PHYS 690C: Homework, set #1

due date: Feb 24th in my mailbox.

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exercise 1: By analogy with the "ABCD" matrix formalism in photonic optics, the trace-space coordinates of a particle at a given location (1), are transformed, at a downstream location (2), following: $\overrightarrow{X}^{(2)} = M^{1\to 2}\overrightarrow{X}^{(1)}$. Here $\overrightarrow{X} \equiv \widetilde{(x, x')}$ (as in the Notes " $\sim$ " stands for the transpose operator), and $M^{1\to 2}$ is the transfer matrix between points (1) and (2) in the trace space (x, x') associated to the transformation. We consider the transformation matrix associated to a free-space drift D and a thin focusing lens F to be:

$$D = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}, \text{ and, } \quad F = \begin{bmatrix} 1 & 0 \\ \kappa & 1 \end{bmatrix}, \tag{1}$$

wherein L is the drift length and κ the focusing strength of the lens. We consider a beam with initial beam matrix Σ_0 that passes through the lens and then the drift. Let Σ_F be the final beam matrix.

1. Find the final beam matrix Σ_F and write it as a function of the beam's initial moments.
2. Deduce an expression for the final squared rms beam size of the beam ($\langle x^2 \rangle$).
3. Explain how a measurement of the final squared rms beam size as a function of the lens strength yields the initial beam matrix elements (and initial beam emittance).

exercise 2: Consider a beam experiencing a force of the form $F_x \propto x$ so that the transverse momentum becomes $p_x \to p_x + \mu x$ where μ is a real scalar. Show that such a force does not change the value of the statistical rms emittance.

exercise 3: Properties of the trace-space ellipse. Consider the trace space ellipse with equation $\gamma x^2 + 2\alpha x x' + \beta x'^2 = \tilde{\varepsilon}$. The parameters α, β and γ are the so-called Twiss parameters, they are related by $\gamma = \frac{1+\alpha^2}{\beta}$.

1. Derive the values of x_{RMS}, x'_{RMS}, x_{INT} and x'_{INT} defined in Fig. **??** as a function of the Twiss parameters.

2. What is the value of the slope defined as $m \equiv \tan(\phi)$?

3. Consider the shearing transformation $x' \rightarrow \hat{x}' = x' + m'x$ what is the required value for m' in order to generate a statistically uncorrelated distribution in (x, x') space (i.e. meaning $\langle x\hat{x}' \rangle = 0$).

4. By identification of m and m', find the expression of α as a function of the beam's second order moments.

5. A shearing transformation does not change the beam's emittance (see exercise 2). By using the results of the previous questions retrieve the statistical emittance definition in the trace-space: $\tilde{\varepsilon} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$.

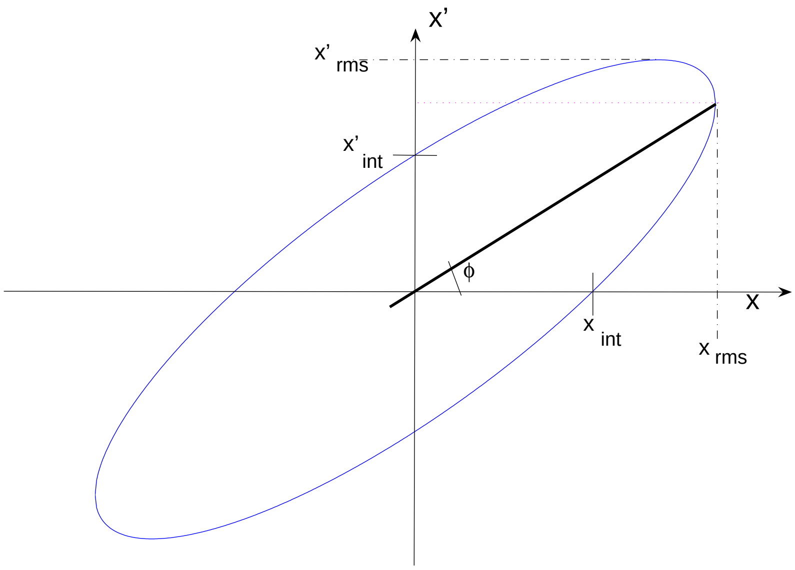

Figure 1: figure of exercise 3.

exercise 4: Show that for a charge distribution that is a uniform ellipsoid in x, y, and z, the second order centered moments are 1/5 of the maximum values.
