

Planetary Motion (con't)

- Experimental observations (made prior to telescopes) were used to understand motion of the planets
- Difference between “apparent motion” and “real” motion understood – key part of modern science.
- Lead to Kepler’s 3 laws of planetary motion
- Provided experimental observations which are later explained by physics developed by Galileo, Newton and others

Brahe and Kepler



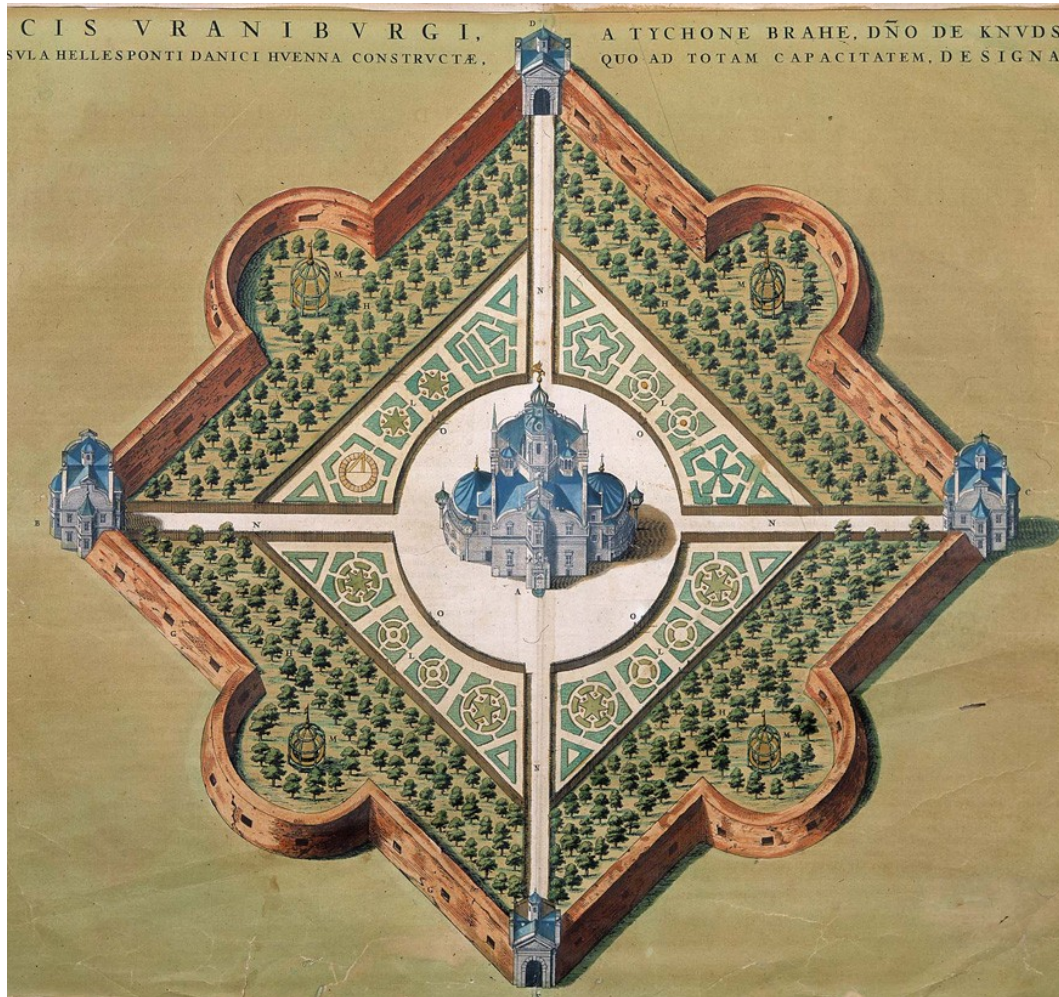
- Brahe led team which collected data on position of planets (1580-1600 no telescopes)
- Kepler (mathematician) hired by Brahe to analyze data. Determined 3 Laws of planetary motion (1600-1630)
- Input - 20 years of data on:
 - angular position of planets
 - approximate distances from Earth (accurate relative distances)
- Few “modern” tools (no calculus, no graph paper, no log tables)

Observations of Brahe 1580-1600

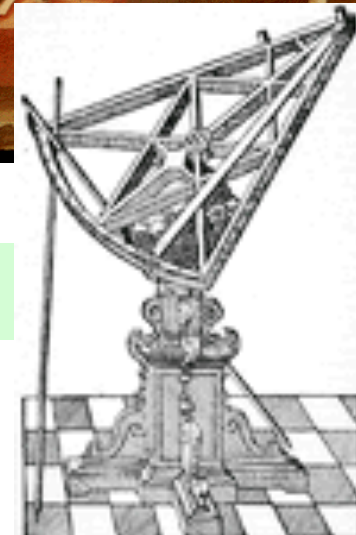
- Brahe was a Danish nobleman who became famous after observing a supernova and showing it was “far away”
- Danish king provided funding and an island where Brahe set up an observatory – no telescopes just (essentially) sextants - that is long sticks to measure angles which could be flipped to measure both E-W and N-S angle at same time



Brahe's Observatory



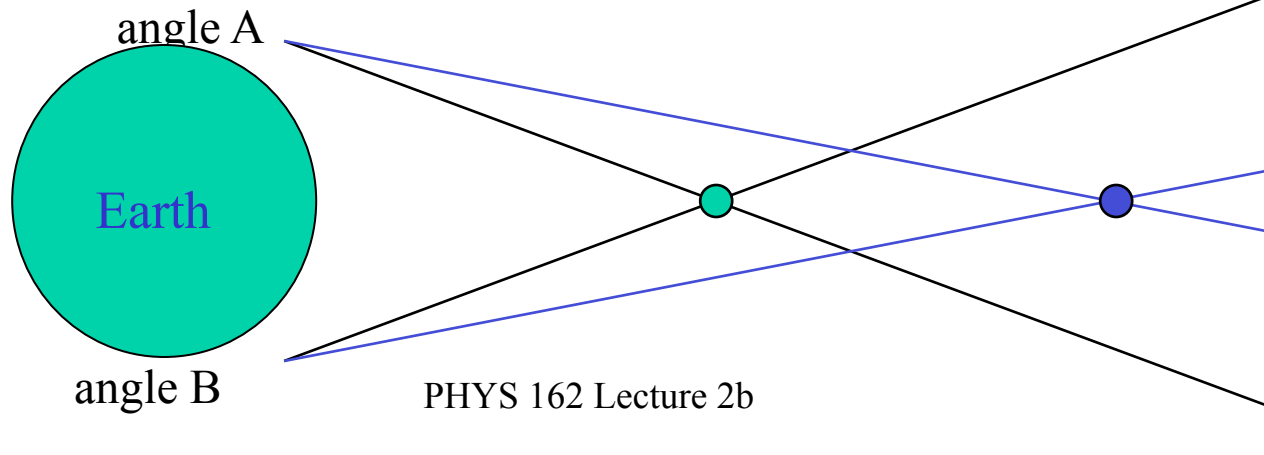
sextant



Apparent Shift = Parallax

- A moving observer sees fixed objects move.
- Near objects appear to move more than far objects
- The effect is due to the change in observation point, and is used by our eyes for depth perception.

Geocentric parallax



Sources of Parallax

- Heliocentric parallax uses the sun as a base.
- Take a photo with telescope at two different seasons → come back to later
- Geocentric parallax uses the earth as a base.
- Make a measurement two or more times in one night.
- Use for planets → Brahe's data also had distances to planets plus position in sky

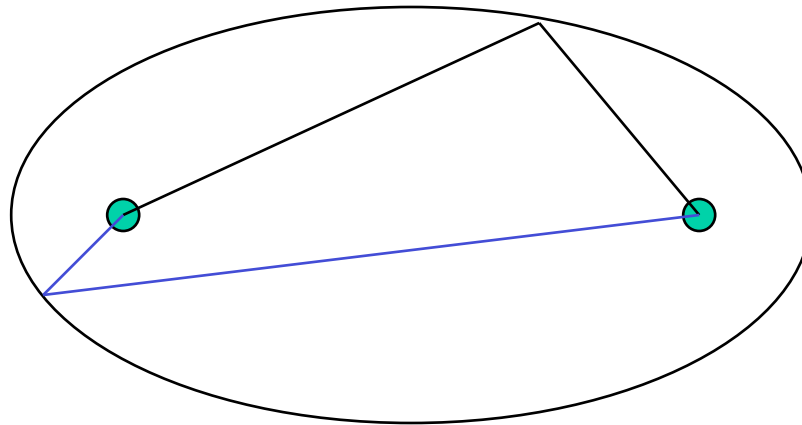
Kepler's Laws of Motion

- Kepler figured out correct orbital shape and determined some relationships between the orbits of different planets
- A big step was realizing that Earth's orbit about the Sun also wasn't a circle – mostly he used relative location of Mars after repeated orbits around the Sun (Mars is close and so most accurate measurements)



Kepler's Laws of Planetary Motion (1630)

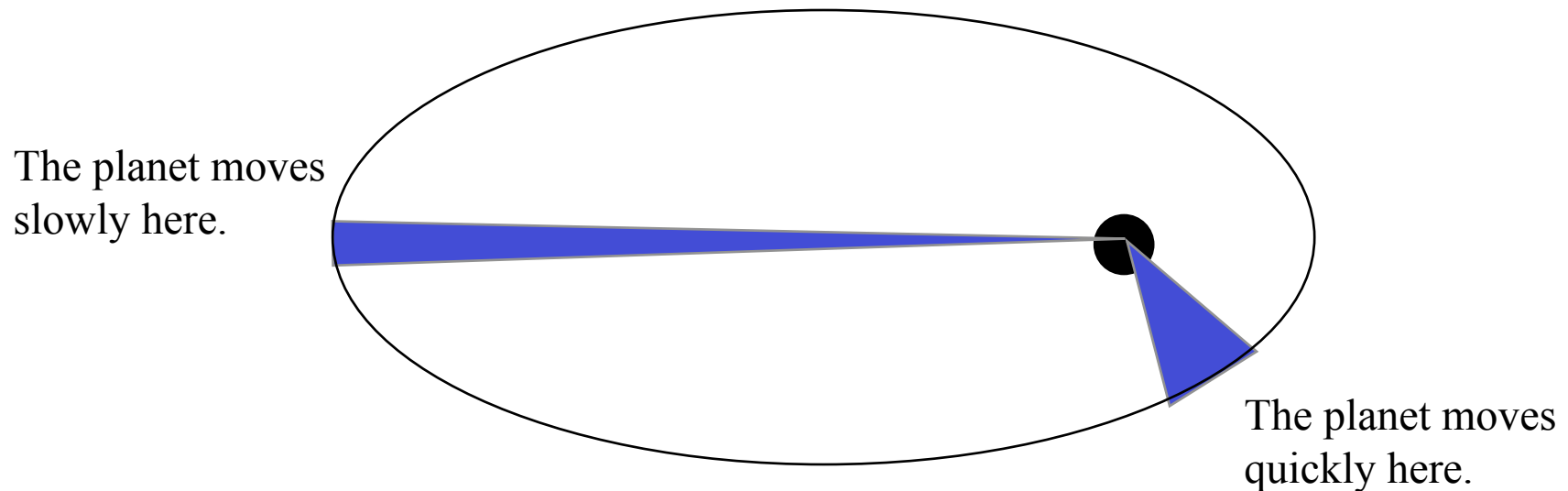
FIRST LAW: The orbit of a planet is an ellipse with the sun at one focus.



A line connecting the two foci in the ellipse always has the same length.

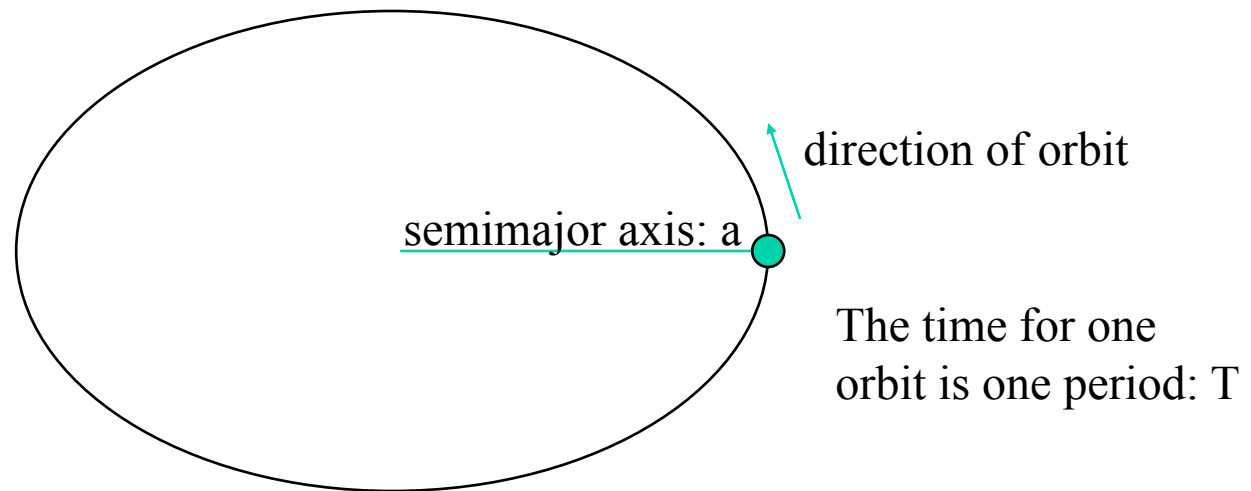
Kepler's Second Law

- The line joining a planet and the sun sweeps equal areas in equal time.



Kepler's Third Law

- The square of a planet's period is proportional to the cube of the length of the orbit's semimajor axis.
- Mathematically, $T^2/a^3 = \text{constant}$. (=1 if use 1 Earth year and 1 AU as units), or $T^2 = \text{const} \times a^3$.
- The constant is the same for all objects orbiting the Sun.



ics

| | Mean Distance from Sun | Sidereal Orbital Period |
|---------|-----------------------------------|------------------------------------|
| | AU | P_e |
| Mercury | 0.387 | 0.241 |
| Venus | 0.723 | 0.615 |
| Earth | 1.000 | 1.000 |
| Mars | 1.524 | 1.881 |
| Jupiter | 5.203 | 11.857 |
| Saturn | 9.537 | 29.424 |
| Uranus | 19.191 | 83.749 |
| Neptune | 30.069 | 163.727 |

Third Law Example

- Jupiter compared to Earth
- If we measure that it takes Jupiter 11.9 years to orbit the Sun then:

$$\text{distance}^3(\text{Jupiter-Sun}) = \text{period}^2$$

$$\text{distance} = \text{period}^{2/3}$$

$$\text{distance} = (11.9 * 11.9)^{1/3}$$

$$\text{distance} = (142)^{1/3} = 5.2 \text{ AU}$$

- Kepler correctly determined the motion of the planets.
- Did not address WHY. Simply what curve best matched orbits and some arithmetical relationships
- The WHY was determined by physicists like Galileo and Newton.
- They needed to develop Physics as a science: understand motion, forces, and gravity

Newton and Kepler

- Newton's First Law of Motion:
 - A body remains at rest or moves in a straight line at a constant speed unless it is acted upon by an outside force
- Newton's Second Law of Motion:
 - If a force, F , works on a body of mass M , then the acceleration, A , is given by
 - $F = M \cdot A$
- Newton's Third Law of Motion:
 - If one body exerts a force on a second body, the second body exerts an equal and opposite force on the first
- Newton's Universal Law of Gravitation
 - $F_{\text{gravity}} = G M_1 M_2 / r^2$
 - $G = 6.67 \times 10^{-11} \text{ meters}^3 \text{ kilograms}^{-1} \text{ seconds}^{-2}$

Newton and Kepler (2)...

- When you combine Newton's gravitation and circular acceleration, which must balance in order for the object to remain in orbit, you get a nice relation between the period, distance, and mass of the central body.
 - $F_{\text{grav}} = F_{\text{cent}} \quad \rightarrow \quad \text{equate gravitational force to centripetal force}$
 - $F_{\text{grav}} = G m_1 m_2 / r^2$
 - $F_{\text{cent}} = m_2 V^2 / r$
- Let the Earth be m_1 and the Moon be m_2 . For **circular motion** the distance r is the semi-major axis A .
 - $V = 2 \pi A / T \quad \rightarrow \quad T = \text{period of moon} \quad \rightarrow \quad G m_1 m_2 / A^2 = m_2 V^2 / A$
 - $G m_1 / A^2 = ((2 \pi A)^2 / T^2) / A \quad \rightarrow \quad \text{note that } m_2 \text{'s cancel!}$
- Rearrange to place all the A terms on the right and all the T terms on the left:
 - $G m_1 / (4 \pi^2) T^2 = A^3 \quad \text{Looks just like } \mathbf{Kepler's Third Law!}$
- To use A and P to solve for mass:
 - $m_1 = A^3 (4 \pi^2 / G) / T^2$