PHYS 690C: Homework, set #4 (all points are bonus)

due date: May 8th in my mailbox.

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exercise 1: Consider the K-V distribution

$$f(x,x',y,y') = f_0 \delta \left[\left(\frac{A_x^2}{\epsilon_x} + \frac{A_y^2}{\epsilon_y} \right) - 1 \right],$$

where $A_i \equiv \frac{i^2}{w_i^2} + (w_i i - w_i' i)^2$ for $i \in \{x, y\}$, and f_0 is a normalization factor. Prove that the KV distribution yields a uniform distribution in the x-y plane and give an expression for the density.

exercise 2: Similarly to the way we derived the rms envelope equation in x, y, derive the envelope equation for the x, y, and z directions. You will assume the velocity of the beam is $\overrightarrow{v} \simeq v\hat{z} \equiv v_z \hat{z}$, and introduce $x' \equiv dx/ds \simeq v_x/v \ll 1$, similarly for y-axis and $z' \equiv dz/ds$. You should recognize that $\overrightarrow{v} = \beta c(x', y', 1 + z')$. In the final results, the moments $\langle xF_x \rangle$, $\langle yF_y \rangle$ and $\langle zF_z \rangle$ should appear (do not try to compute them!). The envelope equation should be of the form $\sigma_i'' + ... = ...$ with $i \in \{x, y, z\}$

exercise 3: Consider a uniform ellipse distribution in the x-y plane with an infinite extent in z-direction. Derive the electrostatic potential produced by such a distribution (I gave the result in class), you can help yourself with a more general derivation given in *R. L. Gluckstern "scalar potential for charge distributions with ellipsoidal symmetry", Fermilab Technical memo TM-1402* (available from Fermilab library website).
