
• The laser amplifier is a distributed-gain device characterized by the gain coefficient
  – When the photon flux is small

\[ \gamma_0(\nu) = N_0 \frac{\lambda^2}{8\pi t_{sp}} g(\nu) \]

  - Wavelength in medium
  - Transition cross section \( \sigma(\nu) \)
  - Equilibrium population density difference
  - Spontaneous lifetime

– When photon flux increases

\[ \gamma(\nu) = \frac{\gamma_0(\nu)}{1 + \phi / \phi_s(\nu)} \]

  - Saturation Photon flux density

\[ (\nu) = (\nu) + 1 + \phi / \phi_s(\nu) \]
Laser amplification II

- Amplification in a medium also introduces a phase shift

\[ \psi = \frac{\nu - \nu_0}{\Delta \nu} \gamma(\nu) \]
Feedback & Loss: Optical resonator

- Optical feedback is achieved by placing the gain medium in an optical resonator.
- Consider a simple resonator with two mirrors separated by $d$ the phase shift per unit length ($= \text{the wave vector}$) is

$$k = \frac{2\pi v}{c}$$

- The resonator also contributes losses in the system the round trip loss is characterized in term of a distributed loss coefficient $r_\alpha$ and is

$$\exp(-2\alpha_r d)$$

this includes mirrors reflection losses and scattering while propagation in the resonator.
Laser threshold I

- Two competing mechanisms
  - Gain in medium
  - Loss in resonator

- The initiation of laser oscillation requires that the small-signal gain coefficient is larger than the loss coefficient:

\[ \gamma_0(\nu) > \alpha_r \]

- Or equivalently that

\[ N_0 > \frac{\alpha_r}{\sigma(\nu)} \equiv N_t \]

\( N_t \) is the threshold population difference. Introducing the photon lifetime \( \alpha_r = 1/(c\tau_p) \).

\[ N_t = \frac{1}{c\tau_p \sigma(\nu)} \]
Laser Threshold II

- $N_t$ is a function of frequency

$$N_t = \frac{8\pi}{\lambda^2 c} t_{sp} \frac{1}{\tau_p g(\nu)}$$

the threshold is lowest where the lineshape function is highest!

- Remember that $g(\nu) = \frac{\Delta \nu / 2\pi}{(\nu - \nu_0)^2 + (\Delta \nu / 2)^2}$

so $g(\nu)$ maximum at $\nu_0$, and

$$N_t = \frac{2\pi}{\lambda^2 c} \frac{2\pi \Delta \nu t_{sp}}{\tau_p}$$

- If we further assume the linewidth is given by the lifetime only then

$$N_t = \frac{2\pi}{\lambda^2 c \tau_p} = \frac{2\pi \alpha_r}{\lambda^2}$$

Typical values are $N_t=10^{5-8}$ cm$^{-3}$

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Phase condition

- The phase shift imparted to the light in the resonator must be a multiple of $2\pi$:

$$2kd + 2\varphi(v)d = 2\pi q$$

Phase shift due to resonator

Medium-induced phase shift

- If $\varphi(v)d$ is small then

$$\nu = \nu_q = q\left(\frac{c}{2d}\right)$$

Already derived many times this is referred to as “cold resonator” condition

- If $\varphi(v)d$ cannot be neglected need to solve the equation

$$\nu + \frac{c}{2\pi} \frac{\nu - \nu_0}{\Delta \nu} \gamma(\nu) = \nu_q$$

This gives rise to a frequency pulling

\( \nu \)

\( \nu_0 \)

\( \nu_q \)

\( \Delta \nu \)

\( \Delta \nu_c \)

Cavity mode

Gain

Laser mode

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Internal photon flux density I

- Provided the threshold and phase conditions are fulfilled, lasing will start.

- Gain initially given by “small-signal” gain, but eventually decrease as photon flux increases.

- As long as gain larger than losses, growth of optical field will prevail.

\[ \Delta \nu_c, \nu_q, \nu_{q+1} \]

\[ \nu_0, \Delta \nu \]

Cold-resonator modes

Laser oscillation modes

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Internal photon flux density II

\[ \frac{\gamma(\nu)}{\gamma_0(\nu)} \]

Laser turn-on

Gain clamping

Steady state

Gain curve

\[ \frac{\phi}{\phi_s(\nu)} \]

\[ \alpha : \text{loss coefficient} \]

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Internal photon flux III

- Equating the large signal gain and loss coefficient gives the photon flux density arising from laser action.

\[ \varphi = \begin{cases} \varphi_s(v) \left( \frac{\gamma_0(v)}{\alpha_r} - 1 \right), & \gamma_0(v) > \alpha_r \\ 0, & \gamma_0(v) \leq \alpha_r \end{cases} \]

- Which can be rewritten in term of \( N_0 \) and \( N_t \) as

\[ \varphi = \begin{cases} \varphi_s(v) \left( \frac{N_0}{N_t} - 1 \right), & N_0 > N_t \\ 0, & N_0 \leq N_t \end{cases} \]
Output photon flux

• Consider the photon to be “out-coupled” from the cavity by having one mirror with transmittance $T_o$

• The output photon flux is then

$$\varphi_o = T_o \varphi$$

• The corresponding optical intensity is

$$I_o = h \nu T_o \varphi$$

• And the beam power would be

$$P_o = I_o A$$

Cross-sectional area of the laser beam