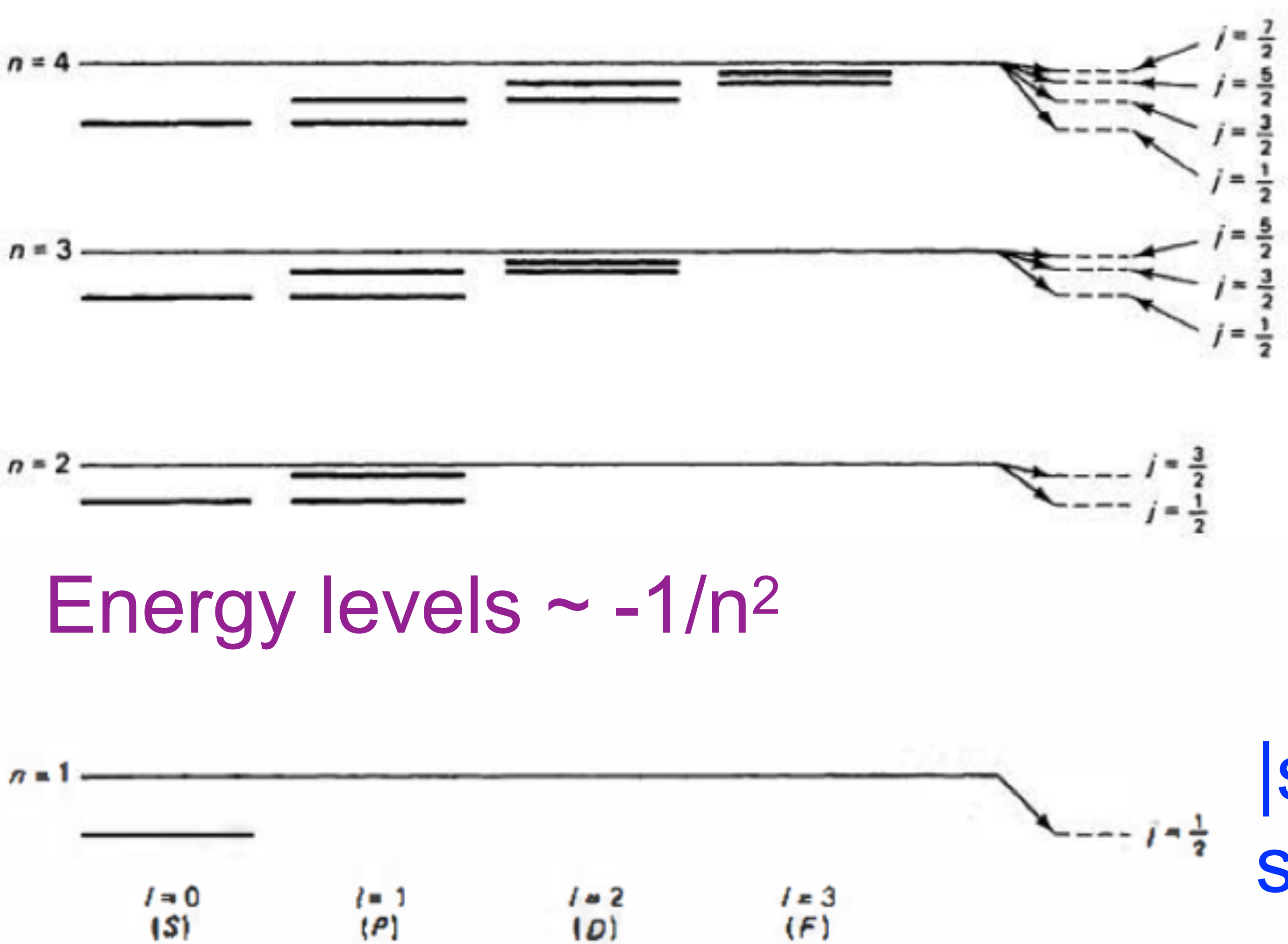
Let's move to Bound States

When we discuss bound states of two objects in central-force potential, kinetic energy and potential energy are ~the same. How does this compare to the rest energy of the objects?

Hydrogen ionization energy: 13.6 eV vs 0.5 MeV rest mass

Masses of b and c quarks are ~relatively large, so we can also consider them non-relativistically (which makes them much easier). NOT true for uds quarks

Briefest of overviews of hydrogen atom



Fine structure from relativistic corrections and spin-orbit coupling

Energy levels ~ $-1/n^2$

$|s(e)| = 1/2$

so $j = L \pm 1/2$

Fig. 5.2 Fine structure in hydrogen. The nth Bohr level (fine line) splits into n sublevels (dashed lines), characterized by $j = \frac{1}{2}, \frac{3}{2}, \ldots, (n - \frac{1}{2})$. Except for the last of these, two different values of l contribute to each level: $l = j - \frac{1}{2}$ and $l = j + \frac{1}{2}$. Spectroscopists' nomenclature – S for $l = 0$, P for $l = 1$, D for $l = 2$, F for $l = 3$ – is indicated. All levels are shifted downward, as shown (the diagram is not to scale, however).

Griffiths

Lamb shift: Led to development of quantum electro-dynamics! QED corrections to the electron-proton interaction break degeneracy of two levels with same n, j but different L (so $2S_{1/2}$ and $2P_{1/2}$ are not fully degenerate)

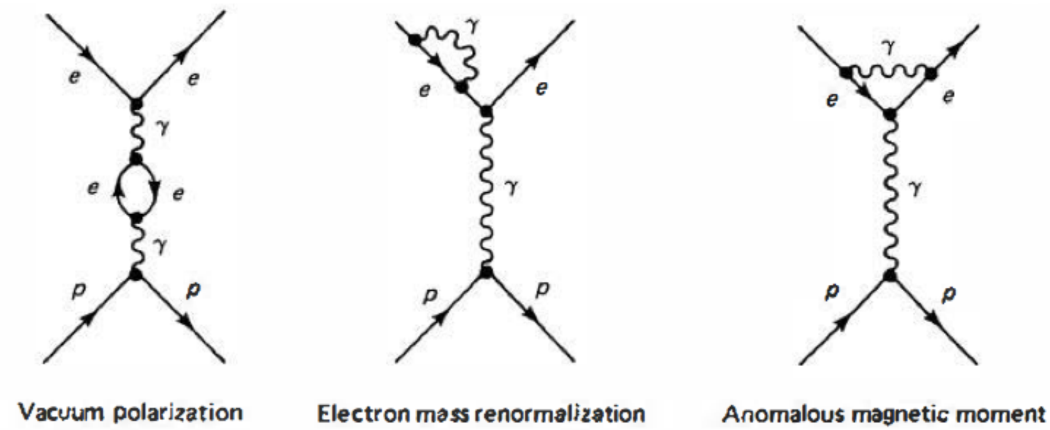
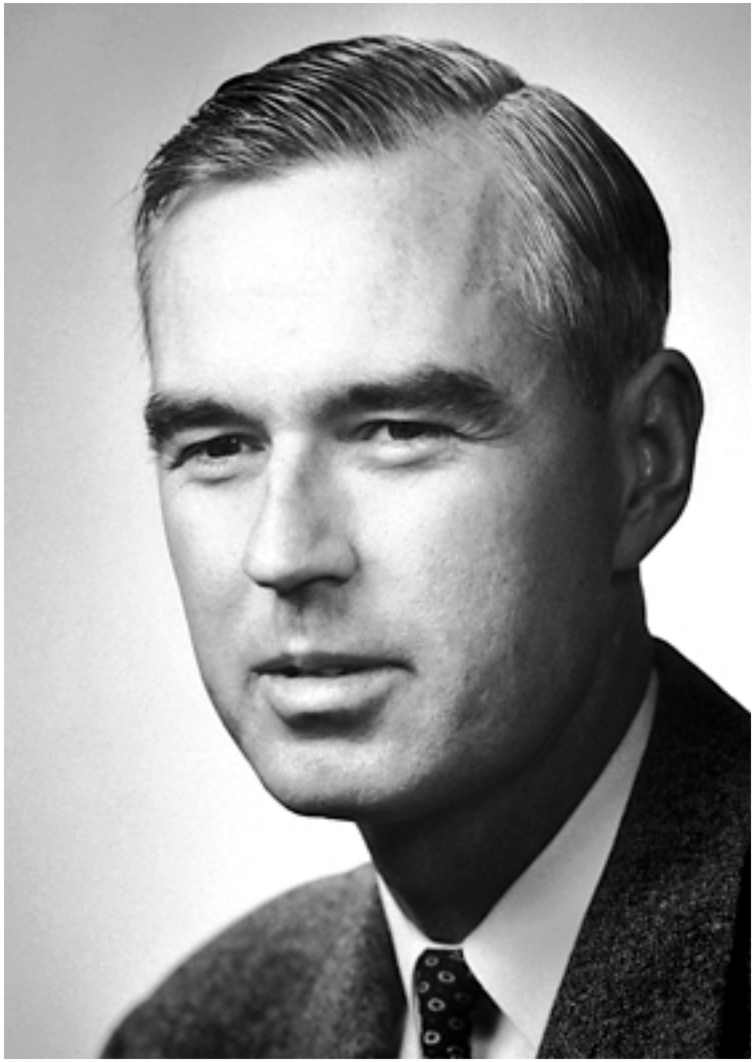


Fig. 5.3 Some loop diagrams contributing to the Lamb shift.

Griffiths



Willis Lamb

Finally, hyperfine splitting

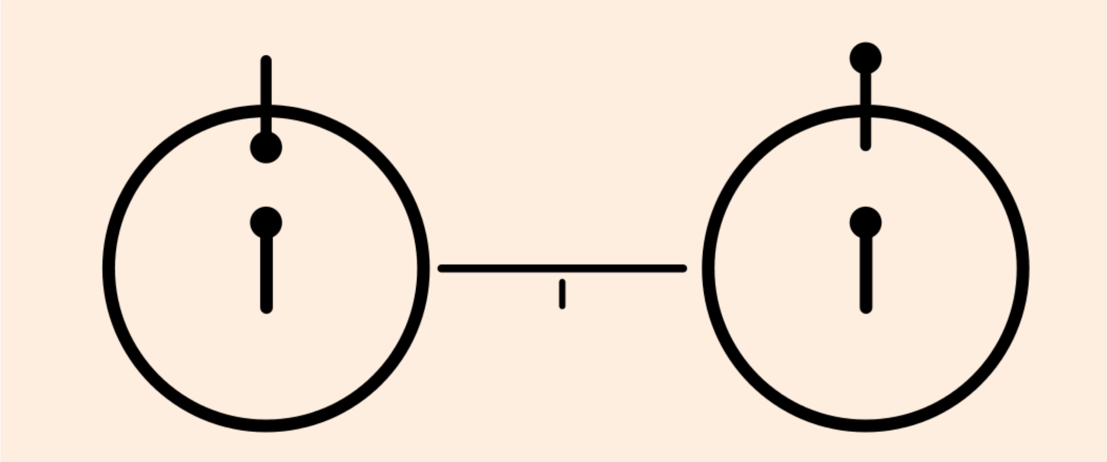
Spin-orbit coupling is principally due to spin of electron interacting with ‘B field’ from nucleus (fine structure). Much smaller is spin of nucleus interacting with ‘B field’ from electron. Goes as $(m_e/m_p)^4$ hence hyperfine (and not fine) splitting.

For n=1, the difference in energy states of proton (e and p spins aligned vs anti-aligned, which is lower) is 5.9 μeV = 1420 MHz = 21 cm. Famous 21 cm line (penetrates dust!)

On the 21 cm line

Lifetime of 21 cm is millions of years! Thankfully, enough hydrogen can provide this transition. Long lifetime = narrow width, so this is excellent for spectroscopy (Doppler shifts)

Used extensively in radio-astronomy, studying the early Universe, galaxy formation, measuring distances to objects, cosmology



Pioneer Plaque: 21 cm line defines distance and unit of time

On to quarkonium

Differences between quarkonium and hydrogen/ positronium: Don't really know the potential (strong force!) Also, interaction between quarks is large. Doesn't work for two light quarks, either

Instead of considering different states as energy levels of an atom, consider different bound states as different particles, each with a different mass. Start with mesons (much easier than baryons)

Q---Q'

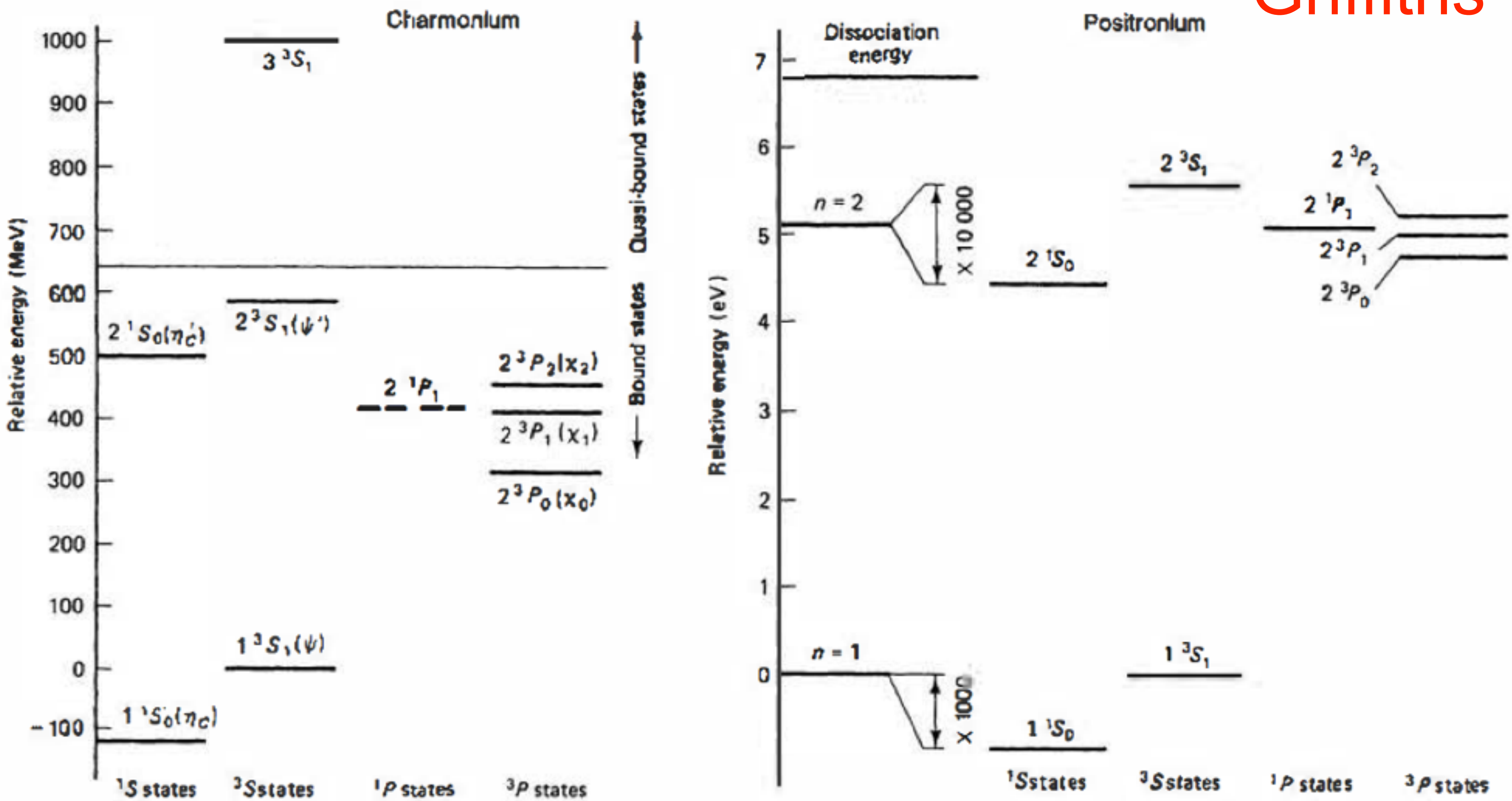
What sorts of potentials can we use?

At short distances, we know that QCD is not a strong force. Reasonable to start with 1/r potential.

At large distances, we know that force grows exponentially. Try V~kr (others could also work). Of course, k can be a function of r too!

Charmonium

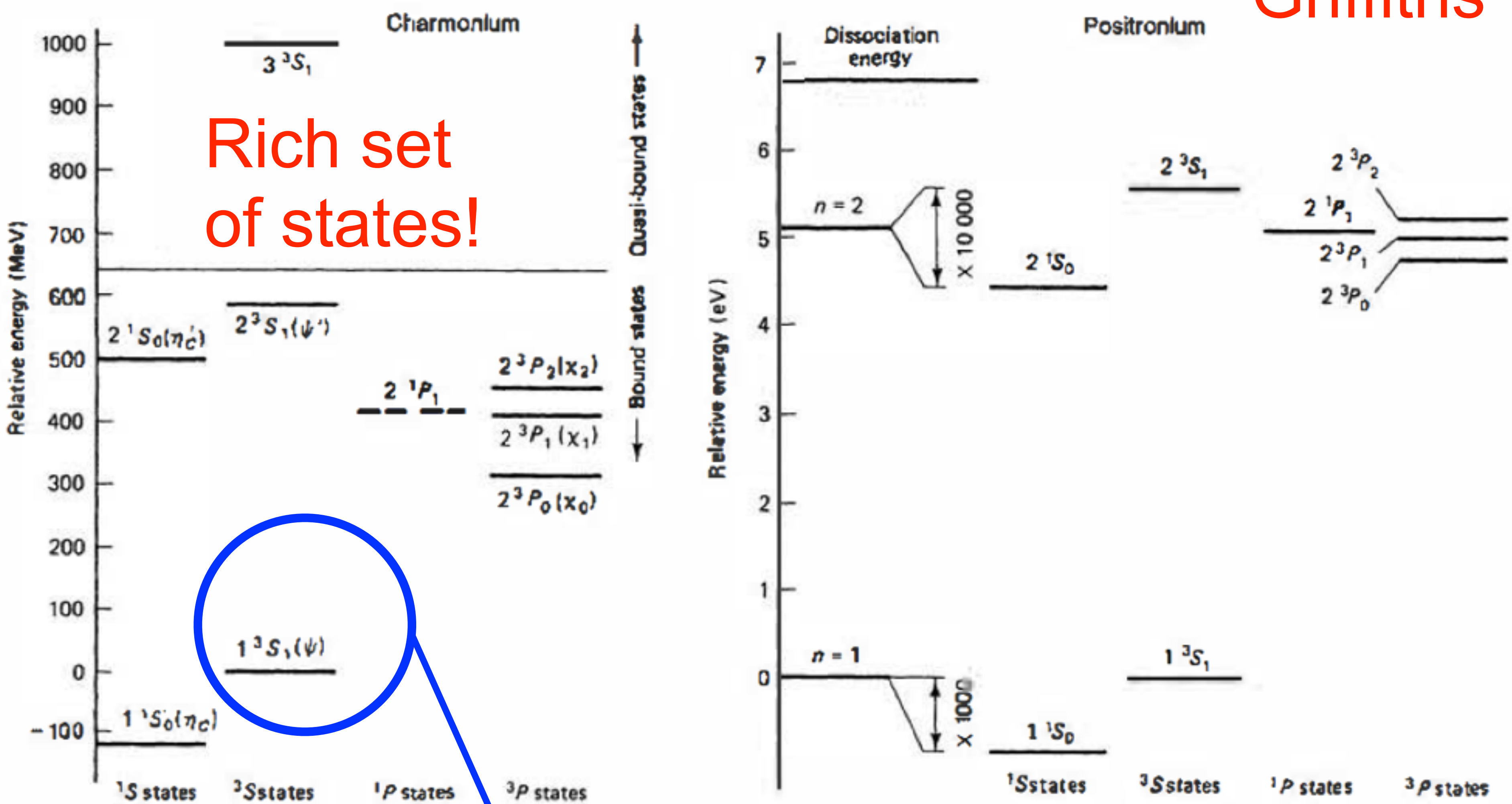
Griffiths



Labeled (confusingly, to me!) as $n^{(2s+1)}L_J$ (L=S,P,D,... for 0,1,2.... and s=0 or 1 for anti-aligned or aligned spins), with J=L+s

Charmonium

Griffiths



Rich set of states!

J/Psi state discovered in 1974

Why was J/Psi discovered first?

η

$I^G(J^{PC}) = 0^+(0^{-+})$

Charged modes

Γ_8	charged modes	(28.06 ± 0.34) %	S=1.2
Γ_9	$\pi^+\pi^-\pi^0$	(22.73 ± 0.28) %	S=1.2
Γ_{10}	$\pi^+\pi^-\gamma$	(4.60 ± 0.16) %	S=2.1
Γ_{11}	$e^+e^-\gamma$	$(6.8\pm0.8)\times10^{-3}$	S=1.7
Γ_{12}	$\mu^+\mu^-\gamma$	$(3.1\pm0.4)\times10^{-4}$	
Γ_{13}	e^+e^-	$< 7.7\times10^{-5}$	CL=90%
Γ_{14}	$\mu^+\mu^-$	$(5.8\pm0.8)\times10^{-6}$	
Γ_{15}	$e^+e^-e^+e^-$	$< 6.9\times10^{-5}$	CL=90%
Γ_{16}	$\pi^+\pi^-e^+e^-$	$(4.2\pm1.2)\times10^{-4}$	
Γ_{17}	$\pi^+\pi^-2\gamma$	$< 2.0\times10^{-3}$	
Γ_{18}	$\pi^+\pi^-\pi^0\gamma$	$< 5\times10^{-4}$	CL=90%
Γ_{19}	$\pi^0\mu^+\mu^-\gamma$	$< 3\times10^{-6}$	CL=90%

From PDG

Due to C-parity!

$J/\psi(1S)$

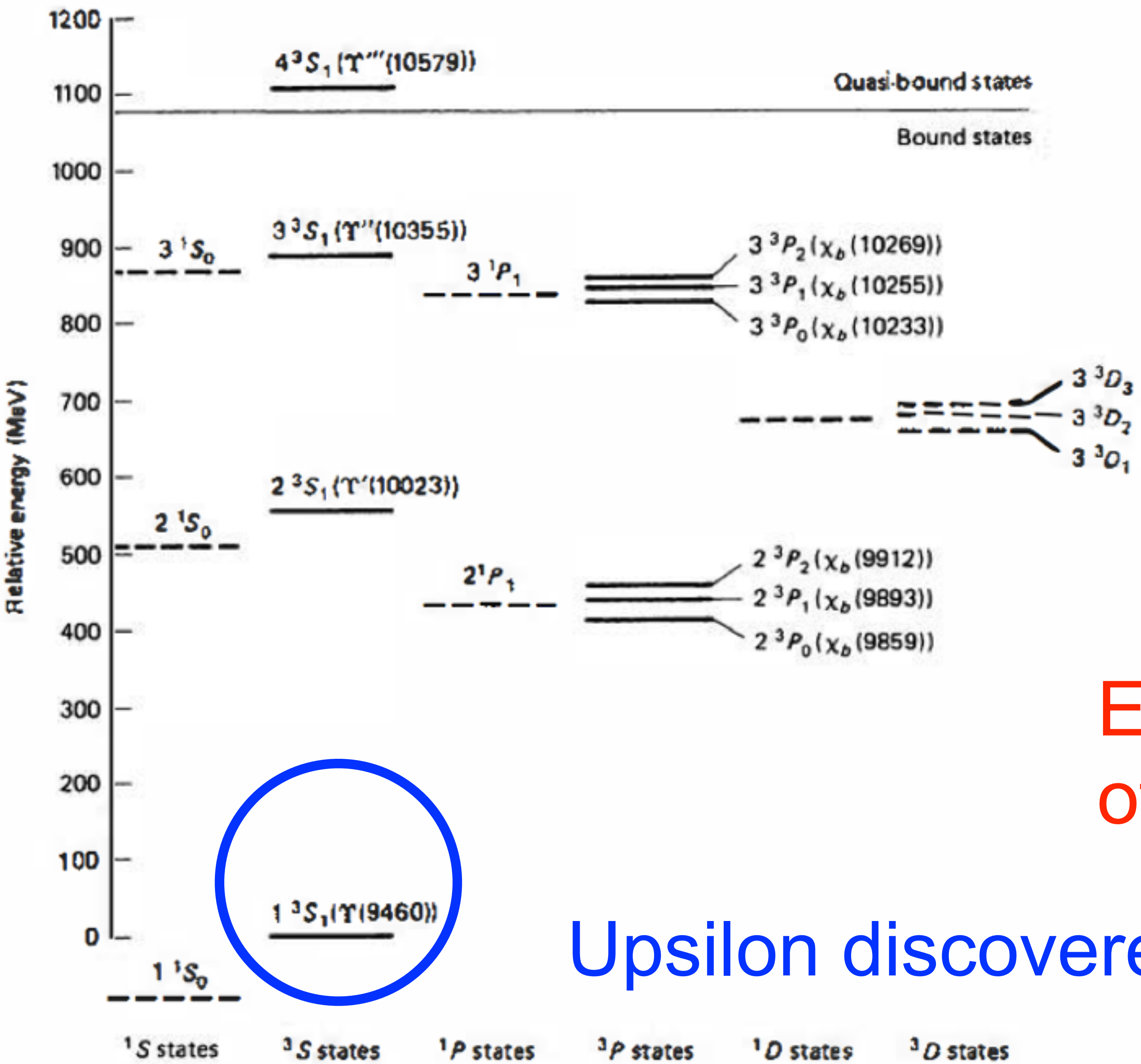
$I^G(J^{PC}) = 0^-(1^{--})$

$J/\psi(1S)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	hadrons	(87.7 ± 0.5) %	
Γ_2	virtual$\gamma\to$ hadrons	(13.50 ± 0.30) %	
Γ_3	ggg	(64.1 ± 1.0) %	
Γ_4	γgg	(8.8 ± 0.5) %	
Γ_5	e^+e^-	(5.94 ± 0.06) %	
Γ_6	$\mu^+\mu^-$	(5.93 ± 0.06) %	

Bottomonium

Griffiths



Even richer set of states!

Upsilon discovered in 1976

How to account for spin in hadron masses?

Recall magnetic moment formula: $\mu = -\frac{e}{m}\mathbf{S}$

Spin-spin interactions in hadrons have two components:

$$\mu_1 \cdot \mathbf{S_2} = -\frac{e}{m_1}\mathbf{S_1} \cdot \mathbf{S_2}$$

$$\mu_2 \cdot \mathbf{S_1} = -\frac{e}{m_2}\mathbf{S_2} \cdot \mathbf{S_1} = -\frac{e}{m_2}\mathbf{S_1} \cdot \mathbf{S_2}$$

Sum is then

$$-e\frac{m_1 + m_2}{m_1 m_2}\left(\mathbf{S_1} \cdot \mathbf{S_2}\right) = A(m_1 + m_2)\frac{1}{m_1 m_2}\left(\mathbf{S_1} \cdot \mathbf{S_2}\right)$$

What is the spin term?

$$(\mathbf{S_1} \cdot \mathbf{S_2})\,, \mathbf{S} = \mathbf{S_1} + \mathbf{S_2}$$

$$\mathbf{S}^2 = \mathbf{S_1}^2 + \mathbf{S_2}^2 + 2\,(\mathbf{S_1} \cdot \mathbf{S_2})$$

$$(\mathbf{S_1} \cdot \mathbf{S_2}) = \frac{1}{2}\left(\mathbf{S}^2 - \mathbf{S_1}^2 - \mathbf{S_2}^2\right)$$

S_1 and $S_2 = \pm 1/2$

$\mathbf{S_1}^2 = \mathbf{S_2}^2 = (1/2)(1/2+1) = 3/4$

$\mathbf{S}^2 = (1)(1+1) = 2$ (spins aligned) or

$\mathbf{S}^2 = (0)(0+1) = 0$ (spins anti-aligned)

So: $\mathbf{S}_1 \cdot \mathbf{S}_2 = 1/4$ (spins aligned)

or: $\mathbf{S}_1 \cdot \mathbf{S}_2 = -3/4$ (spins anti-aligned)

How to account for spin in meson masses?

$$M(m_1 - - - m_2) = m_1 + m_2 + A(m_1 + m_2)\frac{1}{m_1 m_2}(\mathbf{S_1} \cdot \mathbf{S_2})$$

Mass M of meson composed of quarks with mass m_1 and m_2 then generically looks like this

$\mathbf{S}_1 \cdot \mathbf{S}_2 = 1/4$ (spins aligned)
or: $\mathbf{S}_1 \cdot \mathbf{S}_2 = -3/4$ (spins anti-aligned)

Can try something simpler, and assume A is a constant (it surely is not, but maybe that is a reasonable approximation)

Fits for masses (from Griffiths)

Table 5.3 Pseudoscalar and vector meson masses. (MeV/c^2)

Meson	Calculated	Observed
π	139	138
K	487	496
η	561	548
ρ	775	776
ω	775	783
K^*	892	894
ϕ	1031	1020

Very nice agreement! But need to be careful...

For example:

$$\eta = \frac{u\bar{u} + d\bar{d} - 2s\bar{s}}{\sqrt{6}}$$

On to baryons

A lot more complicated - have three quarks, and thus three spins to add together. Most importantly, mesons are always composed of a quark and an anti-quark, ie never contain two of the same particle. In baryons, however (example: proton = uud), this no longer has to be true.

Regardless, though, baryons have half-integer spin (three quarks with s=+/- 1/2 can combine to give s= +/- 1/2 or +/- 3/2 only)

To add three spins together, we first start by adding two of them together. Back to those C-G tables from the PDG ...

Combining two 1/2 x 1/2 particles

ex: J=1

1/2×1/2		1			
		+1	1	0	
+1/2	+1/2	1	0	0	
+1/2	−1/2		1/2	1/2	1
−1/2	+1/2		1/2	−1/2	−1
−1/2	−1/2				1

ex: m_1, m_2= -1/2,+/1-2

$$|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{1}{2}> = |1\ 1>$$

$$|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{-1}{2}> = \sqrt{\frac{1}{2}}|1\ 0> + \sqrt{\frac{1}{2}}|0\ 0>$$

$$|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{1}{2}> = \sqrt{\frac{1}{2}}|1\ 0> - \sqrt{\frac{1}{2}}|0\ 0>$$

$$|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{-1}{2}> = |1\ -1>$$

Let's rearrange

$$|\frac{1}{2}\ \frac{1}{2}>+|\frac{1}{2}\ \frac{1}{2}>=|1\ 1>$$

$$|\frac{1}{2}\ \frac{1}{2}>+|\frac{1}{2}\ \frac{-1}{2}>=\sqrt{\frac{1}{2}}|1\ 0>+\sqrt{\frac{1}{2}}|0\ 0>$$

$$|\frac{1}{2}\ \frac{-1}{2}>+|\frac{1}{2}\ \frac{1}{2}>=\sqrt{\frac{1}{2}}|1\ 0>-\sqrt{\frac{1}{2}}|0\ 0>$$

$$|\frac{1}{2}\ \frac{-1}{2}>+|\frac{1}{2}\ \frac{-1}{2}>=|1\ -1>$$

These are the easy ones

$$|1\ 1>=|\frac{1}{2}\ \frac{1}{2}>+|\frac{1}{2}\ \frac{1}{2}>$$

$$|1\ -1>=|\frac{1}{2}\ \frac{-1}{2}>+|\frac{1}{2}\ \frac{-1}{2}>$$

Let's rearrange (can also use tables for this)

$$|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{1}{2}> = |1\ 1>$$

$$|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{-1}{2}> = \sqrt{\frac{1}{2}}|1\ 0> + \sqrt{\frac{1}{2}}|0\ 0>$$

Add together here

$$|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{1}{2}> = \sqrt{\frac{1}{2}}|1\ 0> - \sqrt{\frac{1}{2}}|0\ 0>$$

$$|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{-1}{2}> = |1\ -1>$$

$$\sqrt{2}|1\ 0> = \left(|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{-1}{2}>\right) + \left(|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|1\ 0> = \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{-1}{2}>\right) + \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{1}{2}>\right)$$

Let's rearrange (can also use tables for this)

$$|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{1}{2}> = |1\ 1>$$

$$|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{-1}{2}> = \sqrt{\frac{1}{2}}|1\ 0> + \sqrt{\frac{1}{2}}|0\ 0>$$

Subtract these

$$|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{1}{2}> = \sqrt{\frac{1}{2}}|1\ 0> - \sqrt{\frac{1}{2}}|0\ 0>$$

$$|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{-1}{2}> = |1\ -1>$$

$$\sqrt{2}|0\ 0> = \left(|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{-1}{2}>\right) - \left(|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|0\ 0> = \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{-1}{2}>\right) - \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{1}{2}>\right)$$

Putting it together

$$|0\ 0> = \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{-1}{2}>\right) - \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|1\ 0> = \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{-1}{2}>\right) + \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|1\ 1> = |\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{1}{2}>$$

$$|1\ -1> = |\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{-1}{2}>$$

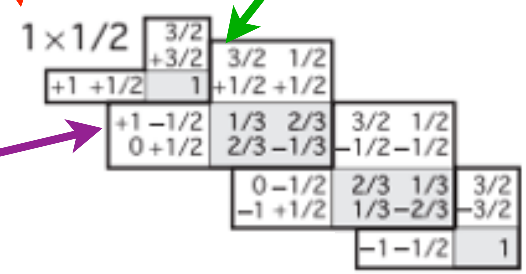
When we add the third quark we will have to add spin 1/2 with either spin 0 or spin 1

Now we add the third one

Combining spin 1 x 1/2 particles

We want the “inverse” of what we have been reading off. Can also use the tables for that!

ex:
J=3/2
m=1/2



ex:
$m_1, m_2 =$
+1,-1/2

Now we add the third one

Combining spin 1 x 1/2 particles

$$|\frac{3}{2}\ \frac{3}{2}>=|1\ 1>+|\frac{1}{2}\ \frac{1}{2}>$$

$$|\frac{3}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{3}}\left(|1\ 1>+|\frac{1}{2}\ \frac{-1}{2}>\right)+\sqrt{\frac{2}{3}}\left(|1\ 0>+|\frac{1}{2}\ \frac{1}{2}>\right)$$

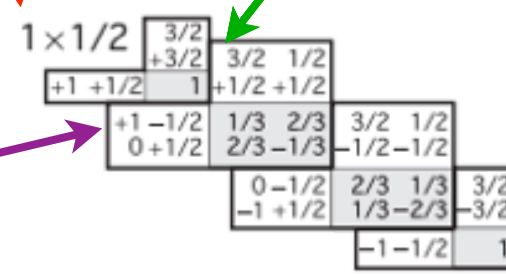
$$|\frac{1}{2}\ \frac{1}{2}>=\sqrt{\frac{2}{3}}\left(|1\ 1>+|\frac{1}{2}\ \frac{-1}{2}>\right)-\sqrt{\frac{1}{3}}\left(|1\ 0>+|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{3}{2}\ \frac{-1}{2}>=\sqrt{\frac{2}{3}}\left(|1\ 0>+|\frac{1}{2}\ \frac{-1}{2}>\right)+\sqrt{\frac{1}{3}}\left(|1\ -1>+|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\sqrt{\frac{1}{3}}\left(|1\ 0>+|\frac{1}{2}\ \frac{-1}{2}>\right)-\sqrt{\frac{2}{3}}\left(|1\ -1>+|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{3}{2}\ \frac{-3}{2}>=|1\ -1>+|\frac{1}{2}\ \frac{-1}{2}>$$

ex:
J=3/2
m=1/2



ex:
$m_1, m_2=$
+1,-1/2

Now we add the third one

Combining spin 0 x 1/2 particles is trivial

$$|\frac{1}{2}\ \frac{1}{2}>=|0\ 0>+|\frac{1}{2}\ \frac{1}{2}>$$

$$|\frac{1}{2}\ \frac{-1}{2}>=|0\ 0>+|\frac{1}{2}\ \frac{-1}{2}>$$

So in total

$$|0\ 0>=\frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{1}{2}>+|\frac{1}{2}\ \frac{-1}{2}>\right)-\frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{-1}{2}>+|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|1\ 0>=\frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{1}{2}>+|\frac{1}{2}\ \frac{-1}{2}>\right)+\frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{-1}{2}>+|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|1\ 1>=|\frac{1}{2}\ \frac{1}{2}>+|\frac{1}{2}\ \frac{1}{2}>$$

$$|1\ -1>=|\frac{1}{2}\ \frac{-1}{2}>+|\frac{1}{2}\ \frac{-1}{2}>$$

From first combination

$$|\frac{3}{2}\ \frac{3}{2}>=|1\ 1>+|\frac{1}{2}\ \frac{1}{2}>$$

$$|\frac{3}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{3}}\left(|1\ 1>+|\frac{1}{2}\ \frac{-1}{2}>\right)+\sqrt{\frac{2}{3}}\left(|1\ 0>+|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{1}{2}\ \frac{1}{2}>=\sqrt{\frac{2}{3}}\left(|1\ 1>+|\frac{1}{2}\ \frac{-1}{2}>\right)-\sqrt{\frac{1}{3}}\left(|1\ 0>+|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{3}{2}\ \frac{-1}{2}>=\sqrt{\frac{2}{3}}\left(|1\ 0>+|\frac{1}{2}\ \frac{-1}{2}>\right)+\sqrt{\frac{1}{3}}\left(|1\ -1>+|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\sqrt{\frac{1}{3}}\left(|1\ 0>+|\frac{1}{2}\ \frac{-1}{2}>\right)-\sqrt{\frac{2}{3}}\left(|1\ -1>+|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{3}{2}\ \frac{-3}{2}>=|1\ -1>+|\frac{1}{2}\ \frac{-1}{2}>$$

$$|0\ 0>+|\frac{1}{2}\ \frac{1}{2}>=|\frac{1}{2}\ \frac{1}{2}>$$

$$|0\ 0>+|\frac{1}{2}\ \frac{-1}{2}>=|\frac{1}{2}\ \frac{-1}{2}>$$

Let's introduce some nicer notation

$$|0\ 0>= \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{-1}{2}>\right) - \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|1\ 0>= \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{-1}{2}>\right) + \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|1\ 1>= |\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{1}{2}>$$

$$|1\ -1>= |\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{-1}{2}>$$

$$|\frac{3}{2}\ \frac{3}{2}>= |1\ 1> + |\frac{1}{2}\ \frac{1}{2}>$$

$$|\frac{3}{2}\ \frac{1}{2}>= \frac{1}{\sqrt{3}}\left(|1\ 1> + |\frac{1}{2}\ \frac{-1}{2}>\right) + \sqrt{\frac{2}{3}}\left(|1\ 0> + |\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{1}{2}\ \frac{1}{2}>= \sqrt{\frac{2}{3}}\left(|1\ 1> + |\frac{1}{2}\ \frac{-1}{2}>\right) - \sqrt{\frac{1}{3}}\left(|1\ 0> + |\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{3}{2}\ \frac{-1}{2}>= \sqrt{\frac{2}{3}}\left(|1\ 0> + |\frac{1}{2}\ \frac{-1}{2}>\right) + \sqrt{\frac{1}{3}}\left(|1\ -1> + |\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{1}{2}\ \frac{-1}{2}>= \sqrt{\frac{1}{3}}\left(|1\ 0> + |\frac{1}{2}\ \frac{-1}{2}>\right) - \sqrt{\frac{2}{3}}\left(|1\ -1> + |\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{3}{2}\ \frac{-3}{2}>= |1\ -1> + |\frac{1}{2}\ \frac{-1}{2}>$$

From first combination

Don't forget: order matters!

$$|0\ 0> + |\frac{1}{2}\ \frac{1}{2}>= |\frac{1}{2}\ \frac{1}{2}>$$

$$|0\ 0> + |\frac{1}{2}\ \frac{-1}{2}>= |\frac{1}{2}\ \frac{-1}{2}>$$

$$|\frac{1}{2}\ \frac{1}{2}>= (\uparrow)$$

$$|\frac{1}{2}\ \frac{-1}{2}>= (\downarrow)$$

Using the notation

$$|\frac{3}{2}\ \frac{3}{2}>=|1\ 1>+(\uparrow)$$

$$|\frac{3}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{3}}(|1\ 1>+(\downarrow))+\sqrt{\frac{2}{3}}(|1\ 0>+(\uparrow))$$

$$|\frac{1}{2}\ \frac{1}{2}>=\sqrt{\frac{2}{3}}(|1\ 1>+(\downarrow))-\sqrt{\frac{1}{3}}(|1\ 0>+(\uparrow))$$

$$|\frac{3}{2}\ \frac{-1}{2}>=\sqrt{\frac{2}{3}}(|1\ 0>+(\downarrow))+\sqrt{\frac{1}{3}}(|1\ -1>+(\uparrow))$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\sqrt{\frac{1}{3}}(|1\ 0>+(\downarrow))-\sqrt{\frac{2}{3}}(|1\ -1>+(\uparrow))$$

$$|\frac{3}{2}\ \frac{-3}{2}>=|1\ -1>+(\downarrow)$$

$$|0\ 0>=\frac{1}{\sqrt{2}}(\uparrow\downarrow)-\frac{1}{\sqrt{2}}(\downarrow\uparrow)$$

$$|1\ 0>=\frac{1}{\sqrt{2}}(\uparrow\downarrow)+\frac{1}{\sqrt{2}}(\downarrow\uparrow)$$

$$|1\ 1>=\uparrow\uparrow$$

$$|1\ -1>=\downarrow\downarrow$$

$$|\frac{1}{2}\ \frac{1}{2}>=|0\ 0>+\uparrow$$

$$|\frac{1}{2}\ \frac{-1}{2}>=|0\ 0>+\downarrow$$

Putting it together

$$|\frac{3}{2}\ \frac{3}{2}>=\uparrow\uparrow\uparrow$$

$$|\frac{3}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow)+\sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{1}{2}\ \frac{1}{2}>=\sqrt{\frac{2}{3}}(\uparrow\uparrow\downarrow)-\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{3}{2}\ \frac{-1}{2}>=\sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)(\downarrow)\right)+\sqrt{\frac{1}{3}}(\downarrow\downarrow\uparrow)$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\downarrow\right)-\sqrt{\frac{2}{3}}(\downarrow\downarrow\uparrow)$$

$$|\frac{3}{2}\ \frac{-3}{2}>=\downarrow\downarrow\downarrow$$

Phew

$$|\frac{1}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\uparrow$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\downarrow$$

How to interpret

$$|\frac{3}{2}\ \frac{3}{2}>=\uparrow\uparrow\uparrow$$

$$|\frac{3}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow)+\sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{1}{2}\ \frac{1}{2}>=\sqrt{\frac{2}{3}}(\uparrow\uparrow\downarrow)-\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{3}{2}\ \frac{-1}{2}>=\sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)(\downarrow)\right)+\sqrt{\frac{1}{3}}(\downarrow\downarrow\uparrow)$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\downarrow\right)-\sqrt{\frac{2}{3}}(\downarrow\downarrow\uparrow)$$

$$|\frac{3}{2}\ \frac{-3}{2}>=\downarrow\downarrow\downarrow$$

$$|\frac{1}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\uparrow$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\downarrow$$

$$|\frac{3}{2}\ \frac{3}{2}>=\uparrow\uparrow\uparrow$$

$$|\frac{3}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow\ +\uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow)$$

$$|\frac{3}{2}\ \frac{-1}{2}>=\frac{1}{\sqrt{3}}(\uparrow\downarrow\downarrow\ +\downarrow\uparrow\downarrow+\downarrow\downarrow\uparrow)$$

$$|\frac{3}{2}\ \frac{-3}{2}>=\downarrow\downarrow\downarrow$$

Spin 3/2 states are easy to interpret: symmetric if we interchange any two quarks

How to interpret

$$|\frac{3}{2}\ \frac{3}{2}>=\uparrow\uparrow\uparrow$$

$$|\frac{3}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow)+\sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{1}{2}\ \frac{1}{2}>=\sqrt{\frac{2}{3}}(\uparrow\uparrow\downarrow)-\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{3}{2}\ \frac{-1}{2}>=\sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)(\downarrow)\right)+\sqrt{\frac{1}{3}}(\downarrow\downarrow\uparrow)$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\downarrow\right)-\sqrt{\frac{2}{3}}(\downarrow\downarrow\uparrow)$$

$$|\frac{3}{2}\ \frac{-3}{2}>=\downarrow\downarrow\downarrow$$

$$|\frac{1}{2}\ \frac{1}{2}>=\sqrt{\frac{2}{3}}(\uparrow\uparrow\downarrow)-\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\downarrow\right)-\sqrt{\frac{2}{3}}(\downarrow\downarrow\uparrow)$$

$$|\frac{1}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\uparrow$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\downarrow$$

Two of spin 1/2 states are asymmetric under interchange of first and second quarks

How to interpret

$$|\frac{3}{2}\ \frac{3}{2}>=\uparrow\uparrow\uparrow$$

$$|\frac{3}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow)+\sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{1}{2}\ \frac{1}{2}>=\sqrt{\frac{2}{3}}(\uparrow\uparrow\downarrow)-\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{3}{2}\ \frac{-1}{2}>=\sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)(\downarrow)\right)+\sqrt{\frac{1}{3}}(\downarrow\downarrow\uparrow)$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\downarrow\right)-\sqrt{\frac{2}{3}}(\downarrow\downarrow\uparrow)$$

$$|\frac{3}{2}\ \frac{-3}{2}>=\downarrow\downarrow\downarrow$$

$$|\frac{1}{2}\ \frac{1}{2}>=\sqrt{\frac{2}{3}}(\uparrow\uparrow\downarrow)-\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\downarrow\right)-\sqrt{\frac{2}{3}}(\downarrow\downarrow\uparrow)$$

These last two spin 1/2 states are symmetric under interchange of first and second quarks

$$|\frac{1}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\uparrow$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\downarrow$$

Back to baryons

We need our 3 quarks to satisfy Fermi-Dirac statistics (must be anti-symmetric under exchange of any two quarks)

For ground state (l=0), space wave function is symmetric. Left off with wave functions for spin, color and flavor. We will see that color wave function is necessarily anti-symmetric. That means that **flavor x spin** combination must be symmetric

Baryon masses

$$M(m_1 m_2 m_e) = m_1 + m_2 + m_3 + A'(\frac{(\mathbf{S_1} \cdot \mathbf{S_2})}{m_1 m_2} + \frac{(\mathbf{S_1} \cdot \mathbf{S_3})}{m_1 m_3} + \frac{(\mathbf{S_2} \cdot \mathbf{S_3})}{m_2 m_3})$$

$\mathbf{S}_i \cdot \mathbf{S}_j$ = 1/4 (spins aligned)
or: $\mathbf{S}_i \cdot \mathbf{S}_j$ = -3/4 (spins anti-aligned)

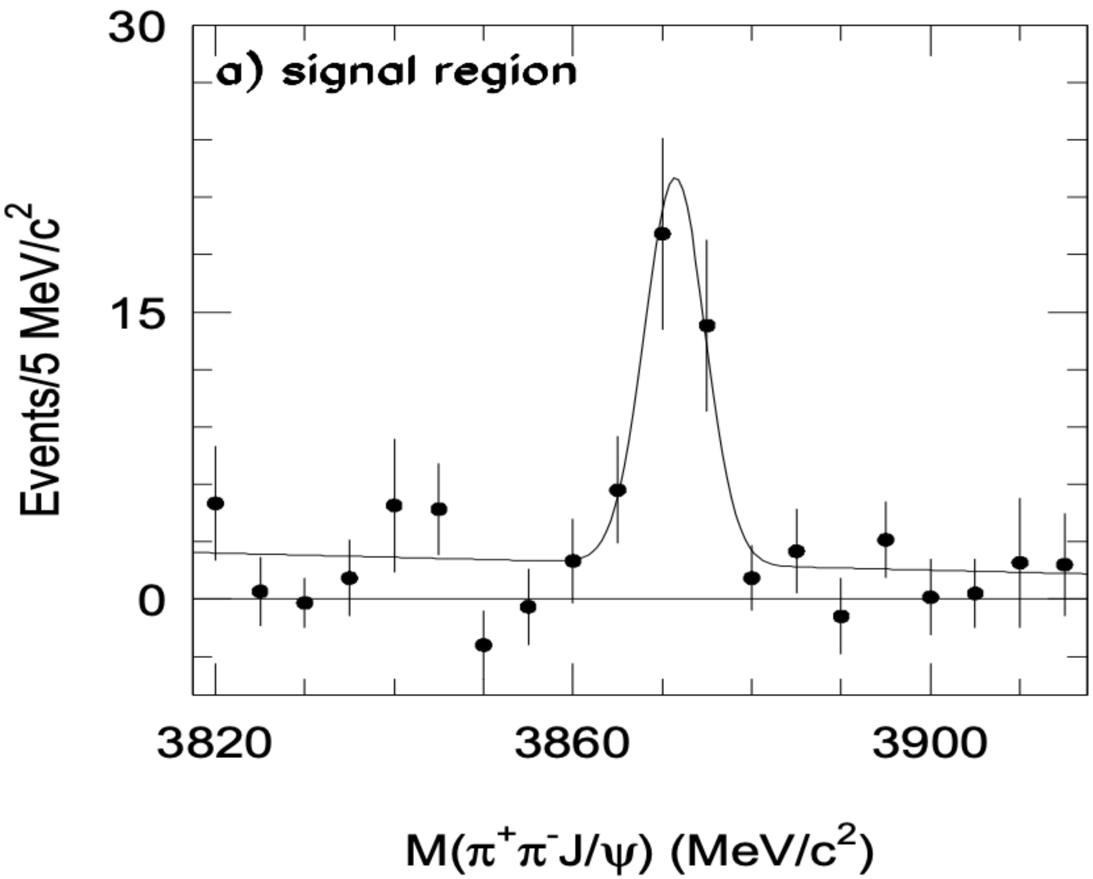
Using same formalism as we used for meson masses. Get a good fit, but find somewhat different quark masses in mesons and baryons (these are effective masses!) Conclusion: Much of the mass in, for example, the proton, comes from the energy of the QCD field and not from constituent quark masses themselves

More exotic hadrons

Why do hadrons come only in meson (q-qbar) and baryon (qqq) form? Why not also combinations of 4 quarks? Or 5 quarks?

You can combine two mesons to form a **tetraquark** that is colorless, or a meson and a baryon to form a colorless **pentaquark**

Such states seemed to come and go in terms of evidence for or against them but these days clearly seem to exist …



X3872 observed by Belle (asymmetric e+e collider): first strong evidence in 2003 for an exotic quark! Intriguingly, mass is very close to the $D^0\overline{D}^{0*}$ mass threshold: is a loosely bound "molecule"?

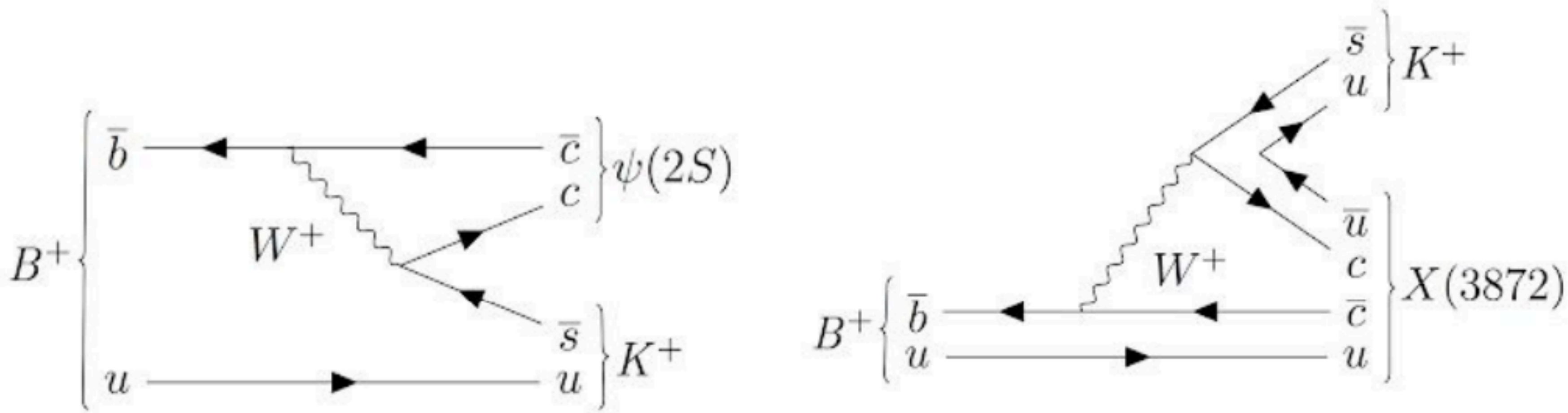


Figure 6. Feynman diagrams showing two different decays of a B^+ meson. On the left is the decay to a final state containing the conventional *ψ(2S)* charmonium state and on the right is the decay to the exotic *X(3872)*. The *ψ(2S)* and *X(3872)* both decay to $J/\psi\pi^+\pi^-$, such that the same particles are in the final state of both B^+ meson decays.

$$\psi(2S) \to \pi^+\pi^- J/\psi \to \to \pi^+\pi^- l^+ l^-$$

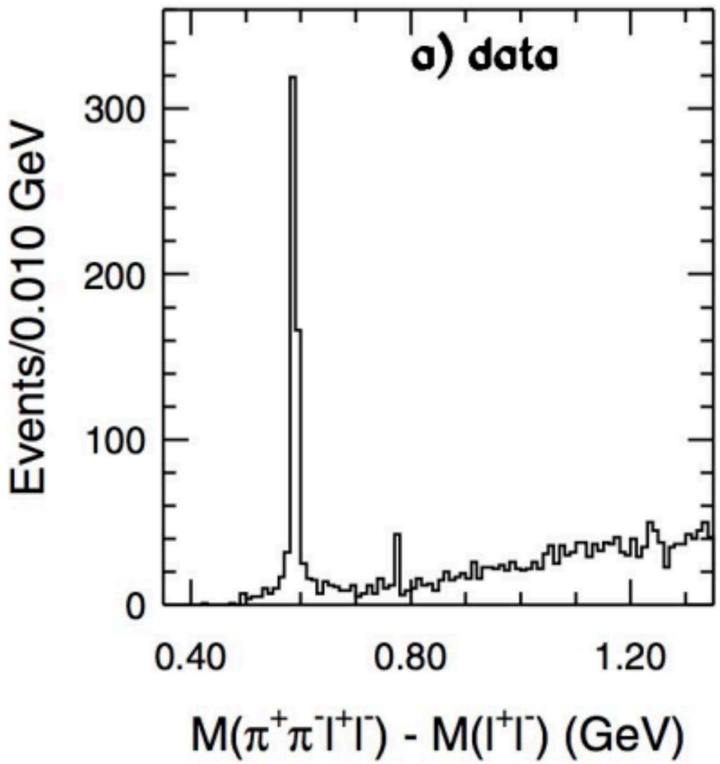
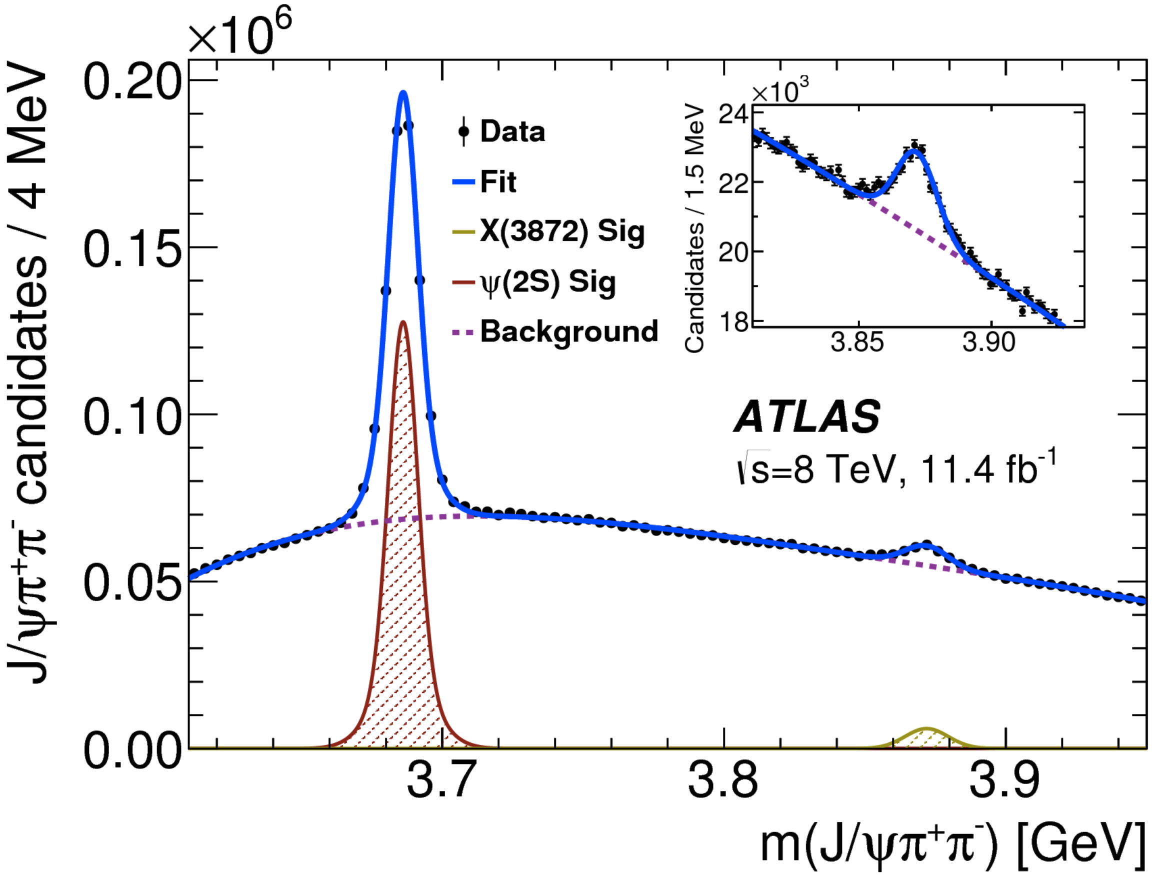


Figure 7. Distribution of the difference between $\pi^+\pi^- l^+ l^-$ and $l^+ l^-$ invariant masses of $B^+ \to J/\psi\pi^+\pi^- K^+$ candidates, where $J/\psi \to l^+ l^-$, l being μ or e. The narrow peaks near 0.6 and 0.8 GeV correspond to the $\psi(2S)$ and *X(3872)* states, respectively. Reproduced from *Phys. Rev. Lett.* **91** (2003) 262001.

X(3872) at the LHC BPHY-2015-03



We can study this at ATLAS and CMS (and of course at LHCb…)

X(3872) in details 2005.13419

$$m_{\chi_{c1}(3872)} = 3871.695 \pm 0.067 \pm 0.068 \pm 0.010\,\text{MeV}$$

Compare with

$$m(D^0\overline{D}^{0*}) = 3871.70 \pm 0.11 \text{ MeV}$$

Quite intriguing!

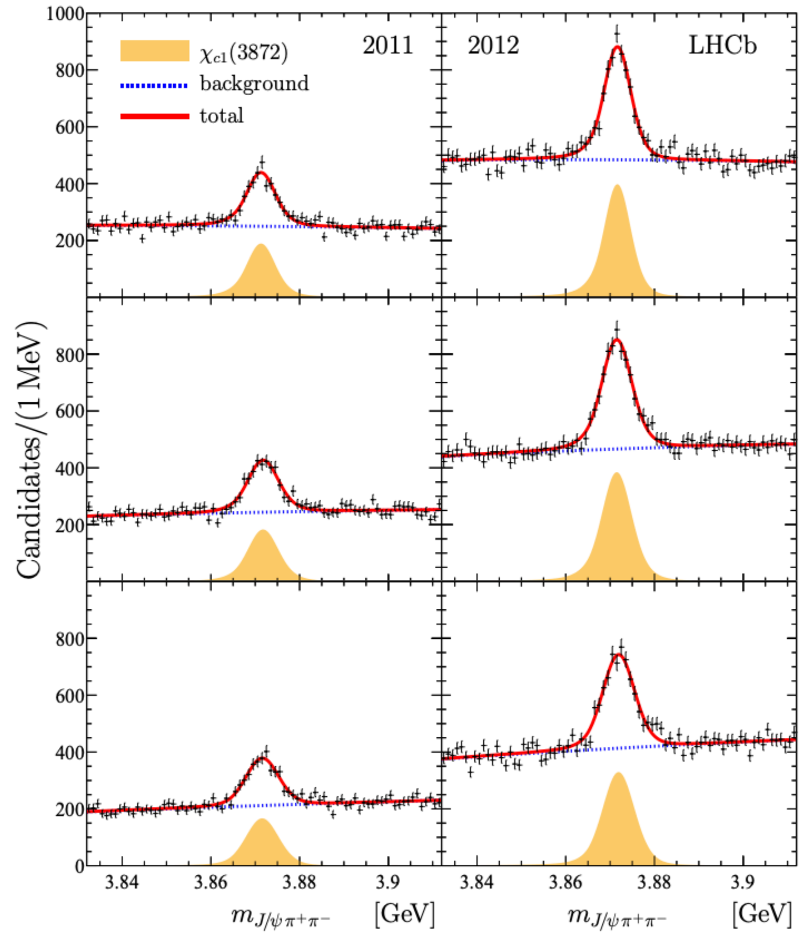


Figure 2: Mass distributions for $J/\psi\pi^+\pi^-$ candidates in the $\chi_{c1}(3872)$ region for (top) the low, (middle) mid and (bottom) high $p_{\pi^+\pi^-}$ bins. The left (right)-hand plot is for 2011 (2012) data. The projection of the fit described in the text is superimposed.

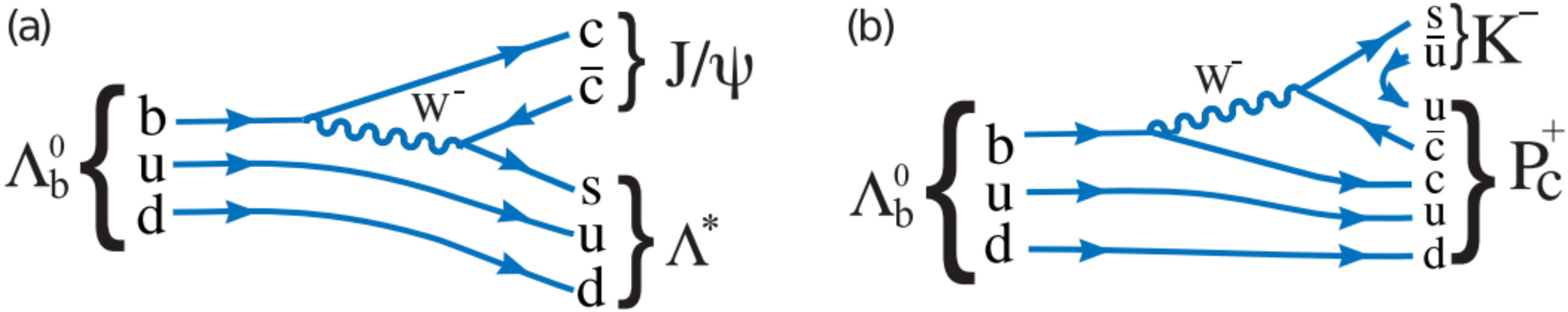
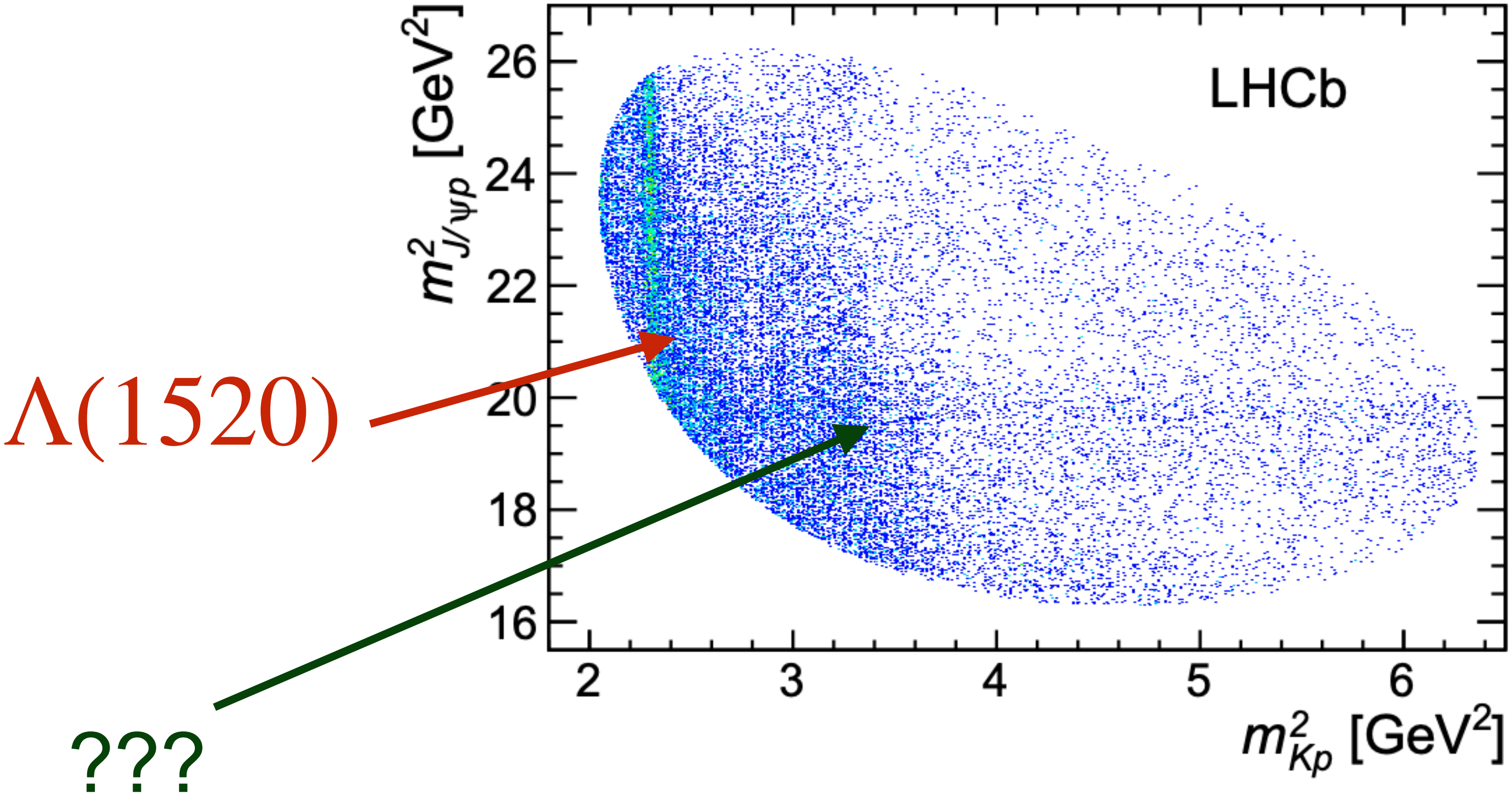


Figure 1: Feynman diagrams for (a) $\Lambda_b^0 \to J/\psi\,\Lambda^*$ and (b) $\Lambda_b^0 \to P_c^+ K^-$ decay.

LHCb studies Λ_b decays ($\Lambda^* \to K^- p$) and found an interesting feature in the J/ψ + p mass distribution ($J/\psi \to \mu^+\mu^-$)

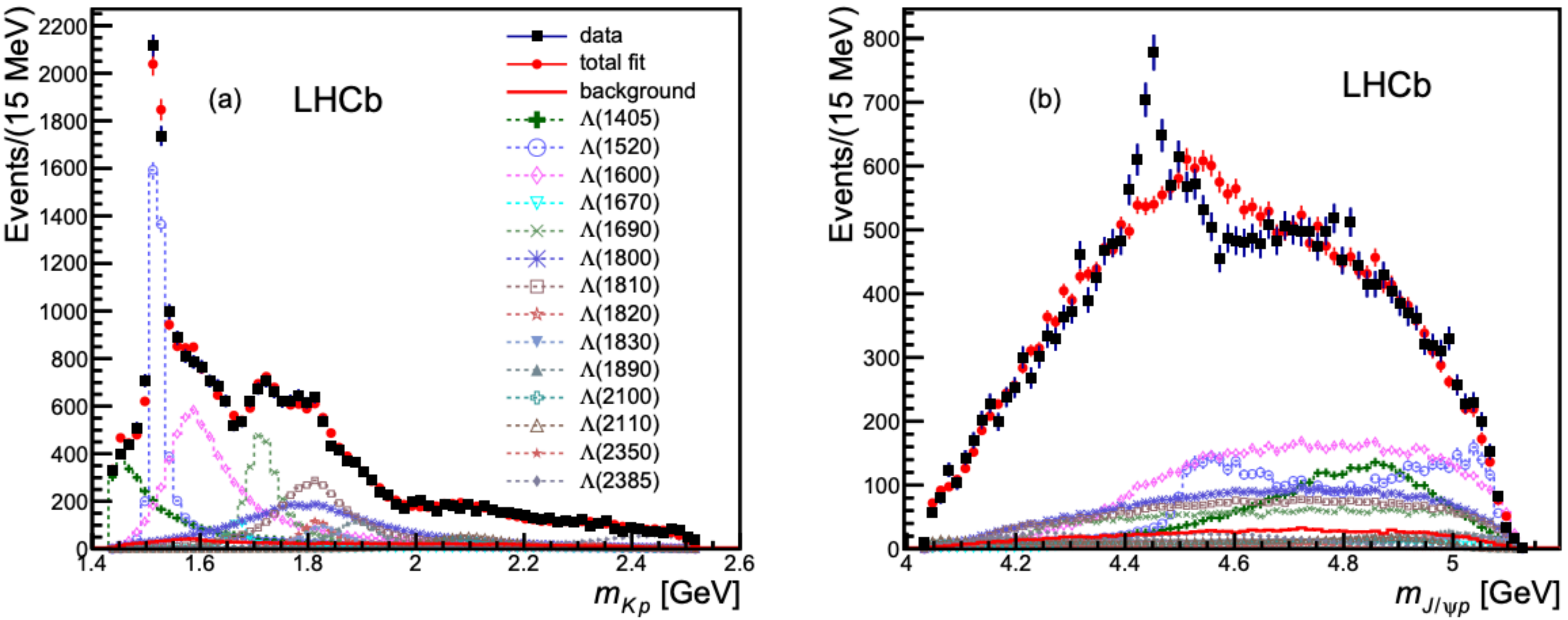
Dalitz plot

1507.03414



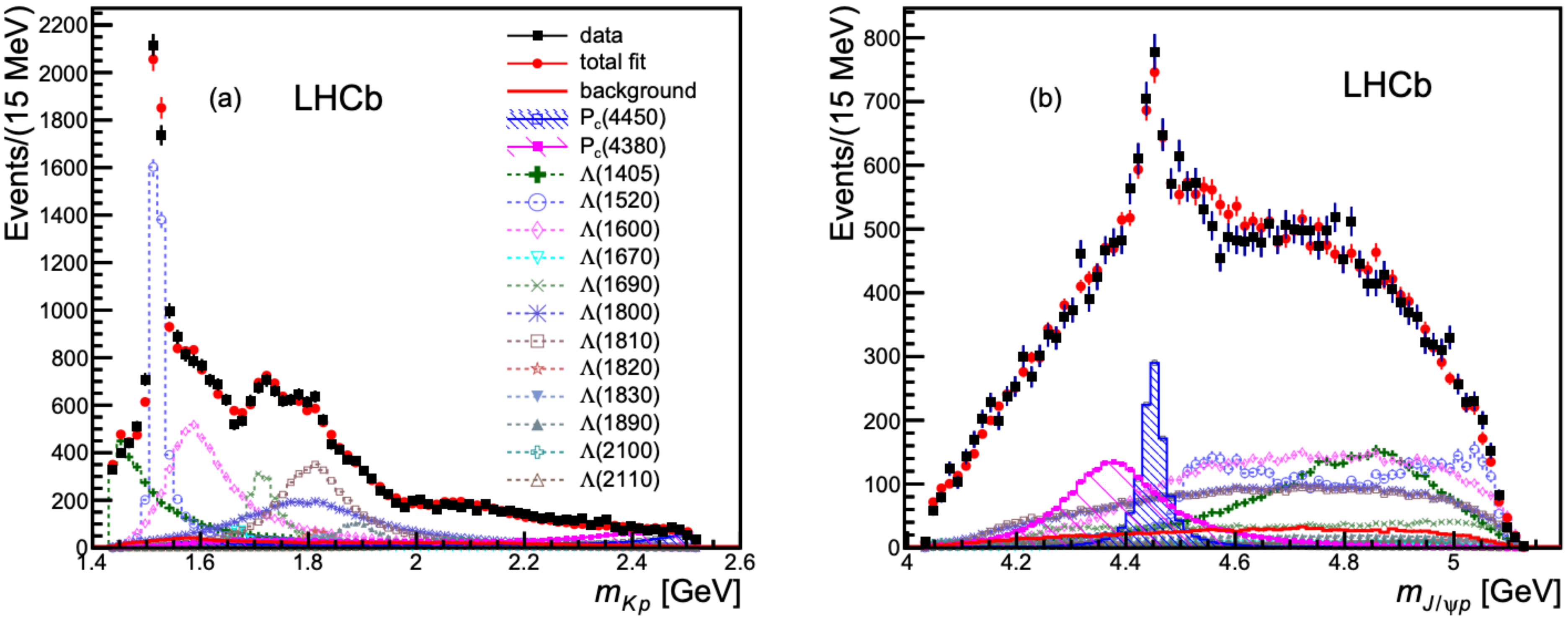
Very useful for finding structure and understanding decay chains!

Mass plots 1507.03414



Need to account for large numbers of Λ states and perform a full analysis accounting for QM interference and angular coefficients (up to J=9/2!). Tricky, as objects near kinematic limits combined with angular effects (and potentially interference) can cause resonant-like peaks that are not from real resonances, just **kinematic reflections**. But clearly the J/ψ + p distribution is not well modeled

More exotic hadrons 1507.03414

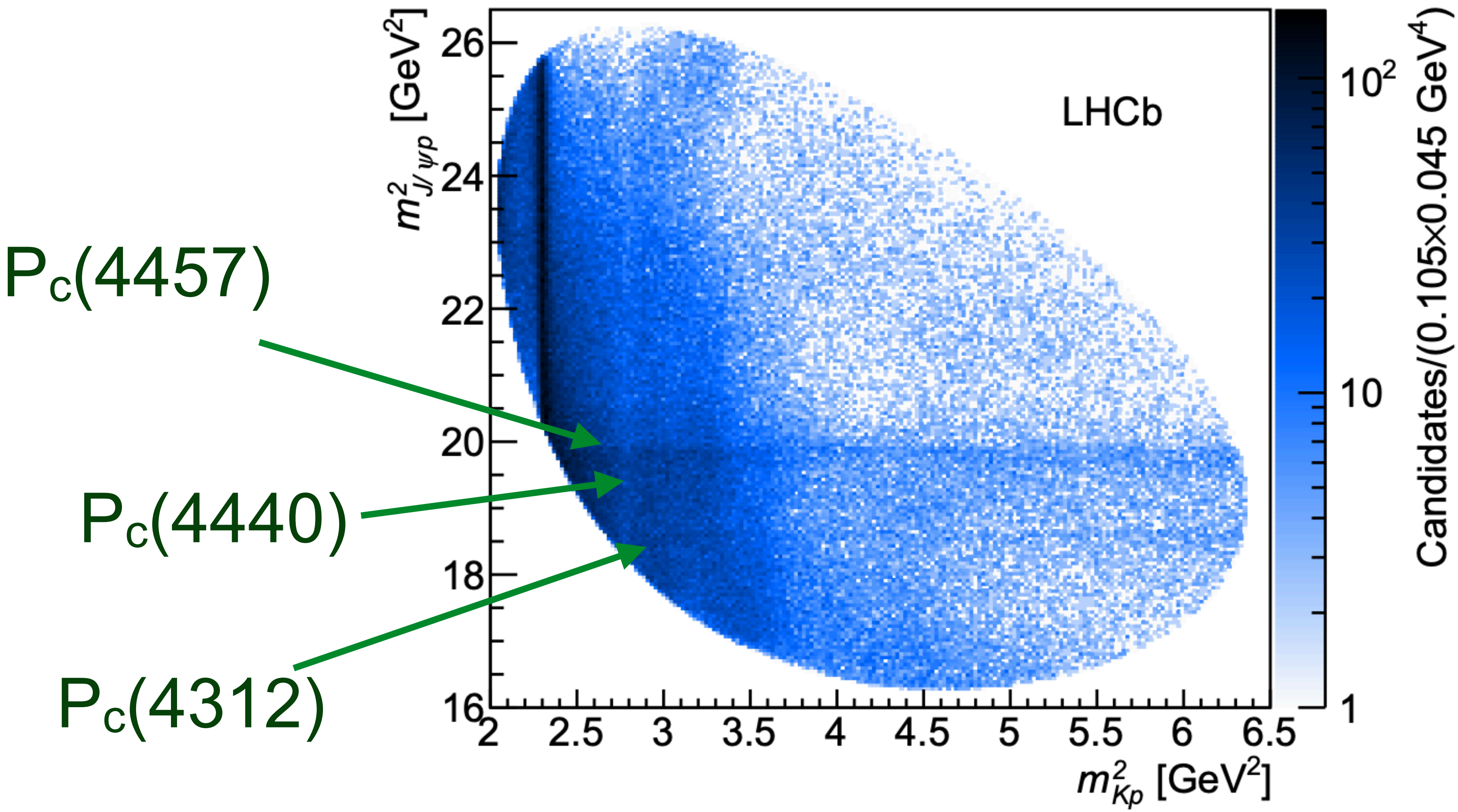


Including two new states gives a much better fit! Two P_c states: Pc(4380), $J^P = (3/2)^-$ and $P_c(4450)$, $J^P = (5/2)^+$

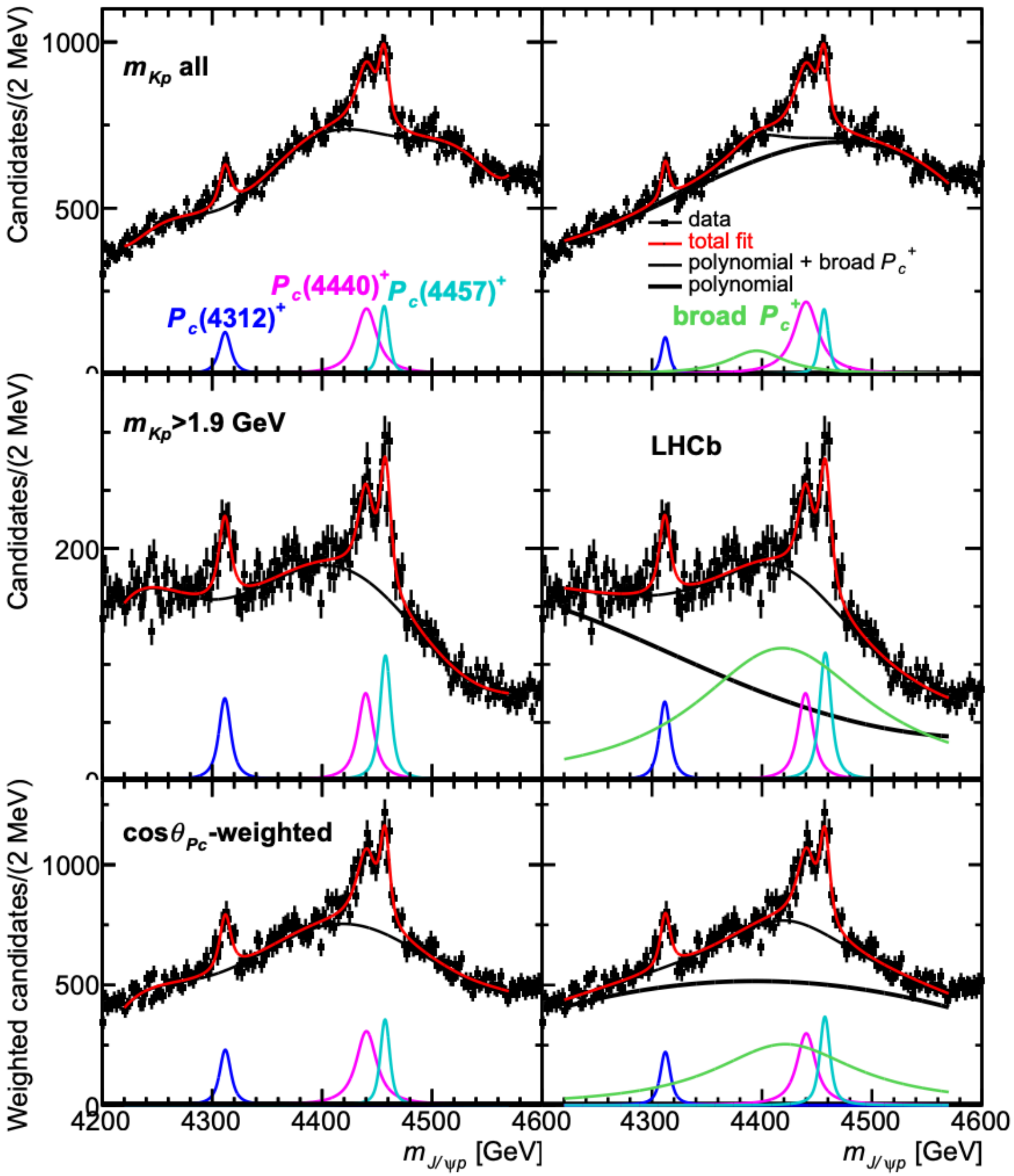
Different binding mechanisms of pentaquark states are possible. Tight-binding was envisioned originally [3,4,35]. A possible explanation is heavy-light diquarks [36]. Examples of other mechanisms include a diquark-diquark-antiquark model [37,38], a diquark-triquark model [39], and a coupled channel model [40]. Weakly bound "molecules" of a baryon plus a meson have been also discussed [41].

More recent updates

1904.03947

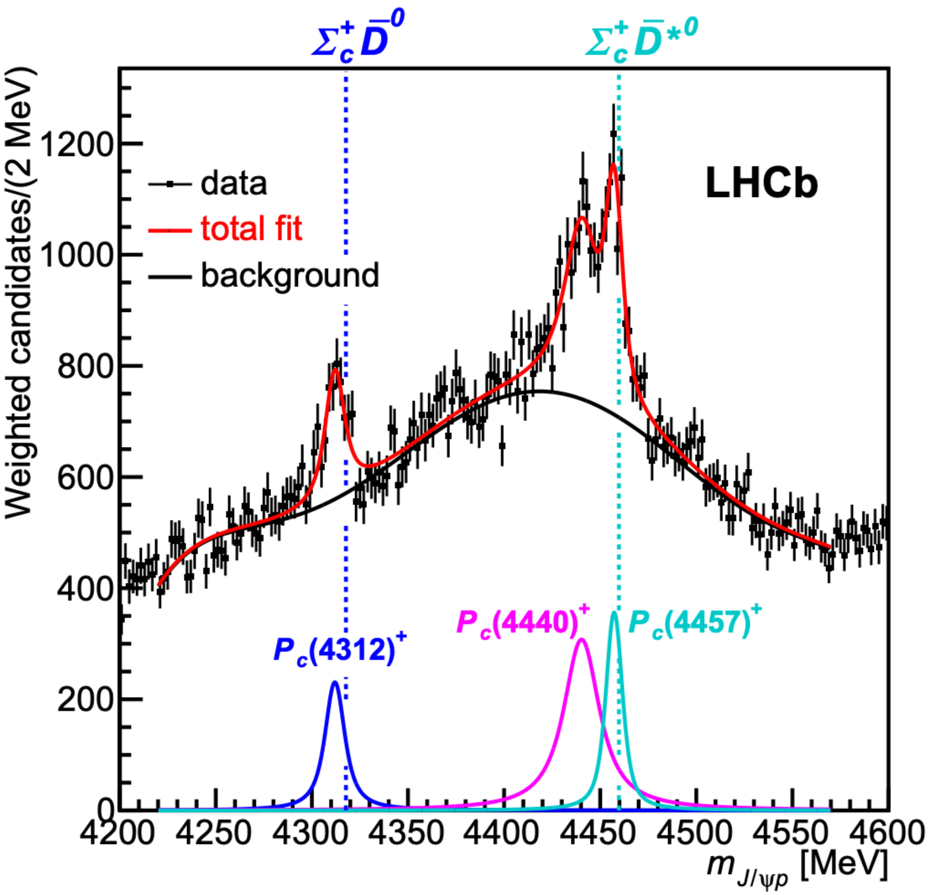


More recent updates 1904.03947



Fit in separate bins of $\cos\theta_{PC}$ (cos of angle between K- and J/ψ) in PC rest frame

More recent updates 1904.03947



Very close to $\Sigma_c \overline{D}^0$ masses, implies baryon-meson bound state?

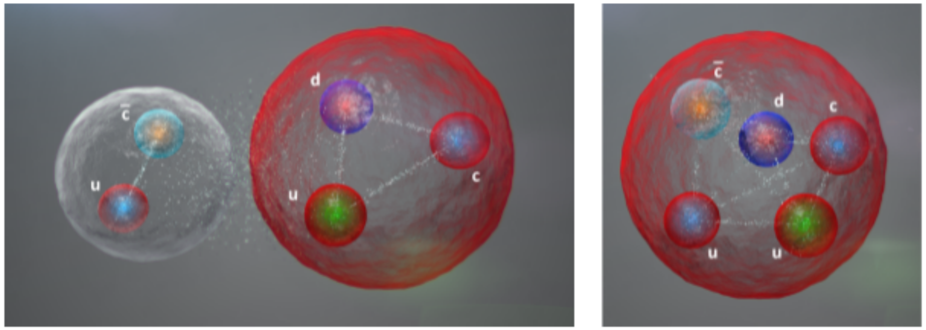
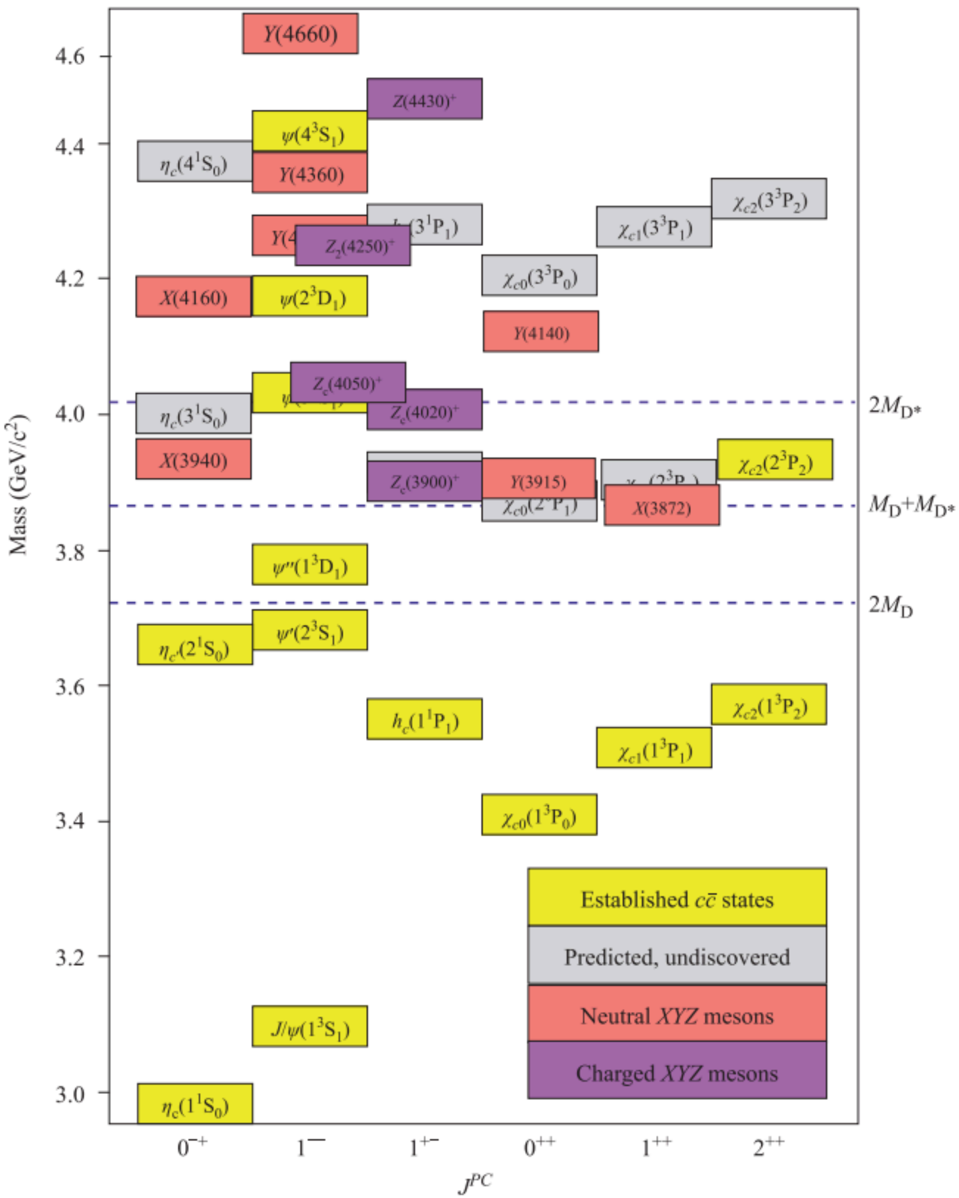


Figure 3. Possible quark combinations to make mesons, baryons and pentaquarks. Copyright CERN.

Summary plot

1411.7738



Not always obvious how the new states fit together. Molecular picture requires something to bind the pieces of the molecule. Pion carries isospin, so it cannot bind hadrons with I ==0. And it is a pseudoscalar, so it cannot bind two other pseudoscalars and conserve parity. Perhaps rho contributes

Fun new tetraquark 2109.01038

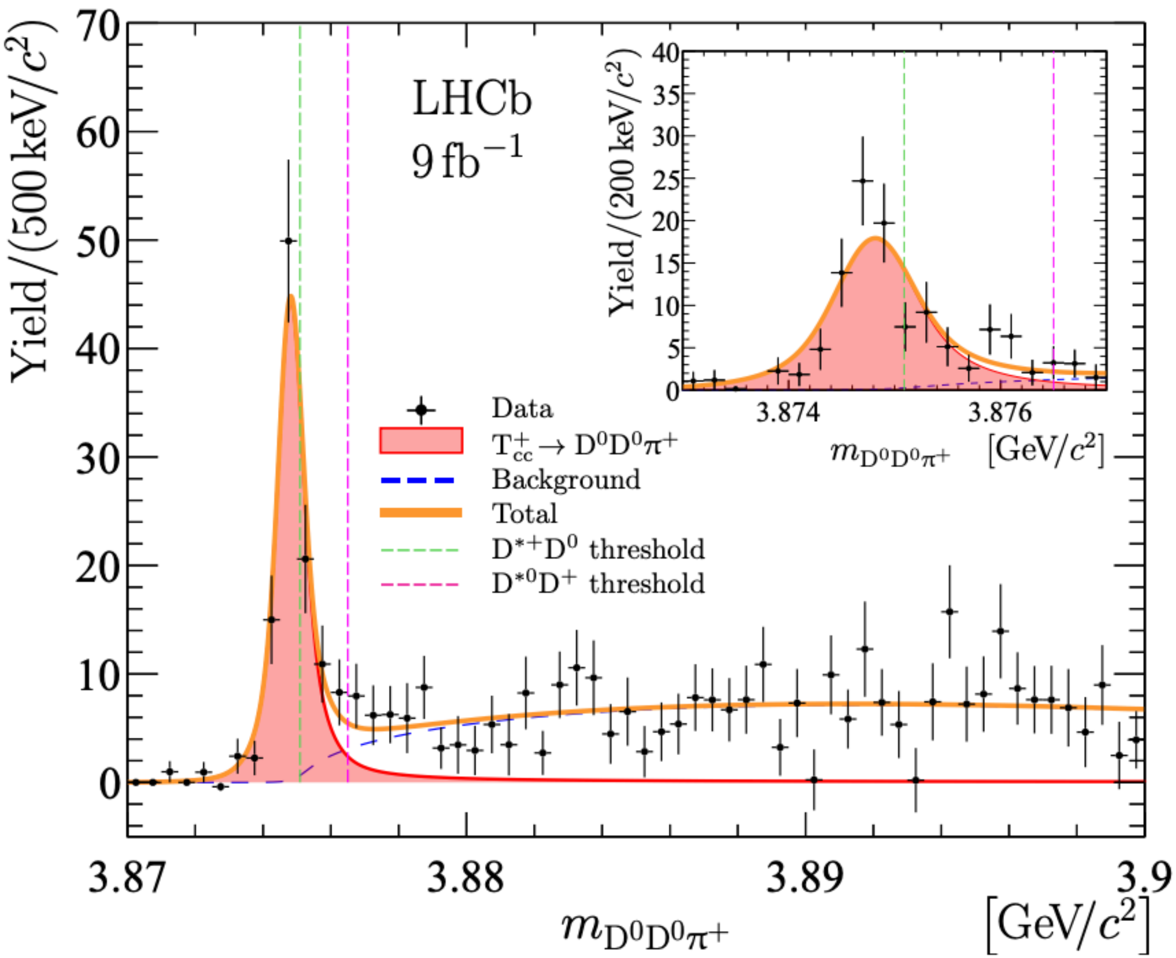
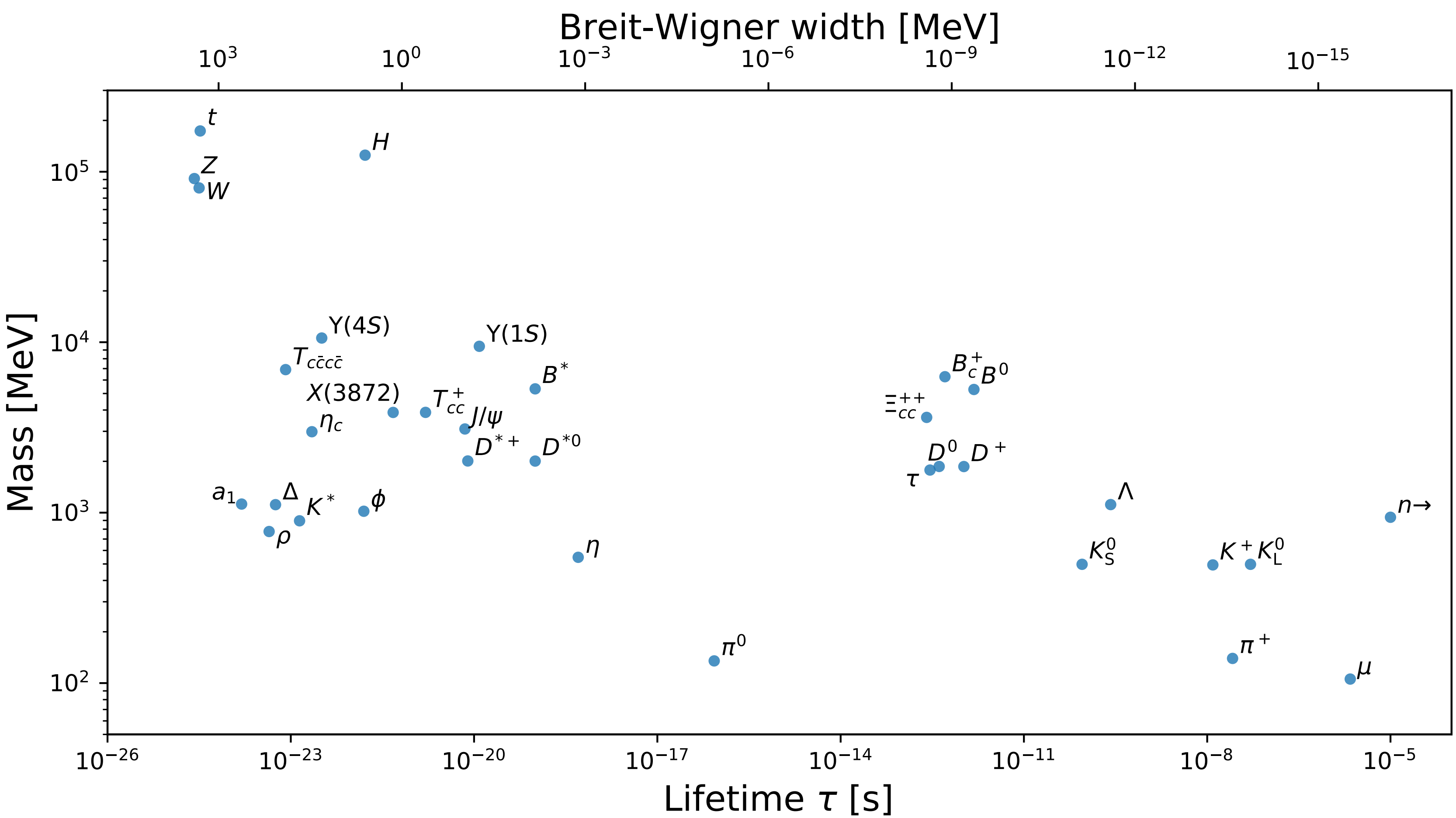


Figure 1: **The $D^0D^0\pi^+$ mass distribution**. The $D^0D^0\pi^+$ mass distribution where the contribution of the non-D^0 background has been statistically subtracted. The result of the fit described in the text is overlaid.

Very close to threshold, so (relatively) long-lived, again supporting the molecular picture. Found using (for both) $D^0 \to K^-\pi^+$



Let's talk about this!

You don’t have Chapter 5 HW, but …

Something new to think about

I'd like for you to begin to think about your final project. You should put in some effort to prepare it, so time to start now. You need to pick a single **analysis** or result to present (something public in the past ~year)

http://arxiv.org/archive/hep-ex

Minos
ATLAS
CMS
LHCb
ALICE
SNO
Belle-2
AMS
LIGO
VIRGO
LUX
Dark Energy Survey
Pierre Auger
X17
Fermi Gamma Ray Telescope

Since you don't have Chapter 5 HW and Chapter 6 is long

Pick an analysis/paper and email it to me for approval no later than 1 week from today. I'll ask for some progress reports in the future, but for now I just want you to pick something interesting (and not what you work on for research, and not what someone else will be talking about). Talk to me if you need help picking a topic!

Some more news on your presentation

Want a 25 minute presentation on the topic! You should be including theory background if possible, as well as information on the detector, the analysis, the background estimation, and the significance of the result. We will all discuss the presentation for 10 minutes after you're done (aka ask you some questions)

I also reserve the right to reject papers that are too broad, too narrow, too old, or too out of focus for this course

Aside on your final presentation

Note 0: It's not first-come/first-served, but instead we will flip coins or play rock-paper-scissors for who gets which topic.

Note 1: If you dropped by my office to discuss the paper, that does not count as fulfilling your homework assignment. Please sent it to me by email

Note 2: This really counts as a homework assignment. So don't miss the deadline! If you do, you get points off - and I get to pick a topic for you :)

Aside on your final presentation

Note 3: Only one topic per person, so you might want to have 1-2 backups in mind (class is small so maybe that's OK). I'll let you know class after it's due where we have duplicates (and alternates due then class after that)

Note 4: You are not covering an "experiment", but rather a single analysis/limit/measurement. So I want you to have the paper you will be reading in mind. If there is no physics result, then this does not count