exercise 1: Consider the K-V distribution

\[ f(x, x', y, y') = f_0 \delta \left( \frac{A_x^2}{\epsilon_x} + \frac{A_y^2}{\epsilon_y} - 1 \right), \]

where \( A_i \equiv \frac{i^2}{w_i} + (w_i - w'_{i})^2 \) for \( i \in \{x, y\} \), and \( f_0 \) is a normalization factor. Prove that the KV distribution yields a uniform distribution in the \( x-y \) plane and give an expression for the density.

exercise 2: Similarly to the way we derived the rms envelope equation in \( x, y \), derive the envelope equation for the \( x, y, \) and \( z \) directions. You will assume the velocity of the beam is \( \overrightarrow{v} \simeq v\hat{z} \equiv v_z\hat{z} \), and introduce \( x' \equiv dx/ds \simeq v_x/v \ll 1 \), similarly for \( y \)-axis and \( z' \equiv dz/ds \). You should recognize that \( \overrightarrow{v} = \beta c (x', y', 1 + z') \). In the final results, the moments \( \langle xF_x \rangle, \langle yF_y \rangle \) and \( \langle zF_z \rangle \) should appear (do not try to compute them!). The envelope equation should be of the form \( \sigma''_i + \ldots = \ldots \) with \( i \in \{x, y, z\} \)

exercise 3: Consider a uniform ellipse distribution in the \( x-y \) plane with an infinite extent in \( z \)-direction. Derive the electrostatic potential produced by such a distribution (I gave the result in class), you can help yourself with a more general derivation given in R. L. Gluckstern “scalar potential for charge distributions with ellipsoidal symmetry”, Fermilab Technical memo TM-1402 (available from Fermilab library website).