Lasers (5-6 lessons)

- Resonator
- Interaction of Atoms and photons
- Amplification
- Generation of laser light
Resonator: introduction

- Resonator are the main ingredient of lasers.

- Used to increase the optical power associated to a mode.

- Boundary conditions implies the existence of “eigenmodes” and eigenfrequencies.

- Simplest model (to understand the physics and the mathematical description) is one-dimensional.
Resonator: standing wave approach

- Take the case of a plane-parallel resonator configuration

- Consider an optical wave with complex amplitude

\[ U(r, t) = U(\vec{r}) e^{2i\pi \nu t} \]

- The boundaries conditions imposed by the planar mirror gives rise to a “quantification” of the wave vector

\[ k_q = q \frac{\pi}{d} \quad \nu_q = q \frac{c}{2d} \]

with associated mode having a complex amplitudes

\[ U(\vec{r}) = \sum_q A_q(\vec{r}) \sin(k_q z) \]

- The mode frequency separation is

\[ \nu_f = \frac{c}{2d} \]
Resonator: traveling wave approach

- Modes can be determined by following a traveling wave as it travels back and forth between two mirrors.

- The phase shift imparted by a round trip is

\[ \phi = 2kd = q2\pi \]
Density of Modes

• The number of modes per unit frequency is the inverse of the frequency spacing between the mode.

• The density of mode is the number of mode per unit of frequency and unit of resonator length.

• The number of mode per unit of frequency is

\[ \frac{1}{\nu_f} = \frac{2d}{c} \]

• For one-dimensional resonator this is

\[ M(\nu) = \frac{4}{c} \]

Factor 2 to account for 2 orthogonal polarization
Losses & resonance spectral width

• In a realistic resonator losses are present (mirror reflection non unity, or due to the medium composing the resonator)

• Consider the loss per round trip to be

\[ h = |r| e^{-i\phi} \quad \phi = 2kd \]

• Then the complex amplitude summed over an infinite number of passes is

\[ U = U_0 + U_1 + U_2 + \ldots \]

\[ = U_0(1 + h + h^2 + \ldots) = \frac{U_0}{1 - h}. \]

• For one-dimensional resonator this is

\[ I = \frac{I_0}{|1 - h|^2} = \frac{I_0}{1 + |r|^2 - 2|r| \cos \phi}. \]
The latter expression can be written

\[ I = \frac{I_{\text{max}}}{1 + (2\mathcal{F}/\pi)^2 \sin^2(\phi/2)}. \]

Where

\[ \mathcal{F} \equiv \frac{\pi \sqrt{|r|}}{1 - |r|}. \]

\[ I_{\text{max}} \equiv \frac{I_0}{(1 - |r|)^2}. \]

Expliciting \( \phi \) gives

\[ I = \frac{I_{\text{max}}}{1 + (2\mathcal{F}/\pi)^2 \sin^2(\pi \nu/\nu_f)}. \]
\[
I/I_{\text{max}} \approx \frac{\nu_f}{\delta \nu}
\]

\[
I_{\text{min}} = \frac{I_{\text{max}}}{1 + (2\mathcal{F}/\pi)^2}
\]
Losses I

• Consider a resonator with losses

\[ |r|^2 = R_1 R_2 e^{-2\alpha_s d} \]

Mirror 1  Mirror 2  Distributed losses

• And rewrite as a distributed loss

\[ |r|^2 = e^{-2\alpha_r d} \]

• Identify to get

\[ \alpha_r = \alpha_s + \frac{1}{2d} \ln \frac{1}{R_1 R_2} \]

\[ \equiv \alpha_s + \alpha_{m1} + \alpha_{m2} \]

where \( \alpha_{mi} \equiv \frac{1}{2d} \ln \frac{1}{R_i} \)

\[ \frac{R_i}{2d} \approx 1 - \frac{1}{R_i} \]

Ignore diffraction losses

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Losses II

• Now specialize to identical mirrors

\[ R_1 = R_2 = R \]

• The finesse is then

\[ \mathcal{F} = \frac{\pi e^{-\alpha_r d/2}}{1 - e^{-\alpha_r d}} \approx \frac{\pi}{\alpha_r d} \]

so finesse inversely proportional to losses

• “Photon lifetime” \( \tau_p \equiv 1/(c\alpha_r) \)

\[ \delta \nu \sim \frac{\nu_f}{\mathcal{F}} \sim \frac{c/2d}{\pi/\alpha_r d} \sim \frac{c\alpha_r}{2\pi} \]
Q-factor

- Figure-of-merit for microwave, optical and electronic oscillator systems

- Defined as

\[ Q = 2\pi \frac{\text{stored energy}}{\text{energy loss per cycle}}. \]

- so

\[ Q = \frac{2\pi}{c\alpha_r \nu_0} = \frac{2\pi \nu_0}{c\alpha_r} \]

\[ = \frac{\nu_0}{\delta\nu} \]

\[ = \frac{\nu_0}{\nu_f} \mathcal{F}. \]

Finesse is proportional to Q

Considered frequency
Spherical-mirror resonator I

- Difference with planar-mirror resonator is transverse focusing effects.

- Naively would think that beam can be over-focused and eventually diverges

- The pass $n$ is related to pass $n-1$ via the matrix equation

$$
\begin{bmatrix}
    x_n \\
    x'_n
\end{bmatrix}
= M
\begin{bmatrix}
    x_{n-1} \\
    x'_{n-1}
\end{bmatrix}
$$

where

$$
M = \begin{bmatrix}
    1 & 0 \\
    \frac{2}{R_1} & 1
\end{bmatrix}
\begin{bmatrix}
    1 & d \\
    0 & 1
\end{bmatrix}
\begin{bmatrix}
    1 & 0 \\
    \frac{2}{R_2} & 1
\end{bmatrix}
\begin{bmatrix}
    1 & d \\
    0 & 1
\end{bmatrix}
$$
The condition for stability is $\text{Tr}(M)<2$ which gives

$$0 \leq \left(1 + \frac{d}{R_1}\right) \left(1 + \frac{d}{R_2}\right) \leq 1$$
Gaussian Modes

- Gaussian beams are the modes associated to spherical mirror resonator
- Given the phase
  \[ \phi(r, z) = kz - \zeta(z) - \frac{kr^2}{2R(z)} \]

- The on-axis phase at the two mirror locations are
  \[ \phi(0, z_1) = kz_1 - \zeta(z_1) \]
  \[ \phi(0, z_2) = kz_2 - \zeta(z_2) \]

so that the total round-trip phase shift is
  \[ \Phi = 2(kd - \Delta\zeta) \]

- From which the resonator frequencies are obtained
  \[ \nu_q = q\nu_f + \frac{\Delta\zeta}{\pi}\nu_f \]
3d rectangular resonators

- Boundary conditions are imposed along all directions

- Resonator frequencies

\[ \nu_q = \sqrt{q_x^2 \nu_{fx}^2 + q_y^2 \nu_{fy}^2 + q_z^2 \nu_{fz}^2} \]

with \[ \nu_{fi} = \frac{c}{2d_i} \]

- Mode density

\[ M(\nu) = \frac{8\pi\nu^2}{c^3} \]
Whispering gallery

- Example of a 2-D micro-resonator

St. Paul Cathedral Church

Hexagonal Prism Blue Laser using Whispering Gallery Mode (WGM) Resonances

WGM (whispering gallery mode)
Resonance path in MQW (active) region

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Whispering mode gallery

- Dynamical resonator using a droplet of oil.