PHYS 690C: Homework, set #2

due date: Mar. 13th in my mailbox.

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exercise 1: The Fermilab 8 GeV proton booster is part of the Tevatron complex. Its parameters are as follows [1]: injection momentum $cp_{inj} = 400$ MeV, maximum momentum $cp_{max} = 8.9$ GeV, harmonic number $h \equiv \frac{1}{2\pi}\omega T_s = 84$ (ω is the rf-frequency and T_s, the revolution period of the synchronous particle), peak rf-voltage $V = 200$ kV, transition "energy" $\gamma_t = 5.4$, and rf-frequency at maximum momentum $f_{rf} = 52.8$ MHz.

1. Calculate the synchrotron oscillation frequency.
2. What are the injection and maximum energies, E_{inj} and E_{max} corresponding respectively to p_{inj} and p_{max}?
3. Calculate and plot the rf and synchrotron oscillation frequency as a function of momentum p with $p_{inj} < p < p_{max}$.
4. what is the energy acceptance ΔE_{max} at injection and maximum energy?
5. How long does the acceleration last?

exercise 2: Consider the synchrotron motion of a particle inside the separatrix for a stationary rf bucket.

1. Derive an expression for the frequency of the synchrotron oscillation as a function of $\Delta\phi_0$, the maximum excursion of the synchrotron phase.
2. Compute the oscillation frequencies in function of the small amplitude synchrotron frequency.

exercise 3: Consider the transformation of a **non-relativistic** particle through an accelerating gap:

$$\begin{aligned} W_f &= W_i + qVT(k)\cos\phi_i \\ \phi_f &= \phi_i \end{aligned}$$

[1] I use the notations of Lecture Notes pg. Long.13; for precise definitions you can also look at http://doc.cern.ch/yellowrep/1994/94-01/p289.pdf

wherein W_i, W_f are respectively the initial and final particle's energies, q the particle's charge, $T(k)$ the transit time factor [2], V the accelerating voltage and $k \equiv \omega/(\beta c)$.

1. Show that the Jacobian of the transformation is not unity.

2. Discuss how to change the above transformation to make the Jacobian unity up to a second order term in $qV/(W_i)$. In particular consider the introduction of a phase jump $\phi_f = \phi_i + C$. Give a possible value for C. This problem was first recognized in the 1960's by Lapostolle and Prome[3]. [hint: note that the second term on the rhs of W_f depends on energy via k; you can introduce $T' = (dT)/(dk)$.]

exercise 4: Consider a traveling wave accelerating structure. The axial electric field is of the form

$$E_z(z,t) = E_o \sin(\omega t - kz + \psi_o), \tag{1}$$

where E_0 is the peak field, k the rf wavenumber and ψ_0 the injection phase of the particle with respect to the rf wave. Let $\psi(z,t) \equiv \omega t - kz + \psi_o$ be the relative phase of the electron w.r.t the wave.

1. Derive the system of coupled first order ODE that describes the longitudinal motion of a particle moving along z in a linear accelerator experiencing the electric field $E_z(z,t)$. You will use γ, ψ as"phase space" variables.

2. Solve for ψ as a function of γ; you need to introduce ψ_0 and γ_0 the initial conditions. Assume the incoming beam is relativistics.

3. Find an expression for the asymptotic value of the phase $\psi_\infty \equiv \lim_{\gamma\to\infty} \psi(\gamma)$ as a function of the initial injection phase.

4. Compute the compression ratio $\mathcal{C} \equiv \frac{\partial \psi_\infty}{\partial \psi_0}$ and discuss the physical meaning of $\mathcal{C}$.

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[2] see Lecture Notes pg. Long 3

[3] see for instance P. M. Lapostolle, "Proton Linear accelerator: A theoretical and historical introduction", report LA-11601-MS July 1989 available from Los Alamos National Lab.