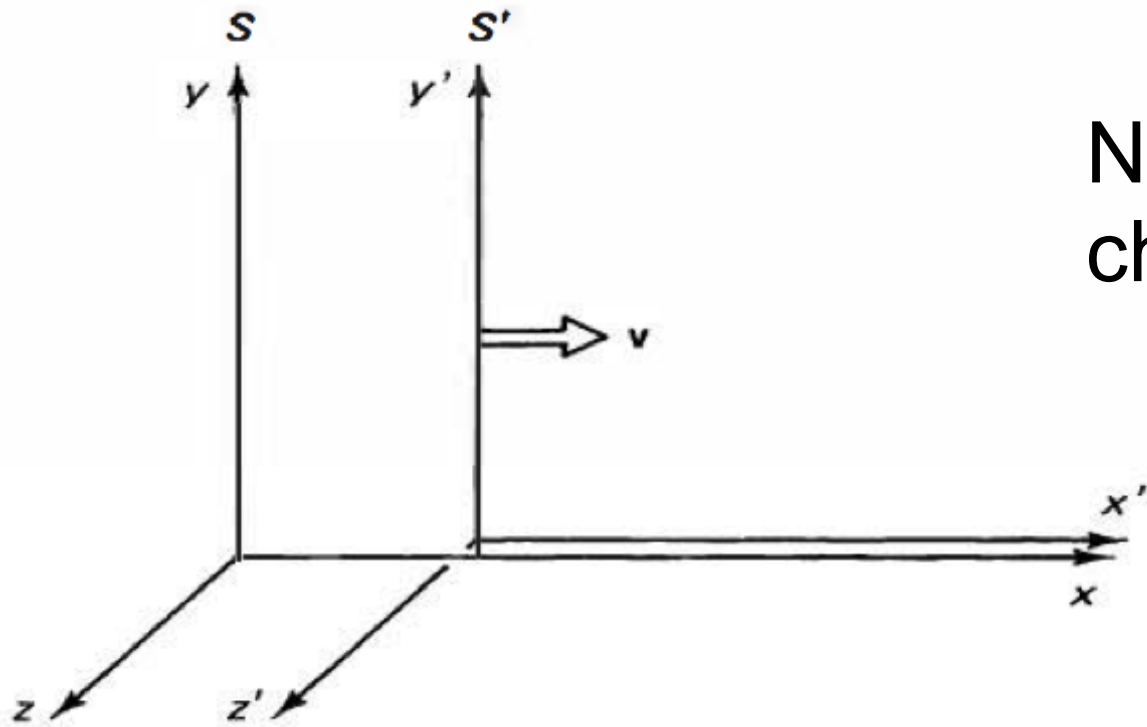


Just a brief review at this stage before we delve in



Note simplicity from  
choice of units!

$$x' = \gamma (x - vt)$$

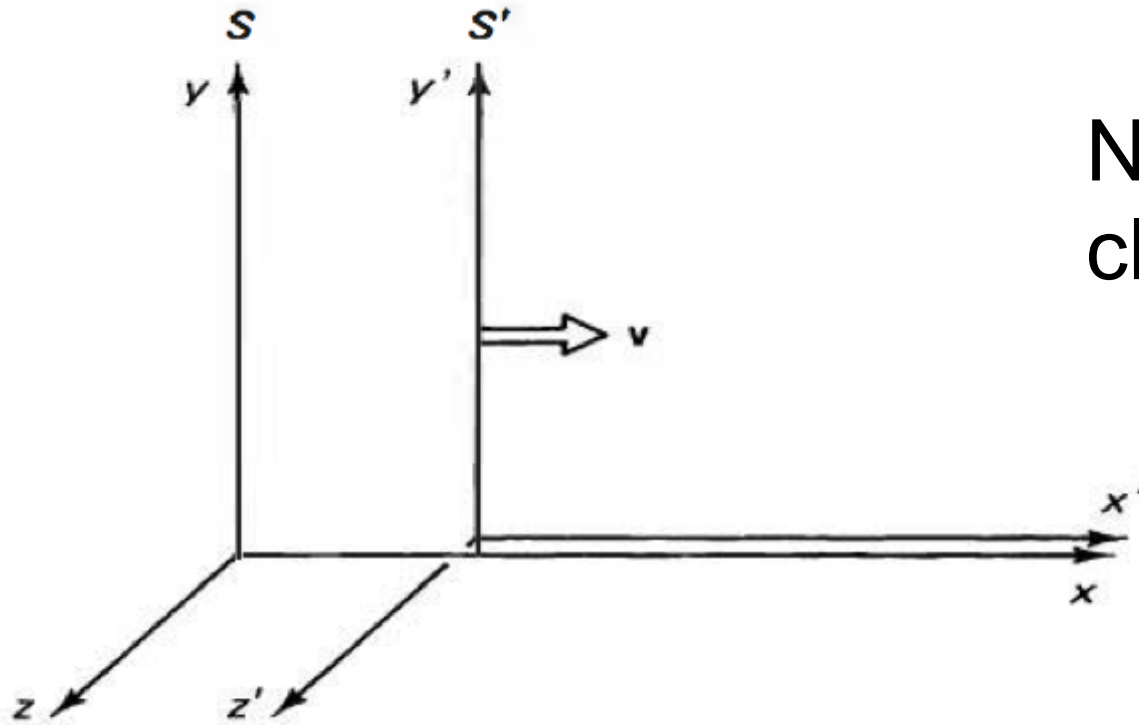
$$\gamma' = \gamma$$

$$z' = z, y' = y$$

$$t' = \gamma (t - vx)$$

$$\gamma = (1 - v^2)^{-1/2}$$

# Inverse transformation



Note simplicity from choice of units!

$$\gamma = (1 - v^2)^{-1/2}$$

$$x = \gamma (x' + vt')$$

$$\gamma = \gamma'$$

$$z = z', y = y'$$

$$t = \gamma (t' + vx')$$

# Consequences

$$t'_A = t'_B + \gamma v (x_B - x_A)$$

Cannot define simultaneity in all reference frames

$$L = \frac{L'}{\gamma}$$

Lorentz Contraction  
(objects shrink)

$$t = \gamma t'$$

Time dilation (moving clocks run slow)

$$u = \frac{u' + v}{1 + u'v}$$

Velocity addition is not as simple as before

What happens if  $u' = \text{speed of light}$  (which is ... what in our units?)

Are folks familiar with four-vector notation?

Get used to them :)

$$x^0 = t$$

$$x^1 = x$$

$$x^2 = y$$

$$x^3 = z$$

All we've done here is label the 0th component of space-time as time, and then the normal (x,y,z)

Note that x in the 4-vector is standard notation, but it is **not** the position x

$$x^{0'} = \gamma (x^0 - vx^1)$$

$$x^{1'} = \gamma (x^1 - vx^0)$$

$$x^{2'} = x^2$$

$$x^{3'} = x^3$$

Introduces a nice symmetry to the transformations, which leads us to re-writing it as...

$$x^{\mu'} = \sum_{\nu=0}^3 \Lambda_{\nu}^{\mu} x^{\nu}$$

Where  $\Lambda$  is a 4x4 matrix. Let's get the coordinates of the matrix on the board...



Hopefully we got this correct :)

$$\Lambda = \begin{bmatrix} \gamma & -\gamma v & 0 & 0 \\ -\gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The symmetry really stands out here

$$x^{\mu'} = \sum_{\nu=0}^3 \Lambda_{\nu}^{\mu} x^{\nu} = \Lambda_{\nu}^{\mu} x^{\nu}$$

Summation notation will be used a LOT

The second form has the sum implicit, and is shorthand to save smart (read: lazy) physicists from typing and writing

Define this **invariant**  $I$ . How does it transform? What is  $I'$  ?

$$I = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2$$

$$I' = (x^{0'})^2 - (x^{1'})^2 - (x^{2'})^2 - (x^{3'})^2$$

$$I' = (x^{0'})^2 - (x^{1'})^2 - (x^2)^2 - (x^3)^2$$

$$I' = \gamma^2 (x^0 - vx^1)^2 - \gamma^2 (x^1 - vx^0)^2 - (x^2)^2 - (x^3)^2$$

$$I' = \gamma^2 ((x^0)^2 - (x^1)^2 + v^2(x^1)^2 - v^2(x^0)^2 - 2vx^0x^1 + 2vx^0x^1) - (x^2)^2 - (x^3)^2$$

$$I' = \gamma^2 ((x^0)^2 (1 - v^2) - (x^1)^2 (1 - v^2)) - (x^2)^2 - (x^3)^2$$

Now use  $\gamma^2 = \frac{1}{1 - v^2}$

$$I = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2$$

$$I' = \gamma^2 ((x^0)^2 (1 - v^2) - (x^1)^2 (1 - v^2)) - (x^2)^2 - (x^3)^2 = I$$

Now we see why we called  $I$   
an **invariant**.

**Define:**


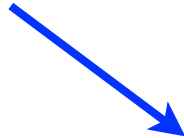
$$g = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

**Rewrite:**

$$I = \sum_{\mu=0}^3 \sum_{\nu=0}^3 g_{\mu\nu} x^\mu x^\nu = g_{\mu\nu} x^\mu x^\nu$$

covariant  
vector (down)

contravariant  
vector (up)


$$x_{\mu} = g_{\mu\nu} x^{\nu}$$

$$I = x_{\mu} x^{\mu} = x_{\nu} x^{\nu}$$

$$x_0 = x^0$$

$$x_1 = -x^1$$

$$x_2 = -x^2$$

$$x_3 = -x^3$$

The names are not quite as important as remembering that they are different (and that it's easy to convert from one to another)

$$a^\mu b_\mu = a_\mu b^\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$

Four-vector version of the dot product (which is also an invariant but only for three spatial dimensions)

$$a^\mu b_\mu = a_\mu b^\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$

Four-vector version of the dot product (which is also an invariant but only for three spatial dimensions)

$$a^\mu b_\mu = a^0 b^0 - \vec{a} \cdot \vec{b}$$

$$a^2 = a^\mu a_\mu = a^0 a^0 - \vec{a}^2$$

$$a^2 = a^\mu a_\mu = a^0 a^0 - \vec{a}^2$$

$a^2 > 0$  time-like

$a^2 = 0$  light-like

$a^2 < 0$  space-like (like normal spatial 3-vectors)

# Let's look at conservation of momentum, classically



Conservation of momentum:

$$m_A v_A + m_B v_B = m_C v_C + m_D v_D$$

Now imagine a frame moving with constant velocity  $V$  with respect to this frame

$$v'_A = v_A - V$$

$$v'_B = v_B - V$$

$$v'_C = v_C - V$$

$$v'_D = v_D - V$$



# Let's look at conservation of momentum

Now imagine a frame moving with constant velocity  $V$  with respect to this frame

$$m_A v'_A + m_B v'_B = m_C v'_C + m_D v'_D$$

$$v'_A = v_A - V$$

$$v'_B = v_B - V$$

$$v'_C = v_C - V$$

$$v'_D = v_D - V$$

$$m_A v_A + m_B v_B - V(m_A + m_B) = m_C v_C + m_D v_D - V(m_C + m_D)$$

And we know  $m_A v_A + m_B v_B = m_C v_C + m_D v_D$

$$V(m_A + m_B) = V(m_C + m_D)$$

$$m_A + m_B = m_C + m_D$$

Classical conservation of momentum (in all inertial reference frames) implies conservation of mass!

$$m_A + m_B = m_C + m_D$$

# Conservation of momentum in SR



Conservation of momentum:

$$m_A v_A + m_B v_B = m_C v_C + m_D v_D$$

Now imagine a frame moving with constant velocity  $V$  with respect to this frame. Note the use of new velocity addition formula!

$$v'_A = \frac{v_A - V}{1 - v_A V}$$

$$v'_B = \frac{v_B - V}{1 - v_B V}$$

$$v'_C = \frac{v_C - V}{1 - v_C V}$$

$$v'_D = \frac{v_D - V}{1 - v_D V}$$

# Conservation of momentum in SR

$$m_A v_A + m_B v_B = m_C v_C + m_D v_D$$

$$m_A v'_A + m_B v'_B = m_C v'_C + m_D v'_D$$

$$v'_A = \frac{v_A - V}{1 - v_A V}$$

$$v'_B = \frac{v_B - V}{1 - v_B V}$$

$$v'_C = \frac{v_C - V}{1 - v_C V}$$

$$v'_D = \frac{v_D - V}{1 - v_D V}$$

$$m_A \frac{v_A - V}{1 - v_A V} + m_B \frac{v_B - V}{1 - v_B V} = m_C \frac{v_C - V}{1 - v_C V} + m_D \frac{v_D - V}{1 - v_D V}$$

Velocities no longer nicely cancel, so we're stuck in the relativistic case! No longer have proof of mass conservation

# Let's rethink conservation of momentum

Nominal definition of momentum  $p = m \frac{dx}{dt}$

Not surprising that we run into trouble - both numerator and denominator are not Lorentz invariant! Let's make an ad hoc try and replace time with **proper time**

$$\tau = t/\gamma$$

$$p = m \frac{dx}{d\tau} = m \frac{dx}{d(t/\gamma)} = \gamma m \frac{dx}{dt}$$

# Conservation momentum with new definition

First frame

$$\frac{m_A v_A}{\sqrt{1 - v_A^2}} + \frac{m_B v_B}{\sqrt{1 - v_B^2}} = \frac{m_C v_C}{\sqrt{1 - v_C^2}} + \frac{m_D v_D}{\sqrt{1 - v_D^2}}$$

New frame

$$\frac{m_A v'_A}{\sqrt{1 - (v'_A)^2}} + \frac{m_B v'_B}{\sqrt{1 - (v'_B)^2}} = \frac{m_C v'_C}{\sqrt{1 - (v'_C)^2}} + \frac{m_D v'_D}{\sqrt{1 - (v'_D)^2}}$$

$$v'_A = \frac{v_A - V}{1 - v_A V}$$

$$v'_B = \frac{v_B - V}{1 - v_B V}$$

$$v'_C = \frac{v_C - V}{1 - v_C V}$$

$$v'_D = \frac{v_D - V}{1 - v_D V}$$

# Some algebra in new frame

$$\frac{m_A v'_A}{\sqrt{1 - (v'_A)^2}} + \frac{m_B v'_B}{\sqrt{1 - (v'_B)^2}} = \frac{m_C v'_C}{\sqrt{1 - (v'_C)^2}} + \frac{m_D v'_D}{\sqrt{1 - (v'_D)^2}}$$

Let's look at first term:

$$\begin{aligned} & \frac{m_A \left( \frac{v_A - V}{1 - v_A V} \right)}{\sqrt{1 - \left( \frac{v_A - V}{1 - v_A V} \right)^2}} = \\ & \frac{m_A \left( \frac{v_A - V}{1 - v_A V} \right)}{\sqrt{\frac{(1 - v_A V)^2 - (v_A - V)^2}{(1 - v_A V)^2}}} = \\ & \frac{m_A (v_A - V)}{\sqrt{(1 - v_A V)^2 - (v_A - V)^2}} = \\ & \frac{m_A (v_A - V)}{\sqrt{1 + v_A^2 V^2 - 2v_A V - v_A^2 - V^2 + 2v_A V}} = \\ & \frac{m_A (v_A - V)}{\sqrt{1 + v_A^2 V^2 - v_A^2 - V^2}} \end{aligned}$$

$$v'_A = \frac{v_A - V}{1 - v_A V}$$

$$v'_B = \frac{v_B - V}{1 - v_B V}$$

$$v'_C = \frac{v_C - V}{1 - v_C V}$$

$$v'_D = \frac{v_D - V}{1 - v_D V}$$

# Some algebra in new frame

$$\frac{m_A v'_A}{\sqrt{1 - (v'_A)^2}} + \frac{m_B v'_B}{\sqrt{1 - (v'_B)^2}} = \frac{m_C v'_C}{\sqrt{1 - (v'_C)^2}} + \frac{m_D v'_D}{\sqrt{1 - (v'_D)^2}}$$

Let's look at first term:

$$\frac{m_A \left( \frac{v_A - V}{1 - v_A V} \right)}{\sqrt{1 - \left( \frac{v_A - V}{1 - v_A V} \right)^2}} =$$

$$\frac{m_A (v_A - V)}{\sqrt{1 + v_A^2 V^2 - v_A^2 - V^2}} =$$

$$\frac{m_A v_A}{\sqrt{(1 - v_A^2)(1 - V^2)}} - \frac{m_A V}{\sqrt{(1 - v_A^2)(1 - V^2)}} =$$

$$\frac{m_A v_A}{\sqrt{1 - v_A^2}} \frac{1}{\sqrt{1 - V^2}} - \frac{m_A}{\sqrt{1 - v_A^2}} \frac{V}{\sqrt{1 - V^2}} =$$

$$v'_A = \frac{v_A - V}{1 - v_A V}$$

$$v'_B = \frac{v_B - V}{1 - v_B V}$$

$$v'_C = \frac{v_C - V}{1 - v_C V}$$

$$v'_D = \frac{v_D - V}{1 - v_D V}$$



# Some algebra in new frame

$$\frac{m_A v'_A}{\sqrt{1 - (v'_A)^2}} + \frac{m_B v'_B}{\sqrt{1 - (v'_B)^2}} = \frac{m_C v'_C}{\sqrt{1 - (v'_C)^2}} + \frac{m_D v'_D}{\sqrt{1 - (v'_D)^2}}$$

First term =  $\frac{m_A v_A}{\sqrt{1 - v_A^2}} \frac{1}{\sqrt{1 - V^2}} - \frac{m_A}{\sqrt{1 - v_A^2}} \frac{V}{\sqrt{1 - V^2}}$

So...

$$\begin{aligned} & \frac{m_A v_A}{\sqrt{1 - v_A^2}} \frac{1}{\sqrt{1 - V^2}} - \frac{m_A}{\sqrt{1 - v_A^2}} \frac{V}{\sqrt{1 - V^2}} + \\ & \frac{m_B v_B}{\sqrt{1 - v_B^2}} \frac{1}{\sqrt{1 - V^2}} - \frac{m_B}{\sqrt{1 - v_B^2}} \frac{V}{\sqrt{1 - V^2}} = \\ & \frac{m_C v_C}{\sqrt{1 - v_C^2}} \frac{1}{\sqrt{1 - V^2}} - \frac{m_C}{\sqrt{1 - v_C^2}} \frac{V}{\sqrt{1 - V^2}} + \\ & \frac{m_D v_D}{\sqrt{1 - v_D^2}} \frac{1}{\sqrt{1 - V^2}} - \frac{m_D}{\sqrt{1 - v_D^2}} \frac{V}{\sqrt{1 - V^2}} \end{aligned}$$

But from original frame

$$\frac{m_A v_A}{\sqrt{1 - v_A^2}} + \frac{m_B v_B}{\sqrt{1 - v_B^2}} = \frac{m_C v_C}{\sqrt{1 - v_C^2}} + \frac{m_D v_D}{\sqrt{1 - v_D^2}}$$

Thus we get

$$\frac{m_A}{\sqrt{1-v_A^2}} \frac{V}{\sqrt{1-V^2}} + \frac{m_B}{\sqrt{1-v_B^2}} \frac{V}{\sqrt{1-V^2}} =$$

$$\frac{m_C}{\sqrt{1-v_C^2}} \frac{V}{\sqrt{1-V^2}} + \frac{m_D}{\sqrt{1-v_D^2}} \frac{V}{\sqrt{1-V^2}}$$

Cancel the common term...

$$\frac{m_A}{\sqrt{1-v_A^2}} + \frac{m_B}{\sqrt{1-v_B^2}} =$$

$$\frac{m_C}{\sqrt{1-v_C^2}} + \frac{m_D}{\sqrt{1-v_D^2}}$$

This quantity  
must be  
conserved then!  
But what is it?

$$\frac{m}{\sqrt{1 - v^2}} = m (1 - v^2)^{-1/2} \sim m(1 + v^2/2)$$

So this is a statement of the conservation of energy! At small velocities, energy is equal to the rest mass plus the kinematic energy (note that when  $v = 0$  the energy is not zero, but  $m$ !)

$$\frac{m}{\sqrt{1 - v^2}} = m (1 - v^2)^{-1/2} \sim m(1 + v^2/2)$$

So this is a statement of the conservation of energy! At small velocities, energy is equal to the rest mass plus the kinematic energy

This way,  $p^2$  is invariant, as it must be

$$p^\mu = (E, p_x, p_y, p_z)$$

$$p^\mu p_\mu = E^2 - \mathbf{p}^2 = m^2$$

For  $m = 0$  (ex for: photons),  $|\mathbf{p}|=E$

$$p^\mu = (E, p_x, p_y, p_z)$$

$$p^\mu p_\mu = E^2 - \mathbf{p}^2 = m^2$$

$$E = \gamma m$$

$$\vec{p} = \gamma m \vec{v}$$

$$\rightarrow \boxed{(\vec{p}/E) = \vec{v}}$$

As expected, for light,  
 $|\mathbf{v}| = 1$  and  $|\mathbf{p}| = E$

We just saw that energy and momentum (correctly defined) are conserved. For process  $A+B \rightarrow C+D$ :

$$E_A + E_B = E_C + E_D$$

$$\vec{p}_A + \vec{p}_B = \vec{p}_C + \vec{p}_D$$

Conveniently written as:  $p_A^\mu + p_B^\mu = p_C^\mu + p_D^\mu$

Mass is clearly not conserved:

$$W^+ \rightarrow e^+ + \nu_e$$

## What about a particle decay?

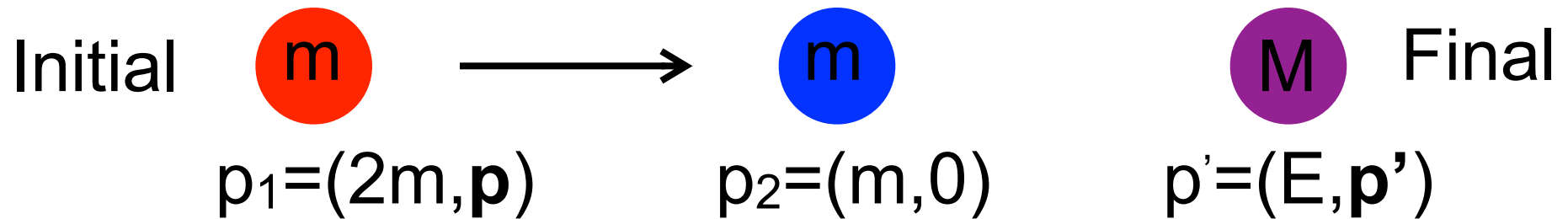
$$A \rightarrow B + C$$

$$A_\mu A^\mu = m_A^2 \rightarrow (B + C)_\mu (B + C)^\mu = (E_B + E_C)^2 - (\mathbf{p}_B + \mathbf{p}_C)^2$$

Lets us estimate the mass of a parent particle from its decay products. Conservation of energy also tells us:

$$m_A = \sqrt{m_B^2 + \mathbf{p}_B^2} + \sqrt{m_C^2 + \mathbf{p}_C^2}$$

Particle decay products cannot be more massive than mass of original particle!  
(Remember - proton stability...)

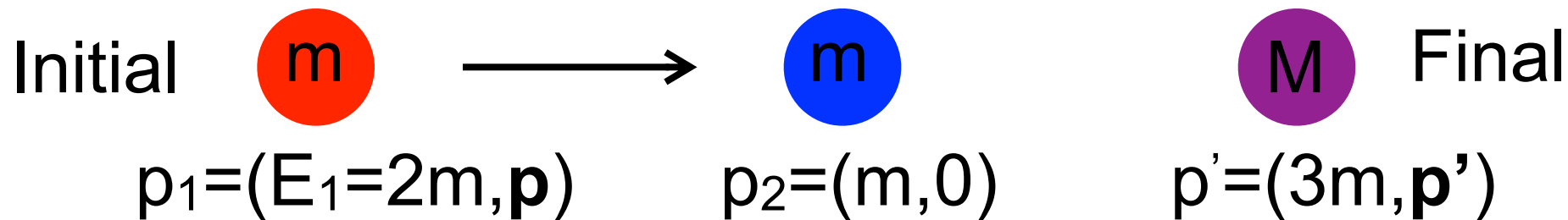


Particle with total energy =  $2m$  collides with an identical particle at rest. They stick together. What is the mass of the resulting lump? What is its velocity?

$$\text{Conservation of } E: 2m + m = 3m = E$$



# One fun example



$$E_1^2 - \mathbf{p}^2 = m^2$$

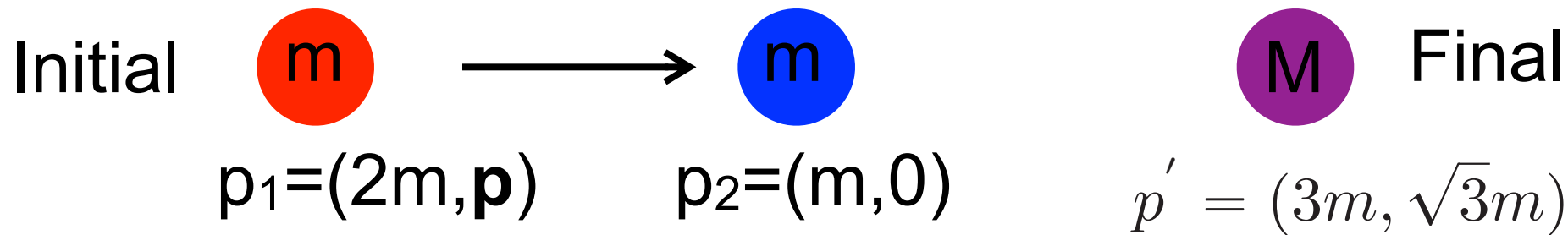
$$(2m^2) - m^2 = \mathbf{p}^2$$

$$\mathbf{p}^2 = 3m^2$$

$$|\mathbf{p}| = \sqrt{3}m$$

$$|\mathbf{p}| = |\mathbf{p}'| = \sqrt{3}m$$

# One fun example



$$E_1^2 - \mathbf{p}^2 = m^2$$

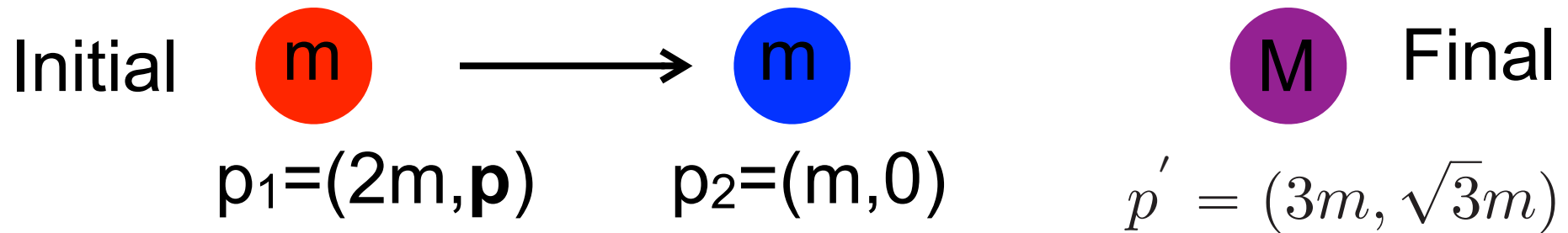
$$(2m)^2 - m^2 = \mathbf{p}^2$$

$$\mathbf{p}^2 = 3m^2$$

$$|\mathbf{p}| = \sqrt{3}m$$

$$|\mathbf{p}| = |\mathbf{p}'| = \sqrt{3}m$$

# One fun example



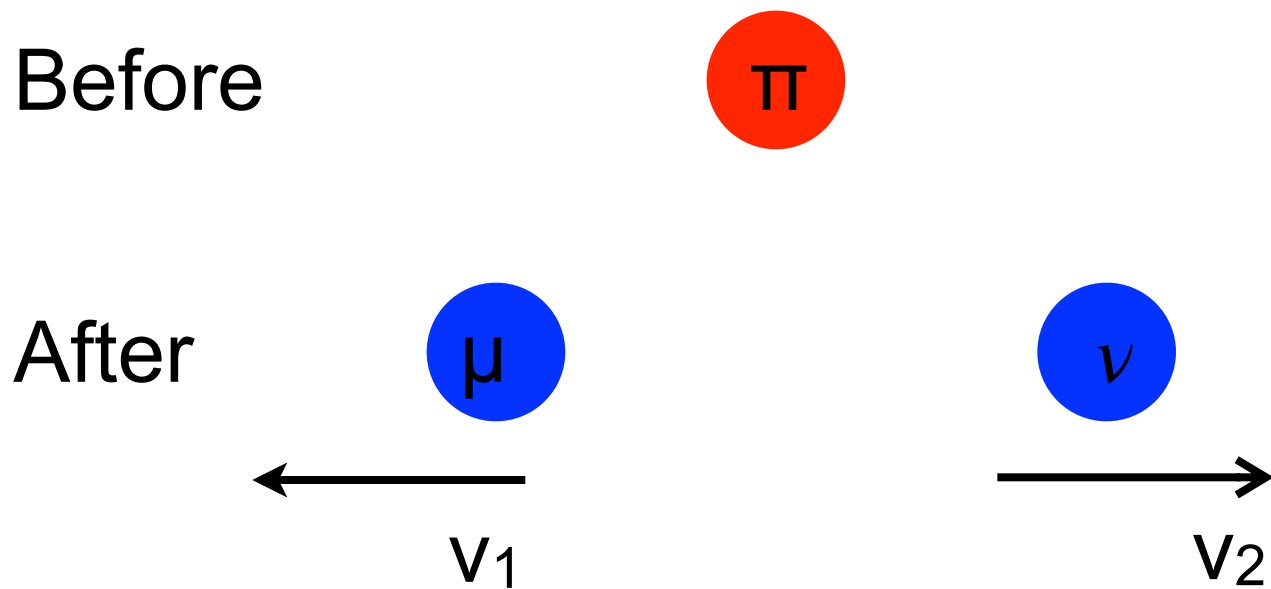
$$M^2 = E^2 - (\mathbf{p}')^2 = 9m^2 - 3m^2$$

$$M = \sqrt{6}m$$

Total mass not  
conserved

$$|v| = \frac{|\mathbf{p}|}{E} = \frac{\sqrt{3}m}{3m} = \frac{1}{\sqrt{3}}$$

A pion at rest decays into a muon and a neutrino. What is the speed of the muon?

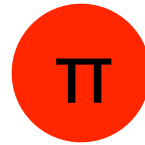


Conservation of momentum:  $\mathbf{p}_\mu = -\mathbf{p}_\nu$

Conservation of energy:  $E_\mu + E_\nu = E_\pi$

So muon and  
neutrino travel in  
opposite directions  
("back to back")

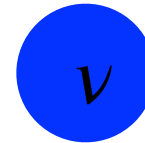
Before



After



$v_1$



$v_2$

Apply conservation of energy:

$$E_{\mu} + E_{\nu} = E_{\pi}$$

$$\sqrt{m_{\mu}^2 + \vec{p}_{\mu}^2} + \sqrt{m_{\nu}^2 + \vec{p}_{\nu}^2} = \sqrt{m_{\pi}^2 + \vec{p}_{\pi}^2}$$

$$\sqrt{m_{\mu}^2 + \vec{p}_{\mu}^2} + |\vec{p}_{\nu}| = m_{\pi}$$

Pion was at rest

Neutrino mass  $\sim 0$

Apply conservation of energy:

$$\sqrt{m_\mu^2 + \vec{p}_\mu^2} + |\vec{p}_\nu| = m_\pi$$

$$\sqrt{m_\mu^2 + \vec{p}_\mu^2} = m_\pi - |\vec{p}_\nu|$$

$$m_\mu^2 + \vec{p}_\mu^2 = m_\pi^2 + |\vec{p}_\nu|^2 - 2m_\pi |\vec{p}_\nu|$$

Equal

$$m_\mu^2 = m_\pi^2 - 2m_\pi |\vec{p}_\nu|$$

$$m_\mu^2 = m_\pi^2 - 2m_\pi |\vec{p}_\mu|$$

$$|\vec{p}_\mu| = \frac{m_\pi^2 - m_\mu^2}{2m_\pi}$$

Equal

## Let's follow Griffiths Ex 3.3

$$|\vec{p}_\mu| = \frac{m_\pi^2 - m_\mu^2}{2m_\pi}$$

$$E_\mu^2 = \vec{p}_\mu^2 + m_\mu^2 = \frac{(m_\pi^2 - m_\mu^2)^2}{4m_\pi^2} + m_\mu^2$$

$$|v| = |p|/E$$

$$E_\mu^2 = \frac{m_\pi^4 + m_\mu^4 - 2m_\pi^2 m_\mu^2 + 4m_\pi^2 m_\mu^2}{4m_\pi^2}$$

$$E_\mu^2 = \frac{m_\pi^4 + m_\mu^4 + 2m_\pi^2 m_\mu^2}{4m_\pi^2}$$

$$|v_\mu| = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2}$$

$$E_\mu^2 = \frac{(m_\pi^2 + m_\mu^2)^2}{4m_\pi^2}$$

$$E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi}$$



Pion mass  $\sim 135$  MeV  
Muon mass  $\sim 106$  MeV

$$|v_\mu| = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2}$$

$v \sim 0.25$   
(25% of the speed of light)

Let's try using four-vector notation (remember that  $p^2=m^2$  but be careful that  $p$  here is the four-vector  $p$ )

# Let's follow Griffiths Ex 3.3 (continued)

$$p_\pi = p_\mu + p_\nu$$

$$p_\nu = p_\pi - p_\mu$$

$$p_\nu^2 = (p_\pi - p_\mu)^2 = m_\nu^2 = 0$$

$$0 = p_\pi^2 + p_\mu^2 - 2p_\pi p_\mu$$

$$0 = m_\pi^2 + m_\mu^2 - 2p_\pi p_\mu$$

$$0 = m_\pi^2 + m_\mu^2 - 2m_\pi E_\mu$$

$$E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi}$$

What is  $p_\pi p_\mu$ ? The pion momentum is zero, so only first component (energy) contributes, and so  $p_\pi p_\mu = E_\pi E_\mu$

Also, since pion was at rest,  $E_\pi = m_\pi$  so  $p_\pi p_\mu = m_\pi E_\mu$

# Let's follow Griffiths Ex 3.3 (continued)

$$p_\pi = p_\mu + p_\nu$$

$$p_\mu = p_\pi - p_\nu$$

$$p_\mu^2 = (p_\pi - p_\nu)^2 = m_\mu^2$$

$$p_\mu^2 = p_\pi^2 + p_\nu^2 - 2p_\pi p_\nu$$

$$m_\mu^2 = m_\pi^2 - 2m_\pi |p_\mu|$$

$$|p_\mu| = \frac{m_\pi^2 - m_\mu^2}{2m_\pi}$$

What is  $p_\pi p_\nu$ ? The pion momentum is zero, so only first component (energy) contributes, and so  $p_\pi p_\nu = E_\pi E_\nu$

Since pion was at rest,  $E_\pi = m_\pi$ . What is  $E_\nu$ ? It is the magnitude of the neutrino momentum, but that is the same as the magnitude of the muon momentum

$pp \rightarrow ppp\bar{p}$ 

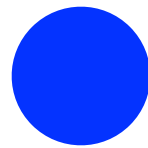
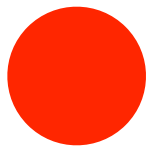
What is the threshold energy for this process?

At threshold, all four final state objects are at rest (if they were not at rest we could reduce the CoM collision energy and reduce their momentum)

Invariant:

$$(p_1 + p_2)^2 = (E + m)^2 - \mathbf{p}^2$$

Initial



$$p_1 = (E, \mathbf{p})$$

$$p_2 = (m, 0)$$

**Invariant:**

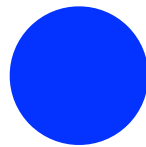
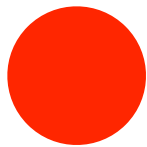
$$(p_1 + p_2)^2 = (E + m)^2 - \mathbf{p}^2$$

$$(p_3 + p_4 + p_5 + p_6)^2 = (4m)^2 = (p_1 + p_2)^2$$

$$16m^2 = (E + m)^2 - \mathbf{p}^2$$

$$\mathbf{p}^2 = E^2 - m^2$$

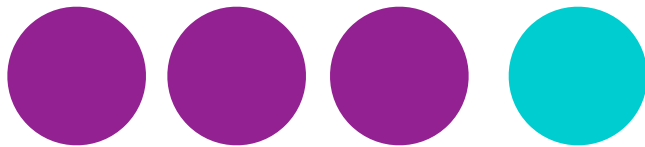
Initial



$$p_1 = (E, \mathbf{p})$$

$$p_2 = (m, 0)$$

Final



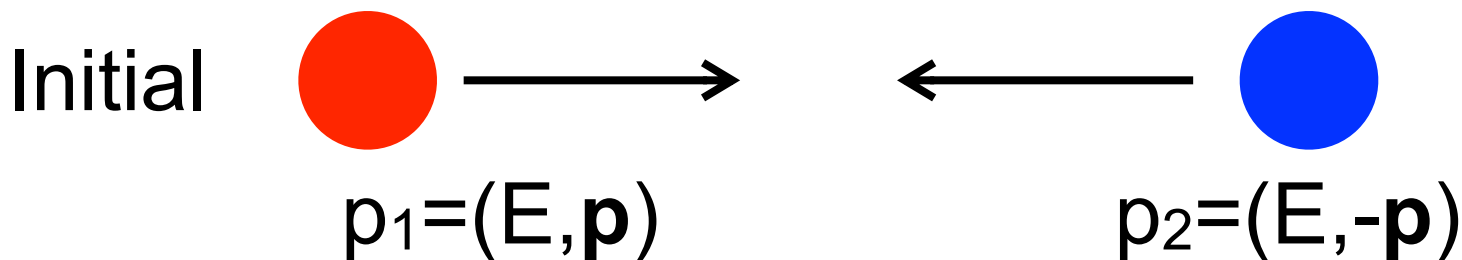
All at rest

$$16m^2 = (E + m)^2 - \mathbf{p}^2 \qquad \mathbf{p}^2 = E^2 - m^2$$

$$16m^2 = E^2 + 2mE + m^2 - E^2 + m^2$$

$$14m^2 = 2mE \rightarrow \boxed{E = 7m}$$

What if we didn't fire at a stationary target?



$$(p_1 + p_2)^2 = (2E)^2 - (\mathbf{p} + -\mathbf{p})^2 = 4E^2$$

$$16m^2 = 4E^2 \rightarrow E = 2m$$

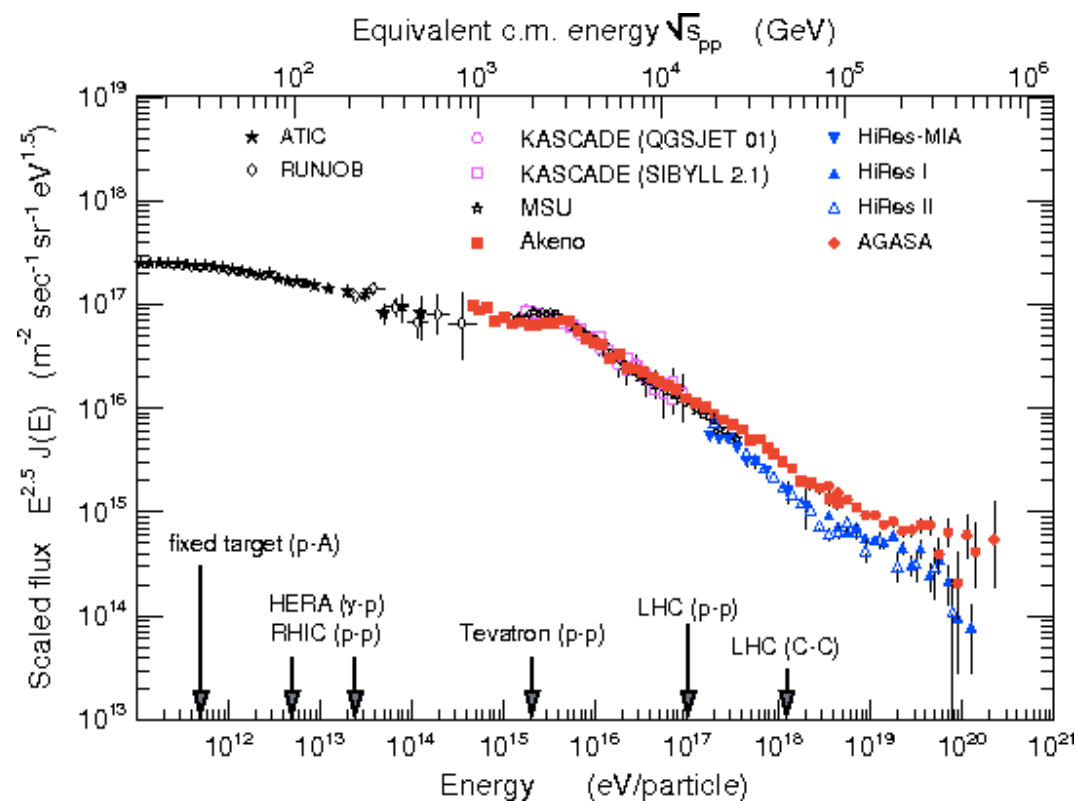
In this example, have to give protons a lot more kinetic energy for the fixed target version! Of course, the accelerator for the beams is much more complex :)



# GZK cutoff

Greisen–Zatsepin–Kuzmin: Over long enough distances, high-energy cosmic rays coming from the Universe should interact with the cosmic microwave background

[http://  
www2.astro.psu.edu  
u/users/nnp/cr.html](http://www2.astro.psu.edu/users/nnp/cr.html)



$$\gamma + p \rightarrow p\pi^0$$

$$p_\gamma + p_p = p_{p'} + p_{\pi^0}$$

$$(p_\gamma + p_p)^2 = (p_{p'} + p_{\pi^0})^2$$

$$p_\gamma^2 + p_p^2 + 2p_\gamma \cdot p_p = p_{p'}^2 + p_{\pi^0}^2 + 2p_{\pi^0} \cdot p_{p'}$$

$$m_p^2 + 2p_\gamma \cdot p_p = m_p^2 + m_{\pi^0}^2 + 2p_{\pi^0} \cdot p_{p'}$$

$$2p_\gamma \cdot p_p = m_{\pi^0}^2 + 2p_{\pi^0} \cdot p_{p'}$$

At **threshold**, pion and proton in final state are at rest and their momenta are zero (and energies equal to their masses)

$$2p_\gamma \cdot p_p = m_{\pi^0}^2 + 2m_{\pi^0}m_p$$

At threshold, have a head-on collision. Assume highly relativistic particles and we get...

$$2p_\gamma \cdot p_p = m_{\pi^0}^2 + 2m_{\pi^0}m_p$$

Head-on collision with energies  $\gg m_p$

$$p_\gamma = (E_\gamma, E_\gamma, 0, 0)$$

$$p_p = (E_p, -E_p, 0, 0)$$

$$2p_p \cdot p_\gamma = 2E_pE_\gamma - 2(-E_p)E_\gamma = 4E_pE_\gamma$$

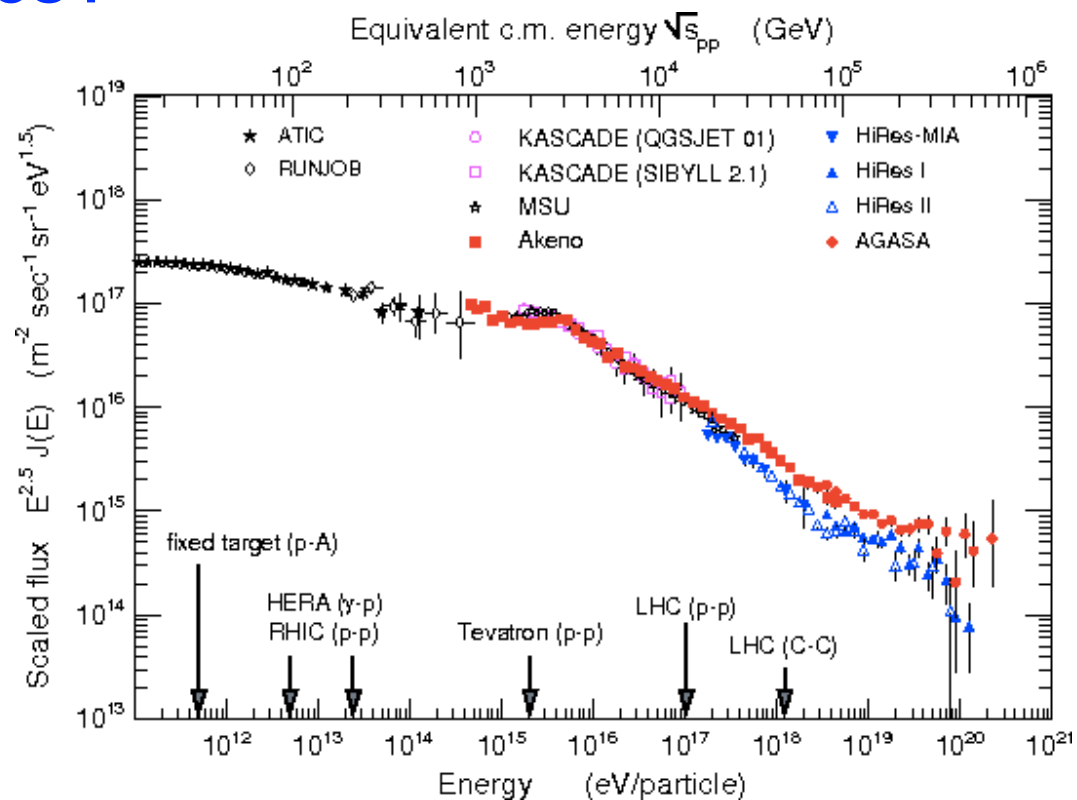
Want to solve for unknown cosmic rate proton energy. Photon energy is CMB ( $6 \times 10^{-4}$  eV). Plug in and we get  $\sim 2 \times 10^{20}$  eV

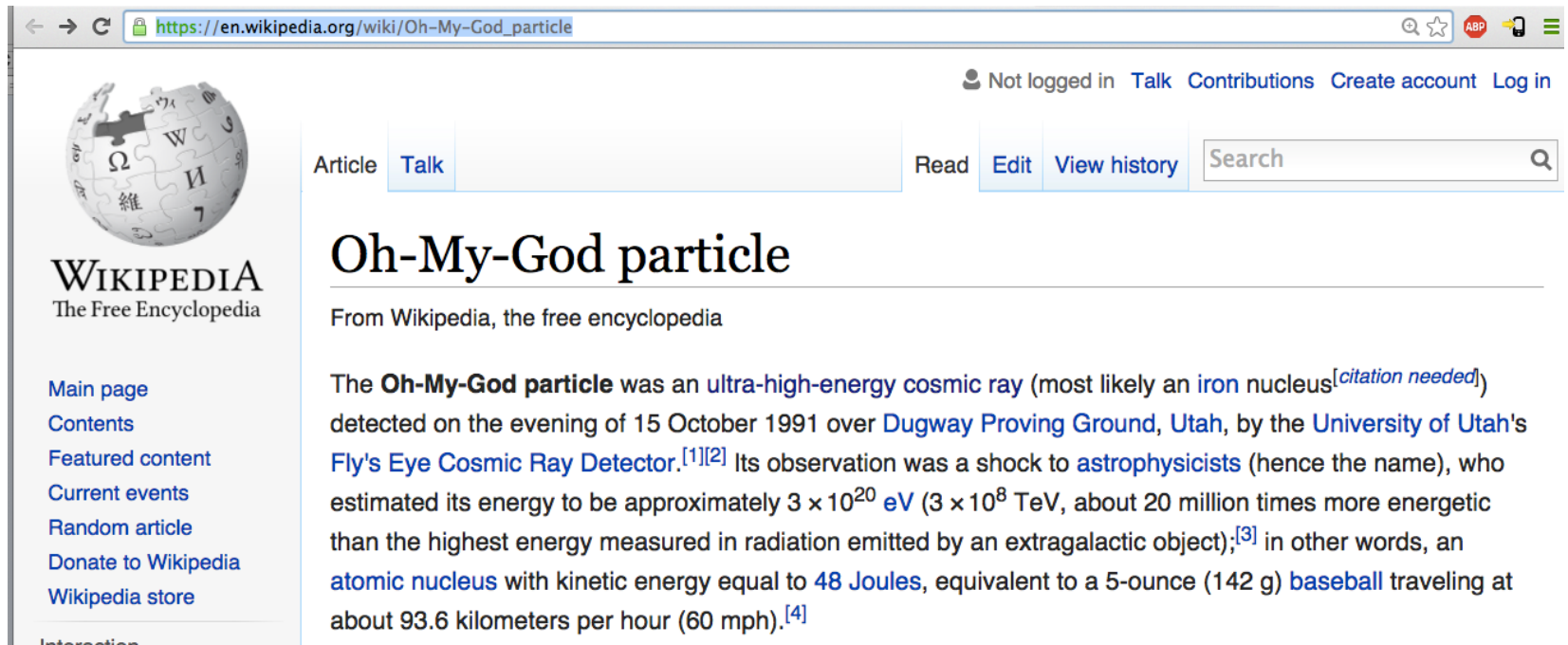
Similarly have  $\gamma + p \rightarrow n\pi^+$

# GZK cutoff

Of course, we used an “average” CMB photon, and ignored a full calculation of the kinematics (which goes through delta resonances) and assumed protons. Nevertheless, distribution doesn't go to zero! Where are these sources coming from? Nearby sources? Or new physics?

[http://  
www2.astro.psu.ed  
u/users/nnp/cr.html](http://www2.astro.psu.edu/users/nnp/cr.html)





The image shows a screenshot of a web browser displaying the Wikipedia article for "Oh-My-God particle". The browser's address bar shows the URL [https://en.wikipedia.org/wiki/Oh-My-God\\_particle](https://en.wikipedia.org/wiki/Oh-My-God_particle). The page layout includes the Wikipedia logo on the left, a navigation menu with links like "Main page", "Contents", and "Featured content", and a main content area. The article title "Oh-My-God particle" is prominently displayed, followed by the subtitle "From Wikipedia, the free encyclopedia". The main text describes the discovery of an ultra-high-energy cosmic ray on October 15, 1991, over the Dugway Proving Ground in Utah, detected by the University of Utah's Fly's Eye Cosmic Ray Detector. It notes that the observation was a shock to astrophysicists and provides details on the particle's energy, estimated at  $3 \times 10^{20}$  eV (3 x 10<sup>8</sup> TeV), which is 20 million times more energetic than the highest energy measured in radiation emitted by an extragalactic object. The particle is described as an atomic nucleus with kinetic energy equal to 48 Joules, equivalent to a 5-ounce (142 g) baseball traveling at about 93.6 kilometers per hour (60 mph).

Article [Talk](#)

[Read](#) [Edit](#) [View history](#)

## Oh-My-God particle

From Wikipedia, the free encyclopedia

The **Oh-My-God particle** was an ultra-high-energy cosmic ray (most likely an iron nucleus<sup>[*citation needed*]</sup>) detected on the evening of 15 October 1991 over [Dugway Proving Ground, Utah](#), by the [University of Utah's Fly's Eye Cosmic Ray Detector](#).<sup>[1][2]</sup> Its observation was a shock to [astrophysicists](#) (hence the name), who estimated its energy to be approximately  $3 \times 10^{20}$  eV ( $3 \times 10^8$  TeV, about 20 million times more energetic than the highest energy measured in radiation emitted by an extragalactic object);<sup>[3]</sup> in other words, an [atomic nucleus](#) with kinetic energy equal to **48 Joules**, equivalent to a 5-ounce (142 g) [baseball](#) traveling at about 93.6 kilometers per hour (60 mph).<sup>[4]</sup>

## And on that note, some homework

Griffiths 3.3, 3.4, 3.6, 3.15, 3.16, 3.17, 3.25 (good for you to start getting familiar with the PDG, I suspect)

Also: At the HERA collider, 27.5 GeV electrons collided head-on with 920 GeV protons. What was the center of mass energy?