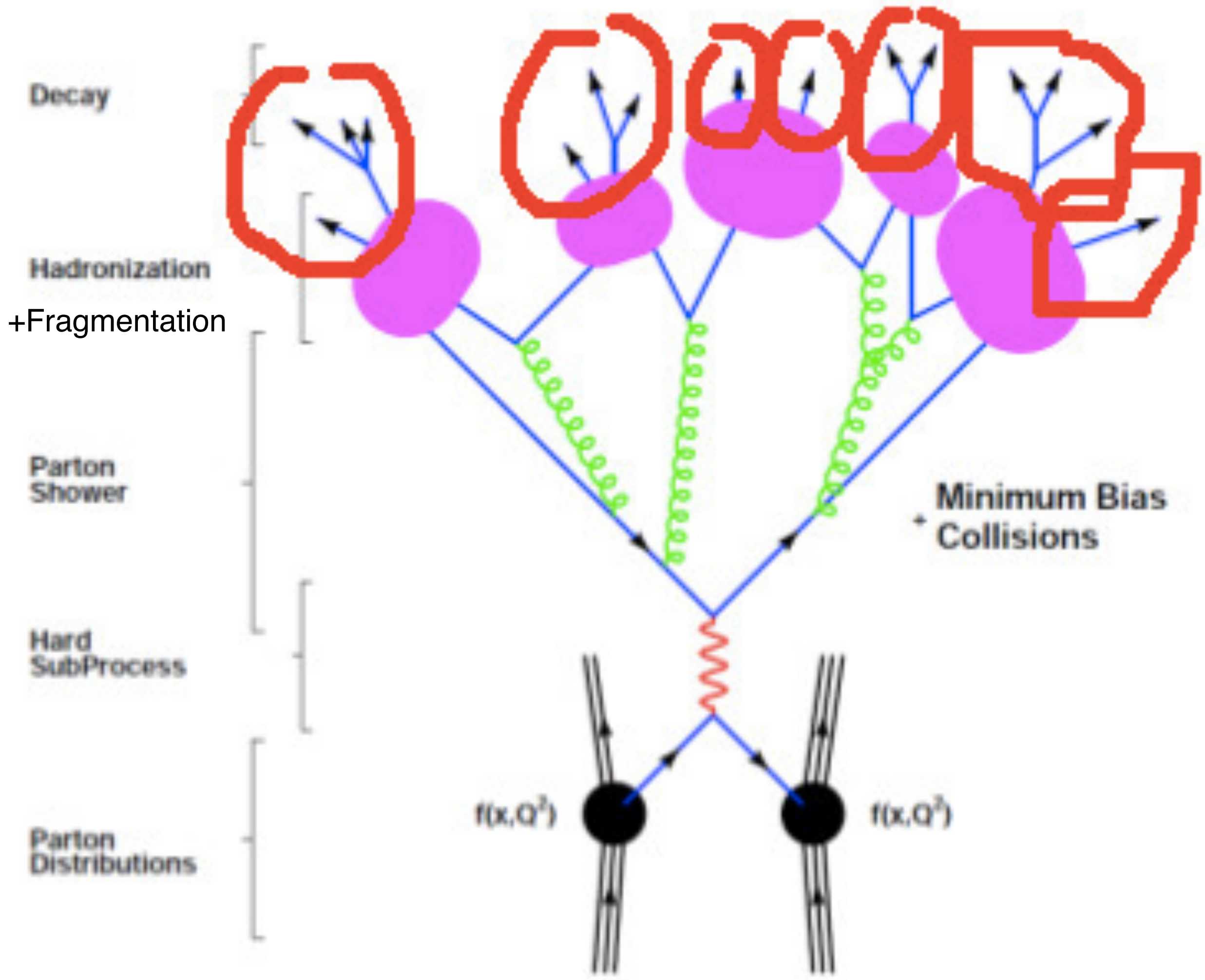
On to QCD

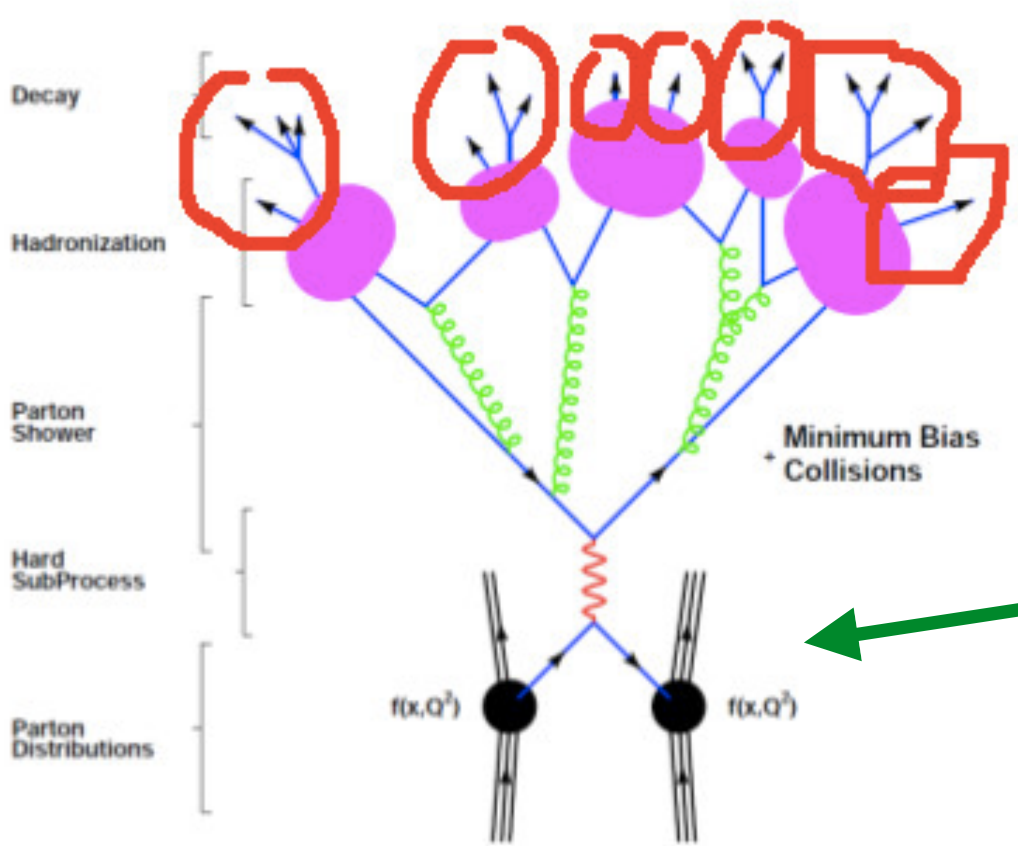


Nice image of what happens when two protons collide

Let's go over this one by one

http://tex.stackexchange.com/questions/128559/draw-randomized-tree-in-tikz

Parton Distribution Functions (PDFs)



No a priori way to predict the probability to find a quark of a **given flavor (or gluon)** containing a **momentum fraction x** when probing the proton with energy scale Q^2

Parton Distribution Functions (PDFs)

arXiv: 1701.05838

Experiment	Beam (E_b) or center-of-mass energy ($\sqrt{s}$)	$\mathcal{L}$ (1/fb)	Process	Kinematic cuts used in the present analysis (cf. orginal references for notations)	Ref.
DIS					
HERA I+II	$\sqrt{s} = 0.225 \div 0.32$ TeV	0.5	$e^\pm p \to e^\pm X$	$2.5 \le Q^2 \le 50000$ GeV^2, $2.5 \cdot 10^{-5} \le x \le 0.65$	[4]
			$e^\pm p \to \overset{(-)}{\nu} X$	$200 \le Q^2 \le 50000$ GeV^2, $1.3 \cdot 10^{-2} \le x \le 0.40$	
BCDMS	$E_b = 100 \div 280$ GeV		$\mu^+ p \to \mu^+ X$	$7 < Q^2 < 230$ GeV^2, $0.07 \le x \le 0.75$	[61]
NMC	$E_b = 90 \div 280$ GeV		$\mu^+ p \to \mu^+ X$	$2.5 \le Q^2 < 65$ GeV^2, $0.009 \le x < 0.5$	[60]
SLAC-49a	$E_b = 7 \div 20$ GeV		$e^- p \to e^- X$	$2.5 \le Q^2 < 8$ GeV^2, $0.1 < x < 0.8$, $W \ge 1.8$ GeV	[54] [62]
SLAC-49b	$E_b = 4.5 \div 18$ GeV		$e^- p \to e^- X$	$2.5 \le Q^2 < 20$ GeV^2, $0.1 < x < 0.9$, $W \ge 1.8$ GeV	[54] [62]
SLAC-87	$E_b = 8.7 \div 20$ GeV		$e^- p \to e^- X$	$2.5 \le Q^2 < 20$ GeV^2, $0.3 < x < 0.9$, $W \ge 1.8$ GeV	[54] [62]
SLAC-89b	$E_b = 6.5 \div 19.5$ GeV		$e^- p \to e^- X$	$2.5 \le Q^2 \le 19$ GeV^2, $0.17 < x < 0.9$, $W \ge 1.8$ GeV	[56] [62]
DIS heavy-quark production					
HERA I+II	$\sqrt{s} = 0.32$ TeV		$e^\pm p \to e^\pm cX$	$2.5 \le Q^2 \le 2000$ GeV^2, $2.5 \cdot 10^{-5} \le x \le 0.05$	[63]
H1	$\sqrt{s} = 0.32$ TeV	0.189	$e^\pm p \to e^\pm bX$	$5 \le Q^2 \le 2000$ GeV^2, $2 \cdot 10^{-4} \le x \le 0.05$	[15]
ZEUS	$\sqrt{s} = 0.32$ TeV	0.354	$e^\pm p \to e^\pm bX$	$6.5 \le Q^2 \le 600$ GeV^2, $1.5 \cdot 10^{-4} \le x \le 0.035$	[16]
CCFR	$87 \lesssim E_b \lesssim 333$ GeV		$\overset{(-)}{\nu} p \to \mu^\pm cX$	$1 \le Q^2 < 170$ GeV^2, $0.015 \le x \le 0.33$	[64]
CHORUS	$\langle E_b \rangle \approx 27$ GeV		$\nu p \to \mu^+ cX$		[18]
NOMAD	$6 \le E_b \le 300$ GeV		$\nu p \to \mu^+ cX$	$1 \le Q^2 < 20$ GeV^2, $0.02 \lesssim x \le 0.75$	[17]
NuTeV	$79 \lesssim E_b \lesssim 245$ GeV		$\overset{(-)}{\nu} p \to \mu^\pm cX$	$1 \le Q^2 < 120$ GeV^2, $0.015 \le x \le 0.33$	[64]
DY					
ATLAS	$\sqrt{s} = 7$ TeV	0.035	$pp \to W^\pm X \to l^\pm \nu X$	$p_T^l > 20$ GeV, $p_T^\nu > 25$ GeV, $m_T > 40$ GeV	[66]
			$pp \to ZX \to l^+ l^- X$	$p_T^l > 20$ GeV, $66 < m_{ll} < 116$ GeV	
	$\sqrt{s} = 13$ TeV	0.081	$pp \to W^\pm X \to l^\pm \nu X$	$p_T^\nu > 25$ GeV, $m_T > 50$ GeV	[26]
			$pp \to ZX \to l^+ l^- X$	$p_T^l > 25$ GeV, $66 < m_{ll} < 116$ GeV	
CMS	$\sqrt{s} = 7$ TeV	4.7	$pp \to W^\pm X \to \mu^\pm \nu X$	$p_T^\mu > 25$ GeV	[24]
	$\sqrt{s} = 8$ TeV	18.8	$pp \to W^\pm X \to \mu^\pm \nu X$	$p_T^\mu > 25$ GeV	[25]
DØ	$\sqrt{s} = 1.96$ TeV	7.3	$\bar{p}p \to W^\pm X \to \mu^\pm \nu X$	$p_T^\mu > 25$ GeV, $\not{E}_T > 25$ GeV	[23]
		9.7	$\bar{p}p \to W^\pm X \to e^\pm \nu X$	$p_T^e > 25$ GeV, $\not{E}_T > 25$ GeV	[22]
LHCb	$\sqrt{s} = 7$ TeV	1	$pp \to W^\pm X \to \mu^\pm \nu X$	$p_T^\mu > 20$ GeV	[19]
			$pp \to ZX \to \mu^+ \mu^- X$	$p_T^\mu > 20$ GeV, $60 < m_{\mu\mu} < 120$ GeV	
	$\sqrt{s} = 8$ TeV	2	$pp \to ZX \to e^+ e^- X$	$p_T^e > 20$ GeV, $60 < m_{ee} < 120$ GeV	[21]
		2.9	$pp \to W^\pm X \to \mu^\pm \nu X$	$p_T^\mu > 20$ GeV	[20]
			$pp \to ZX \to \mu^+ \mu^- X$	$p_T^\mu > 20$ GeV, $60 < m_{\mu\mu} < 120$ GeV	
FNAL-605	$E_b = 800$ GeV		$pCu \to \mu^+ \mu^- X$	$7 \le M_{\mu\mu} \le 18$ GeV	[67]
FNAL-866	$E_b = 800$ GeV		$pp \to \mu^+ \mu^- X$	$4.6 \le M_{\mu\mu} \le 12.9$ GeV	[68]
			$pD \to \mu^+ \mu^- X$		

What sorts of measurements can help **constrain** PDFs, which are parametric models?

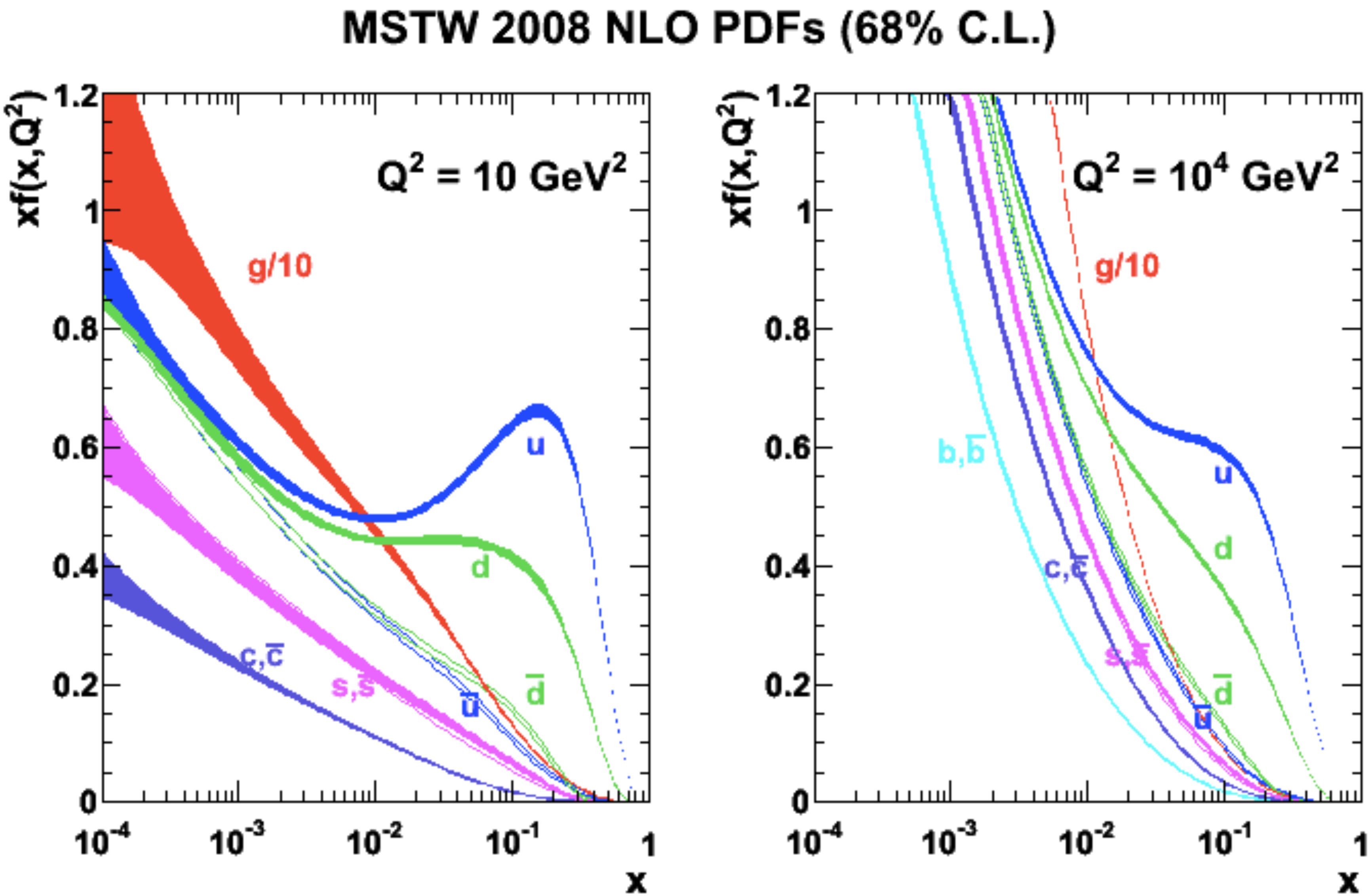
Experiment	ATLAS			CMS			CDF&DØ
$\sqrt{s}$ (TeV)	7	8	13	7	8	13	1.96
Final states	tq	tq	tq	tq	tq	tq	$tq, t\bar{b}$
Reference	[27]	[28]	[29]	[30]	[31]	[32]	[53]
Luminosity (1/fb)	4.59	20.3	3.2	2.73	19.7	2.3	9.7x2
Cross section (pb)	68 ± 8	82.6 ± 12.1	247 ± 46	67.2 ± 6.1	83.6 ± 7.7	232 ± 30.9	$3.30^{+0.52}_{-0.40}$ (sum)

		Cross section (pb)						
$\sqrt{s}$ (TeV)		5	7		8		13	
Experiment		CMS	ATLAS	CMS	ATLAS	CMS	ATLAS	CMS
Decay mode	dilepton + b-jet(s)		183 ± 48 [36]		243 ± 8 [36]		818 ± 36 [37]	792 ± 43 [38]
	dilepton + jets		181 ± 11 [33]	174 ± 6 [34]		245 ± 9 [34]		746 ± 86 [35]
	lepton + jets			162 ± 14 [39]	260 ± 24 [40]	229 ± 15 [39]		836 ± 133 [41]
	lepton + jets, $b \to \mu\nu X$		165 ± 38 [42]					
	lepton + $\tau \to$ hadrons		183 ± 25 [43]	143 ± 26 [44]		257 ± 25 [51]		
	jets + $\tau \to$ hadrons		194 ± 49 [46]	152 ± 34 [47]				
	all-jets		168 ± 60 [48]	139 ± 28 [49]		276 ± 39 [45]		834^{+123}_{-109} [50]
	$e\mu$	82 ± 23 [52]						

Parton Distribution Functions (PDFs)

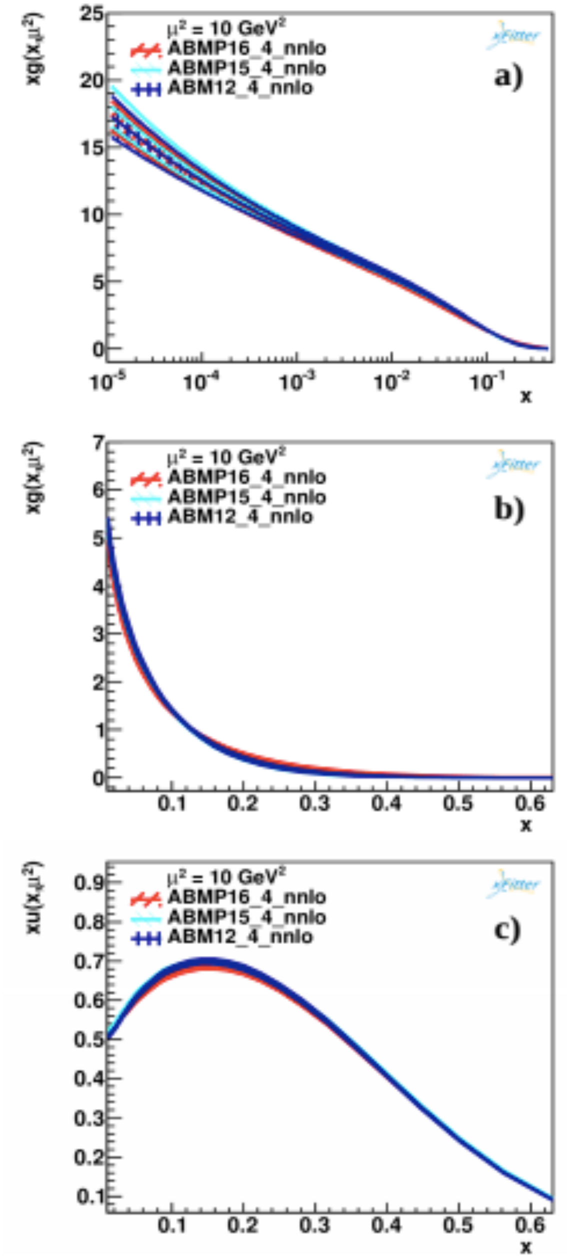
https://mstwpdf.hepforge.org/

Note large uncertainty on gluons and on PDFs at lower x!



What do you think you see if you probe the proton (or an anti-proton) with very low energy?

Can lead to large uncertainties on production/kinematics of many processes at the LHC!



arXiv: 1701.05838

But recent interesting news from LHCb

arXiv: 2109.08084

Study Z+jet production as a function of rapidity in the detector, particularly $\sigma(Zc)/\sigma(Zj)$

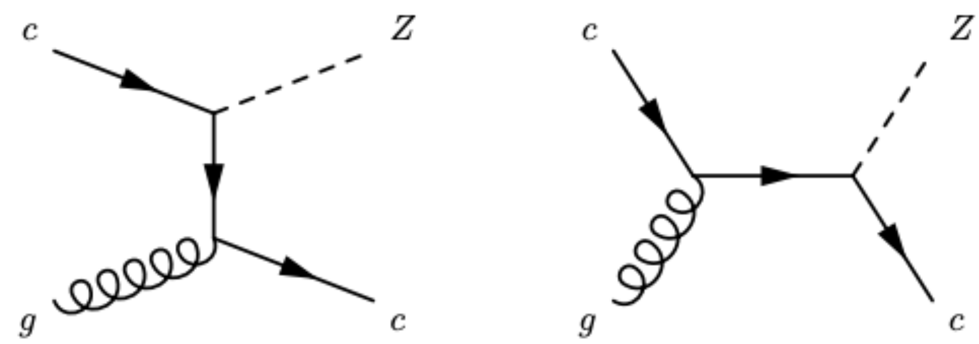


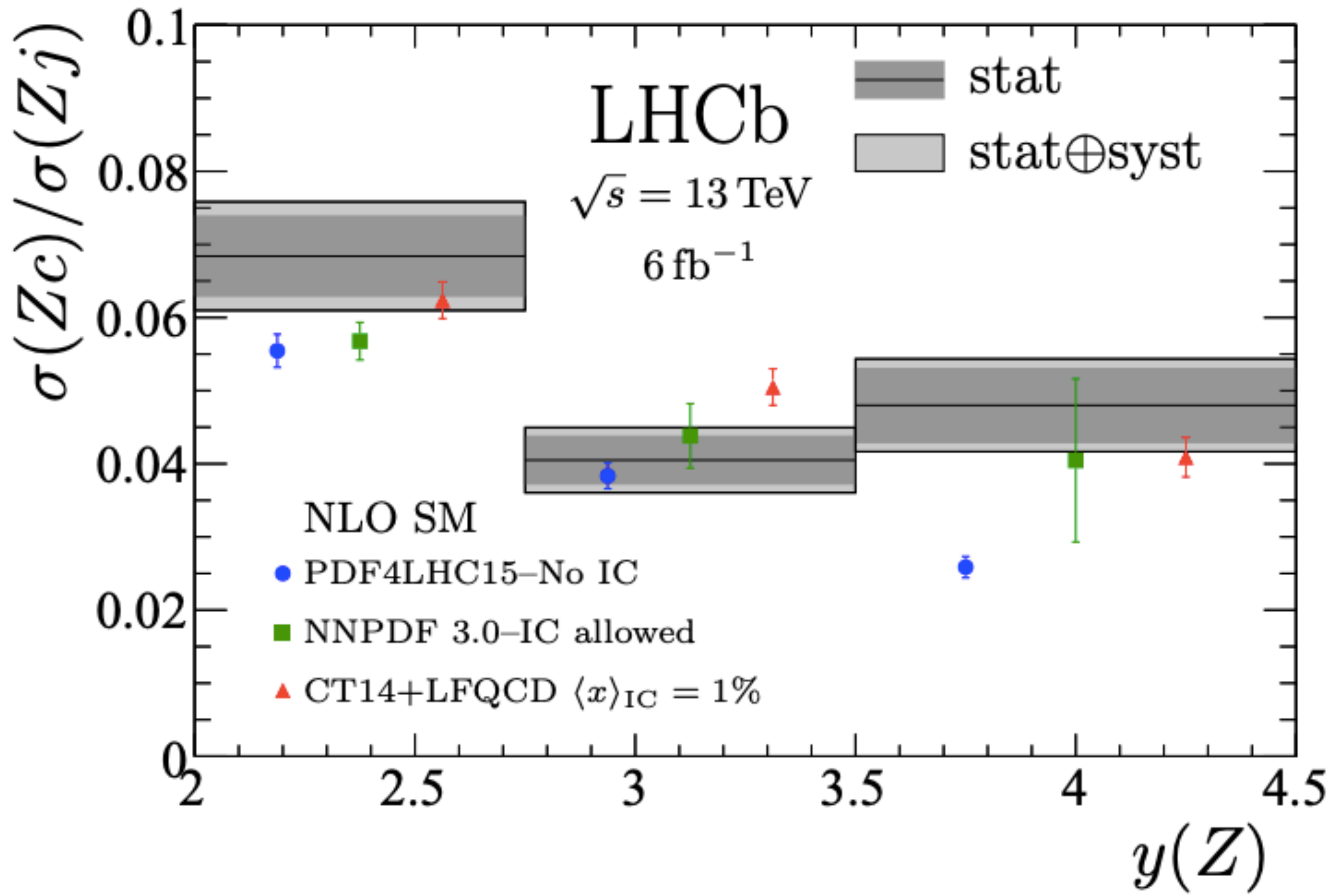
Figure 1: Leading-order Feynman diagrams for $gc \to Zc$ production.

"Intrinsic charm" ($|p>$ contains $|uudc\bar{c}>$ component) would appear as charm enhancement in forward region

Let's talk about why this is tricky to measure

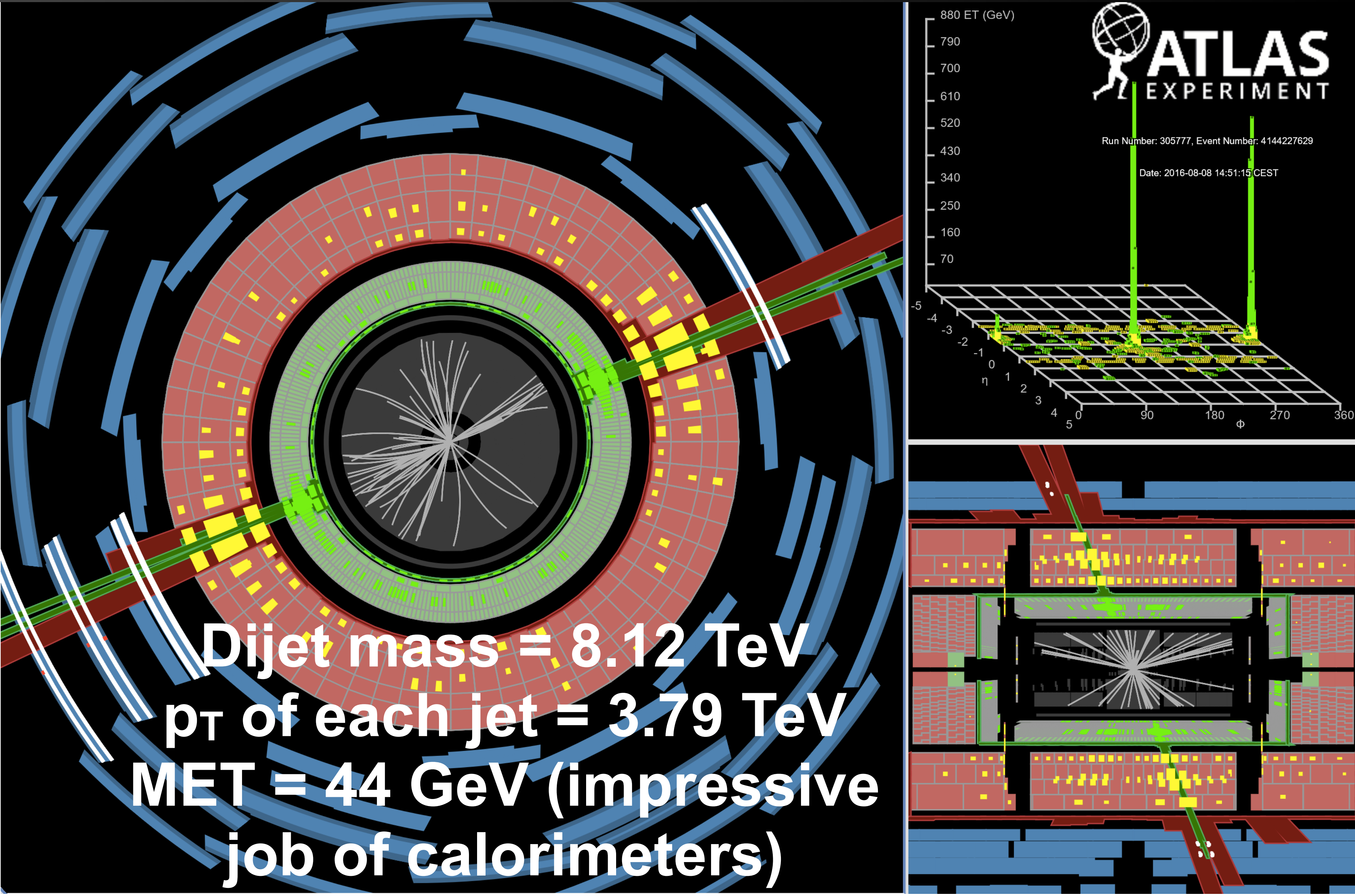
But recent interesting news from LHCb

arXiv: 2109.08084



Evidence for intrinsic charm in the proton!

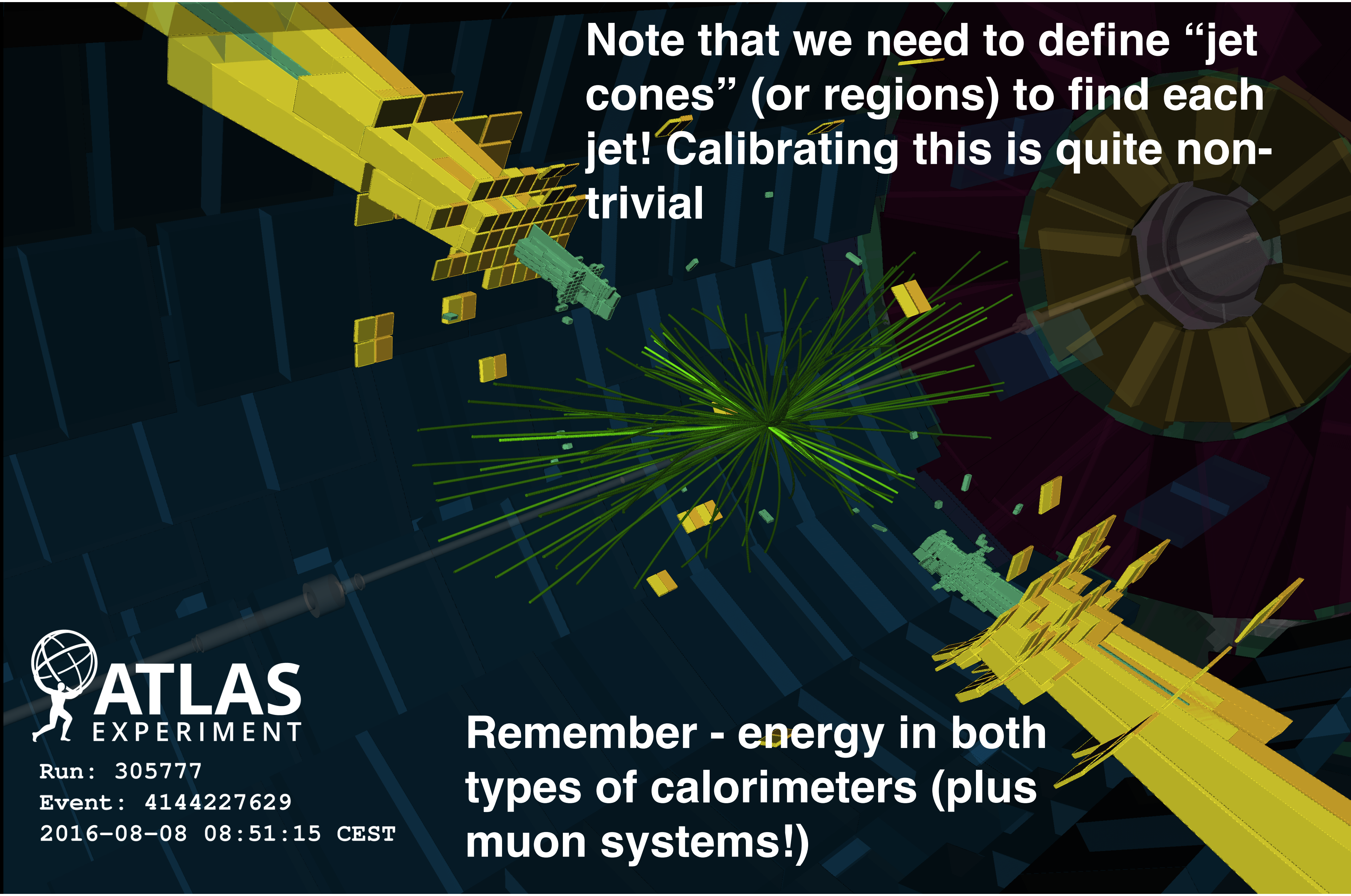
ATLAS dijet event



Dijet mass = 8.12 TeV
p_T of each jet = 3.79 TeV
MET = 44 GeV (impressive job of calorimeters)

https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PAPERS/EXOT-2016-21/

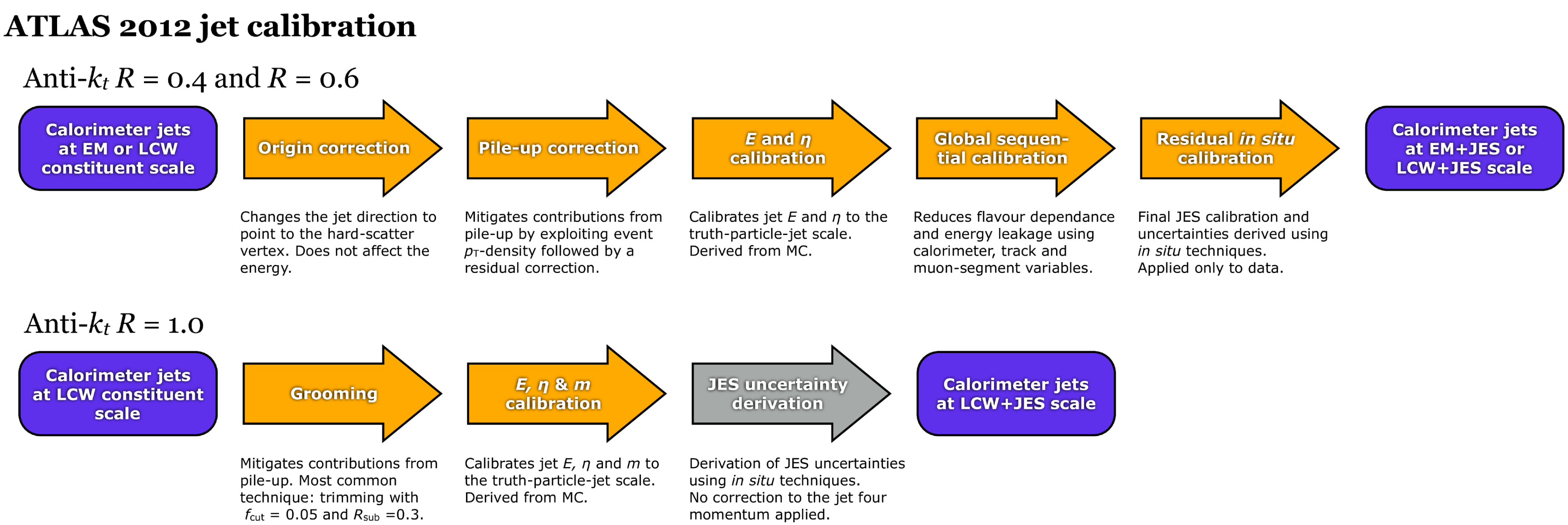
Another view of the event



Note that we need to define "jet cones" (or regions) to find each jet! Calibrating this is quite non-trivial

Remember - energy in both types of calorimeters (plus muon systems!)

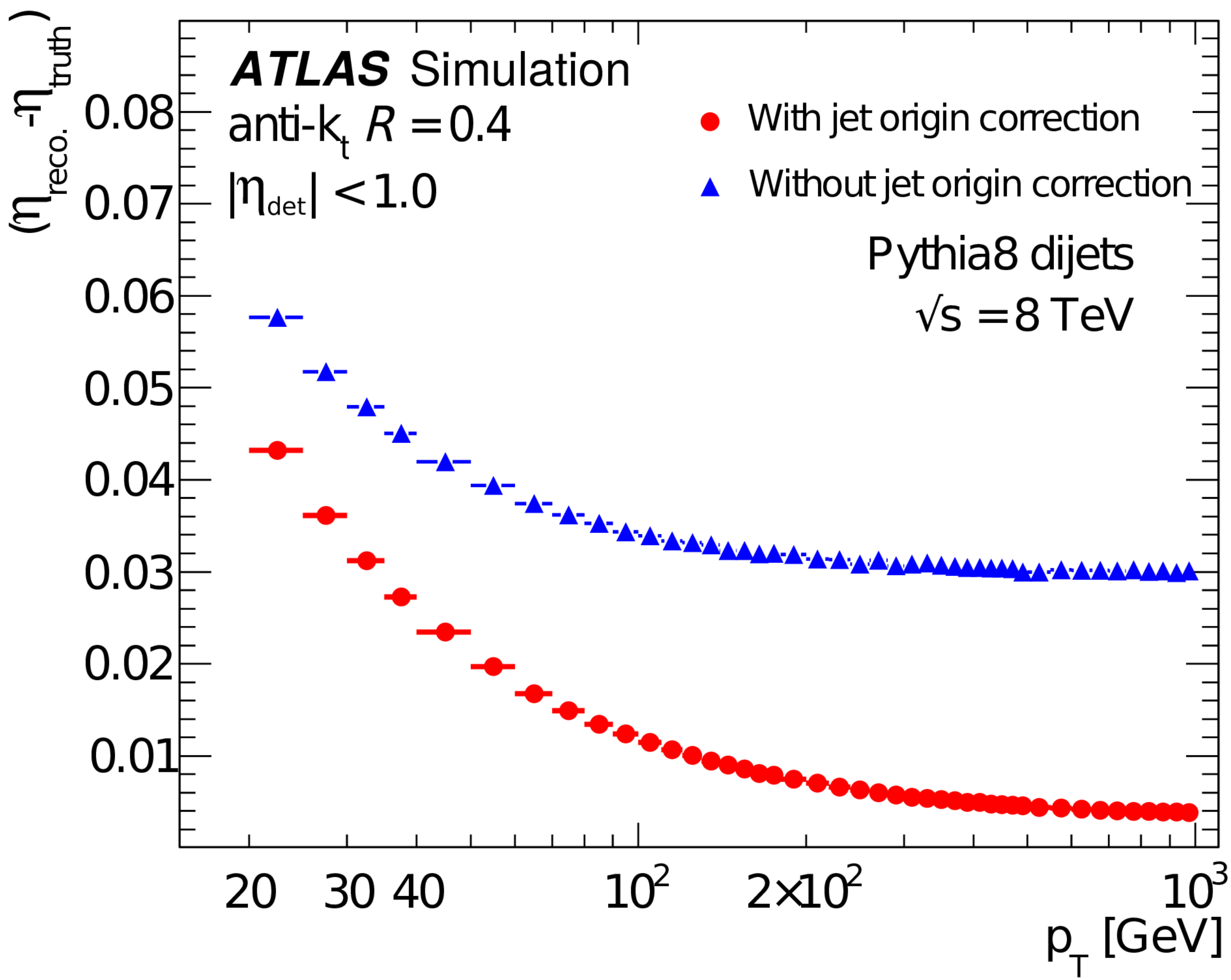
Jet calibration



1910.04482

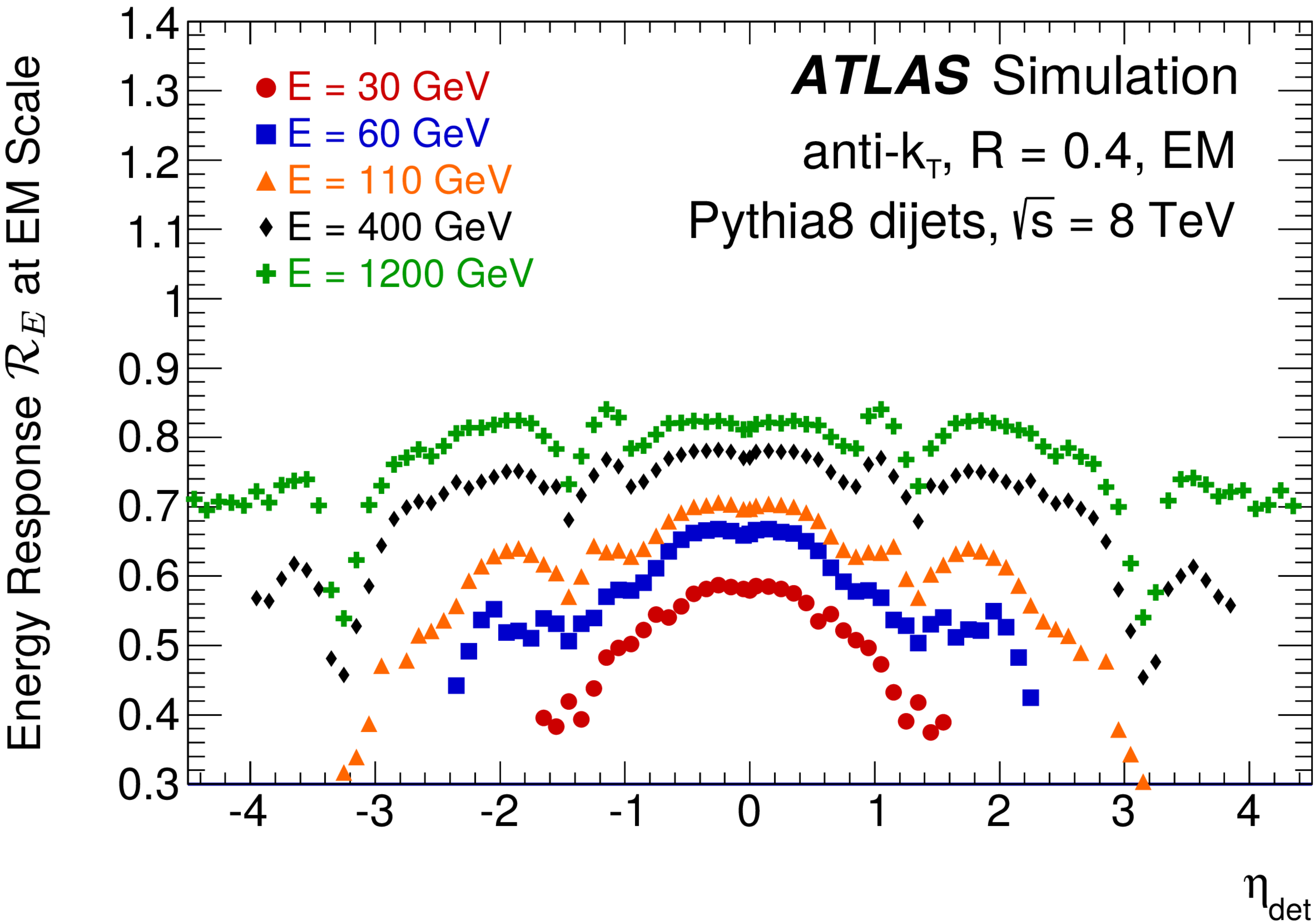
Jet origin choice

1910.04482

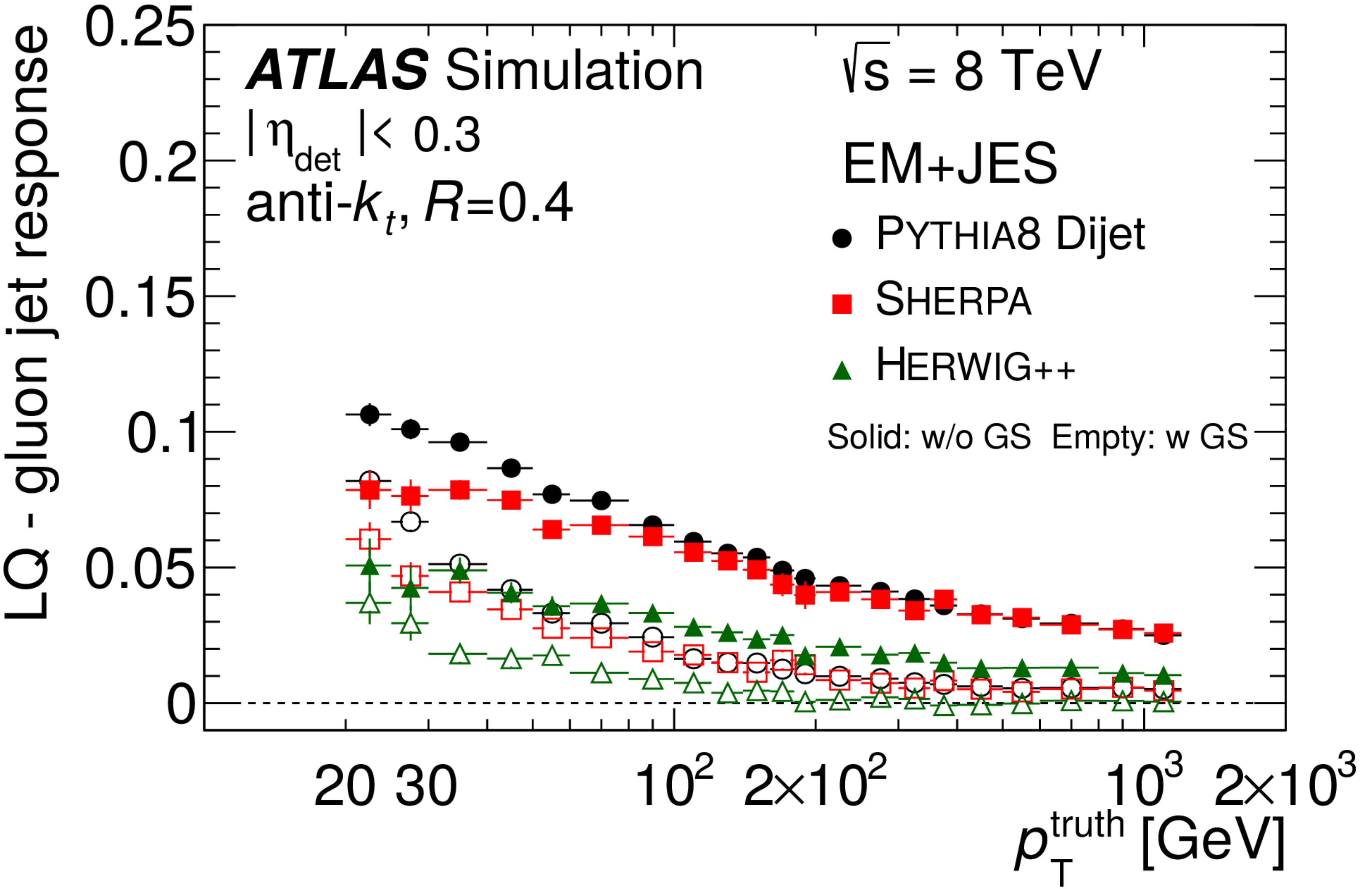


Jet origin response

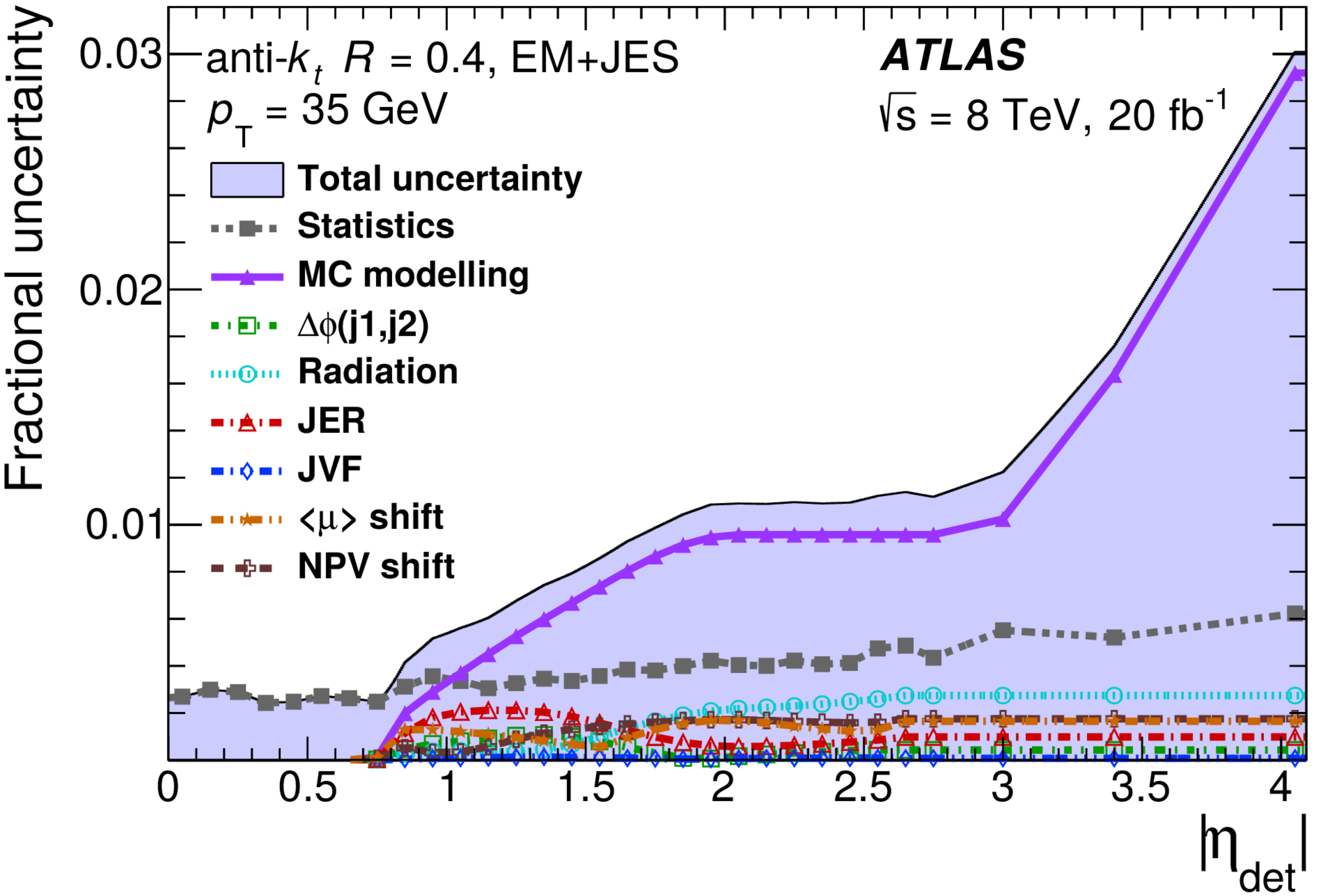
1910.04482



Response for different jets 1910.04482

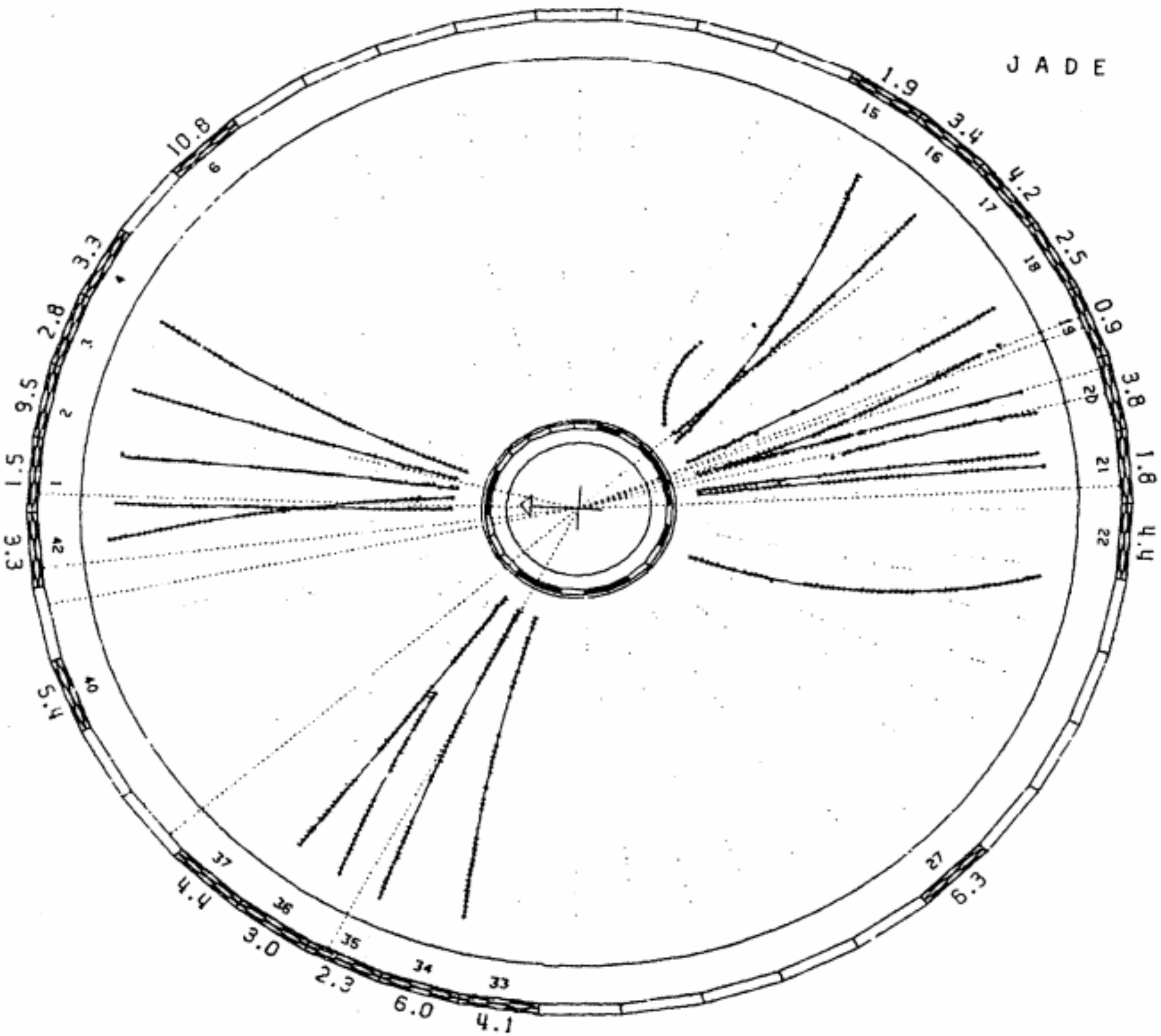


Jet Energy Scale uncertainties 1910.04482



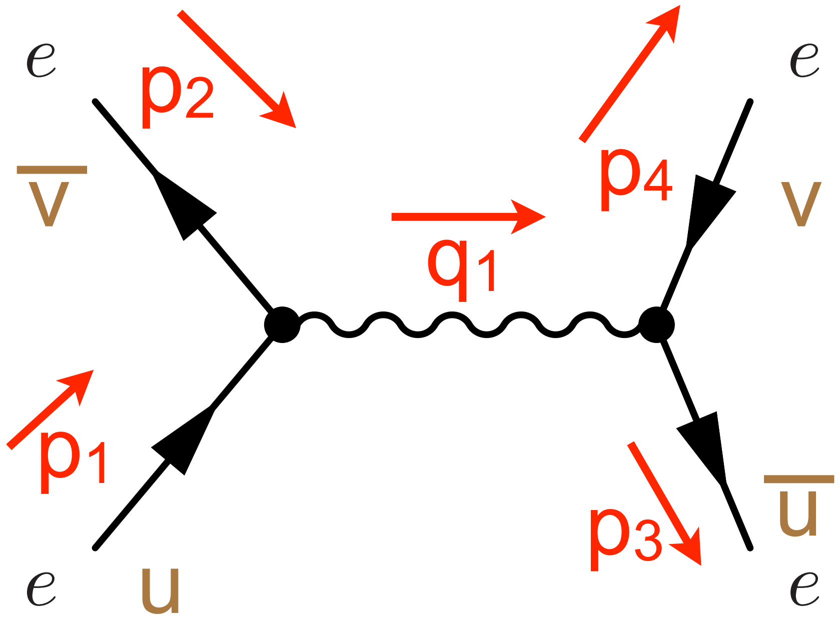
Trijet event

P. Duinker, *"Review of e+e- physics at PETRA,"* Rev. Mod. Phys. 54 (2), 325-387 (1982), also http://www.quantumdiaries.org/2011/07/09/in-a-world-without-color-why-do-i-believe-in-gluons/trijet_topology_rhophi_2/



Since quarks/anti-quarks only come in pairs, tri-jet events can be used as evidence for QCD radiation of gluons

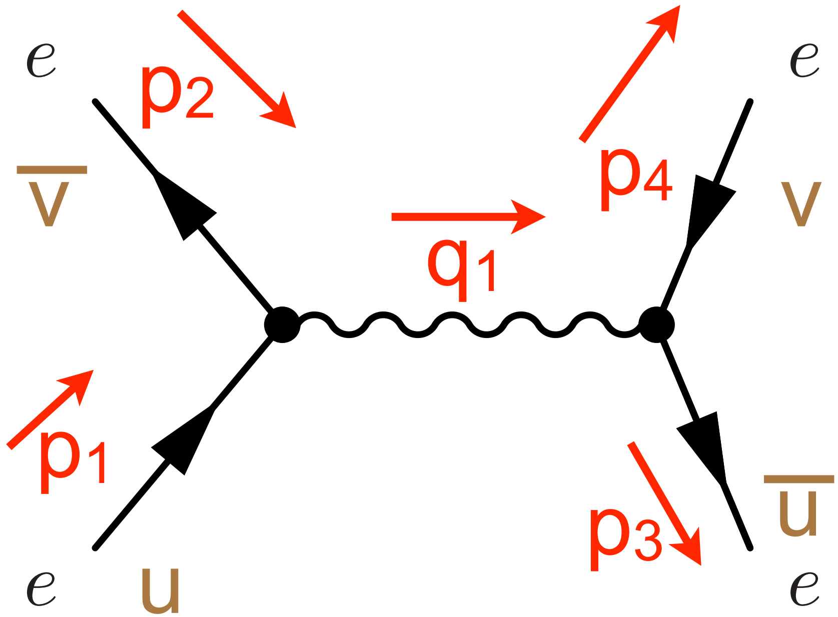
QCD production of quarks



$$-\frac{e^2}{(p_1+p_2)^2}[\overline{u}(3)(\gamma^\mu)v(4)][\overline{v}(2)\gamma_\mu u(1)]$$

We evaluated this diagram when discussing QED. QCD is similar, except that we have to be careful about the charge of quarks

QCD production of quarks



$$\mathcal{M} = -\frac{Qe^2}{(p_1+p_2)^2}[\overline{u}(3)(\gamma^\mu)v(4)][\overline{v}(2)\gamma_\mu u(1)]$$

Each vertex gave us a factor of "e" - but u,c and t quarks have charge Q=2e/3 and d,c,s quarks have charge Q=-e/3

Evaluating matrix element

$$\mathcal{M} = -\frac{Qe^2}{(p_1+p_2)^2}[\overline{u}(3)(\gamma^\mu)v(4)][\overline{v}(2)\gamma_\mu u(1)]$$

$$\mathcal{M}^2 = \frac{1}{4}\left[\frac{Qe^2}{(p_1+p_2)^2}\right]^2 Tr[\gamma^\mu(\not{p}_4 - m_Q)\gamma^\nu(\not{p}_3 + m_Q)]Tr[\gamma_\mu(\not{p}_1 + m_e)\gamma_\nu(\not{p}_2 - m_e)]$$

Where 1/4 comes from averaging over initial spins and m_e is mass of electron and m_Q is mass of quarks being collided. That obvious?

Let's expand that out

$$\mathcal{M}^2 = \frac{1}{4}\left[\frac{Qe^2}{(p_1+p_2)^2}\right]^2 Tr[\gamma^\mu(\not{p}_4 - m_Q)\gamma^\nu(\not{p}_3 + m_Q)]Tr[\gamma_\mu(\not{p}_1 + m_e)\gamma_\nu(\not{p}_2 - m_e)]$$

$$\mathcal{M}^2 = \frac{1}{4}\left[\frac{Qe^2}{(p_1+p_2)^2}\right]^2 Tr\left[\gamma^\mu\not{p}_4\gamma^\nu\not{p}_3 - m_Q^2\gamma^\mu\gamma^\nu + \cancel{m_Q(\gamma^\mu\not{p}_4\gamma^\nu - \gamma^\mu\gamma^\nu\not{p}_3)}\right] \times$$

$$Tr\left[\gamma_\mu\not{p}_1\gamma_\nu\not{p}_2 - m_e^2\gamma_\mu\gamma_\nu + \cancel{m_e(\gamma_\mu\not{p}_1\gamma_\nu - \gamma_\mu\gamma_\nu\not{p}_2)}\right]$$

Product of odd number of gamma matrices is zero

Let's expand that out

$$\mathcal{M}^2 = \frac{1}{4}\left[\frac{Qe^2}{(p_1+p_2)^2}\right]^2 Tr\left[\gamma^\mu \not{p}_4 \gamma^\nu \not{p}_3 - m_Q^2\gamma^\mu\gamma^\nu\right] \times Tr\left[\gamma_\mu \not{p}_1 \gamma_\nu \not{p}_2 - m_e^2\gamma_\mu\gamma_\nu\right]$$

$$\mathcal{M}^2 = \frac{1}{4}\left[\frac{Qe^2}{(p_1+p_2)^2}\right]^2 \left[Tr\ \gamma^\mu \not{p}_4 \gamma^\nu \not{p}_3 - 4m_Q^2 g^{\mu\nu}\right] \times \left[Tr\ \gamma_\mu \not{p}_1 \gamma_\nu \not{p}_2 - 4m_e^2 g_{\mu\nu}\right]$$

$$\mathcal{M}^2 = \frac{1}{4}\left[\frac{Qe^2}{(p_1+p_2)^2}\right]^2 \left[p_{4\lambda}p_{3\sigma} Tr\ \gamma^\mu\gamma^\lambda\gamma^\nu\gamma^\sigma - 4m_Q^2 g^{\mu\nu}\right] \times \left[p_1^\lambda p_2^\sigma Tr\ \gamma_\mu\gamma_\lambda\gamma_\nu\gamma_\sigma - 4m_e^2 g_{\mu\nu}\right]$$

More plug and chug

$$\mathcal{M}^2 = \frac{1}{4}\left[\frac{Qe^2}{(p_1+p_2)^2}\right]^2 \left[p_{4\lambda}p_{3\sigma}Tr\ \gamma^\mu\gamma^\lambda\gamma^\nu\gamma^\sigma - 4m_Q^2 g^{\mu\nu}\right] \times \left[p_1^\lambda p_2^\sigma Tr\ \gamma_\mu\gamma_\lambda\gamma_\nu\gamma_\sigma - 4m_e^2 g_{\mu\nu}\right]$$

$$\mathcal{M}^2 = \frac{1}{4}\left[\frac{Qe^2}{(p_1+p_2)^2}\right]^2 \left[p_{4\lambda}p_{3\sigma}4(g^{\mu\lambda}g^{\nu\sigma} + g^{\mu\sigma}g^{\lambda\nu} - g^{\mu\nu}g^{\lambda\sigma}) - 4m_Q^2 g^{\mu\nu}\right] \times$$
$$\left[p_1^\lambda p_2^\sigma 4(g_{\mu\lambda}g_{\nu\sigma} + g_{\mu\sigma}g_{\lambda\nu} - g_{\mu\nu}g_{\lambda\sigma}) - 4m_e^2 g_{\mu\nu}\right]$$

$$\mathcal{M}^2 = 4\left[\frac{Qe^2}{(p_1+p_2)^2}\right]^2 \left[p_4^\mu p_3^\nu + p_4^\nu p_3^\mu - g^{\mu\nu}p_3\cdot p_4 - m_Q^2 g^{\mu\nu}\right] \times$$
$$\left[p_{1\mu}p_{2\nu} + p_{1\nu}p_{2\mu} - g_{\mu\nu}p_1\cdot p_2 - m_e^2 g_{\mu\nu}\right]$$

More plug and chug

$$\mathcal{M}^2 = 4\left[\frac{Qe^2}{(p_1+p_2)^2}\right]^2 \left[p_4^\mu p_3^\nu + p_4^\nu p_3^\mu - g^{\mu\nu} p_3 \cdot p_4 - m_Q^2 g^{\mu\nu}\right] \times$$
$$\left[p_{1\mu} p_{2\nu} + p_{1\nu} p_{2\mu} - g_{\mu\nu} p_1 \cdot p_2 - m_e^2 g_{\mu\nu}\right]$$

$$\mathcal{M}^2 = 4\left[\frac{Qe^2}{(p_1+p_2)^2}\right]^2 [(p_1\cdot p_4)(p_2\cdot p_3) + (p_2\cdot p_4)(p_1\cdot p_3) - (p_3\cdot p_4)(p_1\cdot p_2) - m_e^2(p_3\cdot p_4) + (p_1\cdot p_3)(p_2\cdot p_4) +$$
$$(p_2\cdot p_3)(p_1\cdot p_4) - (p_3\cdot p_4)(p_1\cdot p_2) - m_e^2(p_3\cdot p_4) - (p_3\cdot p_4)(p_1\cdot p_2) - (p_3\cdot p_4)(p_1\cdot p_2) +$$
$$4(p_3\cdot p_4)(p_1\cdot p_2) + 4m_e^2(p_3\cdot p_4) - m_Q^2(p_1\cdot p_2) - m_Q^2(p_1\cdot p_2) + 4m_Q^2(p_1\cdot p_2) + 4m_Q^2 m_e^2]$$

$$\mathcal{M}^2 = 4\left[\frac{Qe^2}{(p_1+p_2)^2}\right]^2 [2(p_1\cdot p_4)(p_2\cdot p_3) + 2(p_2\cdot p_4)(p_1\cdot p_3) + 2m_e^2(p_3\cdot p_4) + 2m_Q^2(p_1\cdot p_2) + 4m_Q^2 m_e^2]$$

$$\mathcal{M}^2 = 8\left[\frac{Qe^2}{(p_1+p_2)^2}\right]^2 [(p_1\cdot p_4)(p_2\cdot p_3) + (p_2\cdot p_4)(p_1\cdot p_3) + m_e^2(p_3\cdot p_4) + m_Q^2(p_1\cdot p_2) + 2m_Q^2 m_e^2]$$

Now we pick a frame

$$\mathcal{M}^2 = 8\left[\frac{Qe^2}{(p_1+p_2)^2}\right]^2 [(p_1\cdot p_4)(p_2\cdot p_3) + (p_2\cdot p_4)(p_1\cdot p_3) + m_e^2(p_3\cdot p_4) + m_Q^2(p_1\cdot p_2) + 2m_Q^2 m_e^2]$$

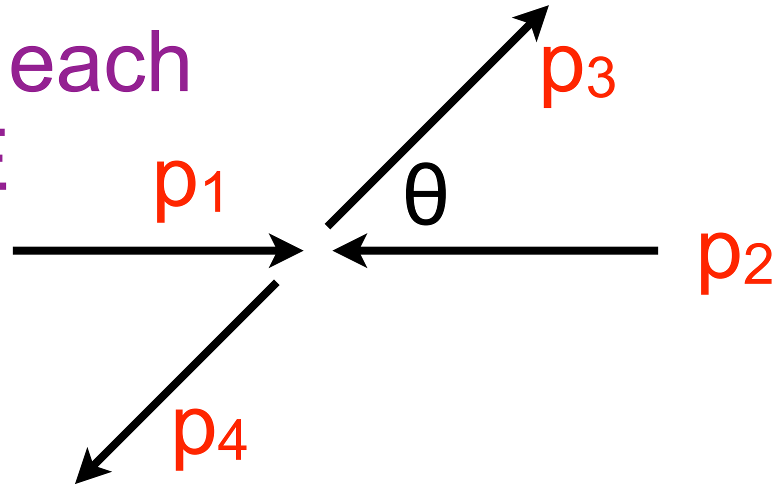
$|\mathbf{p}_1|=|\mathbf{p}_2|=p_i$

$|\mathbf{p}_3|=|\mathbf{p}_4|=p_f$

$$|\mathbf{p_i}| = \sqrt{E^2 - m_e^2}$$

$$|\mathbf{p_f}| = \sqrt{E^2 - m_Q^2}$$

Pick center of mass frame where energy of each object = E



$$p_1\cdot p_2 = E^2 + p_i^2 = 2E^2 - m_e^2$$

$$(p_1+p_2)^2 = p_1^2 + p_2^2 + 2p_1\cdot p_2 = 2m_e^2 + 4E^2 - 2m_e^2 = 4E^2$$

$$p_3\cdot p_4 = E^2 + p_f^2 = 2E^2 - m_Q^2$$

$$p_1\cdot p_3 = E^2 - p_f p_i\cos\theta = p_2\cdot p_4$$

$$p_1\cdot p_4 = E^2 + p_f p_i\cos\theta = p_2\cdot p_3$$

Plugging it in

$$\mathcal{M}^2 = 8\left[\frac{Qe^2}{(p_1+p_2)^2}\right]^2 [(p_1\cdot p_4)(p_2\cdot p_3) + (p_2\cdot p_4)(p_1\cdot p_3) + m_e^2(p_3\cdot p_4) + m_Q^2(p_1\cdot p_2) + 2m_Q^2 m_e^2]$$

$$p_1\cdot p_2 = E^2 + p_i^2 = 2E^2 - m_e^2$$

$$(p_1+p_2)^2 = p_1^2 + p_2^2 + 2p_1\cdot p_2 = 2m_e^2 + 4E^2 - 2m_e^2 = 4E^2$$

$$p_3\cdot p_4 = E^2 + p_f^2 = 2E^2 - m_Q^2$$

$$p_1\cdot p_3 = E^2 - p_f p_i \cos\theta = p_2\cdot p_4$$

$$p_1\cdot p_4 = E^2 + p_f p_i \cos\theta = p_2\cdot p_3$$

$$\mathcal{M}^2 = 8\left[\frac{Qe^2}{4E^2}\right]^2 [(E^2 + p_f p_i\cos\theta)^2 + (E^2 - p_f p_i\cos\theta)^2 + (2E^2 - m_Q^2)m_e^2 + (2E^2 - m_e^2)m_Q^2 + 2m_Q^2 m_e^2]$$

$$\mathcal{M}^2 = \frac{Q^2e^4}{2E^4}[2E^4 + 2p_f^2 p_i^2\cos^2\theta + 2E^2(m_e^2 + m_Q^2)]$$

Plugging it in

$$\mathcal{M}^2 = \frac{Q^2 e^4}{2E^4}[2E^4 + 2p_f^2 p_i^2 \cos^2\theta + 2E^2(m_e^2 + m_Q^2)]$$

$$|\mathbf{p_i}| = \sqrt{E^2 - m_e^2}$$

$$|\mathbf{p_f}| = \sqrt{E^2 - m_Q^2}$$

$$\mathcal{M}^2 = \frac{Q^2 e^4}{2E^4}[2E^4 + 2(E^2 - m_e^2)(E^2 - m_Q^2)\cos^2\theta + 2E^2(m_e^2 + m_Q^2)]$$

$$\mathcal{M}^2 = \frac{Q^2 e^4}{2E^4}[2E^4 + 2\cos^2\theta(E^4 + m_e^2 m_Q^2 - E^2 m_e^2 - E^2 m_Q^2) + 2E^2 m_e^2 + 2E^2 m_Q^2]$$

$$\mathcal{M}^2 = Q^2 e^4 \left[1 + \cos^2\theta \left(1 + \frac{m_e^2 m_Q^2}{E^4} - \frac{m_e^2}{E^2} - \frac{m_Q^2}{E^2}\right) + \frac{m_Q^2 + m_e^2}{E^2}\right]$$

$$\boxed{\mathcal{M}^2 = Q^2 e^4 \left[1 + \cos^2\theta \left(1 - \frac{m_e^2}{E^2}\right)\left(1 - \frac{m_Q^2}{E^2}\right) + \frac{m_Q^2 + m_e^2}{E^2}\right]}$$

Differential cross section

$$\mathcal{M}^2 = Q^2 e^4 \left[1 + \cos^2\theta \left(1 - \frac{m_e^2}{E^2}\right)\left(1 - \frac{m_Q^2}{E^2}\right) + \frac{m_Q^2 + m_e^2}{E^2} \right]$$

Recall that we derived this a long time ago

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{|M|^2}{(E_1 + E_2)^2} \cancel{\frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}}$$

Cancel

In region with energy far about masses

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{1}{4E^2} Q^2 e^4 (1 + \cos^2\theta)$$

$$\frac{d\sigma}{d\Omega} = \frac{Q^2 e^2}{256\pi^2 E^2} (1 + \cos^2\theta)$$

How to get at the total cross section?

$$\mathcal{M}^2 = Q^2 e^4 \left[1 + \cos^2\theta \left(1 - \frac{m_e^2}{E^2}\right)\left(1 - \frac{m_Q^2}{E^2}\right) + \frac{m_Q^2 + m_e^2}{E^2} \right]$$

Start again with

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{|M|^2}{(E_1 + E_2)^2} \frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}$$

$$\sigma = \int \frac{d\sigma}{d\Omega} \sin\theta d\theta d\phi$$

$$\sigma = \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \frac{1}{64\pi^2} \frac{Q^2 e^4 \left[1 + \cos^2\theta (1 - \frac{m_e^2}{E^2})(1 - \frac{m_Q^2}{E^2}) + \frac{m_Q^2 + m_e^2}{E^2}\right]}{4E^2} \sqrt{\frac{E^2 - m_Q^2}{E^2 - m_e^2}} d\theta$$

Total cross section calculation

$$\sigma = \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \frac{1}{64\pi^2} \frac{Q^2 e^4 \left[1 + \cos^2\theta(1 - \frac{m_e^2}{E^2})(1 - \frac{m_Q^2}{E^2}) + \frac{m_Q^2 + m_e^2}{E^2}\right]}{4E^2} \sqrt{\frac{E^2 - m_Q^2}{E^2 - m_e^2}} d\theta$$

$$\sigma = \int_0^{\pi} \sin\theta \frac{1}{32\pi} \frac{Q^2 e^4 \left[1 + \cos^2\theta(1 - \frac{m_e^2}{E^2})(1 - \frac{m_Q^2}{E^2}) + \frac{m_Q^2 + m_e^2}{E^2}\right]}{4E^2} \sqrt{\frac{E^2 - m_Q^2}{E^2 - m_e^2}} d\theta$$

$$\int_0^{\pi} \sin\theta d\theta = [-\cos\theta]_0^{\pi} = (-(-1) + 1) = 2$$

$$\int_0^{\pi} \cos^2 \sin\theta d\theta$$

$$u = -\cos\theta, u^2 = \cos^2\theta, du = \sin\theta d\theta$$

$$\int_0^{\pi} \cos^2 \sin\theta d\theta = \int u^2 du = \frac{1}{3}[-\cos^3\theta]_0^{\pi} = \frac{1}{3}(1+1) = \frac{2}{3}$$

Total cross section calculation

$$\sigma = \int_0^\pi \sin\theta \frac{1}{32\pi} \frac{Q^2 e^4 \left[1 + \cos^2\theta(1 - \frac{m_e^2}{E^2})(1 - \frac{m_Q^2}{E^2}) + \frac{m_Q^2 + m_e^2}{E^2}\right]}{4E^2} \sqrt{\frac{E^2 - m_Q^2}{E^2 - m_e^2}} d\theta$$

$$\sigma = \frac{Q^2 e^4}{128 E^2 \pi} \sqrt{\frac{E^2 - m_Q^2}{E^2 - m_e^2}} \int_0^\pi \sin\theta(1 + \frac{m_Q^2 + m_e^2}{E^2}) + \sin\theta\cos^2\theta(1 - \frac{m_e^2}{E^2})(1 - \frac{m_Q^2}{E^2}) d\theta$$

$$\sigma = \frac{Q^2 e^4}{128 E^2 \pi} \sqrt{\frac{E^2 - m_Q^2}{E^2 - m_e^2}} \left(2 + 2\frac{m_Q^2 + m_e^2}{E^2} + \frac{2}{3}(1 - \frac{m_e^2}{E^2})(1 - \frac{m_Q^2}{E^2})\right)$$

$$\sigma = \frac{Q^2 e^4}{128 E^2 \pi} \sqrt{\frac{E^2 - m_Q^2}{E^2 - m_e^2}} \left(\frac{8}{3} + \frac{4}{3}\frac{m_Q^2 + m_e^2}{E^2} + \frac{2}{3}\frac{m_e^2 m_Q^2}{E^4}\right)$$

$$\boxed{\sigma = \frac{Q^2 e^4}{48 E^2 \pi} \sqrt{\frac{E^2 - m_Q^2}{E^2 - m_e^2}} \left(1 + \frac{m_Q^2}{2E^2}\right)\left(1 + \frac{m_e^2}{2E^2}\right)}$$

Total cross section calculation

$$\sigma = \frac{Q^2 e^4}{48 E^2 \pi} \sqrt{\frac{E^2 - m_Q^2}{E^2 - m_e^2}} \left(1 + \frac{m_Q^2}{2E^2}\right) \left(1 + \frac{m_e^2}{2E^2}\right)$$

Clear that energy can't be less than quark mass or electron mass or calculation makes no sense (good!) If energy is large enough, this is approximated by

$$\sigma = \frac{Q^2 e^4}{48 E^2 \pi}$$

Total cross section calculation

$$\sigma = \frac{Q^2 e^4}{48 E^2 \pi}$$

As we crank up the energy, we expect the ee→qq cross section to be flat until we reach another kinematic regime where a new quark is allowed to be produced. Have to be careful about two things:

1) Q charges not all the same! (Evidence for charges of quarks!)

2) If we compute R, the rate relative to muon-antimuon production, in region where mass effects are unimportant, we need a factor of 3x for color. Evidence for quark color!

up down strange

$$R = 3\left[\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2\right] = 2$$

below charm mass

up down strange charm

$$R = 3\left[\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{2}{3}\right)^2\right] = 3.3$$

when we reach charm threshold

up down charm

$$R = 3\left[\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2\right] = 3.7$$

strange bottom

when we reach bottom threshold

Total cross section ratio from PDG

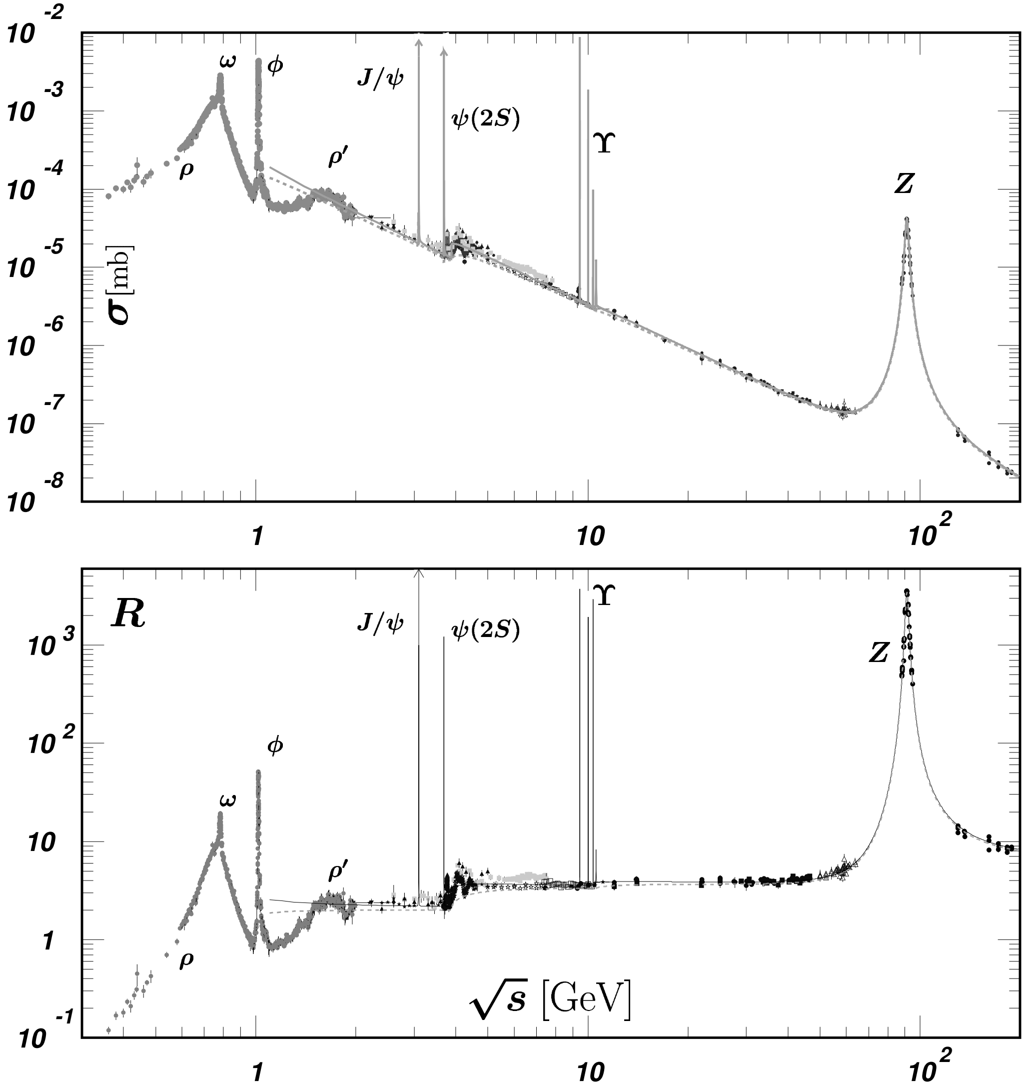


Figure 40.6: World data on the total cross section of $e^+e^- \to hadrons$ and the ratio $R(s) = \sigma(e^+e^- \to hadrons, s)/\sigma(e^+e^- \to \mu^+\mu^-, s)$. $\sigma(e^+e^- \to hadrons, s)$ is the experimental cross section corrected for initial state radiation and electron-positron vertex loops, $\sigma(e^+e^- \to \mu^+\mu^-, s) = 4\pi\alpha^2(s)/3s$. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are an educative guide: the broken one is a naive quark-parton model prediction and the solid one is 3-loop pQCD prediction (see "Quantum chromodynamics" section of this *Review*, Eq. (9.12) or, for more details, K. G. Chetyrkin *et al.*, Nucl. Phys. B **586** (2000) 56 (Erratum *ibid.* B **634** (2002) 413). Breit-Wigner parameterizations of J/ψ, $\psi(2S)$, and $\Upsilon(nS), n = 1,2,3,4$ are also shown. The full list of references to the original data and the details of the R ratio extraction from them can be found in hep-ph/0312114. Corresponding computer-readable data files are available at http://pdg.ihep.su/xsect/contents.html. (Courtesy of the COMPAS(Protvino) and HEPDATA(Durham) Groups, August 2005. Corrections by P. Janot (CERN) and M. Schmitt (Northwestern U.))

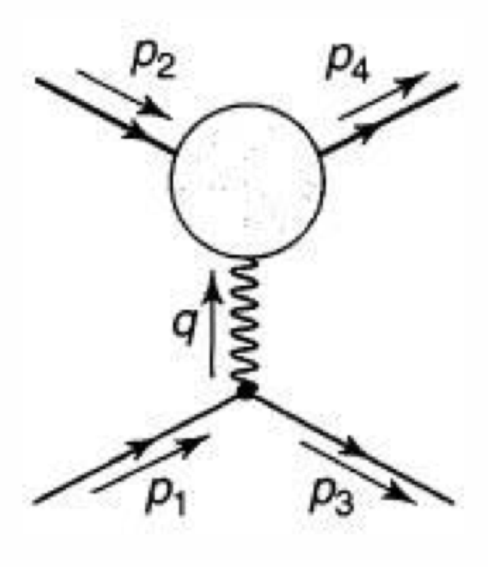
Can see falling ($1/E^2$) cross section, and also evidence for charm quark and then bottom quark!

http://pdg.lbl.gov/2007/hadronic-xsections/hadronicrpp_page6.pdf

Pretty nice agreement between prediction and observation, though this is a simplified, leading order calculation. There are loop/higher-order effects, and the model cannot account for bound states/resonances, for taus, and especially not for Drell-Yan/Z boson production

Form factors

Interesting discussion of form factors in Griffiths, but we'll skip it - hopefully it makes for fun reading :) If we have time we can return to it



We haven't really used it, but as an alternative to electric charge e (or g_e as Griffiths uses), we can define a coupling constant for QCD:

$$e = \sqrt{4\pi\alpha_e}$$

$$g_s = \sqrt{4\pi\alpha_s}$$

$$\alpha_e \sim 1/137$$

$$\alpha_s \sim 1$$

"Strong" force!

Color for quarks

$$u^{(s)}(p) \rightarrow u^{(s)}(p)c$$

$$c(red) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

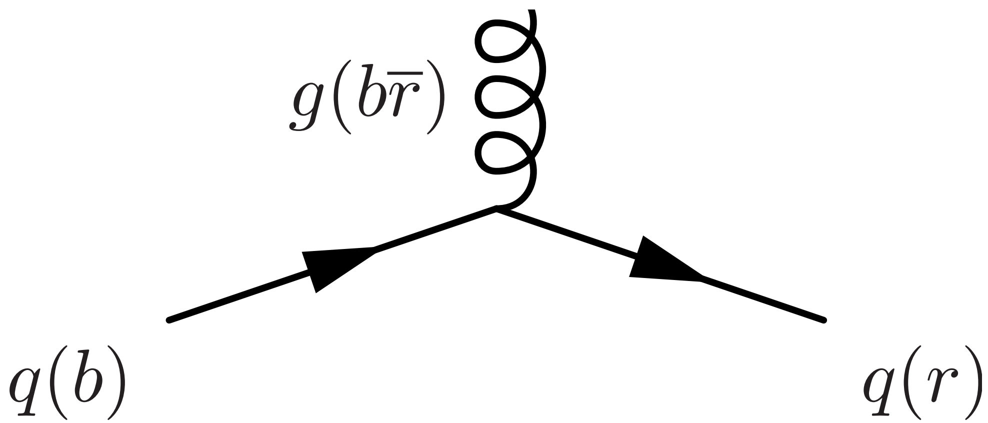
$$c(green) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$c(blue) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Spinors now get an associated color vector!

Of course, remember that "red", "green" and "blue" are just convenient names and nothing more than that

Color for gluons



Gluons are spin-1 bosons and carry two color quantities - one unit of color and one unit of anti-color. Here, a blue quark emits a blue/anti-red gluon, and becomes a red quark (color is then conserved)

Gluons

Naively would predict nine gluons - a color octet and a color singlet. But the singlet has not been observed (only 8 gluons). Difference between SU(3) and U(3) color symmetry

Color octet

$$|1> = (r\bar{b} + b\bar{r})/\sqrt{2} \qquad |5> = -i(r\bar{g} - g\bar{r})/\sqrt{2}$$

$$|2> = -i(r\bar{b} - b\bar{r})/\sqrt{2} \qquad |6> = (b\bar{g} + g\bar{b})/\sqrt{2}$$

$$|3> = (r\bar{r} - b\bar{b})/\sqrt{2} \qquad |7> = -i(b\bar{g} - g\bar{b})/\sqrt{2}$$

$$|4> = (r\bar{g} + g\bar{r})/\sqrt{2} \qquad |8> = (r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6}$$

Color singlet (not observed!)

$$|9> = (r\bar{r} + b\bar{b} + g\bar{g})/\sqrt{3}$$

Range of strong force

Observed states (proton and neutron, for example) are color singlets. If they could exchange color singlet gluons then QCD would be a long-range force!

Color singlet (not observed!)

$$|9>=(r\overline{r}+b\overline{b}+g\overline{g})/\sqrt{3}$$

Color state of gluon

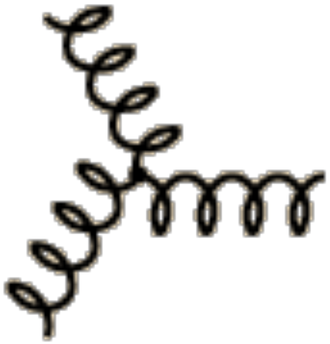
$$|1>= \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$|2>= \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$|5>= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Column vector a represents the color state of the gluon (one of the 8 possible states)

Reminder that gluons self-couple! These are both valid diagrams/vertices



Gell-Mann Lambda matrices

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Compare with...

$$|1> = (r\bar{b} + b\bar{r})/\sqrt{2} \qquad |5> = -i(r\bar{g} - g\bar{r})/\sqrt{2}$$

$$|2> = -i(r\bar{b} - b\bar{r})/\sqrt{2} \qquad |6> = (b\bar{g} + g\bar{b})/\sqrt{2}$$

$$|3> = (r\bar{r} - b\bar{b})/\sqrt{2} \qquad |7> = -i(b\bar{g} - g\bar{b})/\sqrt{2}$$

$$|4> = (r\bar{g} + g\bar{r})/\sqrt{2} \qquad |8> = (r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6}$$

Why are we introducing the lambda matrices?

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

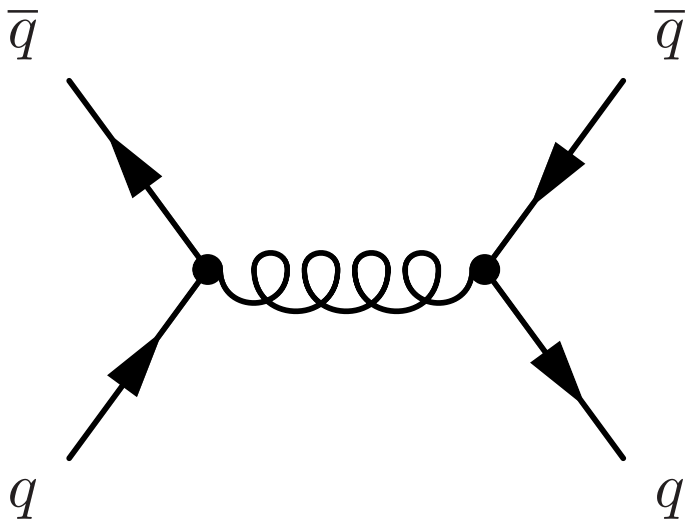
$$[\lambda^\alpha, \lambda^\beta] = 2 i f^{\alpha\beta\gamma} \lambda^\gamma \qquad f^{\alpha\beta\gamma} = -f^{\beta\alpha\gamma} = -f^{\alpha\gamma\beta}$$

$$f^{123} = 1, f^{147} = f^{246} = f^{257} = f^{345} = f^{516} = f^{637} = 1/2$$

$$f^{458} = f^{678} = \sqrt{3}/2$$

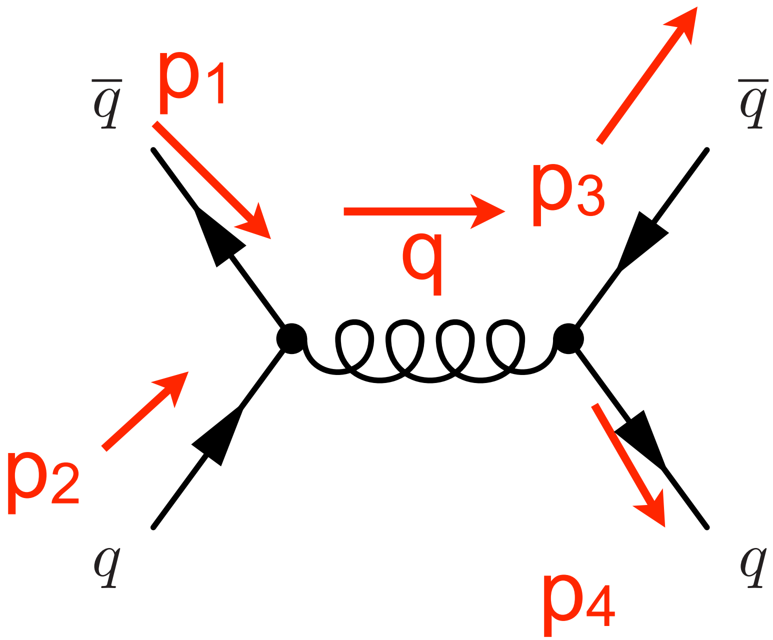
Plus commutations (and rest are zero)

Example QCD diagram



Here we have one of the diagrams contributing to qqbar→qqbar scattering. Quark and anti-quark in initial state must be the same flavor. Same for final state. Obvious why? Let's move on to the matrix element calculation

Feynman rules for QCD (1)

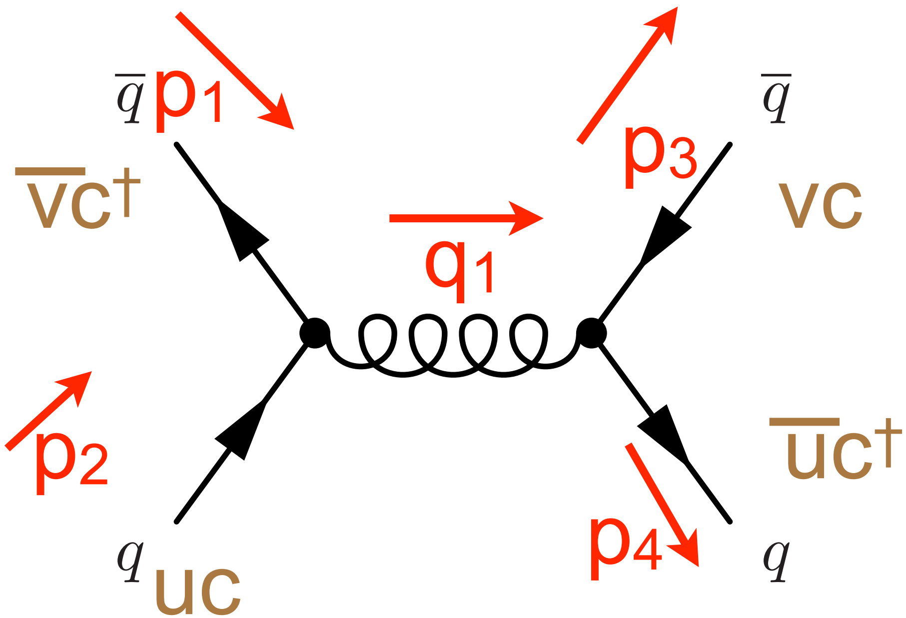


Label all incoming and outgoing lines with p_1, p_2, ... p_n

Internal lines q can go either way

Use arrows to keep track of what is going in and out (of course we have arrows on the anti-particles, but that is labeling something different).

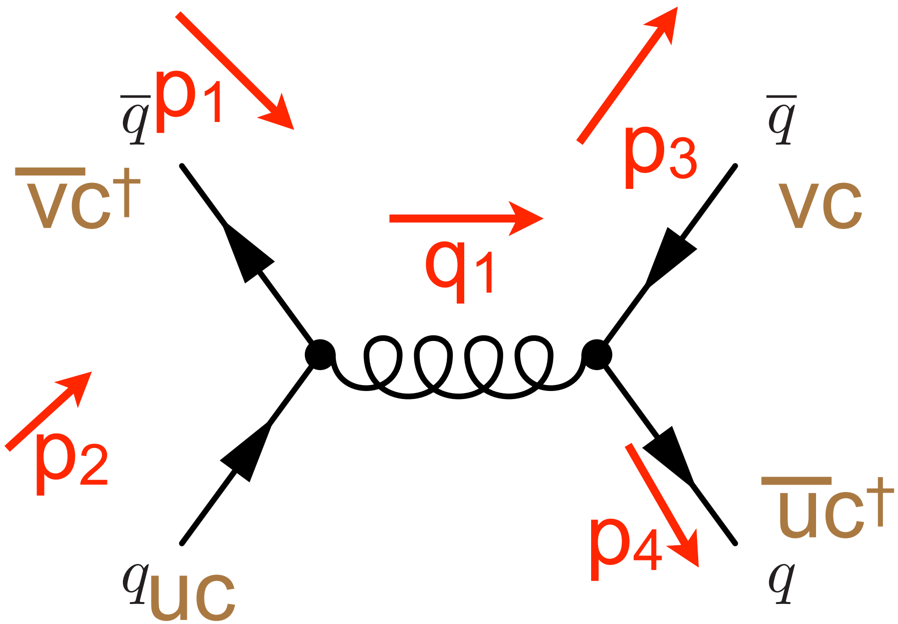
Feynman rules for QCD (2)

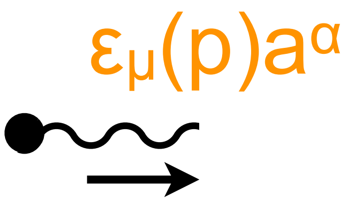


Incoming (outgoing) quarks get a factor of uc($\bar{u}c^\dagger$), outgoing (incoming) anti-quarks get a factor of vc($\bar{v}c^\dagger$). Spin implicit here and not labeled

Color factors to be grouped together!

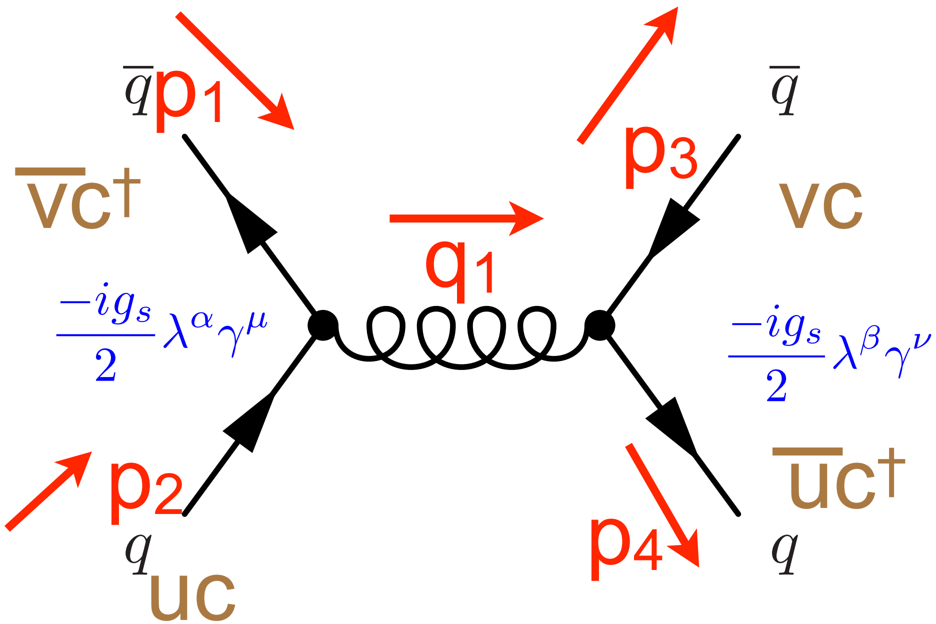
Feynman rules for QCD (3)





Incoming (outgoing) external gluons with color label a get a factor of $\varepsilon_\mu(p)a^\alpha$ $(\varepsilon^*_\mu(p)a^{\alpha *})$

Feynman rules for QCD (4)

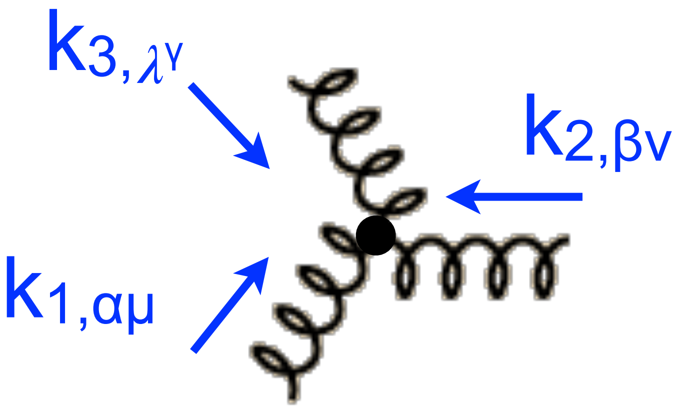


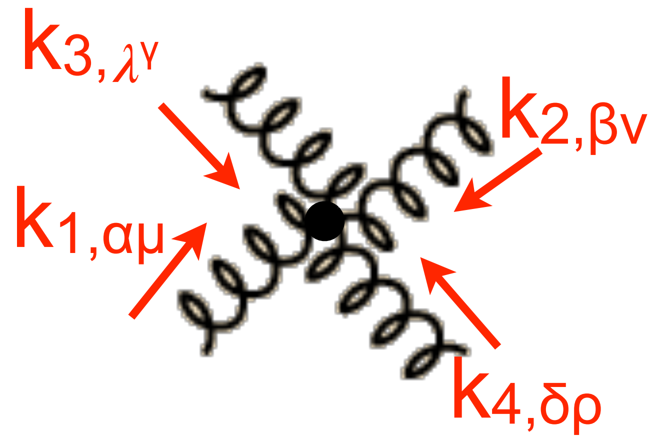
Add factors of

$$\frac{-ig_s}{2}\lambda^\alpha\gamma^\mu$$

at each quark-gluon vertex. Lambda matrices define the gluon that is exchanged (can be any of the 8, though only some will contribute)

Feynman rules for QCD (5)





3 gluon vertex

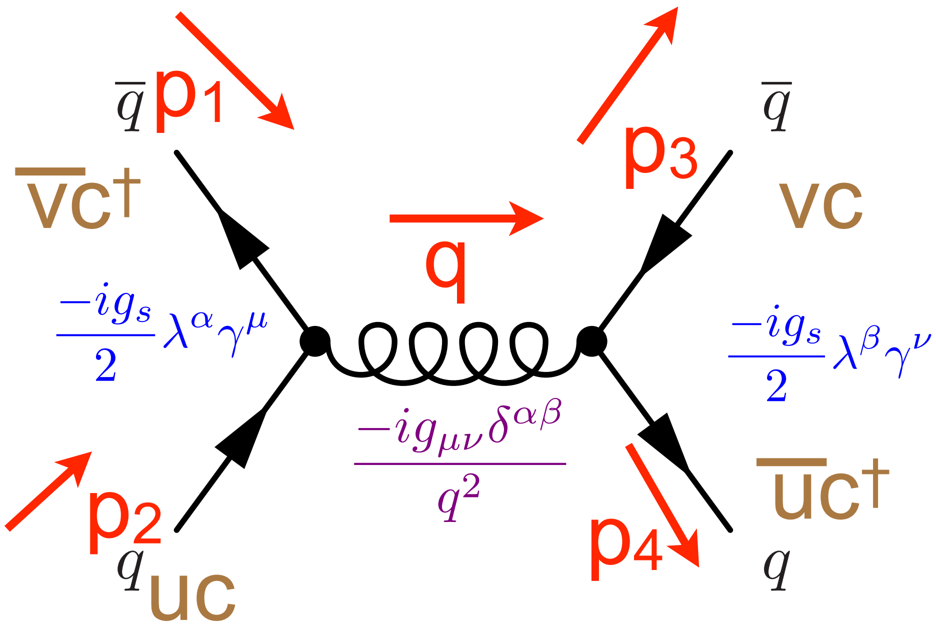
$$-g_s f^{\alpha\beta\gamma}[g_{\mu\nu}(k_1-k_2)_\lambda + g_{\nu\lambda}(k_2-k_3)_\mu + g_{\lambda\mu}(k_3-k_1)_\nu +]$$

4 gluon vertex

$$-ig_s^2\left[f^{\alpha\beta\eta}f^{\gamma\delta\eta}(g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda})+f^{\alpha\delta\eta}f^{\beta\gamma\eta}(g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho})+f^{\alpha\gamma\eta}f^{\delta\beta\eta}(g_{\mu\rho}g_{\nu\lambda}-g_{\mu\nu}g_{\lambda\rho})\right]$$

Can see that QCD can easily get tricky, and that's without loops!

Feynman rules for QCD (6)

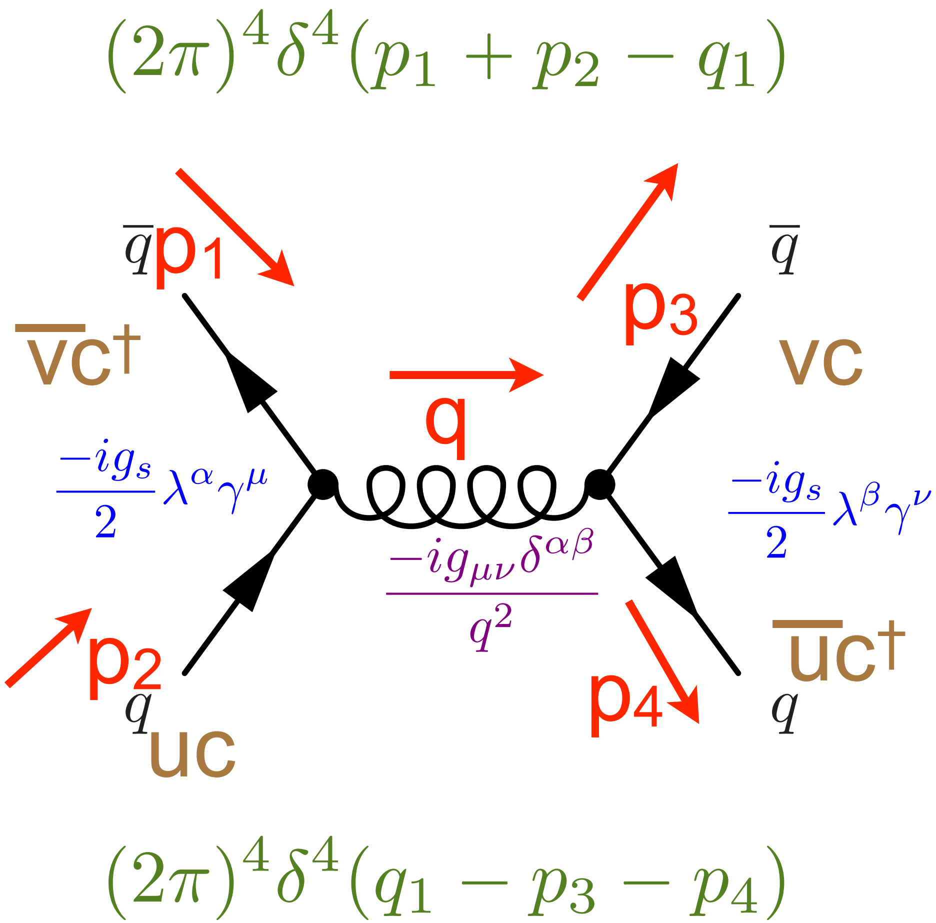


For each internal gluon line add a factor for the propagator (delta function ensures conservation of color!)

$$\frac{i\left(\not{q}+m\right)}{q^2-m^2}$$

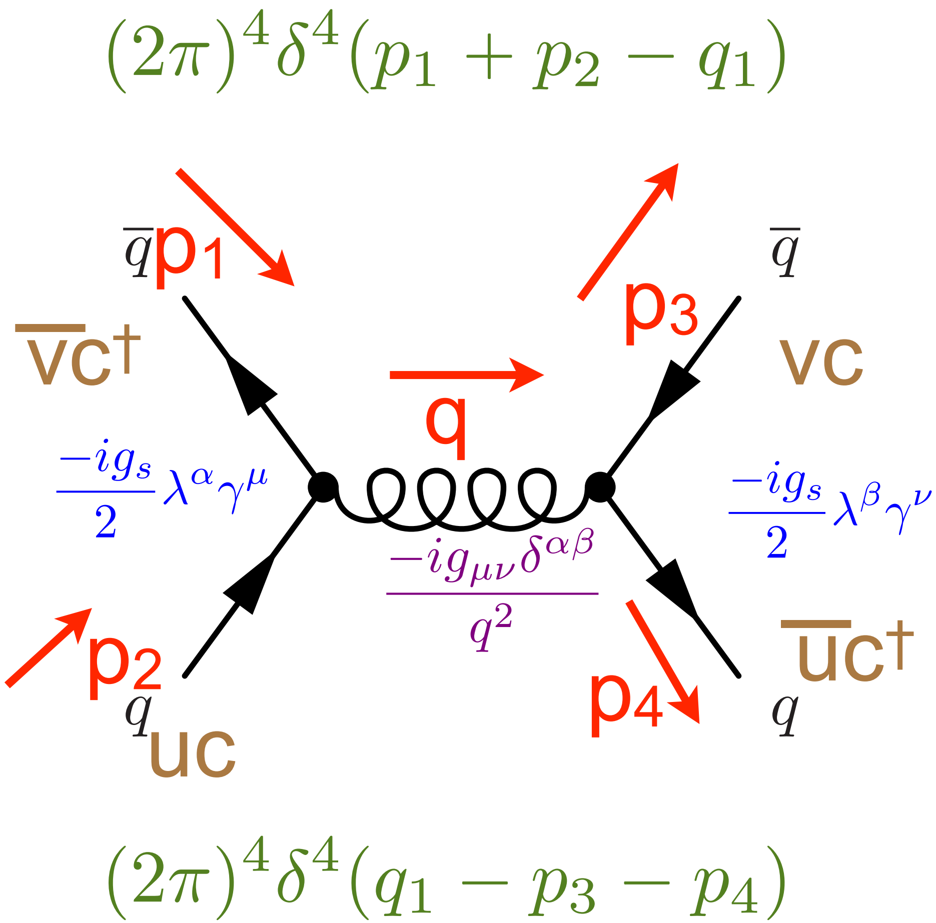
for internal quarks/anti-quarks

Feynman rules for QCD (7)



Impose conservation of energy and momentum at each vertex with 4d Dirac Delta function (with appropriate 2pi normalization)

Feynman rules for QCD (8)

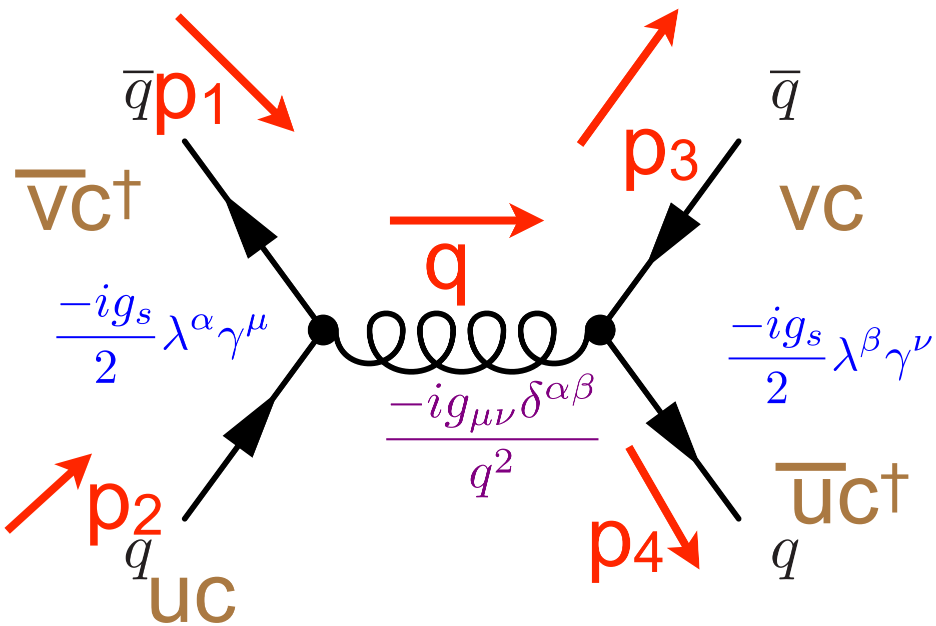


$$\frac{1}{(2\pi)^4}d^4q$$

Integrate over 4-momentum of internal lines with appropriate 2pi normalization factor

Feynman rules for QCD (9)

$$(2\pi)^4\delta^4(p_1+p_2-q_1)$$



$$(2\pi)^4\delta^4(q_1-p_3-p_4)$$

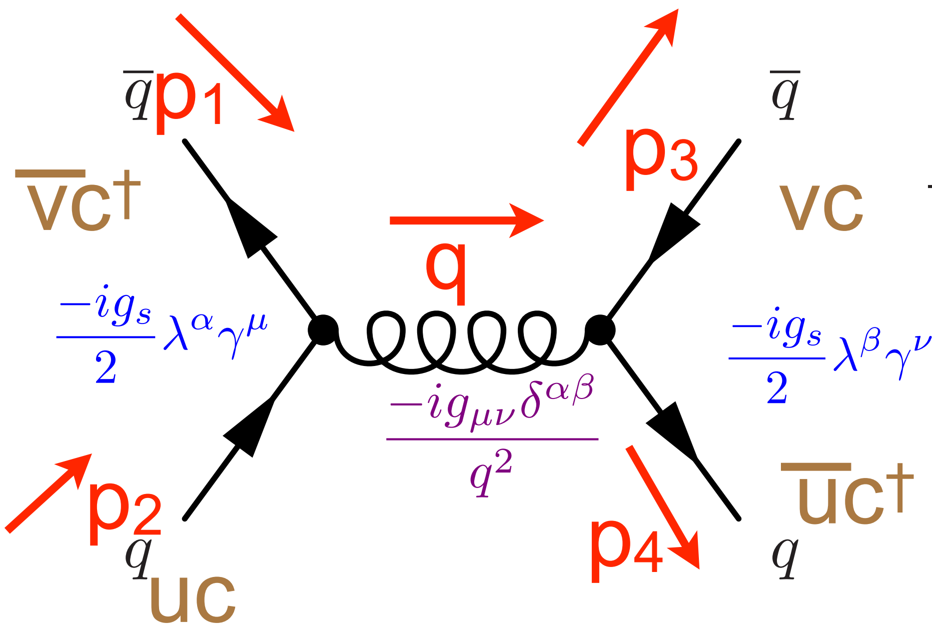
$$i$$

$$\frac{1}{(2\pi)^4}d^4q$$

Cancel remaining delta function and add a factor of i, and you have the matrix element (phew)

Feynman rules for QCD (10)

$$(2\pi)^4\delta^4(p_1+p_2-q_1)$$



$$(2\pi)^4\delta^4(q_1-p_3-p_4)$$

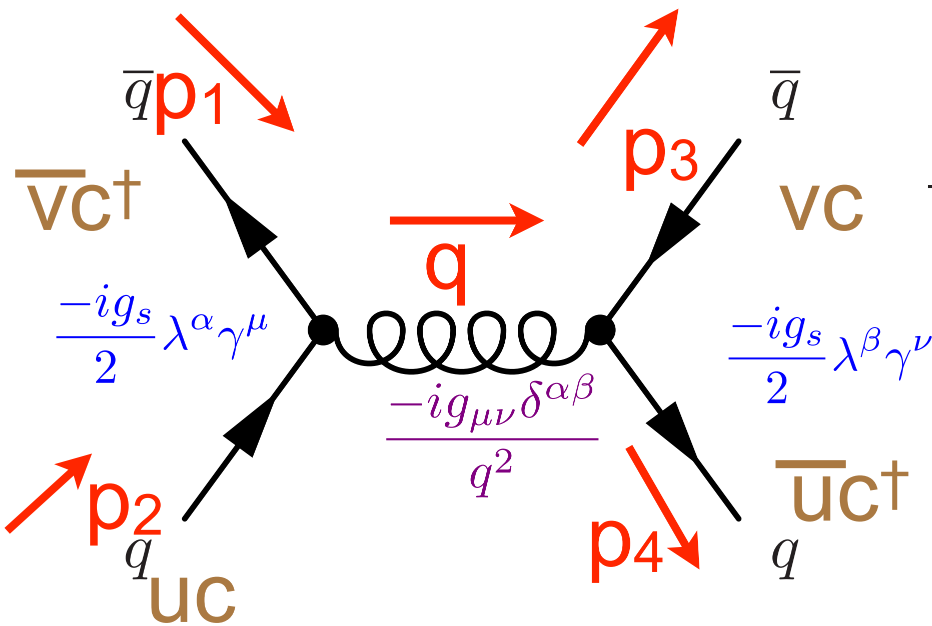
$$i$$

$$\frac{1}{(2\pi)^4}d^4q$$

Add minus sign between diagrams differing only in exchange of two incoming or two outgoing fermions, or incoming fermion and outgoing anti-fermion (or vice versa)

Feynman rules for QCD

$$(2\pi)^4\delta^4(p_1 + p_2 - q_1)$$



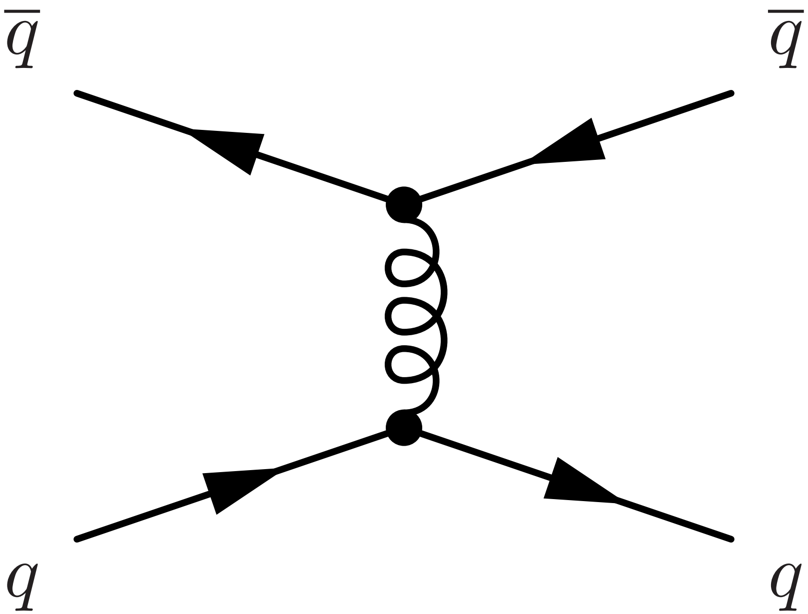
$$(2\pi)^4\delta^4(q_1 - p_3 - p_4)$$

$$i$$

$$\frac{1}{(2\pi)^4}d^4q$$

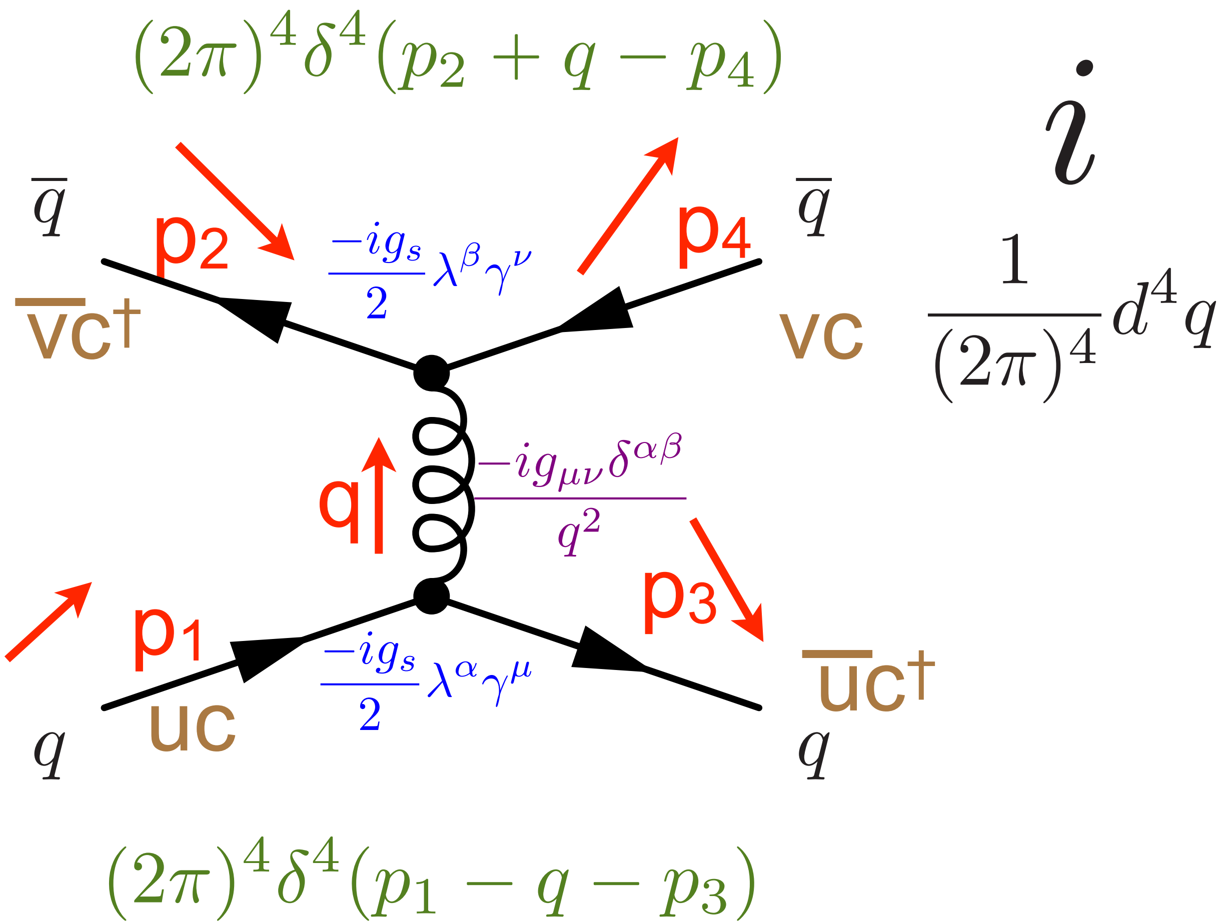
PHEW

Example calculation (qqbar→qqbar)

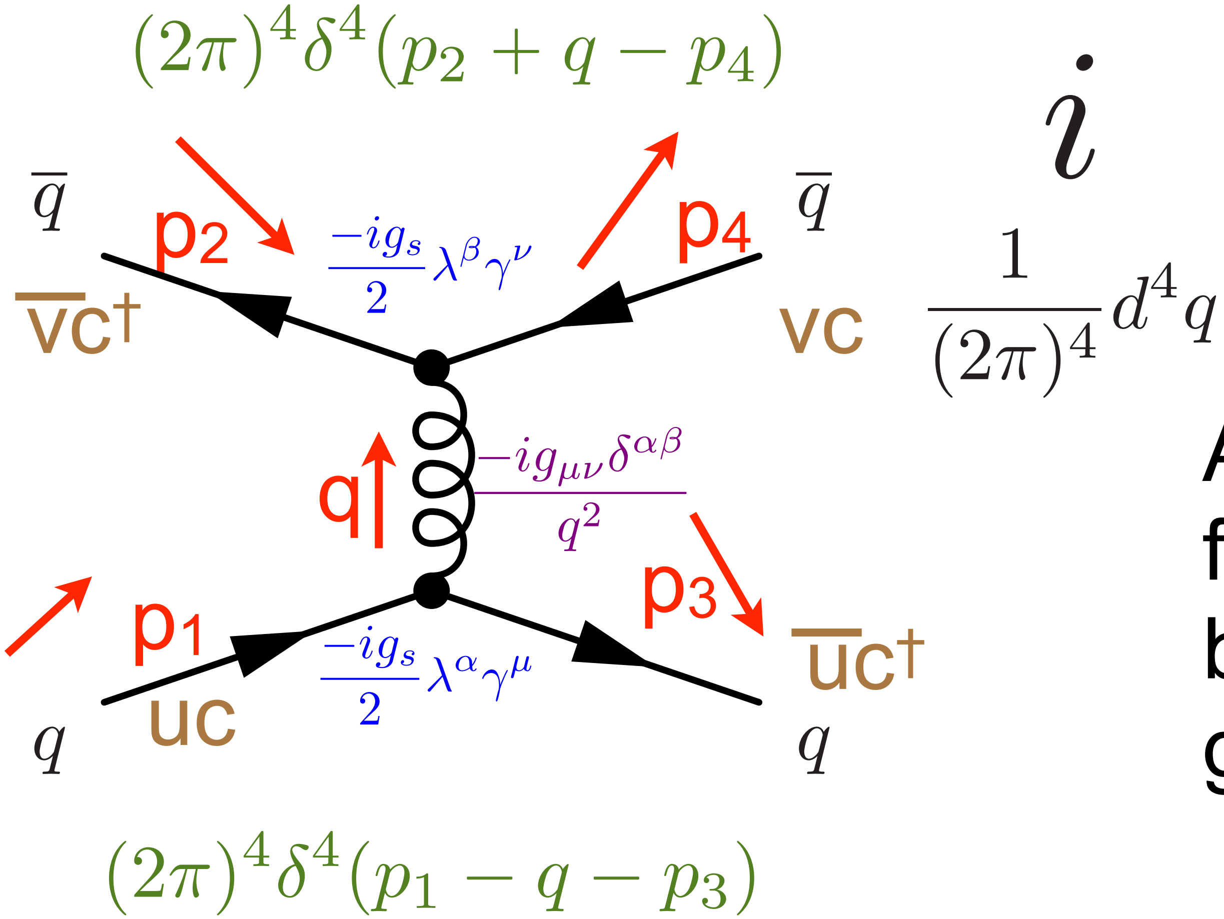


Can't do a full calculation, since we don't have here the ability to calculate the hadronization steps and formation of bound states (not to mention higher order effects). But there are still interesting things to see...

Example calculation (qqbar→qqbar)



Example calculation (qqbar→qqbar)



As always, follow fermion lines backwards to get grouping right!

$$\mathcal{M} = \int i[\overline{u}(3)c_3^\dagger][-i\frac{g_s}{2}\lambda^\alpha\gamma^\mu][u(1)c_1]\frac{-ig_{\mu\nu}\delta^{\alpha\beta}}{q^2}[\overline{v}(2)c_2^\dagger][-i\frac{g_s}{2}\lambda^\beta\gamma^\nu][v(4)c_4]$$
$$(2\pi)^4\delta^4(p_2+q-p_4)(2\pi)^4\delta^4(p_1-q-p_3)\frac{d^4q}{(2\pi)^4}$$

Example calculation (qqbar→qqbar)

$$\mathcal{M} = \int i[\overline{u}(3)c_3^\dagger][-i\frac{g_s}{2}\lambda^\alpha\gamma^\mu][u(1)c_1]\frac{-ig_{\mu\nu}\delta^{\alpha\beta}}{q^2}[\overline{v}(2)c_2^\dagger][-i\frac{g_s}{2}\lambda^\beta\gamma^\nu][v(4)c_4]$$

$$(2\pi)^4\delta^4(p_2+q-p_4)(2\pi)^4\delta^4(p_1-q-p_3)\frac{d^4q}{(2\pi)^4}$$

$$\mathcal{M} = \cancel{\int}\frac{-g_s^2}{4}[\overline{u}(3)c_3^\dagger][\lambda^\alpha\gamma^\mu][u(1)c_1]\frac{\delta^{\alpha\beta}}{(p_1-p_3)^2}[\overline{v}(2)c_2^\dagger][\lambda^\beta\gamma_\mu][v(4)c_4]$$

$$\cancel{(2\pi)^4\delta^4(p_2+q-p_4)\frac{d^4q}{(2\pi)^4}}$$

So matrix element is:

$$\mathcal{M} = \frac{-g_s^2}{4}[\overline{u}(3)c_3^\dagger][\lambda^\alpha\gamma^\mu][u(1)c_1]\frac{\delta^{\alpha\beta}}{(p_1-p_3)^2}[\overline{v}(2)c_2^\dagger][\lambda^\beta\gamma_\mu][v(4)c_4]$$

Or simplifying last delta:

$$\mathcal{M} = \frac{-g_s^2}{4(p_1-p_3)^2}[\overline{u}(3)c_3^\dagger][\lambda^\alpha\gamma^\mu][u(1)c_1][\overline{v}(2)c_2^\dagger][\lambda^\alpha\gamma_\mu][v(4)c_4]$$

Example calculation (qqbar→qqbar)

Matrix element is

$$\mathcal{M} = \frac{-g_s^2}{4(p_1 - p_3)^2}[\overline{u}(3)c_3^\dagger][\lambda^\alpha \gamma^\mu][u(1)c_1][\overline{v}(2)c_2^\dagger][\lambda^\alpha \gamma_\mu][v(4)c_4]$$

Compare with e+ e- scattering:

$$\mathcal{M} = -\frac{e^2}{(p_1 - p_3)^2}[\overline{u}^{(s3)}(p3)\gamma^\mu u^{(s1)}(p1)][\overline{v}^{(s2)}(p2)\gamma_\mu b^{(s4)}(p4)]$$

Similar matrix elements except for (ignoring g_s vs e) a color factor:

$$f = \frac{1}{4}(c_3^\dagger \lambda^\alpha c_1)(c_2^\dagger \lambda^\alpha c_4)$$

What does this tell us?

Compare quark-antiquark scattering vs electron-antielectron scattering. Major difference is the additional factor:

$$f = \frac{1}{4}(c_3^\dagger \lambda^\alpha c_1)(c_2^\dagger \lambda^\alpha c_4)$$

Let's look at color octet case first. Let's pick the incoming quark to be red and the incoming anti-quark to be anti-blue, just as an example. Then outgoing quark must also be red and outgoing anti-quark must be anti-blue (since there is no other source of QCD color)

What does this tell us?

Let's pick the incoming quark to be red and the incoming anti-quark to be anti-blue, and the outgoing quark red and outgoing anti-quark anti-blue

$$f = \frac{1}{4}(c_3^\dagger \lambda^\alpha c_1)(c_2^\dagger \lambda^\alpha c_4)$$

$$c(red) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$c(green) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$c(blue) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$c_2 = c_4 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$c_1 = c_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Calculating color factor

$$f = \frac{1}{4}(c_3^\dagger \lambda^\alpha c_1)(c_2^\dagger \lambda^\alpha c_4)$$

$$c_2 = c_4 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$c_1 = c_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

λ^α is describing the possible types of exchanged gluons (for any of the 8 values of α)

$$f = \frac{1}{4}\left[(1\ 0\ 0)\lambda^\alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right]\left[(0\ 0\ 1)]\lambda^\alpha \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right] = \frac{1}{4}\lambda_{11}^\alpha \lambda_{33}^\alpha$$

Calculating color factor

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$f = \frac{1}{4}\left[(1\ 0\ 0)\lambda^\alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right]\left[(0\ 0\ 1)\lambda^\alpha \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right] = \frac{1}{4}\lambda^\alpha_{11}\lambda^\alpha_{33}$$

λ^8 is only matrix with entries in 11 and 33

Calculating color factor

$$f = \frac{1}{4}\left[(1\ 0\ 0)\lambda^{\alpha}\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[(0\ 0\ 1)\lambda^{\alpha}\begin{pmatrix}0\\0\\1\end{pmatrix}\right] = \frac{1}{4}\lambda^{\alpha}_{11}\lambda^{\alpha}_{33}$$

$$f = \frac{1}{4}\lambda^{8}_{11}\lambda^{8}_{33} = \frac{1}{4}\frac{1}{\sqrt{3}}\frac{-2}{\sqrt{3}} = -\frac{1}{6}$$

Compared to e^+e^- potential, which is attractive, here we have an extra minus sign, indicating that color octet is repulsive! So no binding occurs (pions do not have any color)

Color factor for singlet

$$f = \frac{1}{4}(c_3^\dagger \lambda^\alpha c_1)(c_2^\dagger \lambda^\alpha c_4)$$

Let's switch to the color singlet case:

$$|9> = (r\overline{r} + b\overline{b} + g\overline{g})/\sqrt{3}$$

So out-going q/qbar are in a singlet state, and in-coming quarks are also in a singlet state
Start with incoming ones (c1, c2)

$$f = \frac{1}{4}\frac{1}{\sqrt{3}}\left[c_3^\dagger \lambda^\alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right][(1\ 0\ 0)\lambda^\alpha c_4] + \frac{1}{4}\frac{1}{\sqrt{3}}\left[c_3^\dagger \lambda^\alpha \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right][(0\ 1\ 0)\lambda^\alpha c_4] +$$

$$\frac{1}{4}\frac{1}{\sqrt{3}}\left[c_3^\dagger \lambda^\alpha \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right][(0\ 0\ 1)\lambda^\alpha c_4] +$$

Color factor for singlet

Out-going q-qbar (c3,c4) must also be in a singlet state

$$f = \frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[(1\ 0\ 0)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[(1\ 0\ 0)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right] + \frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[(0\ 1\ 0)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[(1\ 0\ 0)\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right] +$$
$$\frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[(0\ 0\ 1)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[(1\ 0\ 0)\lambda^\alpha\begin{pmatrix}0\\0\\1\end{pmatrix}\right] + \frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[(1\ 0\ 0)\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[(0\ 1\ 0)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right] +$$
$$\frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[(0\ 1\ 0)\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[(0\ 1\ 0)\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right] + \frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[(0\ 0\ 1)\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[(0\ 1\ 0)\lambda^\alpha\begin{pmatrix}0\\0\\1\end{pmatrix}\right] +$$
$$\frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[(1\ 0\ 0)\lambda^\alpha\begin{pmatrix}0\\0\\1\end{pmatrix}\right]\left[(0\ 0\ 1)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right] + \frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[(0\ 1\ 0)\lambda^\alpha\begin{pmatrix}0\\0\\1\end{pmatrix}\right]\left[(0\ 0\ 1)\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right] +$$
$$\frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[(0\ 0\ 1)\lambda^\alpha\begin{pmatrix}0\\0\\1\end{pmatrix}\right]\left[(0\ 0\ 1)\lambda^\alpha\begin{pmatrix}0\\0\\1\end{pmatrix}\right]$$

Each of 9 terms is a multiplication of λ_{ij} and λ_{ji}, which simplifies this

Color factor for singlet

$$f = \frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[(1\ 0\ 0)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[(1\ 0\ 0)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right] + \frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[(0\ 1\ 0)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[(1\ 0\ 0)\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right] +$$

$$\frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[(0\ 0\ 1)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[(1\ 0\ 0)\lambda^\alpha\begin{pmatrix}0\\0\\1\end{pmatrix}\right] + \frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[(1\ 0\ 0)\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[(0\ 1\ 0)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right] +$$

$$\frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[(0\ 1\ 0)\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[(0\ 1\ 0)\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right] + \frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[(0\ 0\ 1)\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[(0\ 1\ 0)\lambda^\alpha\begin{pmatrix}0\\0\\1\end{pmatrix}\right] +$$

$$\frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[(1\ 0\ 0)\lambda^\alpha\begin{pmatrix}0\\0\\1\end{pmatrix}\right]\left[(0\ 0\ 1)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right] + \frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[(0\ 1\ 0)\lambda^\alpha\begin{pmatrix}0\\0\\1\end{pmatrix}\right]\left[(0\ 0\ 1)\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right] +$$

$$\frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[(0\ 0\ 1)\lambda^\alpha\begin{pmatrix}0\\0\\1\end{pmatrix}\right]\left[(0\ 0\ 1)\lambda^\alpha\begin{pmatrix}0\\0\\1\end{pmatrix}\right]$$

Each of 9 terms is a multiplication of λ_{ij} and λ_{ji}, which simplifies this (go ahead and write it out yourself if you want to check)

$$f = \frac{1}{12}\sum_\alpha Tr(\lambda^\alpha\lambda^\alpha)$$

Color factor for singlet

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$f = \frac{1}{12}\sum_\alpha Tr(\lambda^\alpha \lambda^\alpha)$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$(\lambda^1)^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (\lambda^2)^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (\lambda^3)^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

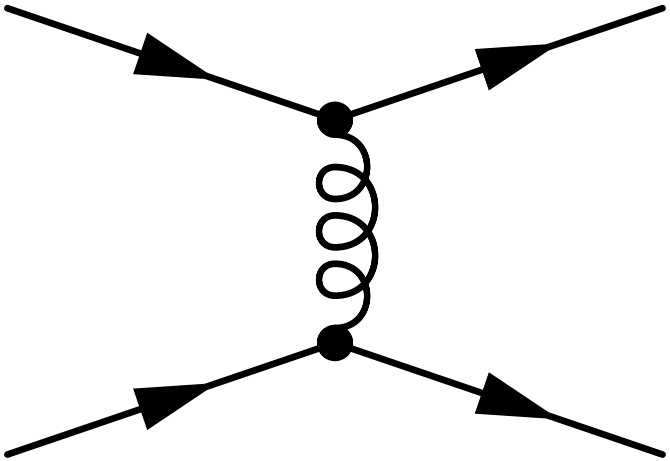
$$(\lambda^4)^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\lambda^5)^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\lambda^6)^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

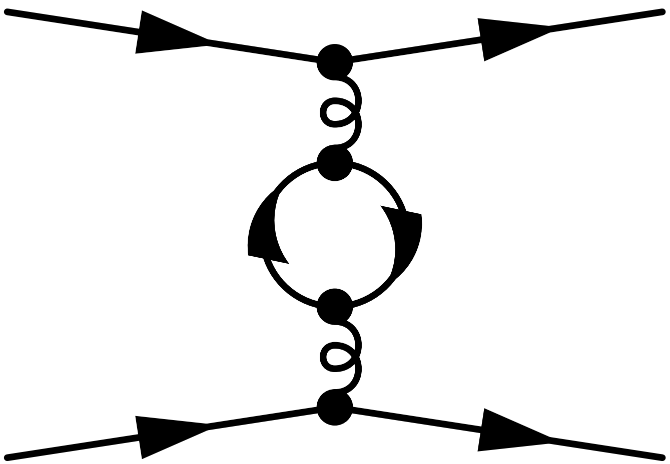
$$(\lambda^7)^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\lambda^8)^2 = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 4/3 \end{pmatrix}$$

$$f = \frac{1}{12}(2 \cdot 8) = 4/3$$

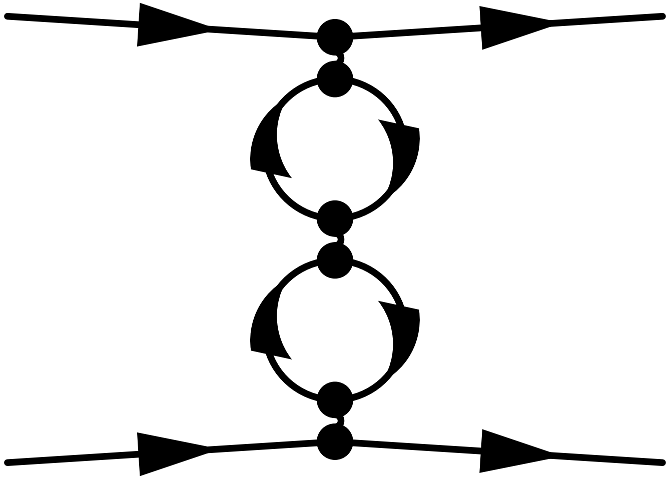
Color singlet is attractive!

Asymptotic Freedom

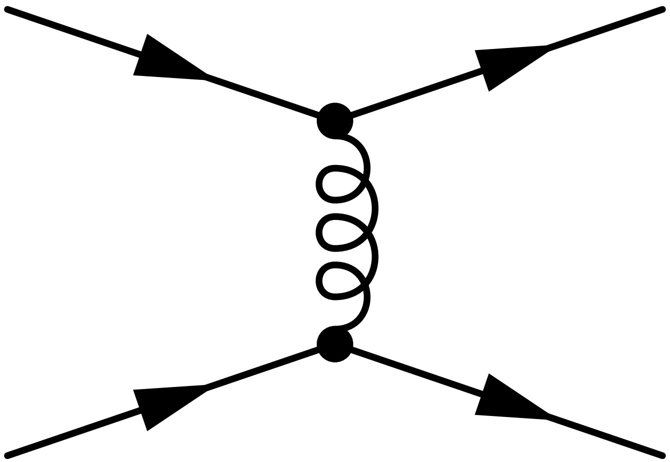


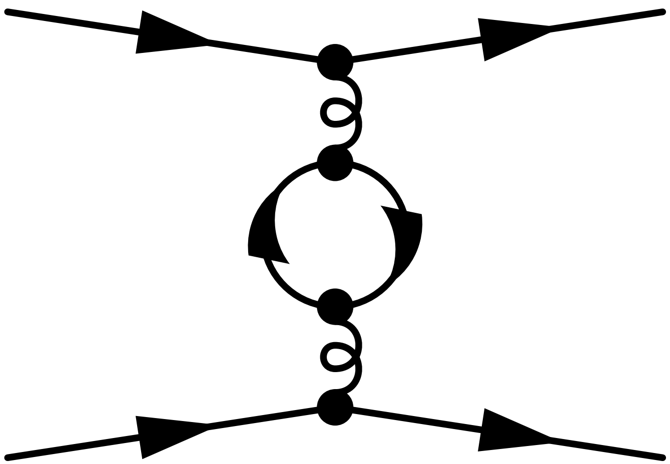


QCD screening effects: QCD coupling varies as a function of the momentum transfer (ie how close you probe the quarks), just like in QED

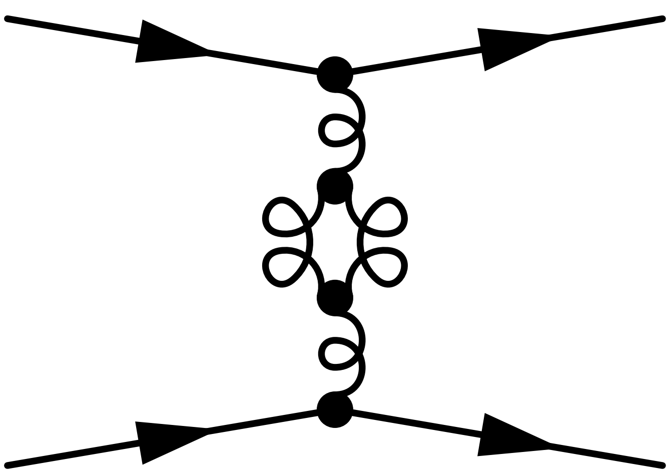


Asymptotic Freedom and gluons





But now we have to account for virtual gluon loops too, since gluons self-couple! These anti-screen the coupling and compete with quark loops



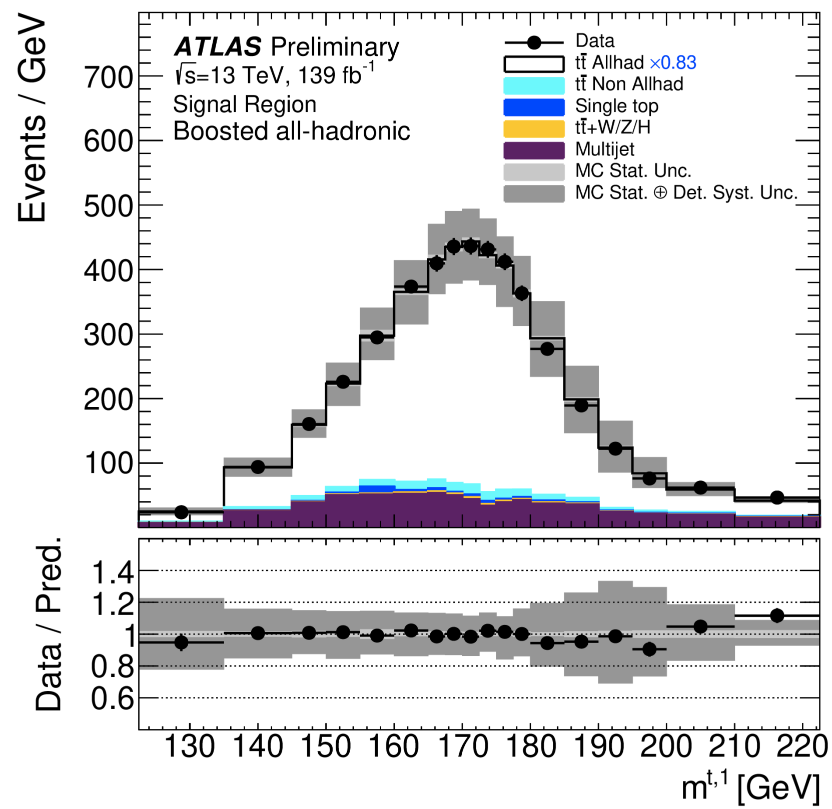
Asymptotic Freedom and gluons

$$\alpha_s(q^2) = \frac{\alpha_s(\mu^2)}{q + \frac{\alpha_s(\mu^2)}{12\pi}(11n - 2f)\ln\left(|q^2|/\mu^2\right)}$$

n=3 is number of colors, f=6 is number of quarks, so anti-screening dominates. Can't use μ=0 as reference, so need it as a parameter that defines the baseline for renormalization!

What happens to the kinematics of the top quark (and its decay products) in pp→X→ttbar if m_X is very large?

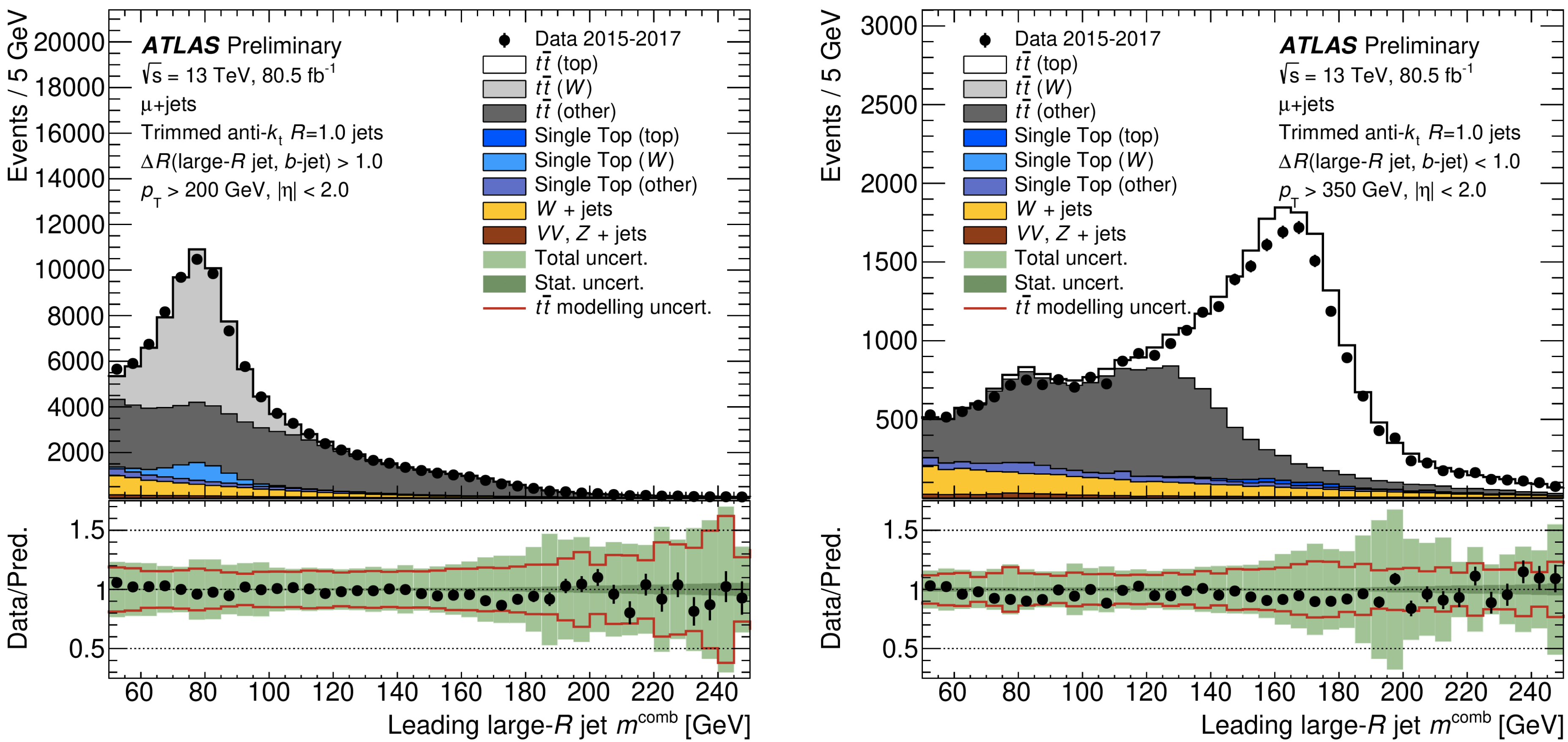
Most jets have very small masses (why?) but if we define a large enough jet (ie a “**fat jet**”) in the detector, we can compute its mass. Why might that be useful?



ATLAS-CONF-2021-050

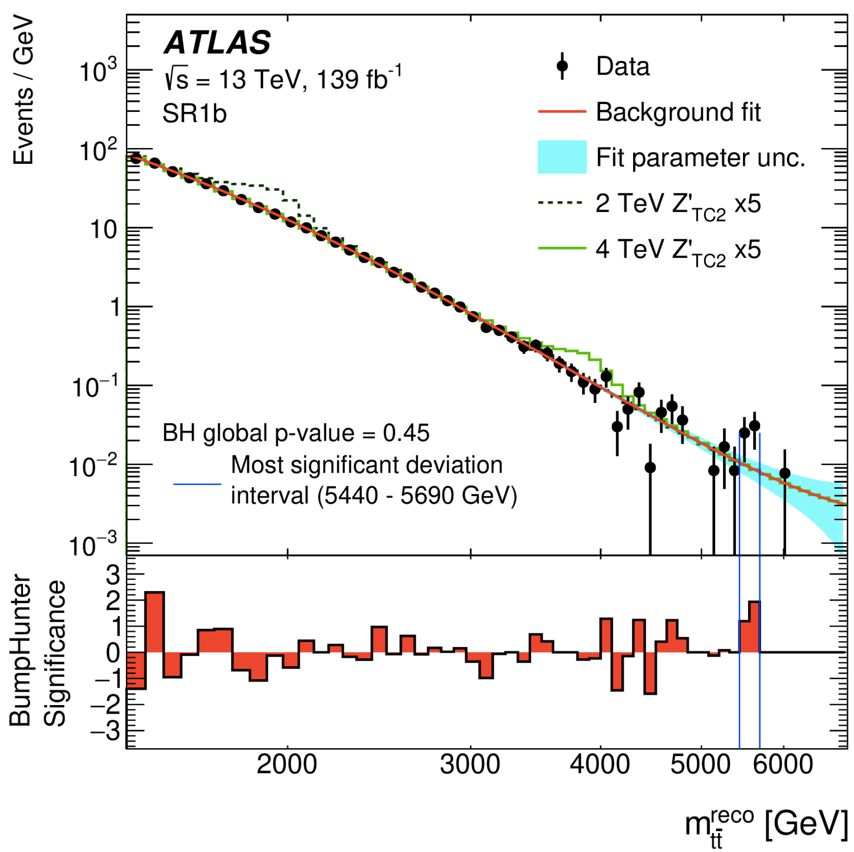
“Fat” and boosted jets

ATL-PHYS-PUB-2020-017



There’s a large program of study to efficiently select “boosted” top and W boson candidate jets while rejecting very large QCD backgrounds - lots of machine learning and advanced data analysis

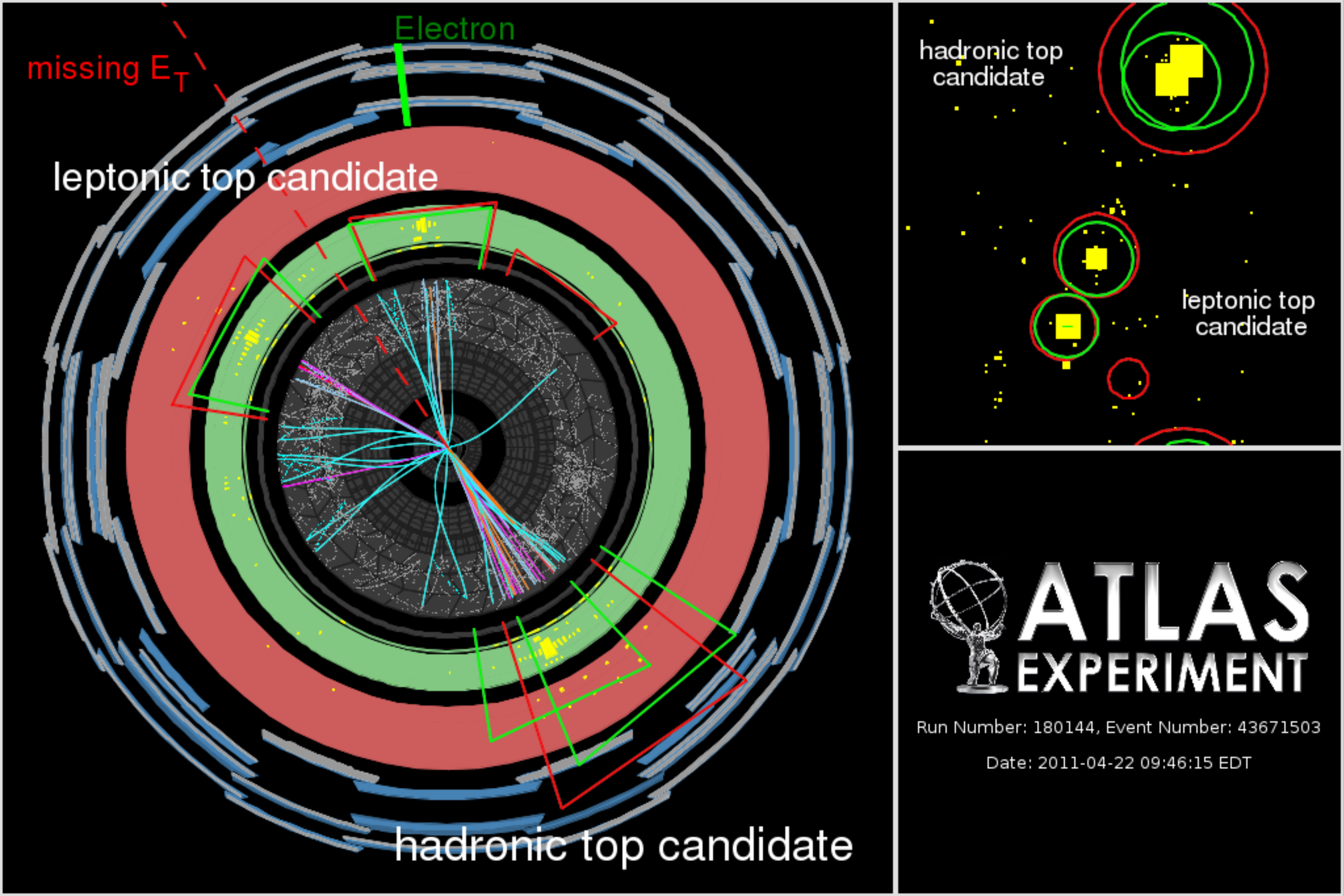
Fat jets useful for finding very massive objects in all-hadronic boosted topologies. Can make use of “sub-jets” to reject background, too!



But gets tricky to calibrate!

arXiv:2005.05138

Event display of boosted ttbar candidate



https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PAPERS/TOPQ-2011-23/fig_01.png

Some homework for you

Due in 1 week: 8.17,8.24. Also, draw the lowest order QCD diagrams for pp->2 jets

Finally, two important question for experimentalists:

1) In what pT range are boosted topologies useful? For example, imagine a very high-pT, boosted W boson decaying to quarks. Show that the jet cone size containing the two quarks from its decay is roughly given by dR (really the cone diameter) ~ 2*m_W / p_T.
Given typically large-R cone sizes of dR ~1, this implies a pT of 160 GeV, non-negligible!

2) Top quarks are typically pair produced at the LHC ($t\bar{t}$). As we will see, the top quarks decay before hadronizing to a W boson and a b quark ($t\bar{t} \rightarrow W^+bW^-\bar{b}$). The experimental signature thus depends on the subsequent W boson decays. W bosons can decay to a charged lepton-neutrino pair or a quark-antiquark pair. They cannot decay back to a top quark. Using only this knowledge and QCD, find the leading order branching ratios for $t\bar{t}$ to decay a) to only quarks/anti-quarks (no leptons, called the "all-hadronic" final state) and b) to a final state with an electron and a muon (include anti-electrons and anti-muons in your accounting)