On to neutrinos!

The pp Chain

Step 1: Two protons make a deuteron

$$p + p \rightarrow d + e^{+} + \nu_e$$
$$p + p + e^{-} \rightarrow d + \nu_e$$

Step 2: Deuteron plus proton makes ^{3}He.

$$d + p \rightarrow {}^{3}\mathrm{He} + \gamma$$

Step 3: Helium-3 makes alpha particle or ^{7}Be.

$$^{3}\mathrm{He} + p \rightarrow \alpha + e^{-} + \nu_e$$
$$^{3}\mathrm{He} + {}^{3}\mathrm{He} \rightarrow \alpha + p + p$$
$$^{3}\mathrm{He} + \alpha \rightarrow {}^{7}\mathrm{Be} + \gamma$$

Step 4: Berillium makes alpha particles.

$$^{7}\mathrm{Be} + e^{-} \rightarrow {}^{7}\mathrm{Li} + \nu_e$$
$$^{7}\mathrm{Li} + p \rightarrow \alpha + \alpha$$
$$^{7}\mathrm{Be} + p \rightarrow {}^{8}\mathrm{B} + \gamma$$
$$^{8}\mathrm{B} \rightarrow {}^{8}\mathrm{Be}^{*} + e^{+} + \nu_e$$
$$^{8}\mathrm{Be}^{*} \rightarrow \alpha + \alpha$$

From Griffiths

Fairly complicated decay chain for nuclear fusion in the sun. Very well modeled and predicted, first by John Bahcall

Ray Davies

NOT our physicist of interest. This is Ray Davies from the Kinks

The (physicist) Ray Davies

Worked with Bahcall on the Homestake experiment in the 1960s. Collected and count solar neutrinos. Need to perform neutrino experiments deep underground (often in mines). Why?

Homestake

Let's double check what is happening in this collision

$$\nu_e + ^{37}Cl \rightarrow ^{37}Ar + e^-$$

https://www.bnl.gov/bnlweb/raydavis/images/hires/1-390-66.jpg

Literally collecting single argon atoms (crazy!) produced every few days, and found only 1/3 of prediction. WHY?!

Explanation: neutrino oscillations

$$\nu_e +^{37} Cl \rightarrow^{37} Ar + e^-$$

Above reaction only occurs for electron neutrinos. If some of the neutrinos en route to Earth convert to muon or tau neutrinos, lepton number says the above doesn't take place!

Possible if the mass eigenstates are not the same as the flavor eigenstates

Two-flavor oscillations

The mixing angle is an unknown parameter to be determined

Imagine that the eigenstates of mass and of the Hamiltonian are

$$\nu_1 = \cos\theta\nu_\mu - \sin\theta\nu_e$$
$$\nu_2 = \sin\theta\nu_\mu + \cos\theta\nu_e$$

Naive quantum mechanics tells us:

$$\nu_1(t) = \nu_1(0)e^{-iE_1 t}$$
$$\nu_2(t) = \nu_2(0)e^{-iE_2 t}$$

If the solar neutrinos are only electron neutrinos:

$$\nu_1(t=0) = -\sin\theta$$
$$\nu_2(t=0) = \cos\theta$$

Two-flavor oscillations

$$\nu_1(t) = \nu_1(0)e^{-iE_1 t} \qquad \nu_1(t=0) = -\sin\theta$$

$$\nu_2(t) = \nu_2(0)e^{-iE_2 t} \qquad \nu_2(t=0) = \cos\theta$$

$$\nu_1(t) = -\sin\theta e^{-iE_1 t}$$

$$\nu_2(t) = \cos\theta e^{-iE_2 t}$$

$$\nu_1 = \cos\theta\nu_\mu - \sin\theta\nu_e$$

$$\nu_2 = \sin\theta\nu_\mu + \cos\theta\nu_e$$

$$\longrightarrow$$

$$\nu_1\cos\theta = \cos^2\theta\nu_\mu - \sin\theta\cos\theta\nu_e$$

$$\nu_2\sin\theta = \sin^2\theta\nu_\mu + \sin\theta\cos\theta\nu_e$$

$$\nu_1\cos\theta + \nu_2\sin\theta = \nu_\mu$$

Two-flavor oscillations

$$\nu_1(t) = -\sin\theta e^{-iE_1 t}$$
$$\nu_2(t) = \cos\theta e^{-iE_2 t}$$

$$\nu_1 \cos\theta + \nu_2 \sin\theta = \nu_\mu$$

$$\nu_1 \cos\theta + \nu_2 \sin\theta = \nu_\mu$$

$$\nu_\mu(t) = -\cos\theta\sin\theta e^{-iE_1 t} + \cos\theta\sin\theta e^{-iE_1 t}$$
$$\nu_\mu(t) = \cos\theta\sin\theta(e^{-iE_2 t} - e^{-iE_1 t})$$

$$|\nu_\mu(t)|^2 = (\cos\theta\sin\theta)^2(e^{-iE_2 t} - e^{-iE_1 t})(e^{iE_2 t} - e^{iE_1 t})$$
$$|\nu_\mu(t)|^2 = \frac{\sin^2 2\theta}{4}(1 + 1 - e^{it(E_2 - E_1)} - e^{it(E_1 - E_2)})$$
$$|\nu_\mu(t)|^2 = \frac{\sin^2 2\theta}{4}(2 - 2\cos{(E_2 - E_1)t})$$

Two-flavor oscillations

$$|\nu_\mu(t)|^2 = \frac{\sin^2 2\theta}{4}(2 - 2\cos{(E_2 - E_1)t})$$

$$|\nu_\mu(t)|^2 = \frac{\sin^2 2\theta}{4}(2 - 2(1 - 2\sin^2\frac{(E_2 - E_1)t}{2})$$

$$|\nu_\mu(t)|^2 = \frac{\sin^2 2\theta}{4}(4\sin^2\frac{(E_2 - E_1)t}{2})$$

$$|\nu_\mu(t)|^2 = \sin^2 2\theta(\sin^2\frac{(E_2 - E_1)t}{2})$$

Probability to have a muon neutrino (we started out with a pure electron neutrino beam) oscillates over time!

Two-flavor oscillations

$$|\nu_\mu(t)|^2 = \sin^2 2\theta (\sin^2 \frac{(E_2 - E_1)t}{2})$$

$$E = \sqrt{|\mathbf{p}|^2 + m^2} = \sqrt{|\mathbf{p}|^2(1 + \frac{m^2}{|\mathbf{p}|^2})}$$

$$E = |\mathbf{p}|\sqrt{(1 + \frac{m^2}{|\mathbf{p}|^2})} \sim |\mathbf{p}|(1 + \frac{m^2}{2|\mathbf{p}|^2})$$

For small particles (like neutrinos)

$$E_2 - E_1 \sim |\mathbf{p_2}|(1 + \frac{m_2^2}{2|\mathbf{p_2}|^2}) - |\mathbf{p_1}|(1 + \frac{m_1^2}{2|\mathbf{p_1}|^2})$$

$$E_2 - E_1 \sim \frac{m_2^2}{2|\mathbf{p}|} - \frac{m_1^2}{2|\mathbf{p_1}|} \sim \frac{m_2^2 - m_1^2}{2|\mathbf{p}|}$$

$$E_2 - E_1 \sim \frac{m_2^2 - m_1^2}{2E}$$

Two-flavor oscillations

$$|\nu_\mu(t)|^2 = \sin^2 2\theta (\sin^2 \frac{(E_2 - E_1)t}{2})$$

$$E_2 - E_1 \sim \frac{m_2^2 - m_1^2}{2E}$$

$$|\nu_\mu(t)|^2 = \sin^2 2\theta (\sin^2 \frac{(m_2^2 - m_1^2)t}{4})$$

Note sensitivity to mass differences, not individual masses!

For us, neutrinos travel at ~speed of light = 1, so d=ct = t

$$|\nu_\mu(t)|^2 = \sin^2 2\theta (\sin^2 \frac{(m_2^2 - m_1^2)d}{4})$$

Sometimes it's convenient to write this differently

$$E_2 - E_1 \sim \frac{m_2^2 - m_1^2}{2E} \qquad |\nu_\mu(t)|^2 = \sin^2 2\theta (\sin^2 \frac{(m_2^2 - m_1^2)d}{4})$$

Make sure to put things in the right units

$$|\nu_\mu(t)|^2 = \sin^2 2\theta \sin^2\left(1.27\Delta m^2 \frac{d}{E}\right)$$

Δm^2 in units of eV^2 and d/E in units km/GeV

Confirmation

Detection Methods

Homestake experiment (1968):

$$\nu_e + {}^{37}\mathrm{Cl} \rightarrow {}^{37}\mathrm{Ar} + e$$

Super-Kamiokande experiment (1998):

$$\nu + e \rightarrow \nu + e$$

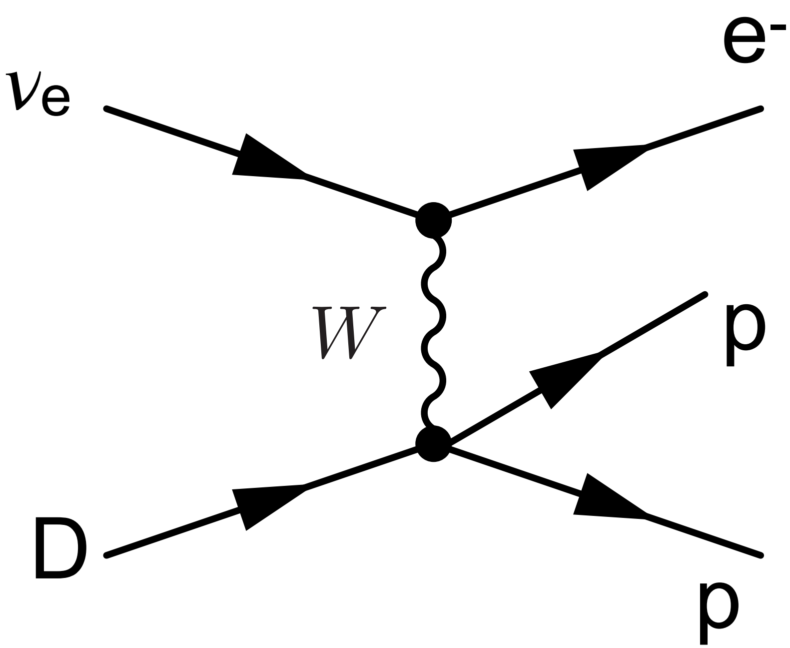
Solar neutrino observatory (2002):

$$\nu_e + d \rightarrow p + p + e$$
$$\nu + d \rightarrow n + p + \nu$$
$$\nu + e \rightarrow \nu + e$$

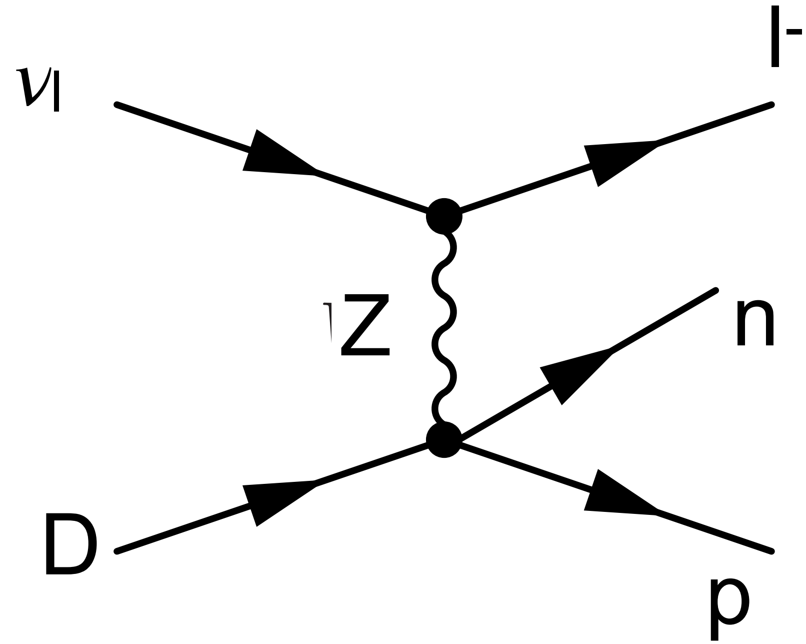
More recently, studies of atmospheric and nuclear reactor neutrinos (different energies and flavors!)

Super K and SNO confirmed this picture

What sorts of Feynman diagrams contribute to e-nu scattering?

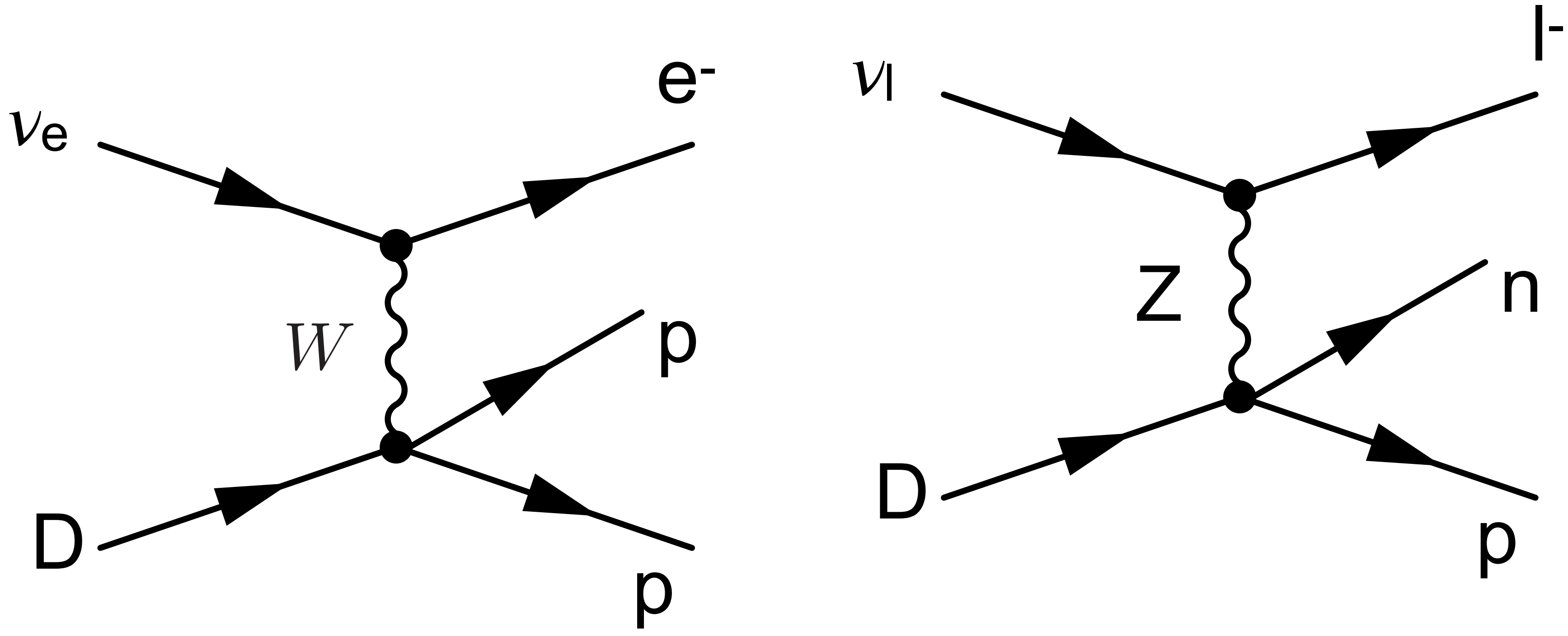


CC interaction



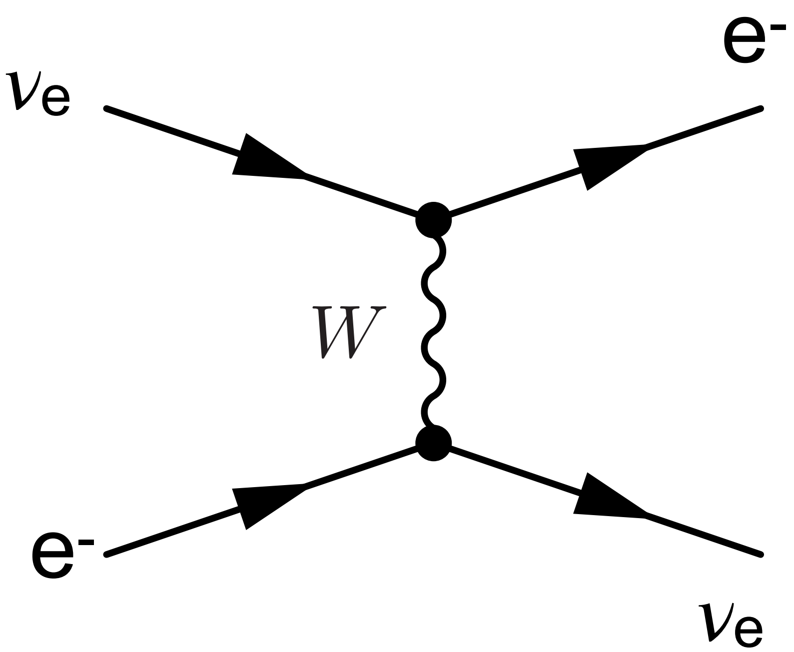
NC interaction

Deuterons have a very small binding energy compared to solar energy of neutrinos, "easy" to break up. But only electrons participate in the CC interaction (muons and taus are "massive")

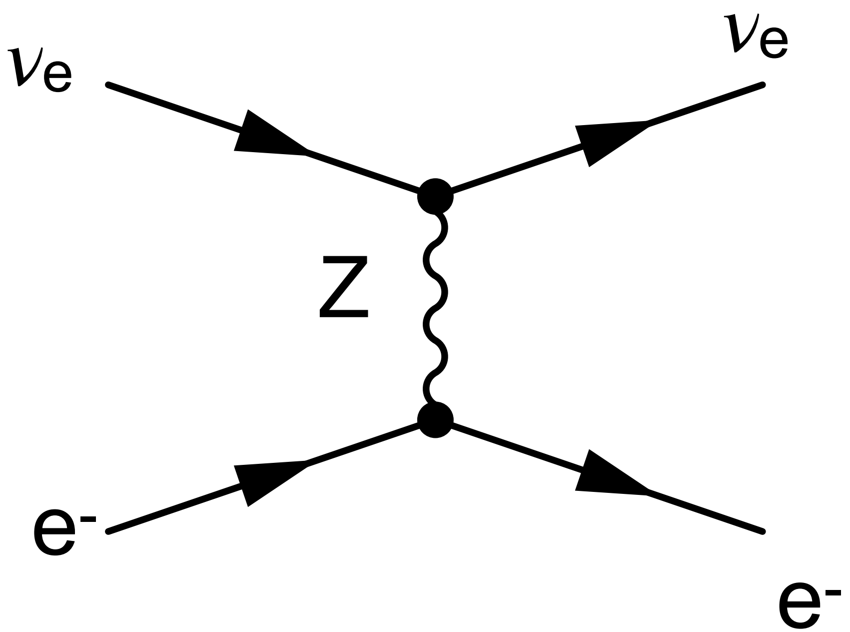


CC interaction NC interaction

Electrons from CC interaction are isotropic. Neutrons from NC interaction gets captured by hydrogen in water, in the process releasing a photon of specific energy



Elastic scattering

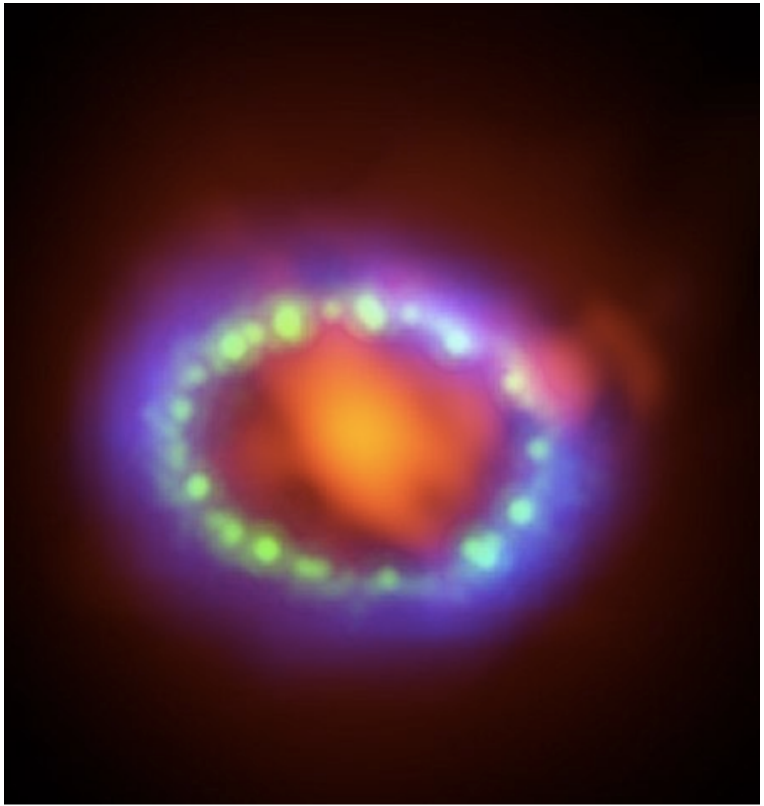


Elastic scattering

Only electron neutrinos can undergo these processes. They also point directly to the sun (lab frame is not the same as the CM frame like on previous slide)

Limits on neutrino masses

Earlier limits from spread of arrival times of neutrinos from SN1987A supernova



More recently, better limits from tritium beta decay (order eV). How?

KATRIN experiment underway

Three-flavor oscillations

Have sensitivity to two mass-squared differences:

$M_{32}=|m^2(3)-m^2(2)|$

$M_{21}=|m^2(2)-m^2(1)|$

Note only two such "differences" for 3 neutrinos

We don't know the order of the (small) neutrino masses, either

3x3 mixing matrix U is the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix, which has three Euler angles and 1 (Dirac) or 3 (Majorana) phases

Three-flavor oscillations

https://www.annualreviews.org/doi/10.1146/annurev-nucl-102014-021939

$$\nu_{lL} = \sum_{j=1}^{3} U_{lj}\nu_{jL},$$

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta_{CP}} \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta_{CP}} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta_{CP}} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} \times \mathrm{diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}).$$

$$P_{ll'} \equiv P(\nu_l \to \nu_{l'}) = \left|\sum_i U_{li}U^*_{l'i}e^{-i(m_i^2/2E)L}\right|^2$$

$$= \sum_i |U_{li}U^*_{l'i}|^2 + \Re\sum_i\sum_{j\neq i} U_{li}U^*_{l'i}U^*_{lj}U_{l'j}e^{i\frac{\Delta m^2_{ij}L}{2E}}$$

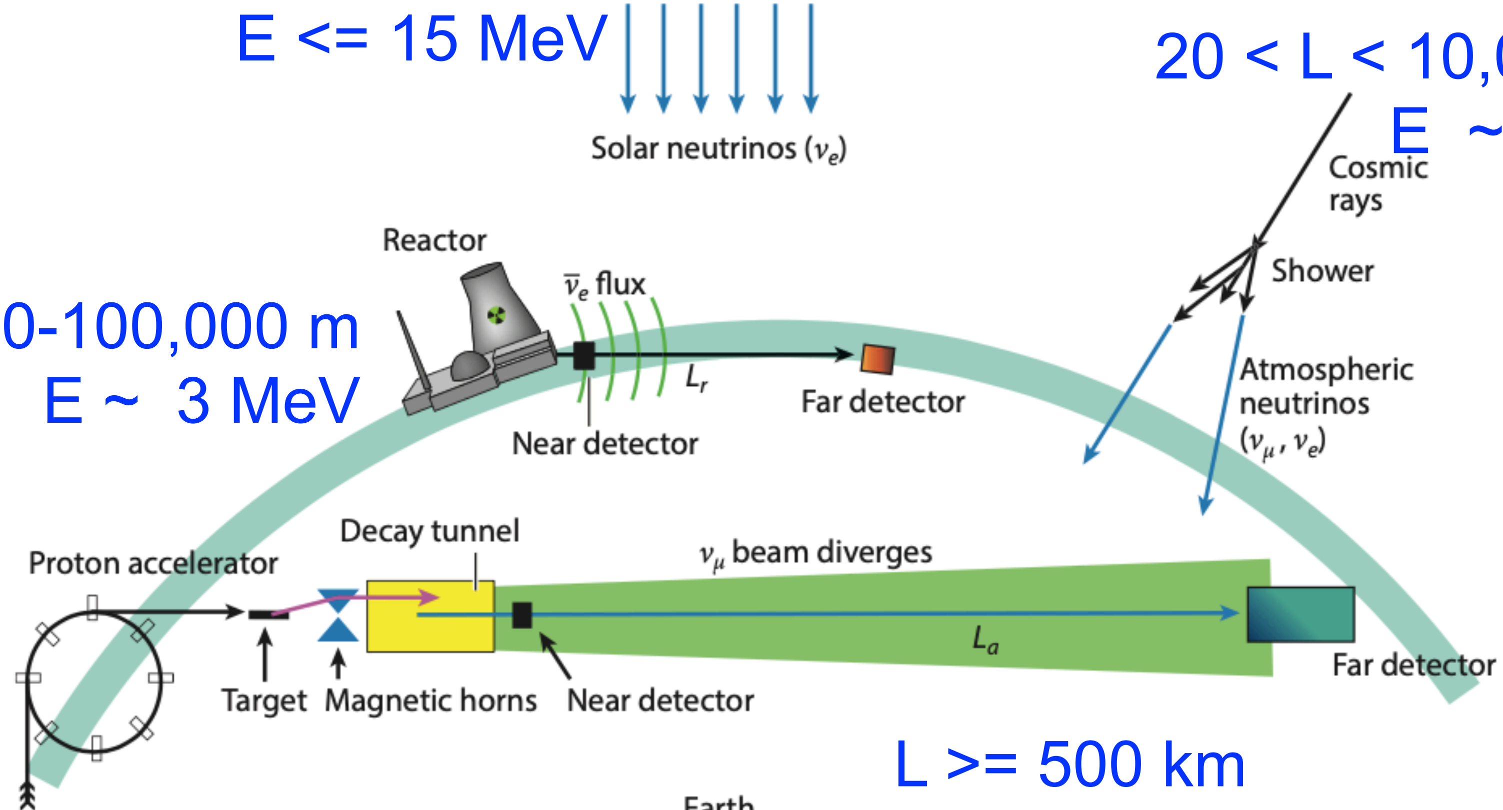
How to study neutrino oscillations these days?

https://www.annualreviews.org/doi/10.1146/annurev-nucl-102014-021939

$L \sim 10^{11}$ m
E <= 15 MeV

20 < L < 10,000 km
E ~ 1 GeV

L ~ 10-100,000 m
E ~ 3 MeV



L >= 500 km
E ~ 1 GeV

Figure 1

Neutrino sources that have contributed to the current understanding of neutrino properties through neutrino oscillation experiments. (*Top*) The Sun produces electron neutrinos (ν_e). (*Right*) Neutrinos of two types, ν_μ and ν_e, and their antiparticles are produced by collisions of high-energy cosmic rays with atoms in the Earth's atmosphere. (*Center*) Nuclear reactors emit electron antineutrinos ($\bar{\nu}_e$) isotropically. (*Bottom*) High-energy proton accelerators produce a beam of neutrinos, predominantly ν_μ or $\bar{\nu}_\mu$, that is directed through the Earth.

Reactor and accelerator experiments can have near + far detectors!

Summary of neutrino experiments

Table 14.1: Characteristic values of L and E for experiments performed using various neutrino sources and the corresponding ranges of $|\Delta m^2|$ to which they can be most sensitive to flavour oscillations in vacuum. SBL stands for Short Baseline and LBL for Long Baseline.

Experiment		L (m)	E (MeV)	$\|\Delta m^2\|$ (eV^2)
Solar		10^{10}	1	10^{-10}
Atmospheric		$10^4 - 10^7$	10^2–10^5	$10^{-1} - 10^{-4}$
Reactor	SBL	$10^2 - 10^3$	1	$10^{-2} - 10^{-3}$
	LBL	$10^4 - 10^5$		$10^{-4} - 10^{-5}$
Accelerator	SBL	10^2	10^3–10^4	> 0.1
	LBL	$10^5 - 10^6$	$10^3 - 10^4$	$10^{-2} - 10^{-3}$

http://pdg.lbl.gov/2019/reviews/rpp2019-rev-neutrino-mixing.pdf

Recent neutrino results from PDG

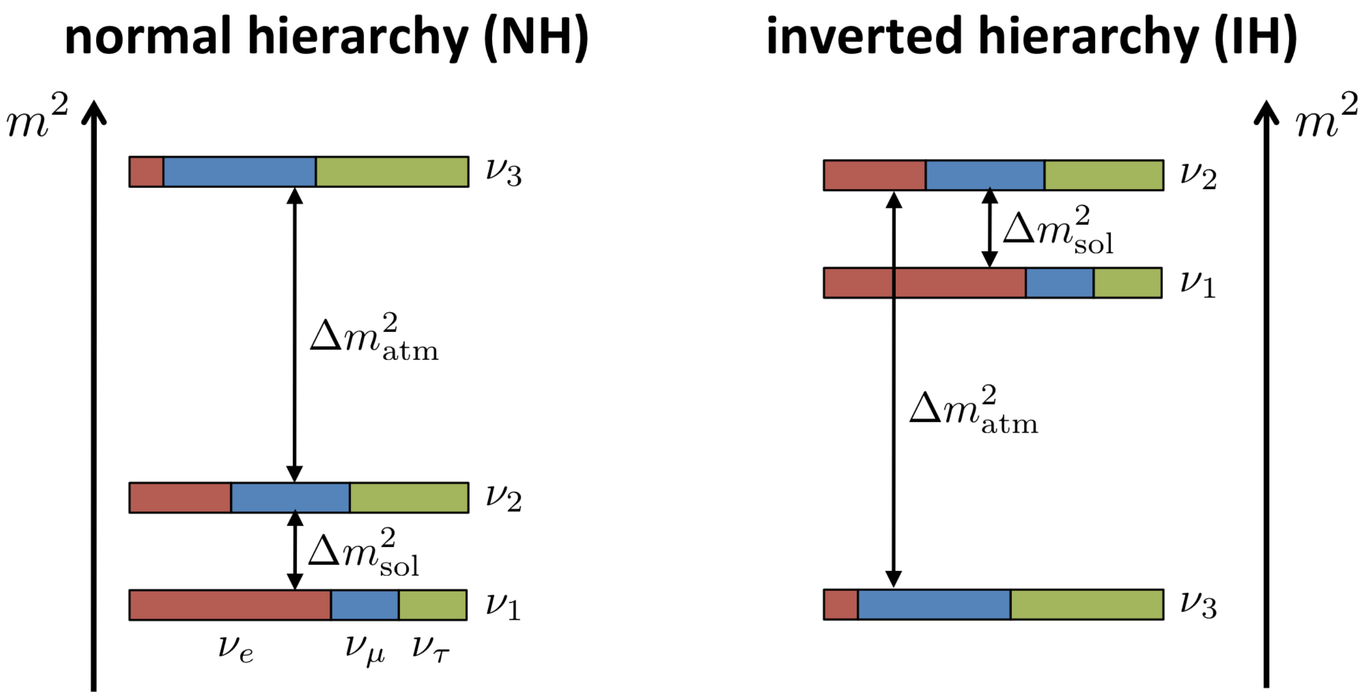
http://pdg.lbl.gov/2019/reviews/rpp2019-rev-neutrino-mixing.pdf

Table 14.7: 3ν oscillation parameters obtained from different global analysis of neutrino data. In all cases the numbers labeled as NO (IO) are obtained assuming NO (IO), *i.e.*, relative to the respective local minimum. SK-ATM makes reference to the tabulated χ^2 map from the Super-Kamiokande analysis of their data in Ref. [94].

	Ref. [188] w/o SK-ATM		Ref. [188] w SK-ATM		Ref. [189] w SK-ATM		Ref. [190] w SK-ATM	
NO	Best Fit Ordering		Best Fit Ordering		Best Fit Ordering		Best Fit Ordering	
Param	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\frac{\sin^2\theta_{12}}{10^{-1}}$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.04^{+0.14}_{-0.13}$	$2.65 \to 3.46$	$3.20^{+0.20}_{-0.16}$	$2.73 \to 3.79$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.46^{+0.87}_{-0.88}$	$30.98 \to 36.03$	$34.5^{+1.2}_{-1.0}$	$31.5 \to 38.0$
$\frac{\sin^2\theta_{23}}{10^{-1}}$	$5.58^{+0.20}_{-0.33}$	$4.27 \to 6.09$	$5.63^{+0.18}_{-0.24}$	$4.33 \to 6.09$	$5.51^{+0.19}_{-0.80}$	$4.30 \to 6.02$	$5.47^{+0.20}_{-0.30}$	$4.45 \to 5.99$
$\theta_{23}/^\circ$	$48.3^{+1.2}_{-1.9}$	$40.8 \to 51.3$	$48.6^{+1.0}_{-1.4}$	$41.1 \to 51.3$	$47.9^{+1.1}_{-4.0}$	$41.0 \to 50.9$	$47.7^{+1.2}_{-1.7}$	$41.8 \to 50.7$
$\frac{\sin^2\theta_{13}}{10^{-2}}$	$2.241^{+0.066}_{-0.065}$	$2.046 \to 2.440$	$2.237^{+0.066}_{-0.065}$	$2.044 \to 2.435$	$2.14^{+0.09}_{-0.07}$	$1.90 \to 2.39$	$2.160^{+0.083}_{-0.069}$	$1.96 \to 2.41$
$\theta_{13}/^\circ$	$8.61^{+0.13}_{-0.13}$	$8.22 \to 8.99$	$8.60^{+0.13}_{-0.13}$	$8.22 \to 8.98$	$8.41^{+0.18}_{-0.14}$	$7.9 \to 8.9$	$8.45^{+0.16}_{-0.14}$	$8.0 \to 8.9$
$\delta_{CP}/^\circ$	222^{+38}_{-28}	$141 \to 370$	221^{+39}_{-28}	$144 \to 357$	238^{+41}_{-33}	$149 \to 358$	218^{+38}_{-27}	$157 \to 349$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.34^{+0.17}_{-0.14}$	$6.92 \to 7.91$	$7.55^{+0.20}_{-0.16}$	$7.05 \to 8.24$
$\frac{\Delta m^2_{32}}{10^{-3}\ \mathrm{eV}^2}$	$2.449^{+0.032}_{-0.030}$	$2.358 \to 2.544$	$2.454^{+0.029}_{-0.031}$	$2.362 \to 2.544$	$2.419^{+0.035}_{-0.032}$	$2.319 \to 2.521$	2.424 ± 0.03	$2.334 \to 2.524$
IO	$\Delta\chi^2 = 6.2$		$\Delta\chi^2 = 10.4$		$\Delta\chi^2 = 9.5$		$\Delta\chi^2 = 11.7$	
$\frac{\sin^2\theta_{12}}{10^{-1}}$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.03^{+0.14}_{-0.13}$	$2.64 \to 3.45$	$3.20^{+0.20}_{-0.16}$	$2.73 \to 3.79$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.75}$	$31.62 \to 36.27$	$33.40^{+0.87}_{-0.81}$	$30.92 \to 35.97$	$34.5^{+1.2}_{-1.0}$	$31.5 \to 38.0$
$\frac{\sin^2\theta_{23}}{10^{-1}}$	$5.63^{+0.19}_{-0.26}$	$4.30 \to 6.12$	$5.65^{+0.17}_{-0.22}$	$4.36 \to 6.10$	$5.57^{+0.17}_{-0.24}$	$4.44 \to 6.03$	$5.51^{+0.18}_{-0.30}$	$4.53 \to 5.98$
$\theta_{23}/^\circ$	$48.6^{+1.1}_{-1.5}$	$41.0 \to 51.5$	$48.8^{+1.0}_{-1.2}$	$41.4 \to 51.3$	$48.2^{+1.0}_{-1.4}$	$41.8 \to 50.9$	$47.9^{+1.0}_{-1.7}$	$42.3 \to 50.7$
$\frac{\sin^2\theta_{13}}{10^{-2}}$	$2.261^{+0.067}_{-0.064}$	$2.066 \to 2.461$	$2.259^{+0.065}_{-0.065}$	$2.064 \to 2.457$	$2.18^{+0.08}_{-0.07}$	$1.95 \to 2.43$	$2.220^{+0.074}_{-0.076}$	$1.99 \to 2.44$
$\theta_{13}/^\circ$	$8.65^{+0.13}_{-0.12}$	$8.26 \to 9.02$	$8.64^{+0.12}_{-0.13}$	$8.26 \to 9.02$	$8.49^{+0.15}_{-0.14}$	$8.0 \to 9.0$	$8.53^{+0.14}_{-0.15}$	$8.1 \to 9.0$
$\delta_{CP}/^\circ$	285^{+24}_{-26}	$205 \to 354$	282^{+23}_{-25}	$205 \to 348$	247^{+26}_{-27}	$193 \to 346$	281^{+23}_{-27}	$202 \to 349$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.34^{+0.17}_{-0.14}$	$6.92 \to 7.91$	$7.55^{+0.20}_{-0.16}$	$7.05 \to 8.24$
$\frac{\Delta m^2_{32}}{10^{-3}\ \mathrm{eV}^2}$	$-2.509^{+0.032}_{-0.032}$	$-2.603 \to -2.416$	$-2.510^{+0.030}_{-0.031}$	$-2.601 \to -2.419$	$-2.478^{+0.035}_{-0.033}$	$-2.577 \to -2.375$	$-2.50\pm^{+0.04}_{-0.03}$	$-2.59 \to -2.39$

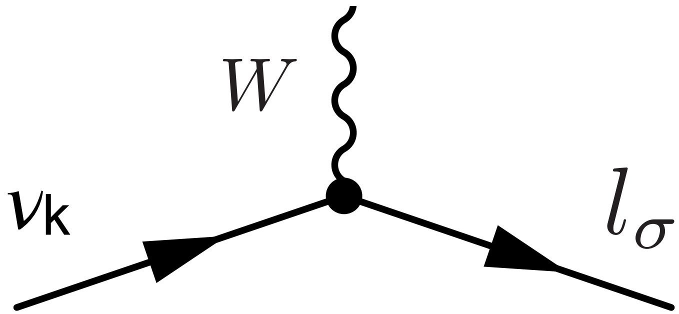
Hierarchy of neutrino masses

http://www.staff.uni-mainz.de/wurmm/juno.html



Don't know yet which of these is correct

New vertex for Feynman diagrams

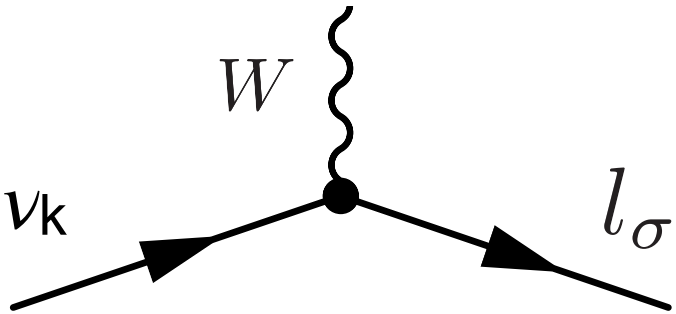


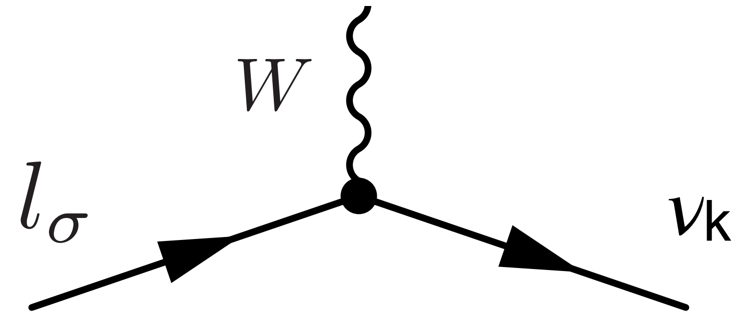
$$\frac{-ig_w}{2\sqrt{2}}\bar{e}\gamma^\mu(1-\gamma^5)\nu_e$$

$$\frac{-ig_w}{2\sqrt{2}}\overline{l_\sigma}\gamma^\mu(1-\gamma^5)U_{\sigma k}\nu_k$$

U is the PMNS mixing matrix that converts between the neutrino flavor and mass eigenstates

New vertex for Feynman diagrams





$$\frac{-ig_w}{2\sqrt{2}}\overline{l_\sigma}\gamma^\mu(1-\gamma^5)U_{\sigma k}\nu_k \qquad \frac{-ig_w}{2\sqrt{2}}U_{\sigma k}\overline{\nu_k}\gamma^\mu(1-\gamma^5)l_\sigma$$

U is the PMNS mixing matrix that converts between the neutrino flavor and mass eigenstates

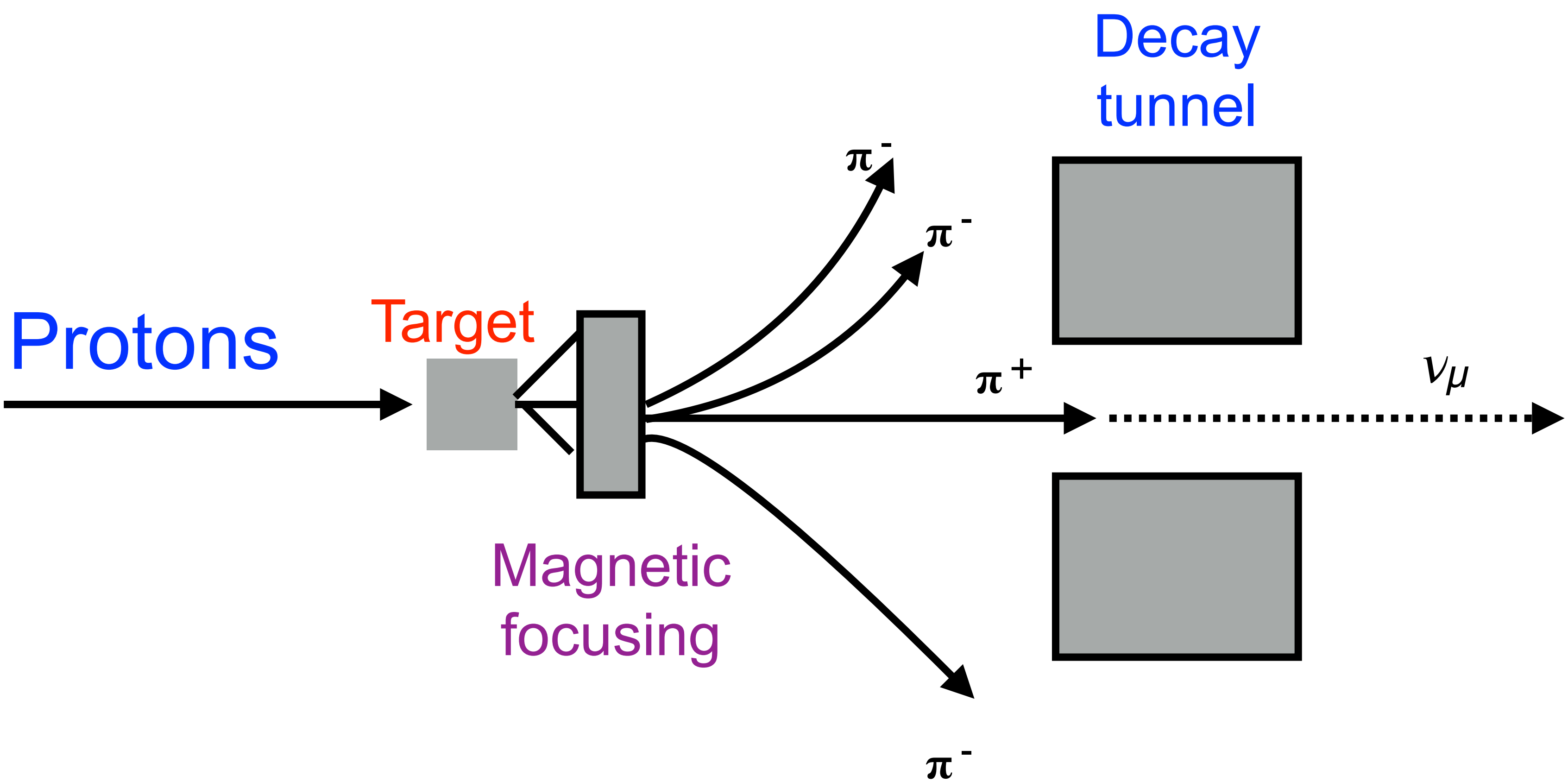
CP violation

CP violation is required to explain the excess of matter over anti-matter in the Universe. And of the known forces, it must come from the weak sector since QED and QCD conserve C and P separately. Can neutrinos contribute?

$$P(\nu_e \to \nu_\mu) \xrightarrow[=]{CP} P(\overline{\nu_e} \to \overline{\nu_\mu}) \quad ?$$

These two probabilities are not equal (and thus there is CP violation) if the mixing matrix U has imaginary elements! Still a possibility

How to produce a neutrino beam?



Magnet can be selected to focus π^+ (π^-), these decay to a μ^+(μ^-) and ν_μ ($\overline{\nu_\mu}$)

Additional complication for accelerator neutrinos

What happens if neutrinos are traveling a long distance through dense matter? We know that in any small chunk of matter the interaction probability is small, but what happens over large distances? Electrons and anti-electron neutrinos have CC and NC interactions with matter, whereas other neutrinos only have NC interactions

This can lead to significant changes in neutrino oscillation behavior and must be accounted for (in for example, the sun or traveling long distances through the Earth)

MSW (Mikheyev–Smirnov–Wolfenstein) effect

MSW effect: Changes in energy (changes in potential differences) between electron/anti-electron neutrinos and all other neutrinos. The more electrons the neutrinos pass by, the more this effect is important

$V_e = \pm\sqrt{2}G_f N_e$: leads to an effective mass difference and an effective mixing angle different from the ones in vacuum!

$$A = \pm\frac{2\sqrt{2}G_F N_e E}{\Delta m^2}$$

$$\Delta m_m^2 = C\Delta m^2,$$

$$sin2\theta_m = \frac{sin2\theta}{C},$$

$$C = \sqrt{(cos2\theta - A)^2 + sin^2 2\theta}.$$

https://hepwww.pp.rl.ac.uk/users/Ricciardi/Lectures/MSW-1.pdf

MSW (Mikheyev–Smirnov–Wolfenstein) effect

$$A = \pm \frac{2\sqrt{2} G_F N_e E}{\Delta m^2}$$

$$\Delta m_m^2 = C \Delta m^2,$$

$$sin2\theta_m = \frac{sin2\theta}{C},$$

$$C = \sqrt{(cos2\theta - A)^2 + sin^2 2\theta}.$$

Key features of the MSW effect:

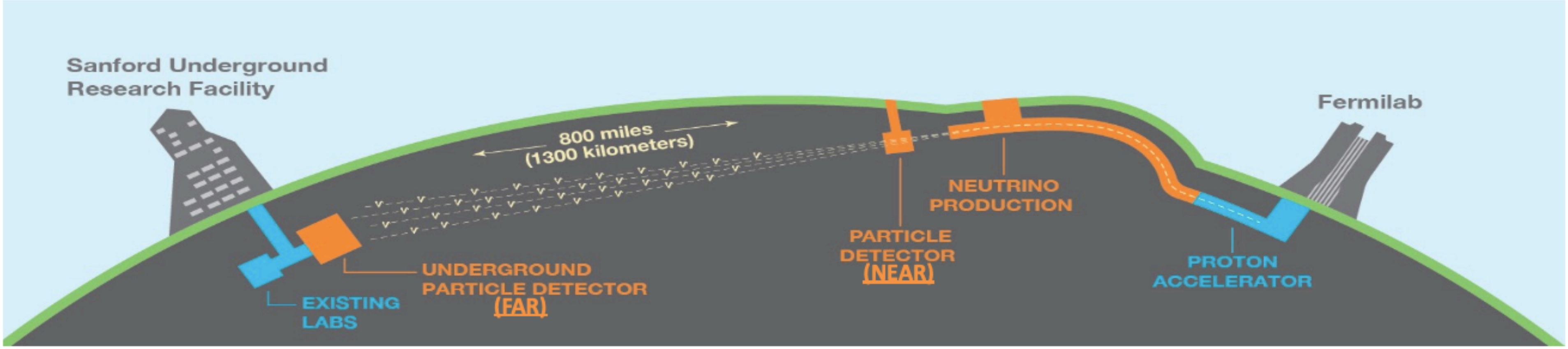
1) Without long distances (through lots of matter) or high matter densities, MSW effect cannot be observed. The MSW effect is critical to understand for solar neutrinos!
2) For $\cos 2\theta = A$, oscillations can be significantly enhanced
3) Oscillations differ for neutrinos and anti-neutrinos due to the $\pm$ sign in A, even without CP violation!
4) Can be used to help break the degeneracy in the mass hierarchy

https://hepwww.pp.rl.ac.uk/users/Ricciardi/Lectures/MSW-1.pdf

Deep Underground Neutrino Experiment

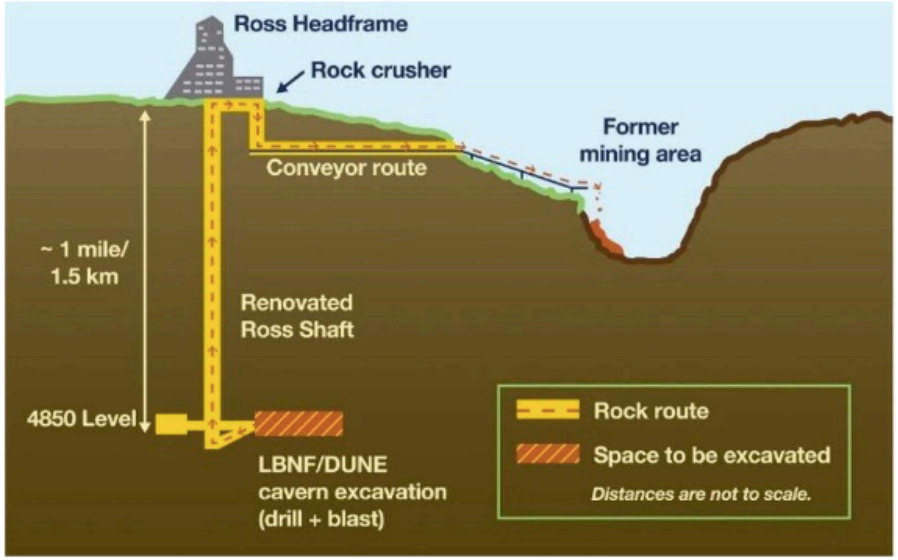
- Long-baseline (LB - 1300 km) experiment:
  - Neutrino and antineutrino beams
- ~ 70 kton volume far detector, 1.5 km underground, divided in 4 modules
- Multi-technology Near Detector, focused on beam characterization and physics
- > 20 years foreseen life span

- **Primary physics goals**:
  - 3-neutrino oscillations parameters: $\nu_\mu/\overline{\nu_\mu}$ disappearance, $\nu_e/\overline{\nu_e}$ appearance
  - δ_{CP} ; mass hierarchy
- **SuperNova** burst neutrinos
- **Beyond-Standard-Model physics**: baryon number violation, sterile neutrinos, non-standard interactions, etc.



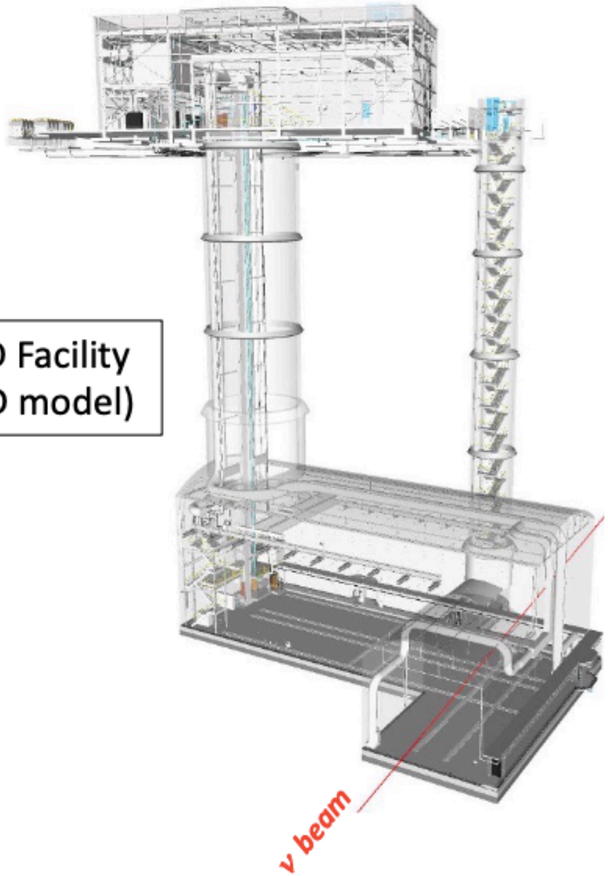
Long Baseline Neutrino Facilities

- **Infrastructures**
  - Excavation works at SURF for Far Detector (FD) caverns
  - Design work for Near Detector (ND) site, due to submission for approval within few months



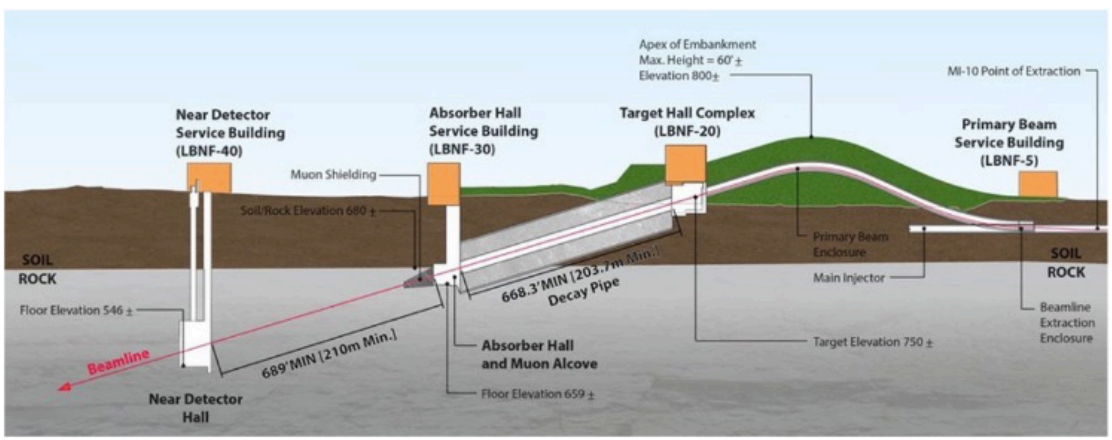
FD excavation sketch

ND Facility (3D model)



Long Baseline Neutrino Facilities

- **Beam line** design under way
  - 60-120 GeV proton beam
  - 5.8 degree vertical bend, to reach SURF
  - 1.2 MW by late 2020's, upgradable to 2.4 MW
  - Assumed minimum uptime of 55%



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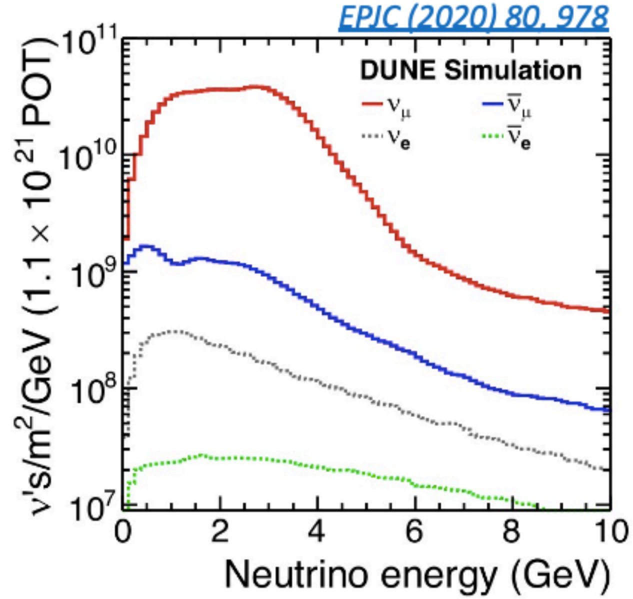
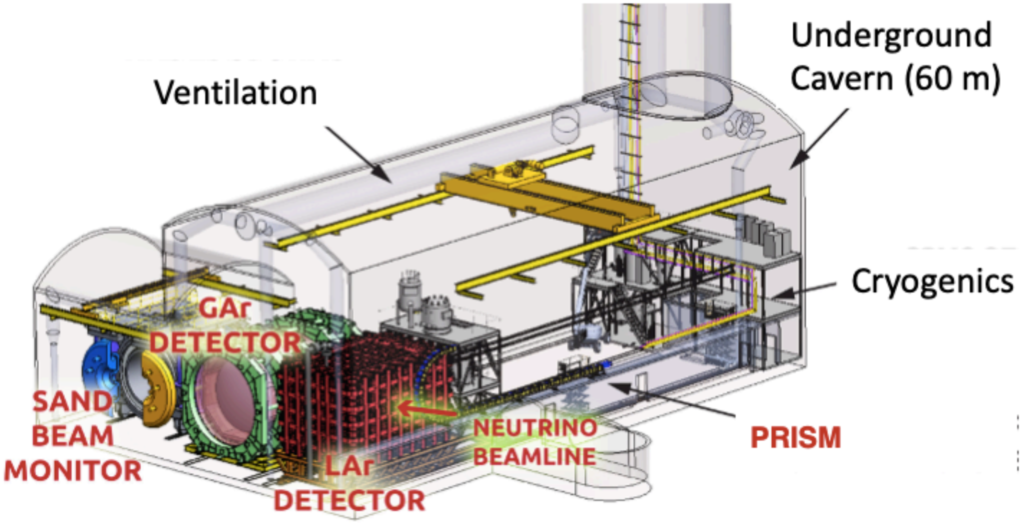
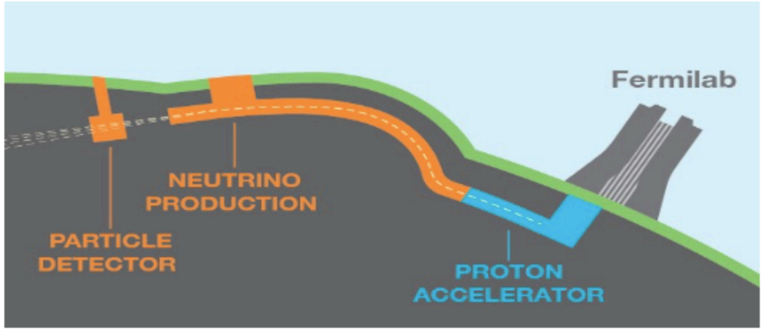


(1.1-1.9) x 10^{21} POT*/y @ 1.2 MW
10 μs pulse duration
*Protons On Target

The Near Detector Station

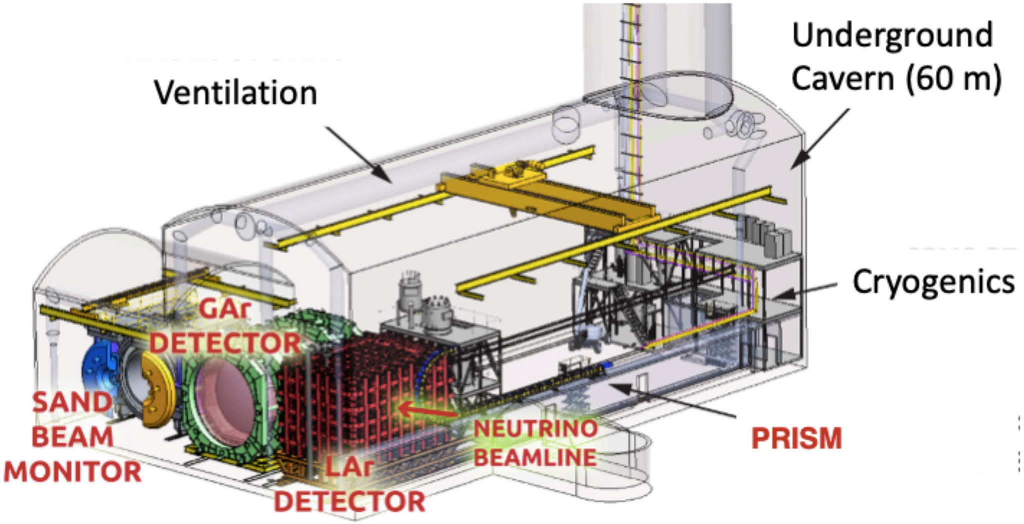
- **DUNE ND complex**
  - Located 574 m from proton beam target
  - Precise characterization of neutrino beam
  - Limitation of cross-section uncertainties for LB neutrino oscillation measurements





The Near Detector Station

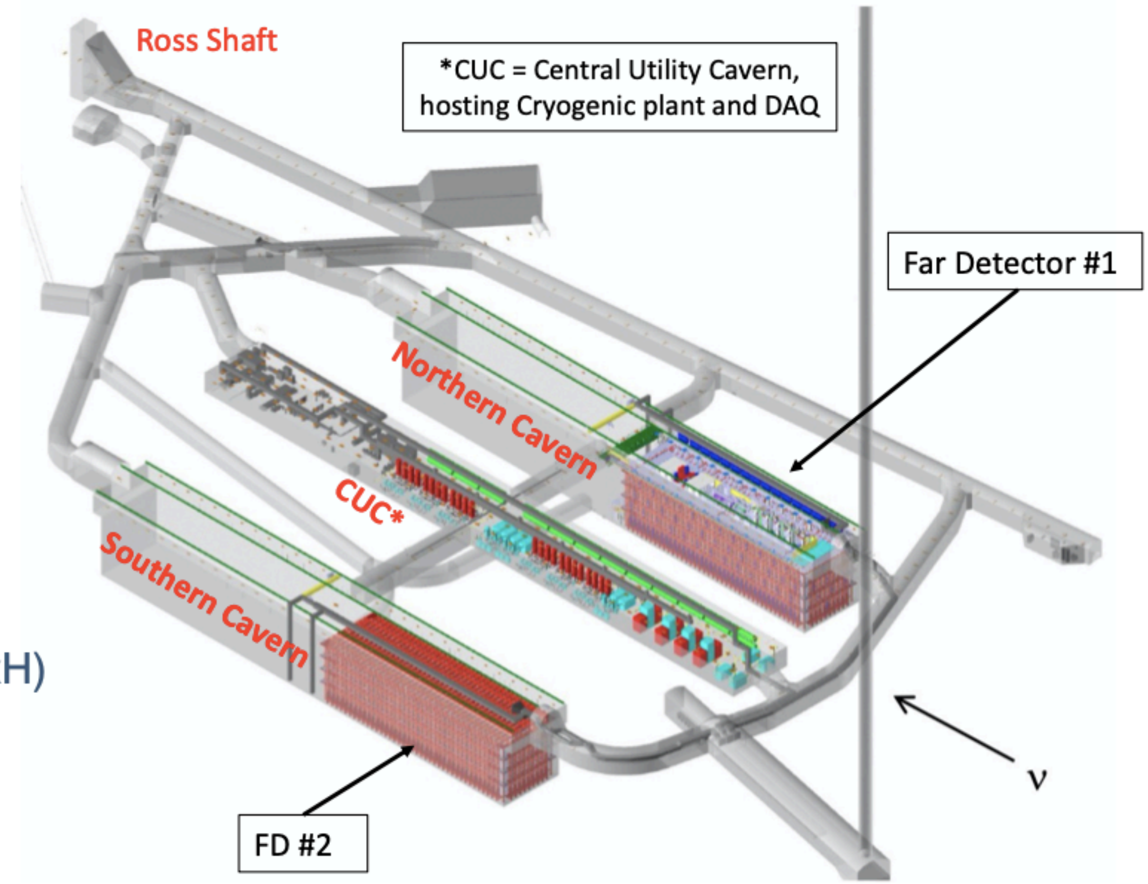
- Multiple complementary systems:
  - **ND-LAr** primary target, modular, pixelated charge read-out LAr-TPC (300 ton)
    - Module 0 successfully tested at Univ. Bern
  - **ND-GAr**: high-pressure GAr-TPC, surrounded by ECAL and magnet
    - intercepts muons escaping LAr-TPC
    - Muon spectrometer; nuclear interaction model constraints
    - Will come at a later stage. A Temporary Muon Spectrometer (**TMS**) will be installed at Day 1
  - **SAND**: inner tracker surrounded by 100 ton ECal and SC magnet (0.6 T)
    - On-axis beam monitor (spectrum/stability)



PRISM: ND-LAr and TMS/ND-GAr can move up to 30 m Off-Axis for beam characterization and lower-energy ν detection

Far Detector Site - SURF

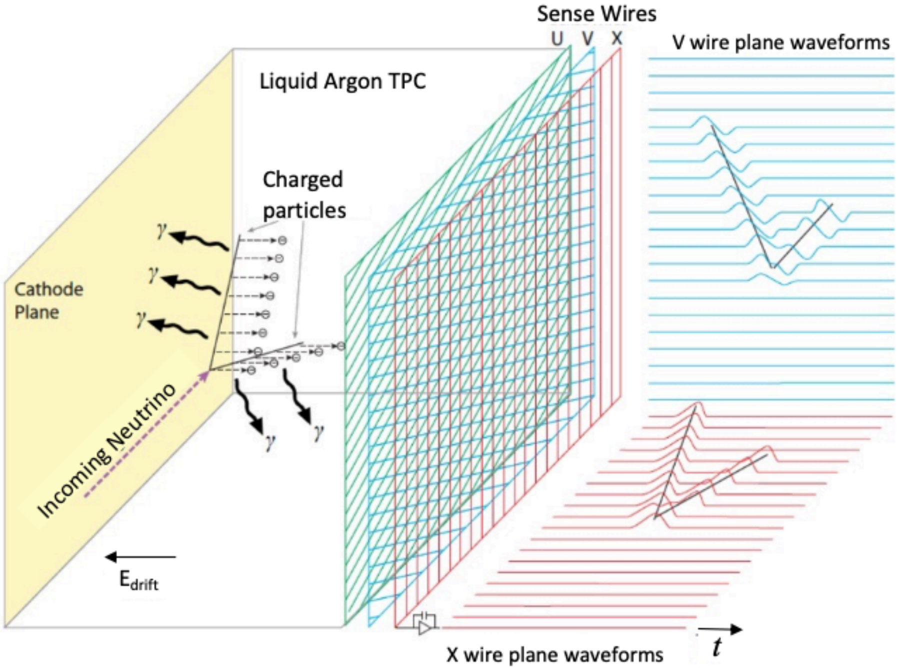
- 4 Detector modules, ~17 kton total volume each
  - Construction in stages
- **FD #1, #2** will be **single-phase (SP) LAr-TPCs,** with Horizontal Drift (HD) and Vertical Drift (VD), respectively
- FD #1 construction starts in mid 2020's
- Maximal cryostat external dimensions: ~ 66 x 19 x 18 m (LxWxH)



LAr-TPC technology

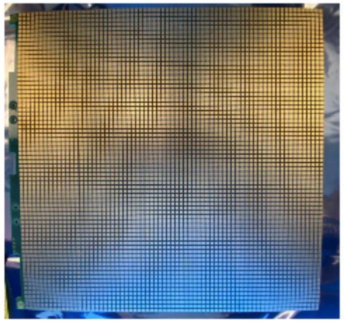
- Charge/light production and collection with wire read-out (HD technology)

- Mature, reliable technology (ICARUS, MicroBooNE)
- Fully compatible with very-long expected life span of the detectors



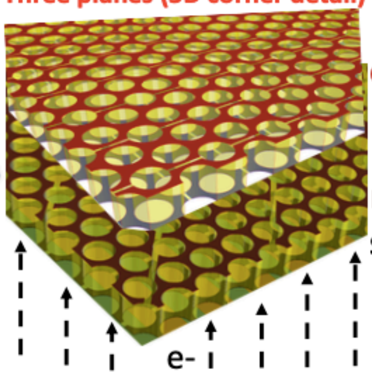
Other read-out solutions:

- Pixels (ND LAr-TPC)
- Perforated PCBs (Vertical Drift)



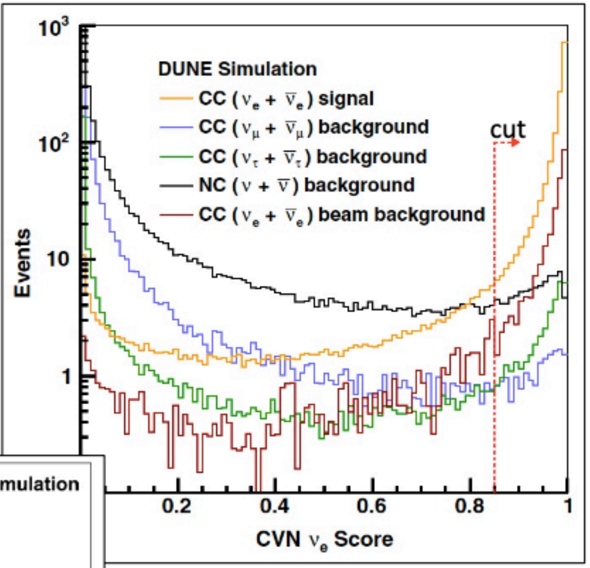
LArPix anode
32 x 32 cm
4.9k pixels

Three planes (3D corner detail)

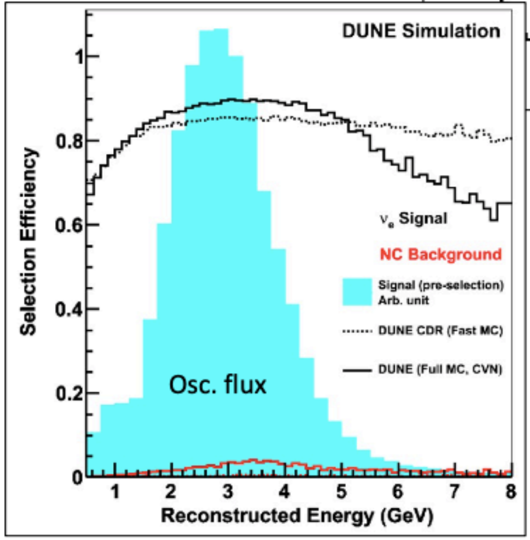


Neutrino Reco/Identification

- Algorithms trained on Convolutional Neural Networks (CNN)
- Hit identification on 2D views and identification of distinct tracks/showers (clustering) with *Pandora*
  - 3D events produced from matching of 2D hits
- Neutrino event reconstruction from 2D images is the perfect input for machine learning / image analysis techniques
  - CNNs trained on, and aiming to classify, images (TPC views) -> Convoluted Visual Network (CVN)
    - 80-90% recognition efficiency for both ν_μ and ν_e
    - low mis-identification rates



ν-beam mode
3.5 y sim. exposure



- ν_e CNN score (top)
- Resulting ν_e selection efficiency with top cut (left)

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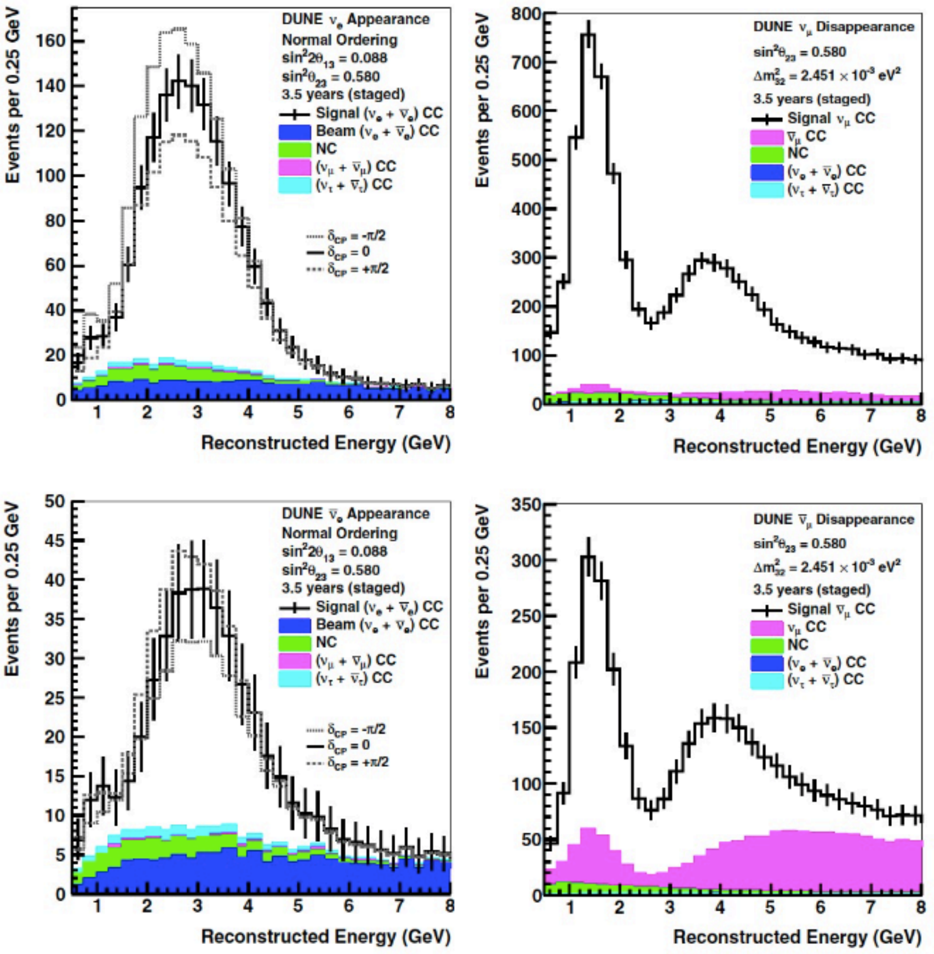
Neutrino Oscillations

- Projected results for $\nu_\mu/\overline{\nu_\mu}$ disappearance and $\nu_e/\overline{\nu}_e$ appearance, assuming:
  - normal ordering
  - 7 staged years (3.5 y ν-beam mode + 3.5 y $\bar{\nu}$-beam mode)

- Measurement and simultaneous fit of oscillation parameters over the four components of FD data
- Sensitivity assessment includes full FD systematics treatment (flux, cross-section, and detector) and ND constraints



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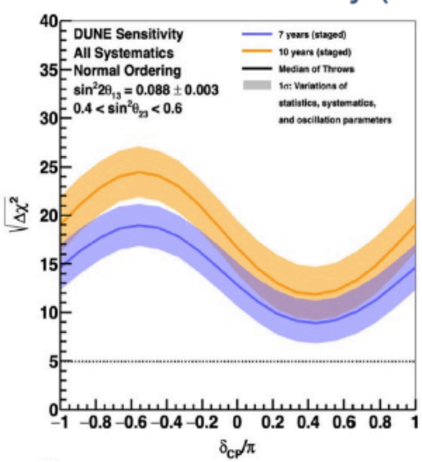
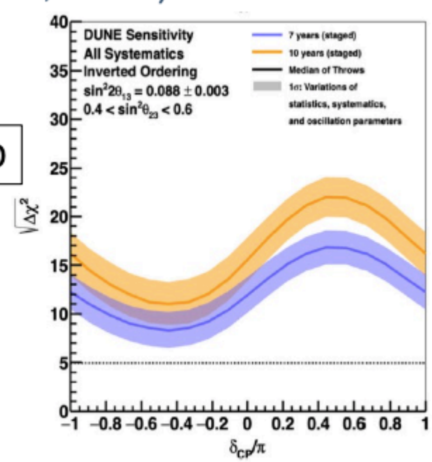
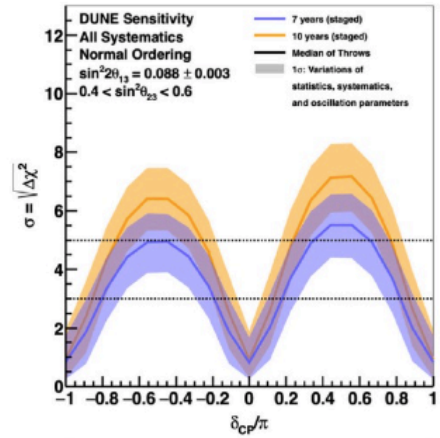
DUNE sensitivity

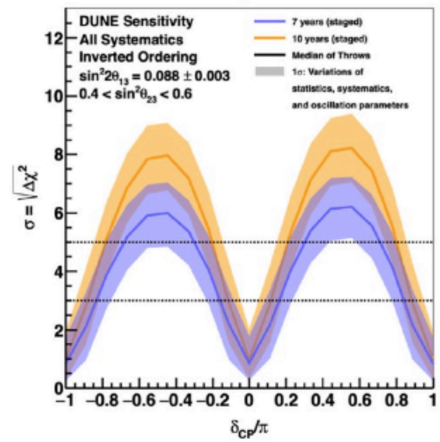
- Assumed staged running as in Technical Design Report (summing ν-beam mode and $\bar{\nu}$-beam mode)
- Potential of CP-violation (δ_{CP}) discovery in 7-10 years (left)
- 2-3 years to unambiguously determine mass hierarchy (NO vs IO, below)

MO



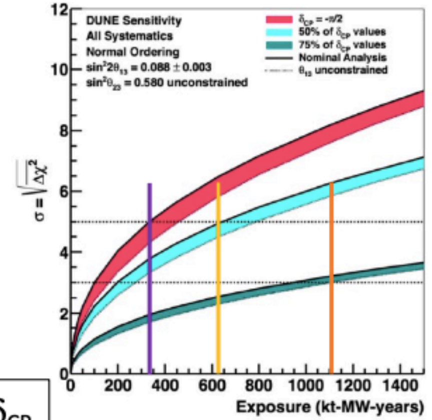


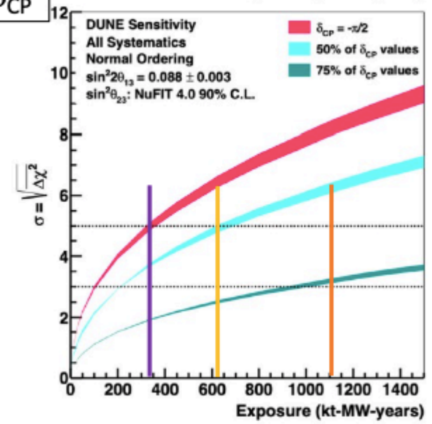




[EPJC (2020) 80, 978](https://doi.org/10.1140/epjc/s10052-020-08456-z)

δ_{CP}





= 7 years (staged)
= 10 years (staged)
= 15 years (staged)

Cosmic Neutrinos

- The DUNE FD will be sensitive to cosmic neutrinos from MeV to tens of GeV in energy
  - Stellar core-collapse supernova (SN) neutrinos
  - Solar neutrinos?
- For a **galactic SN**, DUNE expects to observe up to thousands of ν interactions over the duration of the burst
- High reconstruction efficiency for SN neutrino energy range, 15-20% expected energy resolution with both TPC and PDS

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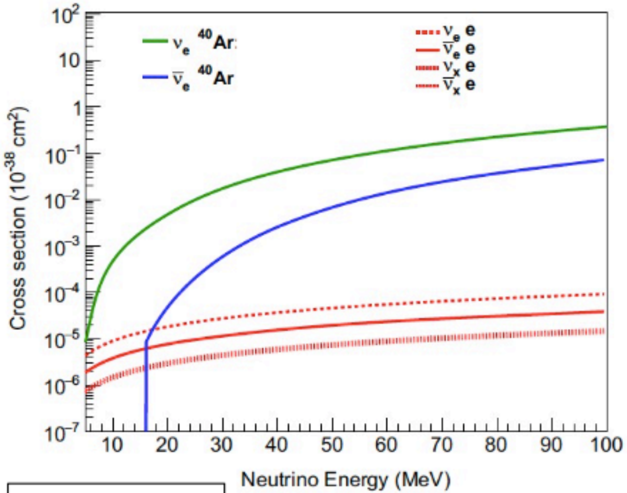
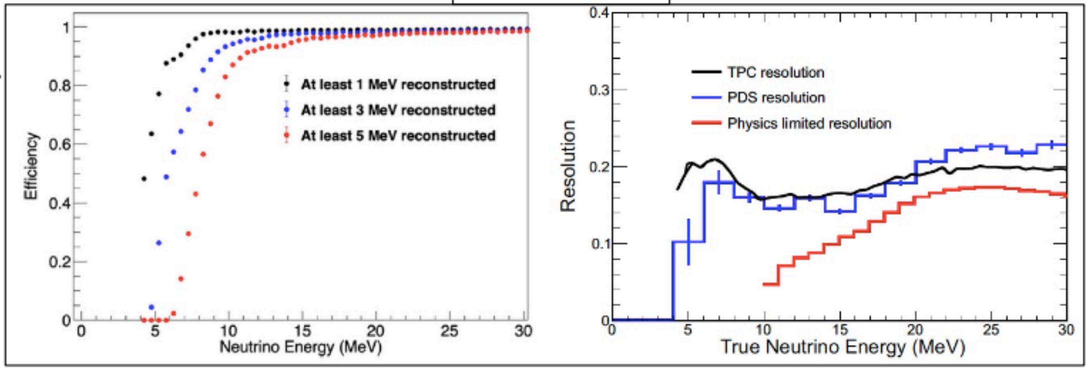
$\nu_e + {}^{40}Ar \rightarrow e^- + {}^{40}K^*$ *(dominant)*

$\bar{\nu}_e + {}^{40}Ar \rightarrow e^+ + {}^{40}Cl^*$



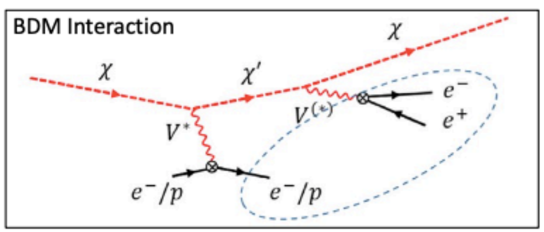
SN sensitivity



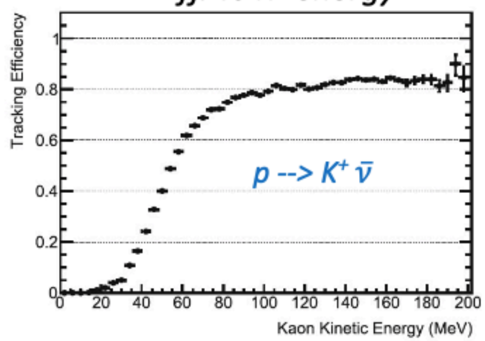
- **Solar neutrino** detection candidates:
  - from ^{8}B, hep (10 < endpoint < 20 MeV)
  - Background limited (detector materials)
  - Feasibility studies underway

BSM Physics

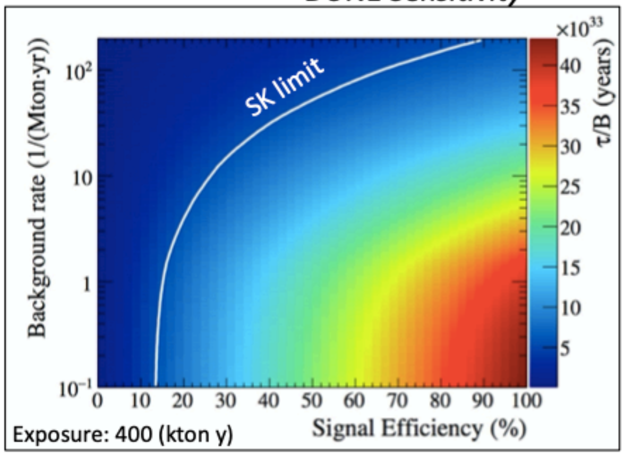
- DUNE can probe several sources of new physics
  - Sterile ν-mixing
  - Non-standard ν interactions
  - Barion number violation
  - **Nucleon decay**
  - Low-mass Dark Matter (@ ND)
  - **(in-)elastic Boosted Dark Matter - BDM (@ FD)**
  - … *EPJC (2021) 81, 322*

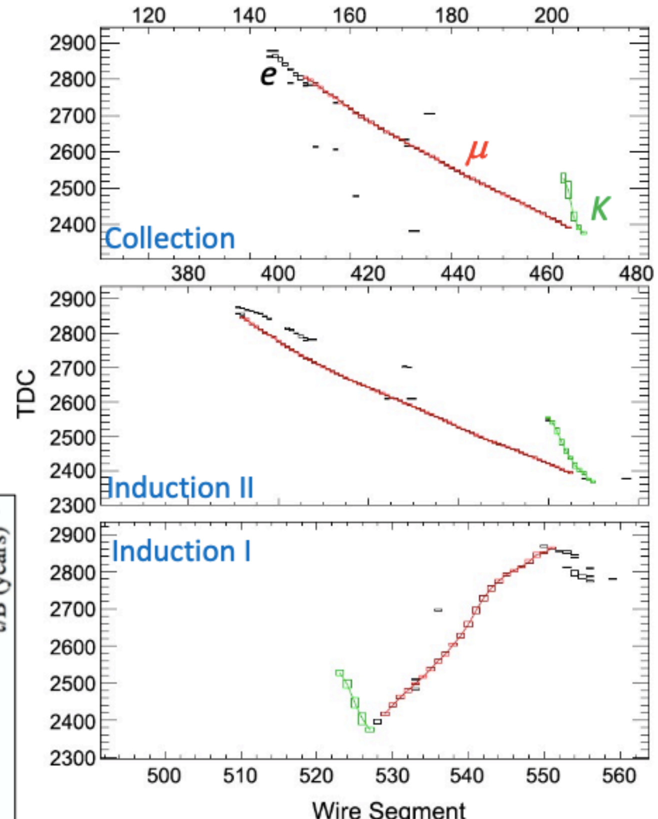


Eff. vs K^+ energy



DUNE Sensitivity





BKG: $\nu_\mu n \dashrightarrow \mu^- p$,
if p is mis-identified
as K (atmospheric)

Switching topics….

Indirectly looking for new physics

2103.11769

Remember that the SM has lepton universality built into it. The leptons all have the same interaction strengths (Higgs Yukawa couplings are an exception!), though not due to any built-in symmetries of the theory

Conveniently, through the use of virtual particles in Feynman diagrams we can indirectly probe much higher energies than we can directly probe

QCD shouldn't affect the ratio of these decays: R=

$(B^+ \to K^+\mu^+\mu^-)/(B^+ \to K^+e^+e^-)$

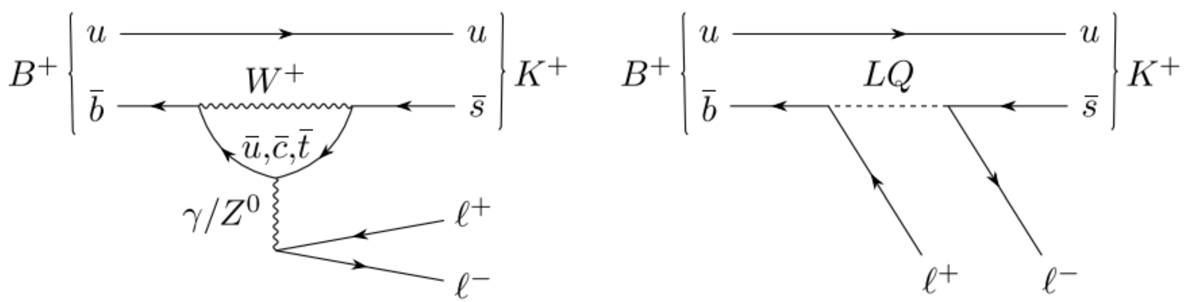


Figure 1: Fundamental processes contributing to $B^+ \to K^+\ell^+\ell^-$ decays in the SM and possible new physics models. A B^+ meson, consisting of $\bar{b}$ and u quarks, decays into a K^+, containing $\bar{s}$ and u quarks, and two charged leptons, $\ell^+\ell^-$. (Left) The SM contribution involves the electroweak bosons γ, W^+ and Z^0. (Right) A possible new physics contribution to the decay with a hypothetical leptoquark (LQ) which, unlike the electroweak bosons, could have different interaction strengths with the different types of leptons.

 2103.11769

Subtle point #1: If q^2 is large enough that phase space effects are important, we need to be careful and more accurately calculate R. So we can measure it in bins of mass squared

Subtle point #2: Need to very carefully measure efficiencies to reconstruct electrons and muons! So measure a double ratio …

$$R_K = \frac{\mathcal{B}(B^+ \to K^+\mu^+\mu^-)}{\mathcal{B}(B^+ \to J/\psi(\to \mu^+\mu^-)K^+)} \bigg/ \frac{\mathcal{B}(B^+ \to K^+e^+e^-)}{\mathcal{B}(B^+ \to J/\psi(\to e^+e^-)K^+)}$$

Indirectly looking for new physics

2103.11769

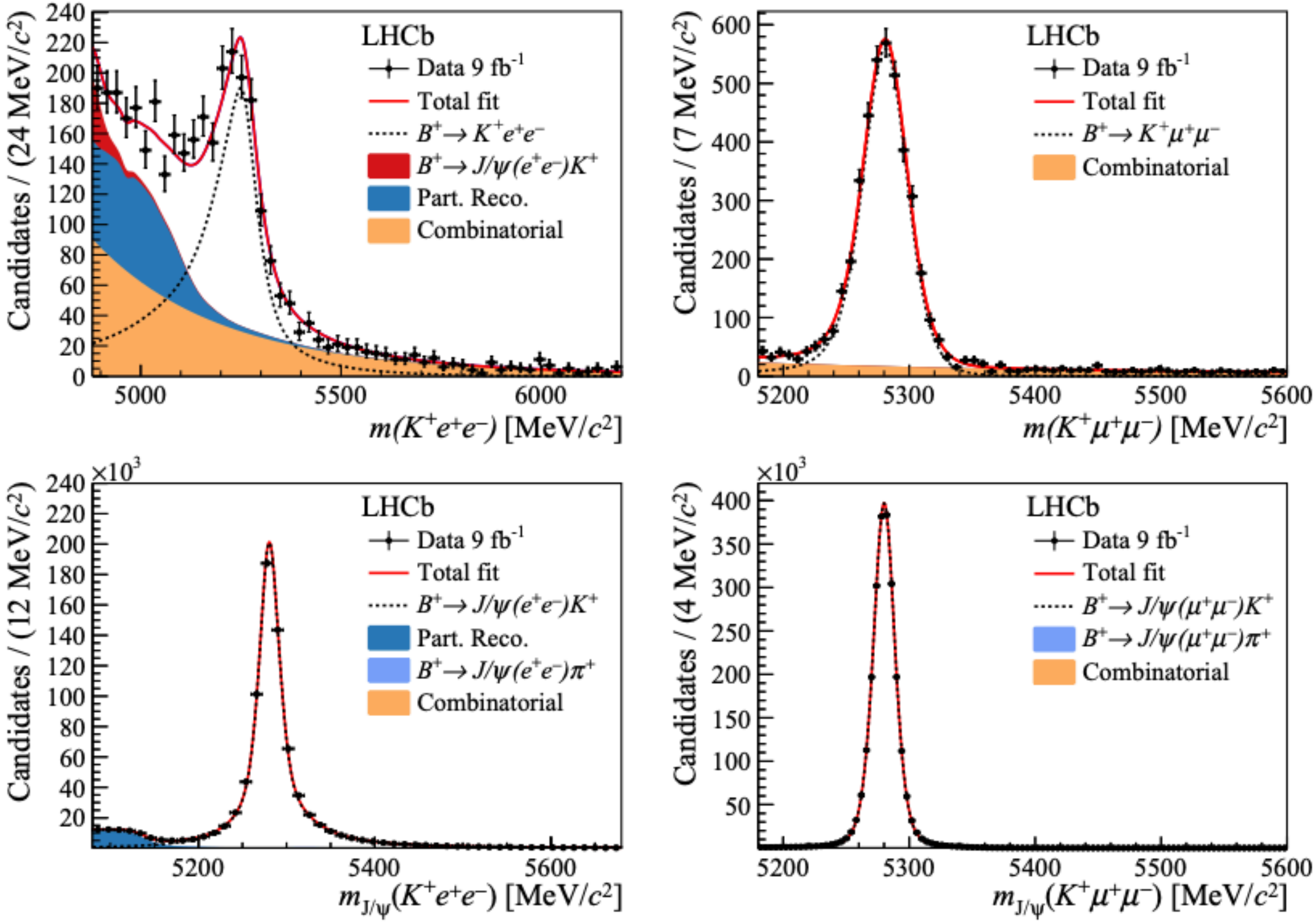


Figure 2: Candidate invariant mass distributions. Distribution of the invariant mass $m_{(J/\psi)}(K^+\ell^+\ell^-)$ for candidates with (left) electron and (right) muon pairs in the final state for the (top) nonresonant $B^+ \to K^+\ell^+\ell^-$ signal channels and (bottom) resonant $B^+ \to J/\psi\,(\to \ell^+\ell^-)K^+$ decays. The fit projection is superimposed. In the resonant-mode distributions, some fit components are too small to be visible.

Indirectly looking for new physics

2103.11769

$$R_K(1.1 < q^2 < 6.0\,\mathrm{GeV}^2/c^4) = 0.846\,^{+0.042}_{-0.039}\,^{+0.013}_{-0.012}$$

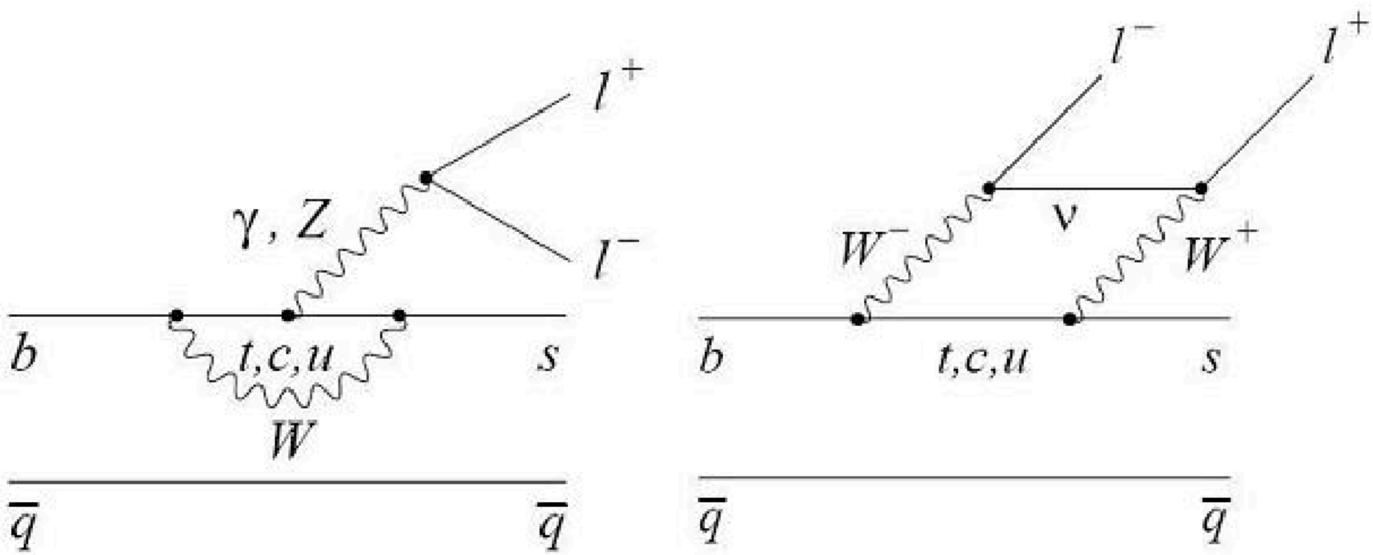
Consistent with the SM only at the level of 0.1% (3.1 sigma evidence for lepton universality violation!)

Other ways to indirectly look

2003.04831

Study angular distributions in $B^0 \rightarrow K^{*0}\mu^+\mu^-$ decays (with $K^{*0} \rightarrow K^+\pi^-$)

Final state determined by q² of muon system, $m(K^+\pi^-)$ and 3 angles, angular momentum of the $(K^+\pi^-)$ system, polariation fraction of K* and the forward-backward asymmetry of dimuon system



Other ways to indirectly look

2003.04831

Not trivial!!! Try and combine these angles to form observables that you can compare to predictions

$$\frac{1}{d(\Gamma+\bar{\Gamma})/dq^2}\frac{d^4(\Gamma+\bar{\Gamma})}{dq^2 d\vec{\Omega}}\bigg|_P = \frac{9}{32\pi}\bigg[\frac{3}{4}(1-F_L)\sin^2\theta_K + F_L\cos^2\theta_K + \frac{1}{4}(1-F_L)\sin^2\theta_K\cos 2\theta_l - F_L\cos^2\theta_K\cos 2\theta_l + S_3\sin^2\theta_K\sin^2\theta_l\cos 2\phi + S_4\sin 2\theta_K\sin 2\theta_l\cos\phi + S_5\sin 2\theta_K\sin\theta_l\cos\phi + \frac{4}{3}A_{FB}\sin^2\theta_K\cos\theta_l + S_7\sin 2\theta_K\sin\theta_l\sin\phi + S_8\sin 2\theta_K\sin 2\theta_l\sin\phi + S_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi\bigg], \quad (1)$$

where F_L is the fraction of the longitudinal polarization of the K^{*0} meson, A_{FB} is the forward-backward asymmetry of the dimuon system, and S_i are other CP-averaged observables [1]. The $K^+\pi^-$ system can also be in an S-wave configuration, which modifies the angular distribution to

$$\frac{1}{d(\Gamma+\bar{\Gamma})/dq^2}\frac{d^4(\Gamma+\bar{\Gamma})}{dq^2 d\vec{\Omega}}\bigg|_{S+P} = (1-F_S)\frac{1}{d(\Gamma+\bar{\Gamma})/dq^2}\frac{d^4(\Gamma+\bar{\Gamma})}{dq^2 d\vec{\Omega}}\bigg|_P + \frac{3}{16\pi}F_S\sin^2\theta_l + \frac{9}{32\pi}(S_{11} + S_{13}\cos 2\theta_l)\cos\theta_K + \frac{9}{32\pi}(S_{14}\sin 2\theta_l + S_{15}\sin\theta_l)\sin\theta_K\cos\phi + \frac{9}{32\pi}(S_{16}\sin\theta_l + S_{17}\sin 2\theta_l)\sin\theta_K\sin\phi,$$

Other ways to indirectly look

2003.04831

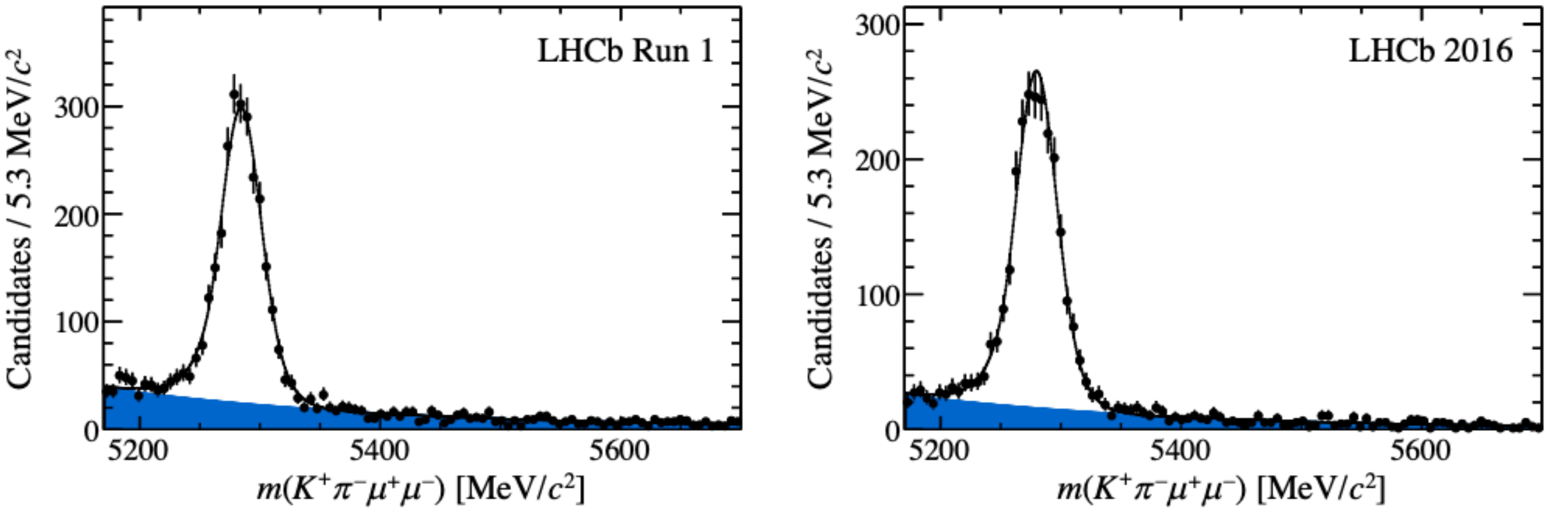


FIG. 1. The $K^+\pi^-\mu^+\mu^-$ mass distribution of candidates with $0.1 < q^2 < 19.0\ \mathrm{GeV}^2/c^4$, excluding the $\phi(1020)$ and charmonium regions, for the (left) Run 1 data and (right) 2016 data. The background is indicated by the shaded region.

What was found?

2003.04831

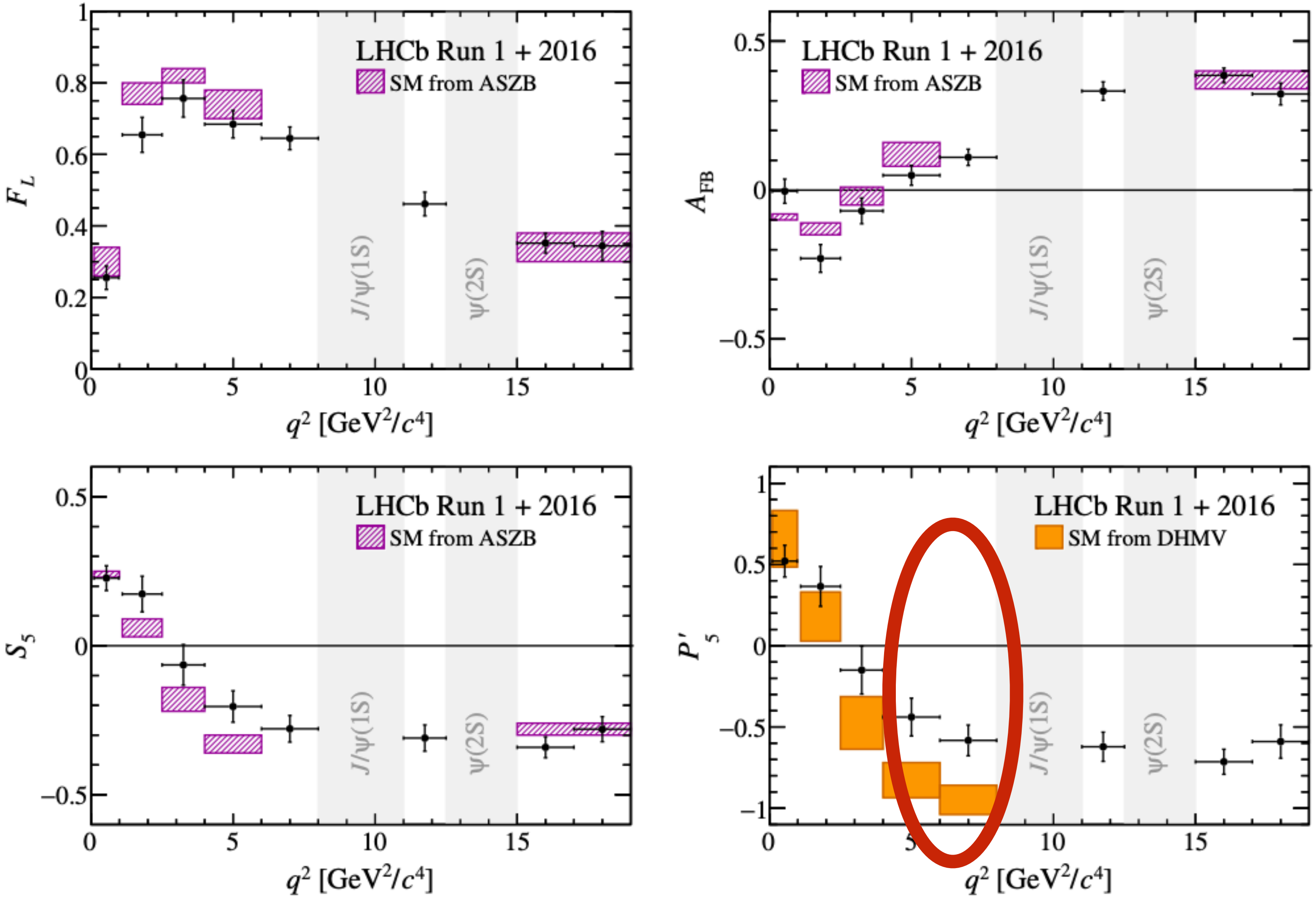


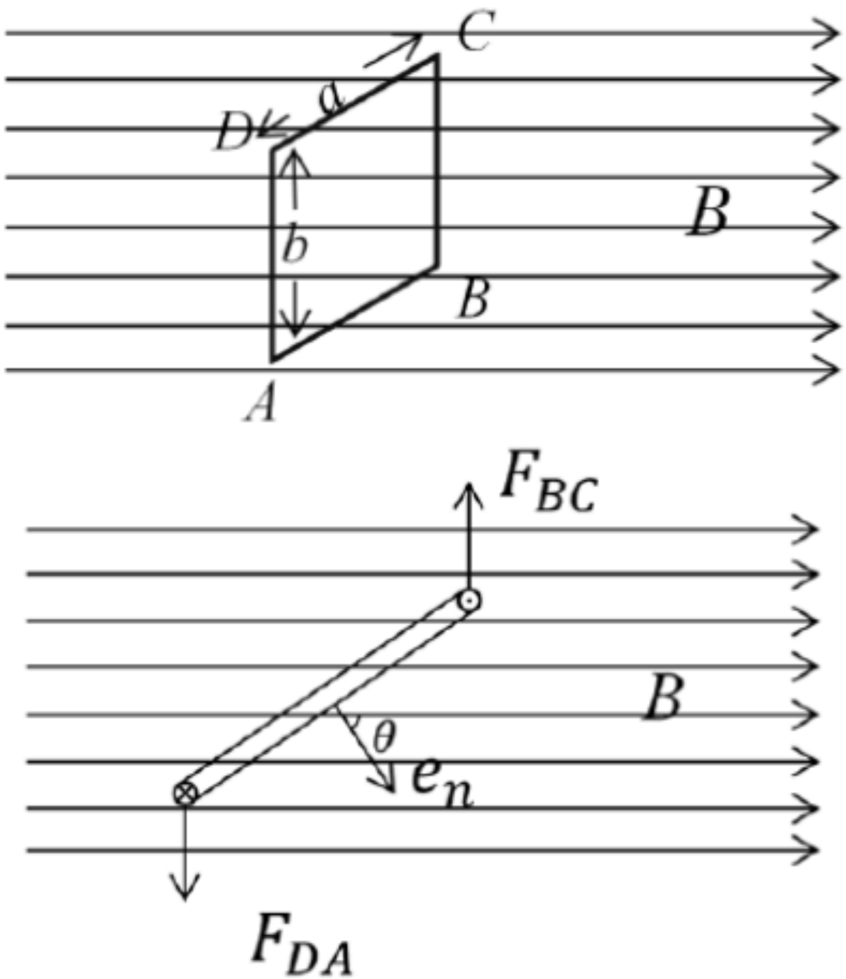
FIG. 2. Results for the CP-averaged angular observables F_L, A_{FB}, S_5, and P'_5 in bins of q^2. The data are compared to SM predictions based on the prescription of Refs. [43,44], with the exception of the P'_5 distribution, which is compared to SM predictions based on Refs. [73,74].

Locally 2.5 and 2.9 sigma discrepancies

Switching topics again….

That brings us to g-2

Following 2110.04673



Current I running through the loop. Biot-Savart law tells us that there is a force F on each of the b length arms, with magnitude bIB, and these are in opposite direction so there is a torque τ on the coil. Perpendicular distance between the two forces is $a \sin\theta$, so $\tau = abIB\sin\theta = AIB\sin\theta$ (A=ab)

$|\tau| = IA\vec{e}_n \times \vec{B} = \vec{M} \times \vec{B}$

$\vec{M} = IA\vec{e}_n$ with $\vec{e}_n$ the unit vector normal to the coil

M for charged lepton as classic rigid body

Following 2110.04673

Imagine we have a charged lepton (like a muon!) as a charged rigid body, with charge density proportional to mass density, so

$\rho_e = \alpha\rho_m, e = \alpha m$

Let our classical rigid body rotate around the z axis with angular velocity ω and assume its charge and mass densities have no angular dependence

d(Charge)

$$M = \int A dI = \int \pi r^2 dI = \int \pi(x^2 + y^2)\frac{\rho_e(\vec{r})dV}{2\pi/\omega}$$

time

$$M = \frac{\omega}{2}\int (x^2 + y^2)\rho_e(\vec{r})dV = \frac{\omega\alpha}{2}\int (x^2 + y^2)\rho_m(\vec{r})dV = \frac{\omega\alpha}{2}\int r^2 dm = \frac{\omega\alpha}{2}I$$

$$M = \frac{\omega\alpha}{2}I = \frac{\alpha}{2}I\omega = \frac{\alpha}{2}L = \frac{e}{2m}L$$

What if we want to do this in QED?

Following 2110.04673 and Thomson

A reminder that we can write the Lagrangian for an electron in an arbitrary E&M field as:

$$\mathscr{L} = \frac{1}{2}m|\dot{\vec{q}}|^2 + e\phi - e\vec{A}\cdot\dot{\vec{q}}$$

We can write to Euler-Lagrange equations in the usual way. For a free particle, $\frac{\partial \mathscr{L}}{\partial \dot{\vec{q}}} = m\dot{\vec{q}}$ is the regular momentum we are used to thinking about. Here instead we get $\frac{\partial \mathscr{L}}{\partial \dot{\vec{q}}} = m\dot{\vec{q}} - e\vec{A}$, which is the canonical momentum. We can plug use this canonical momentum in the Dirac equation (p_μ goes from $i\partial_\mu$ to $i\partial_\mu - eA_\mu$)

With no E&M field (reminder), Dirac equation is $i\gamma^\mu\partial_\mu\psi - m\psi = 0$

With E&M field we get :

$$\left[\gamma^\mu\left(i\partial_\mu - eA_\mu\right) - m\right]\psi = 0$$

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

$$A^\mu = (\phi, \vec{A}), A_\mu = (\phi, -\vec{A})$$

What if we want to do this in QED?

$$\left[\gamma^{\mu}\left(i\partial_{\mu} - eA_{\mu}\right) - m\right]\psi = 0$$ and let's try standard solutions

$$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} e^{-iEt}$$

Reminder: these two each have two components!

$$\left[\gamma^{\mu}\left(i\partial_{\mu} - eA_{\mu}\right) - m\right]\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} e^{-iEt} = 0$$

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$i = 1, 2, 3$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

What if we want to do this in QED?

$$\left[\gamma^\mu\left(i\partial_\mu - eA_\mu\right) - m\right]\begin{pmatrix}\psi_+ \\ \psi_-\end{pmatrix} e^{-iEt} = 0 \qquad \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$i = 1, 2, 3$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$(i\partial_t - eA_0 - m)(\psi_+ e^{-iEt}) + \vec{\sigma} \cdot (i\vec{\nabla} + e\vec{A})(\psi_- e^{-iEt}) = 0$$

Let's work through this together if it's not clear!

$$(-i\partial_t + eA_0 - m)(\psi_- e^{-iEt}) - \vec{\sigma} \cdot (i\vec{\nabla} + e\vec{A})(\psi_+ e^{-iEt}) = 0$$

$$(E - e\phi - m)\psi_+ + \vec{\sigma} \cdot (i\vec{\nabla} + e\vec{A})\psi_- = 0$$

$$(-E + e\phi - m)\psi_- - \vec{\sigma} \cdot (i\vec{\nabla} + e\vec{A})\psi_+ = 0$$

What if we want to do this in QED?

$$(E - e\phi - m)\psi_+ + \vec{\sigma} \cdot (i\vec{\nabla} + e\vec{A})\psi_- = 0$$

$$(-E + e\phi - m)\psi_- - \vec{\sigma} \cdot (i\vec{\nabla} + e\vec{A})\psi_+ = 0$$

Rewrite second equation:

$$(E - e\phi + m)\psi_- + \vec{\sigma} \cdot (i\vec{\nabla} + e\vec{A})\psi_+ = 0$$

Let's evaluate this (a choice!) for a constant electric potential:

$$(E - m)\psi_+ + \vec{\sigma} \cdot (i\vec{\nabla} + e\vec{A})\psi_- = 0$$

$$(E + m)\psi_- + \vec{\sigma} \cdot (i\vec{\nabla} + e\vec{A})\psi_+ = 0$$

And let's consider this in a non-relativistic regime, so E~m. Then second equation gives us:

$$(2m)\psi_- + \vec{\sigma} \cdot (i\vec{\nabla} + e\vec{A})\psi_+ = 0 \rightarrow \psi_- = \frac{-1}{2m}\vec{\sigma} \cdot (i\vec{\nabla} + e\vec{A})\psi_+$$

What if we want to do this in QED?

$$(E-m)\psi_+ + \vec{\sigma}\cdot(i\vec{\nabla}+e\vec{A})\psi_- = 0$$

Plug in: $\psi_- = \dfrac{-1}{2m}\vec{\sigma}\cdot(i\vec{\nabla}+e\vec{A})\psi_+$

$$(E-m)\psi_+ + \vec{\sigma}\cdot(i\vec{\nabla}+e\vec{A})\left[\frac{-1}{2m}\vec{\sigma}\cdot(i\vec{\nabla}+e\vec{A})\psi_+\right] = 0$$

$$(E-m)\psi_+ - \frac{1}{2m}\left[\vec{\sigma}\cdot(i\vec{\nabla}+e\vec{A})\right]^2\psi_+ = 0$$

Useful identity (Appendix C, from Pauli matrices): $(\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{b}) = \vec{a}\cdot\vec{b} + i\vec{\sigma}\cdot(\vec{a}\times\vec{b})$

Use a=b: $\left[\vec{\sigma}\cdot(i\vec{\nabla}+e\vec{A})\right]^2 = (i\vec{\nabla}+e\vec{A})^2 + i\vec{\sigma}\cdot\left[(i\vec{\nabla}+e\vec{A})\times(i\vec{\nabla}+e\vec{A})\right]$

$$\left[\vec{\sigma}\cdot(i\vec{\nabla}+e\vec{A})\right]^2 = (i\vec{\nabla}+e\vec{A})^2 + ie\vec{\sigma}\cdot\left[i\vec{\nabla}\times\vec{A}+i\vec{A}\times\vec{\nabla}\right]$$

What if we want to do this in QED?

$$(E-m)\psi_+ - \frac{1}{2m}\left[\vec{\sigma}\cdot(i\vec{\nabla}+e\vec{A})\right]^2\psi_+ = 0$$

$$\left[\vec{\sigma}\cdot(i\vec{\nabla}+e\vec{A})\right]^2 = (i\vec{\nabla}+e\vec{A})^2 + ie\vec{\sigma}\cdot\left[i\vec{\nabla}\times\vec{A}+i\vec{A}\times\vec{\nabla}\right]$$

$$\left[\vec{\sigma}\cdot(i\vec{\nabla}+e\vec{A})\right]^2 = (i\vec{\nabla}+e\vec{A})^2 + -e\vec{\sigma}\cdot\left[\vec{\nabla}\times\vec{A}+\vec{A}\times\vec{\nabla}\right]$$

Reminder that the derivative term is an operator, so this is NOT zero!

$$(E-m)\psi_+ - \frac{1}{2m}\left[(i\vec{\nabla}+e\vec{A})^2 + -e\vec{\sigma}\cdot\left[\vec{\nabla}\times\vec{A}+\vec{A}\times\vec{\nabla}\right]\right]\psi_+ = 0$$

$$(E-m)\psi_+ - \frac{1}{2m}\left[(i\vec{\nabla}+e\vec{A})^2\psi_+ - e\vec{\sigma}\cdot\left[\vec{\nabla}\times(\vec{A}\psi_+)+\vec{A}\times(\vec{\nabla}\psi_+)\right]\right] = 0$$

$$(E-m)\psi_+ - \frac{1}{2m}\left[(i\vec{\nabla}+e\vec{A})^2\psi_+ - e\vec{\sigma}\cdot\left[(\vec{\nabla}\times\vec{A})\psi_+ + (\vec{\nabla}\psi_+)\times\vec{A}+\vec{A}\times(\vec{\nabla}\psi_+)\right]\right] = 0$$

What if we want to do this in QED?

$$(E-m)\psi_+ - \frac{1}{2m}\left[(i\vec{\nabla}+e\vec{A})^2\psi_+ - e\vec{\sigma}\cdot\left[(\vec{\nabla}\times\vec{A})\psi_+ + (\vec{\nabla}\psi_+)\times\vec{A} + \vec{A}\times(\vec{\nabla}\psi_+)\right]\right] = 0$$

$$(E-m)\psi_+ - \frac{1}{2m}\left[(i\vec{\nabla}+e\vec{A})^2\psi_+ - e\vec{\sigma}\cdot\left[(\vec{\nabla}\times\vec{A})\psi_+\right]\right] = 0$$

$$(E-m)\psi_+ - \frac{1}{2m}\left[(i\vec{\nabla}+e\vec{A})^2\psi_+ - e\vec{\sigma}\cdot\vec{B}\psi_+\right] = 0$$

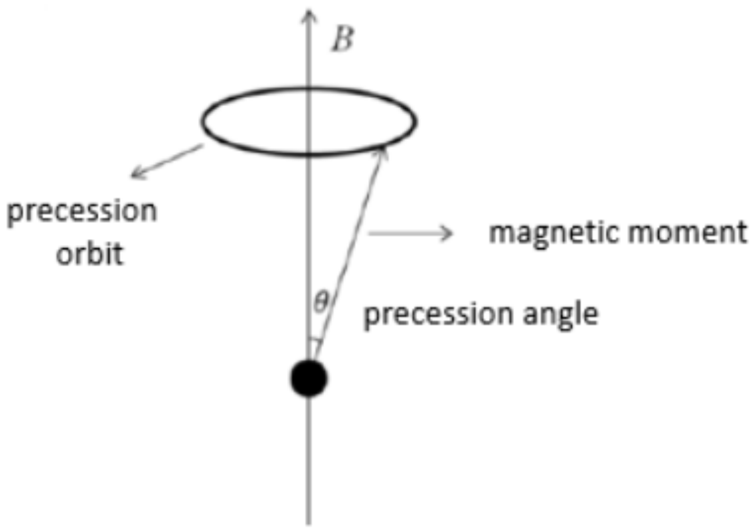
$$(E-m)\psi_+ = \frac{1}{2m}\left[(i\vec{\nabla}+e\vec{A})^2 - e\vec{\sigma}\cdot\vec{B}\right]\psi_+$$

Extra interaction term ~ $-\frac{e\vec{\sigma}\cdot\vec{B}}{2m} = -\frac{e\vec{S}\cdot\vec{B}}{m} = -g_e\frac{e\vec{S}\cdot\vec{B}}{2m}$

$g_e = 2$!!!!!!

How do we measure this

Following 2110.04673



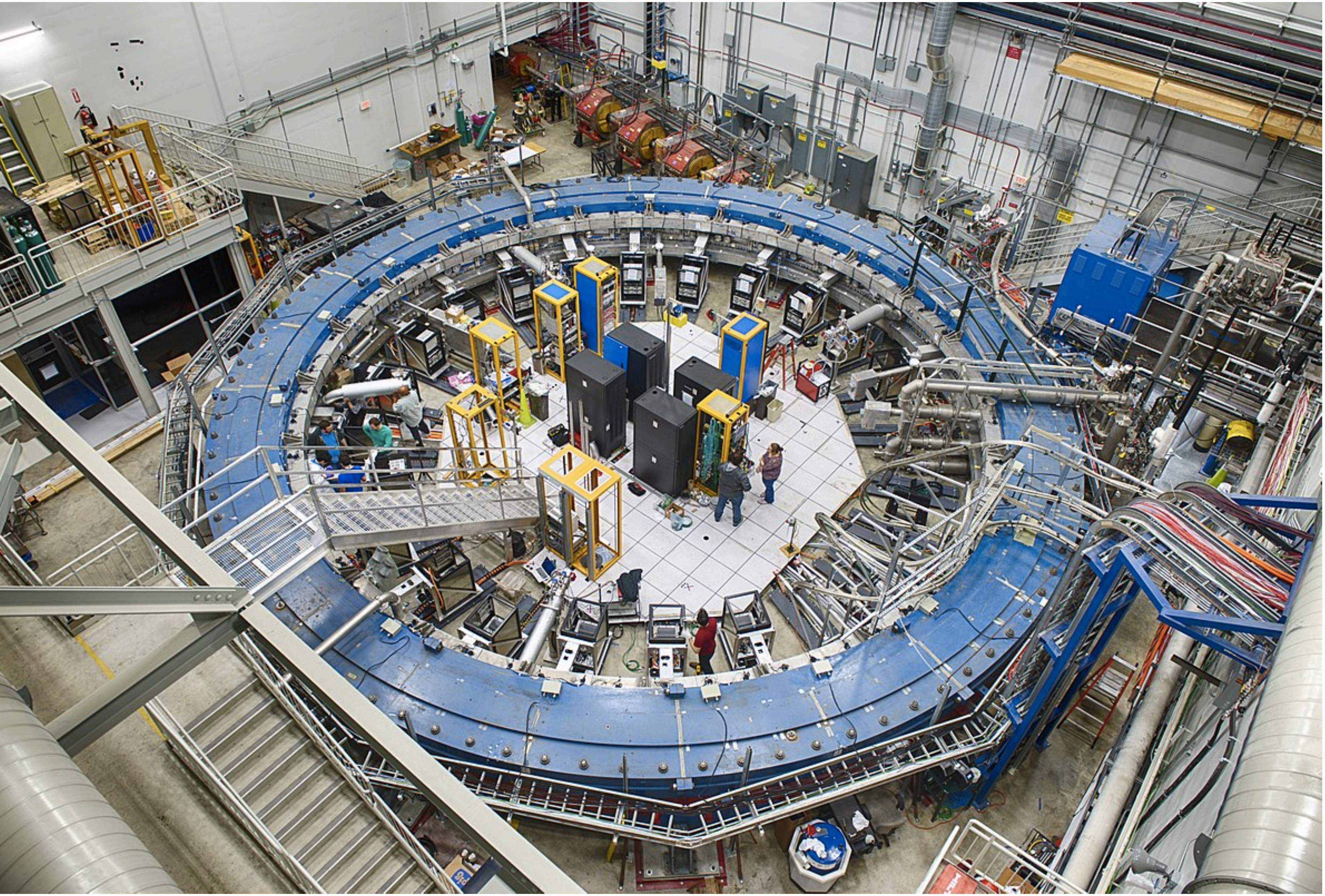
Analogous to a spinning top - spins rotates about the magnetic field

$$\frac{d\vec{S}}{dt} = \vec{L} = \vec{M} \times \vec{B} = g\frac{e}{2m}\vec{S} \times \vec{B}.$$

$$\omega = g\frac{e}{2m}B.$$

Challenge - we are in the lab frame, not the rest frame of the muon!

g-2 at FNAL



How do we measure this, more fully?

Following 2110.04673

$$\frac{d\vec{S}}{dt} = (\vec{\omega}_c + \vec{\omega}_a) \times \vec{S},$$

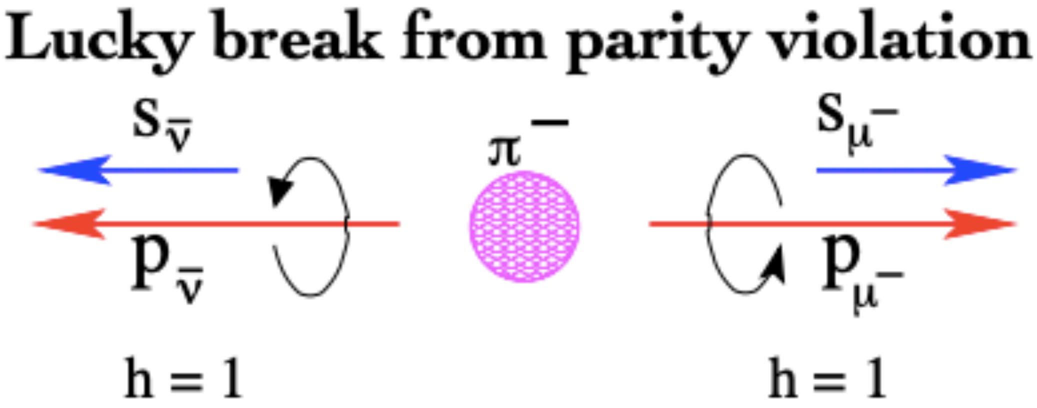
$$\vec{\omega}_c = -\frac{e}{\gamma m}\left(\vec{B} + \frac{\gamma^2}{\gamma^2 - 1}\frac{\vec{E} \times \vec{v}}{c^2}\right),$$

$$\vec{\omega}_a = -\frac{e}{m}\left[a\vec{B} - a\left(\frac{\gamma}{\gamma+1}\frac{\vec{v}\cdot\vec{B}}{c^2}\right)\vec{v} + \left(a - \frac{1}{\gamma^2-1}\right)\frac{\vec{E}\times\vec{v}}{c^2}\right]$$

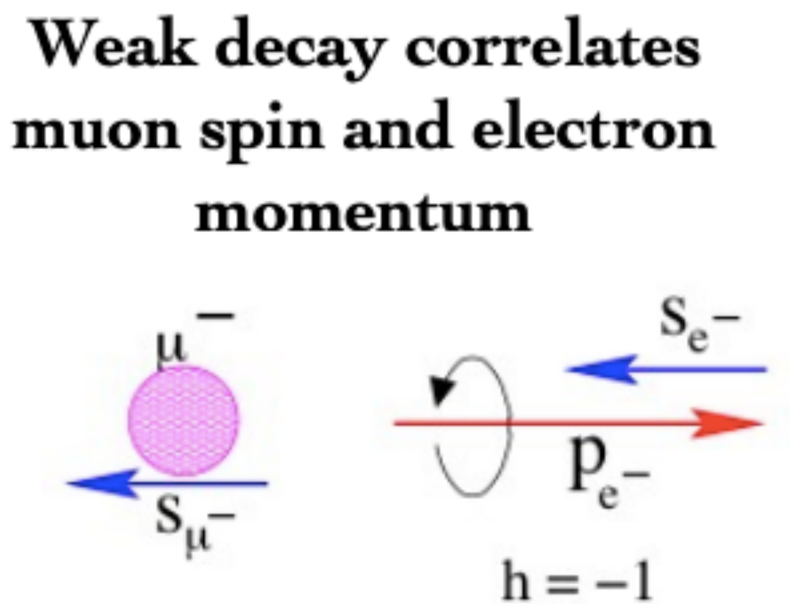
Pick the velocity giving “magic γ factor” to simplify ω_a. The other challenge is precisely measuring the **B** field and ensuring that it is uniform. Not easy! Use careful NMR probes and other tools

Muons come from pion decays. Thanks to parity violation, these are fully polarized!

https://indico.cern.ch/event/276476/contributions/1620139/attachments/501953/693162/g_minus_2_fewbuildins.pdf

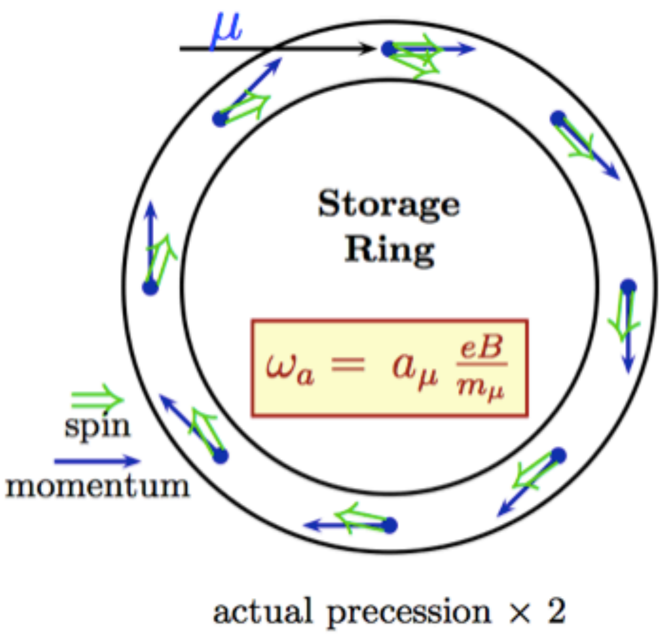


And there is a correlation between muon's spin and electron momentum from its decay



g-2 at FNAL

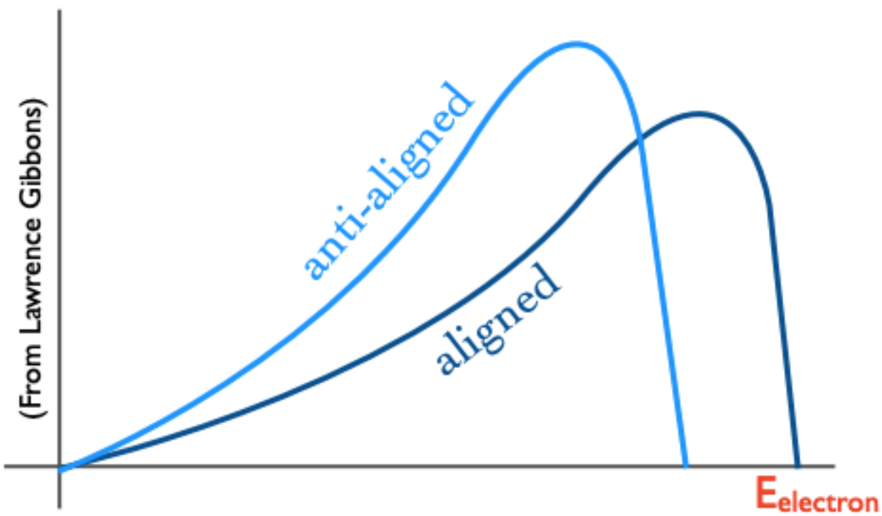
Spin precesses because $g_\mu \neq 2$



https://indico.cern.ch/event/276476/contributions/1620139/attachments/501953/693162/g_minus_2_fewbuildins.pdf

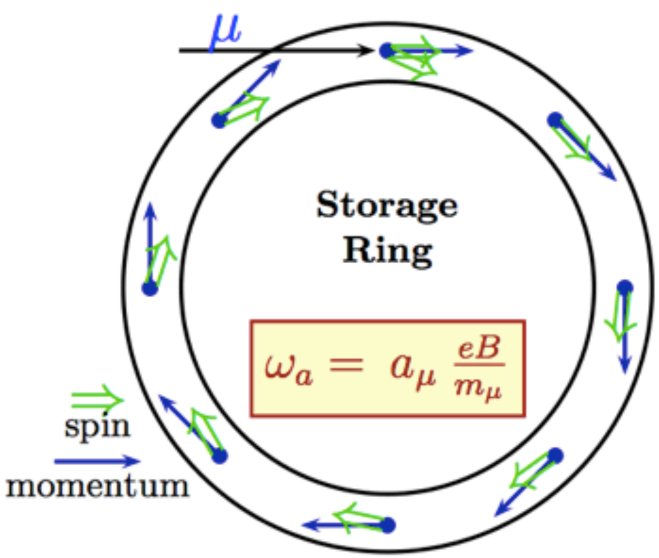
3. Measure Muon Spin Precession → **3. Measure Electron Energy**

Harder electron spectrum when spin and momentum are aligned



g-2 at FNAL

Spin precesses because $g_\mu \neq 2$

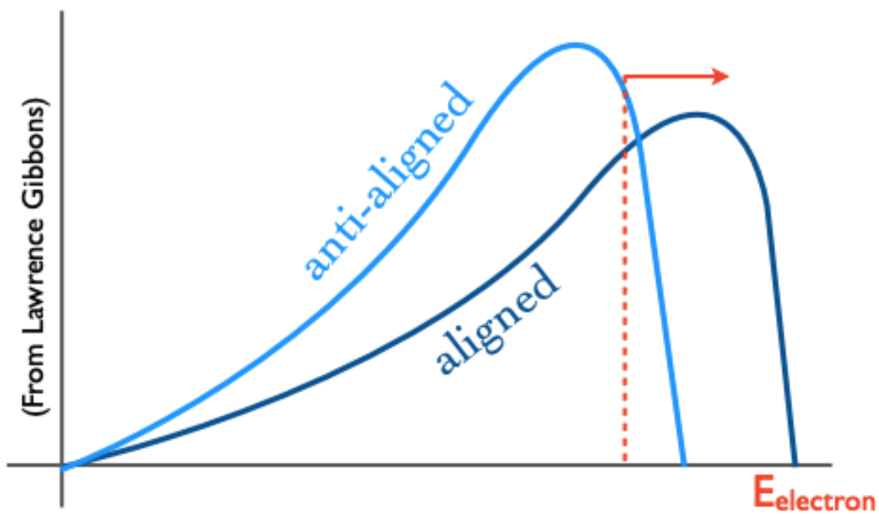


actual precession × 2

Count N(e) above fixed threshold.
Oscillation rate ∝ a_μ

3. Measure Muon Spin Precession → **3. Measure Electron Energy**

Harder electron spectrum when spin and momentum are aligned



https://indico.cern.ch/event/276476/contributions/1620139/attachments/501953/693162/g_minus_2_fewbuildins.pdf

Not an easy thing to calculate at higher orders!

1805.01944

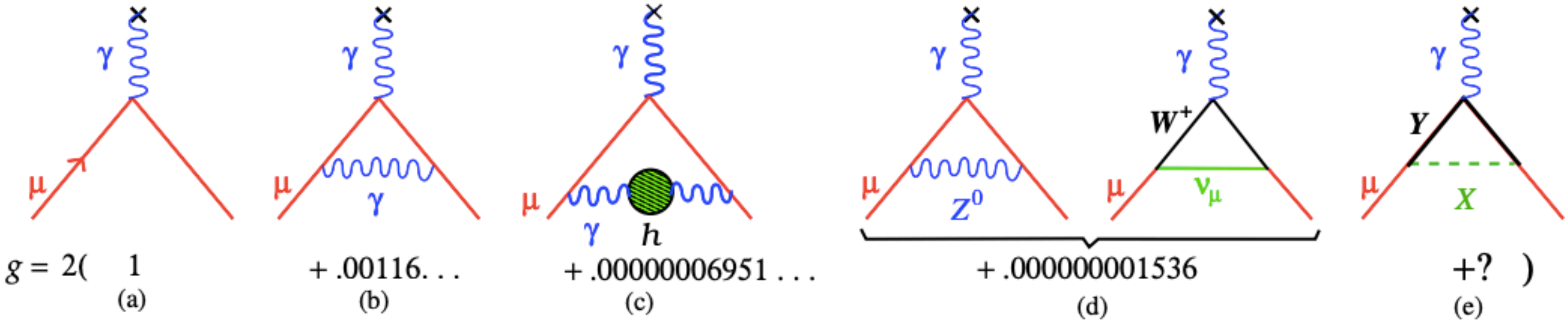


Figure 1: The Feynman graphs showing contributions to g for each of the Standard Model forces, ordered by size: (a) The Dirac interaction. (b) The lowest-order QED term $\alpha/2\pi$, which dominates the value of the anomaly. (c) The hadronic vacuum polarization contribution. (d) The lowest-order electroweak contributions. (The one-loop Higgs contribution is negligible.) (e) Potential contribution from new BSM particles X and Y.

Big challenge for evaluating the potential for BSM physics!
Measurement already accurate to 0.14 ppm!

Following 2110.04673

3.2 The results of the measurements

The BNL E821 result is [2]

$$a_\mu^{\rm BNL} = 116592091(63) \times 10^{-11}. \tag{3.9}$$

The Fermilab result combined with the BNL result is [1]

$$a_\mu^{\rm Exp} = a_\mu^{\rm BNL+FNAL} = 116592061(41) \times 10^{-11}. \tag{3.10}$$

The SM predition

$$a_\mu^{\rm SM} = 116591810(43) \times 10^{-11}, \tag{3.11}$$

consists of the following contributions [3–6]

$$a_\mu^{\rm QED} = 116584718.9(1) \times 10^{-11}, \tag{3.12}$$

5 loops!

$$a_\mu^{\rm EW} = 153.6(1) \times 10^{-11}, \tag{3.13}$$

$$a_\mu^{\rm HVP,\ LO} = 6931(40) \times 10^{-11}, \tag{3.14}$$

$$a_\mu^{\rm HVP,\ NLO} = -98.3(7) \times 10^{-11}, \tag{3.15}$$

$$a_\mu^{\rm HVP,\ NNLO} = 12.4(1) \times 10^{-11}, \tag{3.16}$$

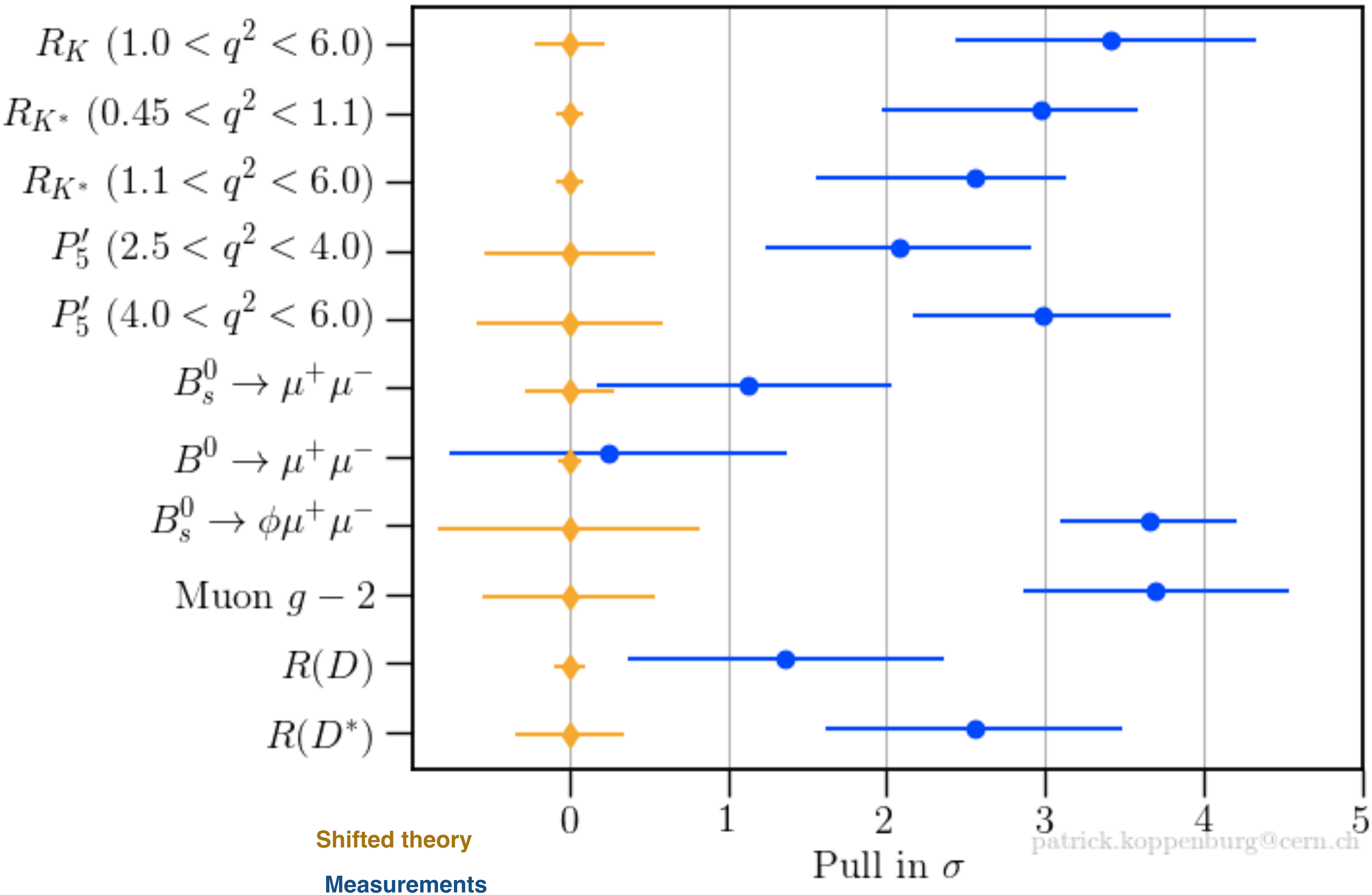
$$a_\mu^{\rm HLBL} + a_\mu^{\rm HLBL,\ NLO} = 92(18) \times 10^{-11}, \tag{3.17}$$

where the main uncertainties come from the hadronic contributions in HVP and HLBL. The deviation between the experiment and SM is

4.2σ !

$$a_\mu^{\rm Exp} - a_\mu^{\rm SM} = 251(59) \times 10^{-11}, \tag{3.18}$$

Another interesting plot from Patrick K



Homework (if we get to this)

The T2K experiment in Japan uses an off-axis beam of muon neutrinos from pion decays. Consider the case where the pion has velocity β along the z-direction in the lab frame and a neutrino with energy E^* is produced at an angle θ^* with respect to the z'-axis in the pion rest frame.

a) Show that the neutrino energy in the pion rest frame is $p^* = (m_\pi^2 - m_\mu^2)/(2m_\pi)$
b) Using a Lorentz transformation, show that the energy E and angle of production θ of the neutrino in the lab frame are: $E = \gamma E^*(1+\beta\cos\theta^*)$ and $E\cos\theta = \gamma E^*(\cos\theta^*+\beta)$, where $\gamma=E_\pi/m_\pi$
c) Show that $1 = \gamma^2(1-\beta\cos\theta)(1+\beta\cos\theta^*)$
d) Show that the maximum value of θ in the frame is $1/\gamma$
e) In the limit $\theta<<1$, show that $E\sim0.43E_\pi/(1+\beta\gamma^2\theta^2)$, and therefore on-axis ($\theta=0$), the neutrino energy spectrum follows that of the pions
f) Assuming the pions have a flat energy spectrum in the range 1-5 GeV, sketch the form of the resulting neutrino energy at the T2K far detector, which is off-axis at $\theta=2.5$ degrees. The detector is located 295 km from the beam where the first oscillation occurs, but being off-axis provides a single benefit. Explain what that is