Interaction of photon with atoms

- Consider an atom located in an optical cavity and consider two of its energy levels to be E_1 and E_2 (assume $E_1<E_2$)

- The cavity support an optical mode with frequency ν_0

- Chose v_0 such that

$$h\nu = E_2 - E_1$$

  the photon energy matches the energy-level difference

- Three types of mechanism are possible:
  - Spontaneous emission
  - Absorption
  - Stimulated emission

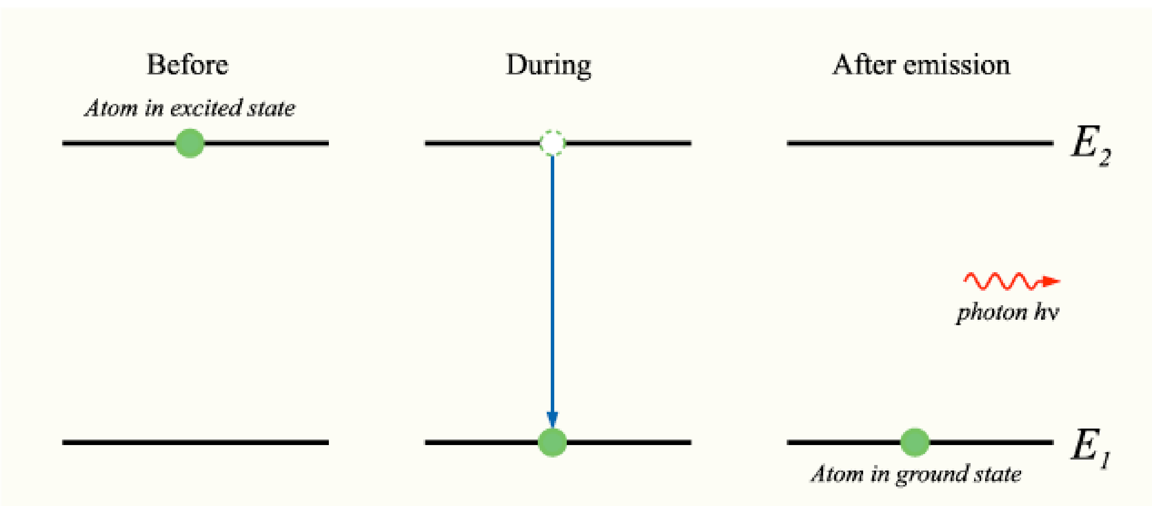
Spontaneous emission

- Atom is initially in "excited" state E2
- Atom decays spontaneously and add the energy hv to the optical mode
- The process is independent of the number of photon already in the optical mode
- The probability density is $p_{sp} = \frac{c}{V}\sigma(\upsilon)$

[s^{-1}]

Volume of cavity

Transition cross section [m^2]



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Absorption

- Atom is initially in state E_1
- Process is induced by a photon: the photon is annihilated and the atom go into excited state E_2
- The process is governed by same law as in spontaneous emission

$$p_{ab} = \frac{c}{V}\sigma(\upsilon)$$

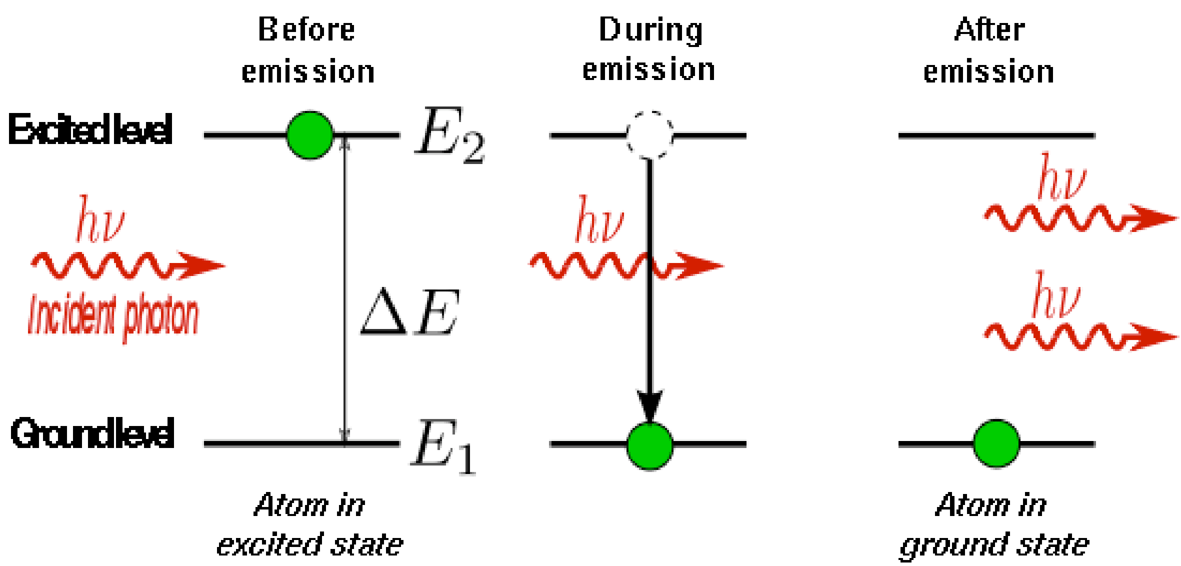
- However is there are n photons in the optical mode the probability is increased by a factor n so

$$P_{ab} = \frac{nc}{V}\sigma(\upsilon)$$

this is the probability of absorption of one photon from a mode with n photons

Stimulated emission

- Atom is initially in "excited" state E2 the optical mode contain a photon
- Atom may be induced to emit another photon into the same mode
- This is the inverse of the absorption process
- The presence of a photon in the mode stimulates the emission of a "clone" photon



$$E_2 - E_1 = \Delta E = h\nu$$

Stimulated emission

- The probability density of stimulated emission is same law that governs spontaneous emission and absorption

$$p_{st} = \frac{c}{V}\sigma(\upsilon)$$

- If the mode has originally n photon, then the probability to stimulate emission of one photon is n times larger

$$P_{st} = \frac{nc}{V}\sigma(\upsilon)$$

- Note that total probability for an atom to emit a photon is

$$P_{st} + p_{sp} = \frac{(n+1)c}{V}\sigma(\upsilon)$$

- Since $P_{ab}=P_{st}$ there are usually written as W_i the probability density of stimulated emission and absorption

Lineshape function

- The transition cross section characterizes the interaction of the atom with the radiation its area

$$S = \int_0^\infty \sigma(\upsilon) d\upsilon$$

is called transition (or oscillator) strength and its shape gives the dependence of the magnitude of the interaction on frequencies

- The line-shape function g(ν) is

$$g(\upsilon) = \frac{\sigma(\upsilon)}{S}$$

It is normalized to unity, centered around the resonance frequency has units of Hz^{-1}
its width is ~ the inverse of resonance bandwidth

Spontaneous Emission

- The equation

$$p_{sp} = \frac{c}{V}\sigma(\upsilon)$$

gives the probability density for spontaneous emission into one mode of frequency n

- The density of mode for a 3d resonator is

$$M(\upsilon) = \frac{8\pi\nu^2}{c^3}$$

Number of mode of frequency n per unit of volume of the cavity per unit of bandwidth

- An atom may emit one photon in any of these modes
- The overall spontaneous-emission probability is

$$P_{sp} = \int_0^\infty [M(\nu)V]\frac{c}{V}\overline{\sigma}(\nu)d\nu = c\int_0^\infty M(\nu)\overline{\sigma}(\nu)d\nu$$

Spontaneous Emission

- The overall spontaneous-emission probability is

$$P_{sp} = c\int_0^\infty M(\nu)\overline{\sigma}(\nu)d\nu$$

$\sigma(\nu)$ is sharply peaked at $\nu=\nu_0$, and $M(\nu)$ is slowly varying

$$\approx cM(\nu_0)\int_0^\infty \overline{\sigma}(\nu)d\nu \equiv cM(\nu_0)\overline{S} = \frac{8\pi\overline{S}}{\lambda^2}$$

- A "spontaneous lifetime" can be defined as

$$t_{sp} = \frac{1}{P_{sp}} \quad \text{and therefore} \quad \frac{1}{t_{sp}} = \frac{8\pi\overline{S}}{\lambda^2}$$

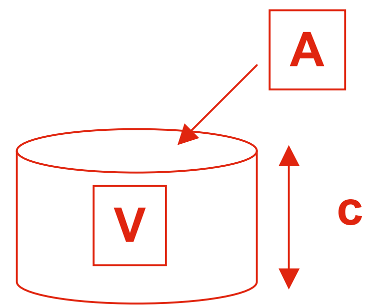
- One can infer the transition strength from a measurement of the spontaneous lifetime
- The average cross section is related the the lineshape function via

$$\overline{\sigma}(\nu) = \frac{\lambda^2}{8\pi t_{sp}} g(\nu)$$

Stimulated emission & absorption

- Consider the interaction of a **single-mode** light with an atom
- Light characterized by its mean photon flux (photon/m^2/s)

$$\varphi = \frac{I}{h\nu}$$



- The photon flux is $\Phi = \varphi A$
- So $\varphi = n\frac{c}{V}$
- And the probability of stimulated emission is

$$W_i = \varphi\sigma(\nu)$$

so σ is a coefficient of proportionality between the probability of an induced transition and the photon flux

Transition induced by broad light I

- Consider the interaction of a **polychromatic** light with an atom
- The light spectral energy density (energy per unit volume per unit banwidth) is $\rho(\nu)$
- So average photon in [ν,ν+dν] is $\rho(\nu)V\frac{d\nu}{h\nu}$
- Each photon has the probability $\frac{c}{V}\sigma(\nu)$ of initiating a transition
- So the overall probability

$$W_i = \int_0^\infty \frac{\rho(\nu)V}{h\nu}\left[\frac{c}{V}\sigma(\nu)\right]d\nu$$

$$\approx \frac{\rho(\nu_0)}{h\nu_0}c\int_0^\infty \sigma(\nu)d\nu = \frac{\rho(\nu_0)}{h\nu_0}cS$$

Transition induced by broad light II

- So $W_i = \dfrac{\lambda^3}{8\pi h t_{sp}} \rho(\nu_0)$
- Defining the mean number of photon per mode to be

$$\bar{n} = \frac{\lambda^3}{8\pi h} \rho(\nu_0)$$

- We have

$$W_i = \frac{\bar{n}}{t_{sp}}$$

- Einstein A and B coefficients

$$P_{sp} \equiv A$$

$$W_i \equiv B\rho(\nu_0)$$

Line broadening

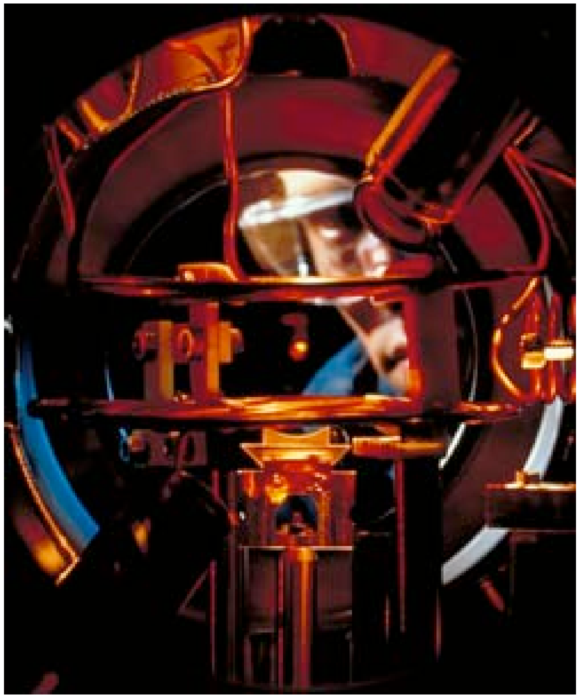
- We did not yet specified the lineshape function
- It usually follows a Lorentzian distribution of the form

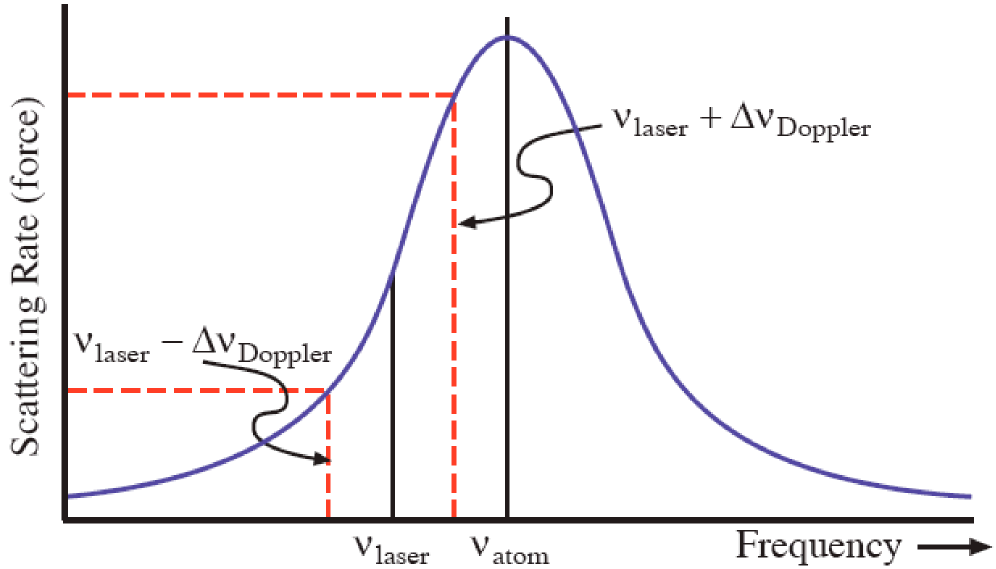
$$g(\nu) = \frac{\Delta\nu / 2\pi}{(\nu - \nu_0)^2 + (\Delta\nu / 2)^2}$$

- Origin of bandwidth:
  - Collision broadening
  - Doppler effects

Example: application of Doppler broadening

- Laser cooling of atom is based on Doppler broadening
- Laser tuned to a frequency slightly below transition frequency
  - Only atom with velocity matching the Doppler shifted frequency will be excited (counter propagating atoms)
  - Spontaneous emission velocity component along the laser direction decreases
  - Use high intensity laser to induce significant damping

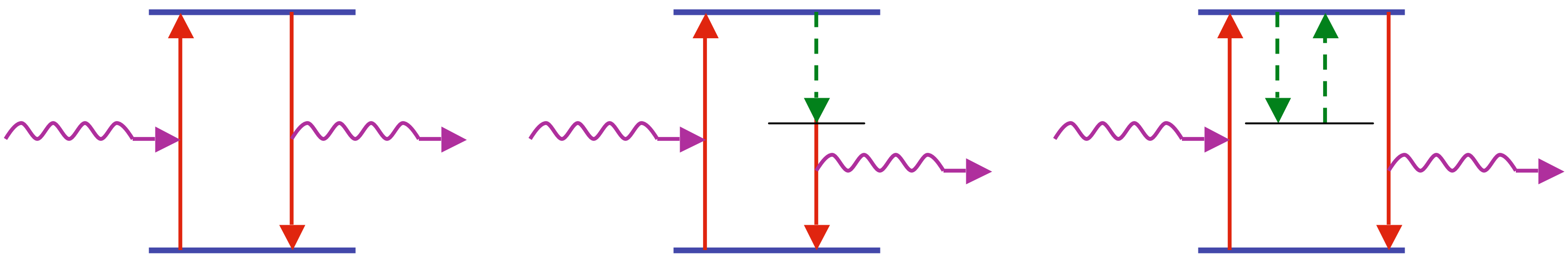




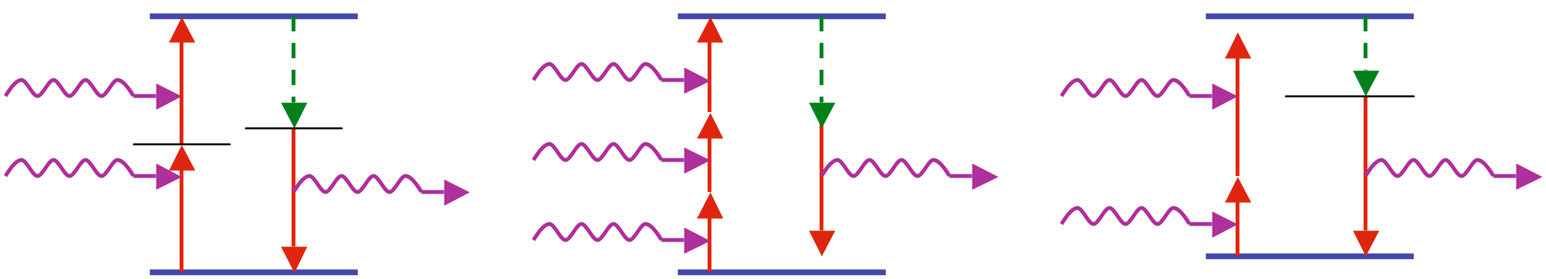
http://focus.aps.org/story/v21/st11

Photoluminescence

- System excited to higher energy level by absorption and decay



Single-photon photoluminescence



Two-photon *Three-photon* *Upconversion*

Multiple-photon photoluminescence