# A brief detour to Lagrangians



You've all seen this before, hopefully more than a few times

$$\mathcal{L} = T - U$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} = \frac{\partial \mathcal{L}}{\partial q}$$

Now we are dealing with fields! Which are themselves functions of x,y,z .. and also t. Define a "Lagrangian density" that is a function of these fields and their derivatives:

Does analogy make sense?

$$\partial_\mu \phi_i = \frac{\partial \phi_i}{\partial x^\mu}$$

$$\partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \right) = \frac{\partial \mathcal{L}}{\partial \phi_i}$$

# Relating Lagrangian density to more familiar Lagrangian

$$\mathcal{L} = T - U$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} = \frac{\partial \mathcal{L}}{\partial q}$$

$$L = \int \mathcal{L}\ d^3\mathbf{x}$$

Lagrangian density: generalized coordinates $q_i$ are replaced by the field themselves, and time derivates $dq_i/dt$ are replaced by derivatives of the fields with respect to each space-time coordinate

# Simplest Lagrangian (Klein-Gordon for scalar)



$$\partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \right) = \frac{\partial \mathcal{L}}{\partial \phi_i}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2$$

$$\mathcal{L} = \frac{1}{2}[\partial_0 \phi \partial_0 \phi - \partial_1 \phi \partial_1 \phi - \partial_2 \phi \partial_2 \phi - \partial_3 \phi \partial_3 \phi - m^2 \phi^2]$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_0 \phi)} = \partial_0 \phi = \partial^0 \phi$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_1 \phi)} = -\partial_1 \phi = \partial^1 \phi$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \partial^\mu \phi$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = -m^2 \phi$$

# Simplest Lagrangian (Klein-Gordon for scalar)



$$\partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \right) = \frac{\partial \mathcal{L}}{\partial \phi_i}$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \partial^\mu \phi \qquad \frac{\partial \mathcal{L}}{\partial \phi} = -m^2 \phi$$

$$\boxed{\partial_\mu \partial^\mu \phi = -m^2 \phi}$$

Klein-Gordon equation describing the field of a spin-0 particle with mass m

# Dirac Lagrangian for spinor field



$$\mathcal{L} = i\overline{\psi}\gamma^\mu\partial_\mu\psi - m^2\overline{\psi}\psi$$

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\overline{\psi})} = 0 \qquad \frac{\partial\mathcal{L}}{\partial\overline{\psi}} = i\gamma^\mu\partial_\mu\psi - m^2\psi$$

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)} = i\overline{\psi}\gamma^\mu \qquad \frac{\partial\mathcal{L}}{\partial\psi} = -m^2\overline{\psi}$$

Implies the Dirac equation!

$$i\gamma^\mu\partial_\mu\psi - m^2\psi = 0$$

$$i\gamma^\mu\partial_\mu\overline{\psi} + m^2\overline{\psi} = 0$$

# Proca Lagrangian for vector field



$$\mathcal{L} = \frac{-1}{16\pi}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{m^2}{8\pi} A^\nu A_\nu$$

Define $$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

And can work through algebra (we won't!) to get:

$$\partial_\mu F^{\mu\nu} + m^2 A^\nu = 0$$

Think back to classical mechanics and an object of mass m orbiting in a gravitational field produced by a second mass M

$$L = T - V = \frac{1}{2}mv^2 + \frac{GMm}{r^2}$$

$$L = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\phi}^2 + \frac{GMm}{r^2}$$

No Φ dependence (Lagrangian is **invariant** under the transformation Φ⟶ Φ+δΦ), so

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) = 0 \quad \longrightarrow \quad \frac{\partial L}{\partial \dot{\phi}} = J = mr^2\dot{\phi}$$ **constant**

# Noether's Theorem and symmetries

We like transformation that don't change our Lagrangian (we like symmetries!) If we can identify a symmetry, then we can say that something is conserved. But we need to expand what sorts of things we examine…

# Local Gauge invariance



$$\mathcal{L} = i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - m^2\overline{\psi}\psi$$

Global gauge invariance (Lagrangian doesn't change due to overall phase)

$$\psi \rightarrow e^{i\theta}\psi$$
$$\overline{\psi} \rightarrow e^{-i\theta}\overline{\psi}$$

**Look at the difference carefully here!**

What about local phase transformations?

$$\psi(x) \rightarrow e^{i\theta(x)}\psi(x)$$
$$\overline{\psi}(x) \rightarrow e^{-i\theta(x)}\overline{\psi}(x)$$

# Local Gauge invariance

$$\mathcal{L} = i\overline{\psi}\gamma^\mu\partial_\mu\psi - m^2\overline{\psi}\psi$$

$$\psi(x) \rightarrow e^{i\theta(x)}\psi(x)$$

$$\overline{\psi}(x) \rightarrow e^{-i\theta(x)}\overline{\psi}(x)$$

$$\mathcal{L}' = ie^{-i\phi(x)}\overline{\psi}\gamma^\mu\partial_\mu(e^{i\theta(x)}\psi) - m^2\overline{\psi}\psi$$

$$\partial_\mu(e^{i\theta(x)}\psi) = i[\partial_\mu\theta(x)]e^{i\theta(x)}\psi + e^{i\theta(x)}\partial_\mu\psi$$

$$\mathcal{L}' = i\overline{\psi}\gamma^\mu\partial_\mu\psi - m^2\overline{\psi}\psi - \overline{\psi}\gamma^\mu[\partial_\mu\theta]\psi$$

So Lagrangian not generally invariant under this transformation! But we can demand it

# Let's try and fix that!

Add a vector field A to the Lagrangian:

$$\mathcal{L} = i\overline{\psi}\gamma^\mu\partial_\mu\psi - m^2\overline{\psi}\psi - (q\overline{\psi}\gamma^\mu\psi)A_\mu$$

$$A_\mu \rightarrow A_\mu + \partial_\mu\lambda$$

$$\mathcal{L}' = i\overline{\psi}\gamma^\mu\partial_\mu\psi - m^2\overline{\psi}\psi - \overline{\psi}\gamma^\mu[\partial_\mu\theta]\psi - q\overline{\psi}\gamma^\mu\psi A_\mu - q\overline{\psi}\gamma^\mu\psi\partial_\mu\lambda$$

We want these to cancel

So ..

$$\lambda = \frac{-\theta}{q}$$

# But we need to add the free term for A

$$\mathcal{L} = i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - m^2\overline{\psi}\psi - (q\overline{\psi}\gamma^{\mu}\psi)A_{\mu} \qquad \lambda = \frac{-\theta}{q}$$

$$A_{\mu} \rightarrow A_{\mu} + \partial_{\mu}\lambda$$

$$\mathcal{L}^{\mathcal{A}} = \frac{-1}{16\pi}(\partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu})(\partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}) + \frac{m^2}{8\pi}A^{\nu}A_{\nu}$$

How do these new terms transform under our gauge transformation?

$$\mathcal{L}^{\mathcal{A}'} = \frac{-1}{16\pi}\left[\partial^{\mu}(A^{\nu} + \partial^{\nu}\lambda) - \partial^{\nu}(A^{\mu} + \partial^{\mu}\lambda)(\partial_{\mu}(A_{\nu} + \partial_{\nu}\lambda) - \partial_{\nu}(A_{\mu} + \partial_{\mu}\lambda)\right] + \frac{m^2}{8\pi}(A^{\nu} + \partial^{\nu}\lambda)(A_{\nu} + \partial_{\nu}\lambda)$$

# How are the new pieces of our Lagrangian transforming?

$$\mathcal{L}^{\mathcal{A}} = \frac{-1}{16\pi}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{m^2}{8\pi} A^\nu A_\nu$$

$$\mathcal{L}^{\mathcal{A}'} = \frac{-1}{16\pi}[\partial^\mu(A^\nu + \partial^\nu\lambda) - \partial^\nu(A^\mu + \partial^\mu\lambda)(\partial_\mu(A_\nu + \partial_\nu\lambda) - \partial_\nu(A_\mu + \partial_\mu\lambda)] + \frac{m^2}{8\pi}(A^\nu + \partial^\nu\lambda)(A_\nu + \partial_\nu\lambda)$$

$$\mathcal{L}^{\mathcal{A}'} = \frac{-1}{16\pi}[\partial^\mu A^\nu + \partial^\mu\partial^\nu\lambda - \partial^\nu A^\mu - \partial^\nu\partial^\mu\lambda)(\partial_\mu A_\nu + \partial_\mu\partial_\nu\lambda - \partial_\nu A_\mu - \partial_\nu\partial_\mu\lambda)]+$$
$$\frac{m^2}{8\pi}(A^\nu A_\nu + \partial^\nu\lambda A_\nu + A^\nu\partial_\nu\lambda + \partial^\nu\lambda\partial_\nu\lambda)$$

$$\partial^\mu\partial^\nu\lambda = \partial^\nu\partial^\mu\lambda$$

$$\Delta(\mathcal{L}^{\mathcal{A}'}) = \frac{m^2}{8\pi}(\partial^\nu\lambda A_\nu + A^\nu\partial_\nu\lambda + \partial^\nu\lambda\partial_\nu\lambda)$$

So this is only invariant if m = 0!

# Can we make this more general?



Physicists like theories with symmetries (see chapter 4!) and local gauge invariance is a nicely general one! How to build on it?

$$\partial_\mu(e^{i\theta(x)}\psi) = i[\partial_\mu\theta(x)]e^{i\theta(x)}\psi + e^{i\theta(x)}\partial_\mu\psi$$

$$\partial_\mu(e^{-i\lambda q}\psi) = -i[\partial_\mu(\lambda q)]e^{-i\lambda q}\psi + e^{-i\lambda q}\partial_\mu\psi$$

$$\partial_\mu(e^{-i\lambda q}\psi) = e^{-i\lambda q}[\partial_\mu - iq(\partial_\mu\lambda)]\psi$$

# Can we make this more general?

$$\partial_\mu(e^{-i\lambda q}\psi) = e^{-i\lambda q}[\partial_\mu - iq(\partial_\mu\lambda)]\psi$$

OK if we replace:

$$\partial_\mu \to \mathcal{D}_\mu = \partial_\mu + iqA_\mu$$

U(1) gauge invarariance

$$\psi \to U\psi, U^\dagger U = 1, U = e^{i\theta}$$

Covariant derivative

If you take field theory, they will become your friend

The price we pay is that we must introduce a new massless vector field

## Two spin-1/2 non-interacting particles

$$\mathcal{L} = i\overline{\psi}_1\gamma^\mu\partial_\mu\psi_1 - m^2\overline{\psi_1}\psi_1 + i\overline{\psi}_2\gamma^\mu\partial_\mu\psi_2 - m^2\overline{\psi_2}\psi_2$$

Write this more cleanly as:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$\overline{\psi} = (\overline{\psi}_1 \ \overline{\psi}_2)$$

$$\mathcal{L} = i\overline{\psi}\gamma^\mu\partial_\mu\psi - M^2\overline{\psi}\psi$$

$$M = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

# Let's discuss particle masses in Lagrangians

Classical $\mathcal{L} = T - U$

More field theory inspired

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2$$

$$\mathcal{L} = \frac{1}{2}[\partial_0\phi\partial_0\phi - \partial_1\phi\partial_1\phi - \partial_2\phi\partial_2\phi - \partial_3\phi\partial_3\phi - m^2\phi^2]$$

Can think of the mass term as the "field squared" term in the Lagrangian. The first piece (with derivatives) is the kinetic energy term (we know that kinetic energy involves derivatives)

# How do we know it is that term?

Could the mass be the k term?

[Field*m]$^2$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2 - k^4\phi^4$$

[Field*GeV]$^2$

[Field*k]$^4$

| | Not our choice | In our choice of natural units |
|---|---|---|
| Energy | GeV | GeV |
| Momentum | GeV/c | GeV |
| Mass | GeV/c$^2$ | GeV |
| Time | hbar/GeV | GeV$^{-1}$ |
| Length | c*hbar/GeV | GeV$^{-1}$ |
| Area | (c*hbar/GeV)$^2$ | GeV$^{-2}$ |

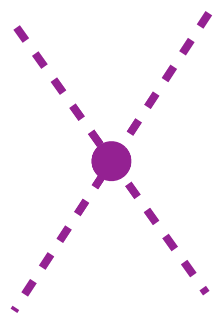
All 3 terms must have the same units!
m has units of GeV that we want!

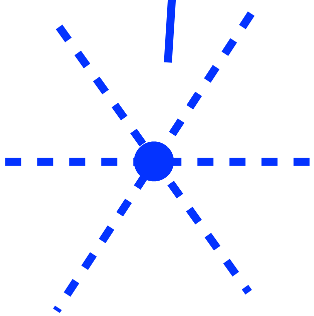
# What does a Lagrangian with higher order terms represent?

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2 - k^4\phi^4 - g^6\phi^6$$

Kinetic terms of the fields

Mass terms





Additional terms represent new couplings, of more objects to a single vertex

# What about this Lagrangian?

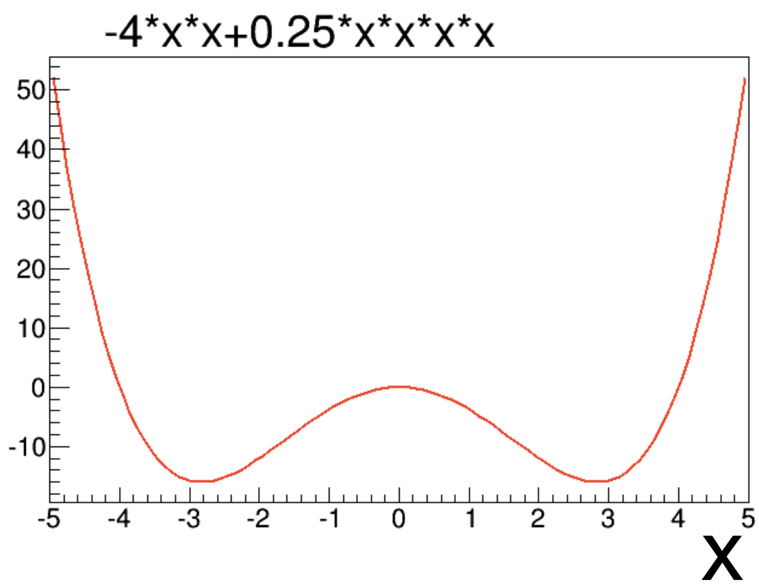


$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}m^2\phi^2 - \frac{1}{4}k^2\phi^4$$

$$U = -\frac{1}{2}m^2\phi^2 + \frac{1}{4}k^2\phi^4$$

Mass term has wrong sign! That is because our calculations are really a fancy version of perturbation theory. In most theories, the ground state of the "potential" has the field at zero (the ground state of the E&M field has no E&M waves or photons!)

Minimum not at Φ=0 but rather Φ=±m/k



# Expanding about the real ground state

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}m^2\phi^2 - \frac{1}{4}k^2\phi^4 \qquad \eta = \phi \pm \frac{m}{k} \qquad \phi = \eta \pm \frac{m}{k}$$

$$\phi^2 = \eta^2 + \frac{m^2}{k^2} \pm 2\frac{m}{k}\eta$$

$$\phi^4 = \eta^4 + \frac{m^4}{k^4} + 4\frac{m^2}{k^2}\eta^2 + 2\frac{m^2}{k^2}\eta^2 \pm 4\frac{m}{k}\eta^3 \pm 4\frac{m^3}{k^3}\eta$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) + \frac{1}{2}m^2(\eta^2 + \frac{m^2}{k^2} \pm 2\frac{m}{k}\eta) - \frac{1}{4}k^2(\eta^4 + \frac{m^4}{k^4} + 4\frac{m^2}{k^2}\eta^2 + 2\frac{m^2}{k^2}\eta^2 \pm 4\frac{m}{k}\eta^3 \pm 4\frac{m^3}{k^3}\eta)$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) + \eta(\pm\frac{m^3}{k} \mp \frac{m^3}{k}) + \eta^2(\frac{m^2}{2} - m^2 - \frac{m^2}{2}) \mp mk\eta^3 - \frac{k^2}{4}\eta^4 + C$$

Ignore constant term and cancel terms...

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - m^2\eta^2 \pm mk\eta^3 - \frac{k^2}{4}\eta^4$$

# New Lagrangian



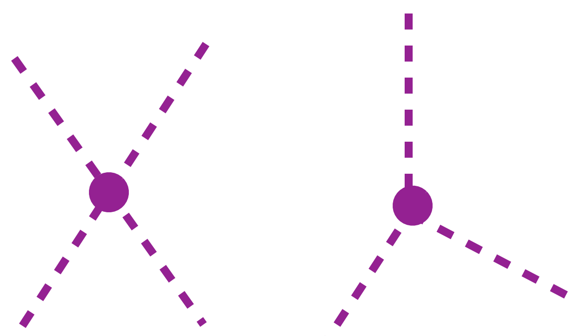
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - m^2\eta^2 \pm mk\eta^3 - \frac{k^2}{4}\eta^4$$

Compare with original one before adding new terms

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2$$

Mass of particle is sqrt(2)*m

And we have these two new interactions



Reminder: description in terms of new variable **must be** the same. Choice of vacuum ground state breaks symmetries

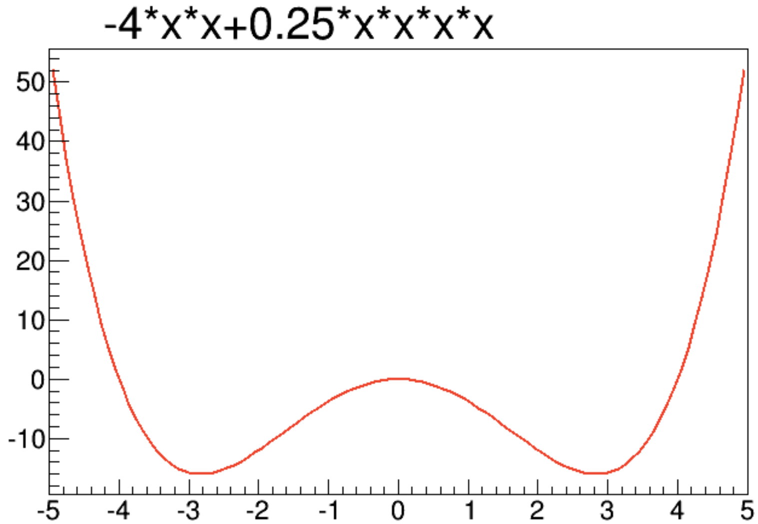
# Spontaneous symmetry breaking



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2} m^2 \phi^2 - \frac{1}{4} k^2 \phi^4$$

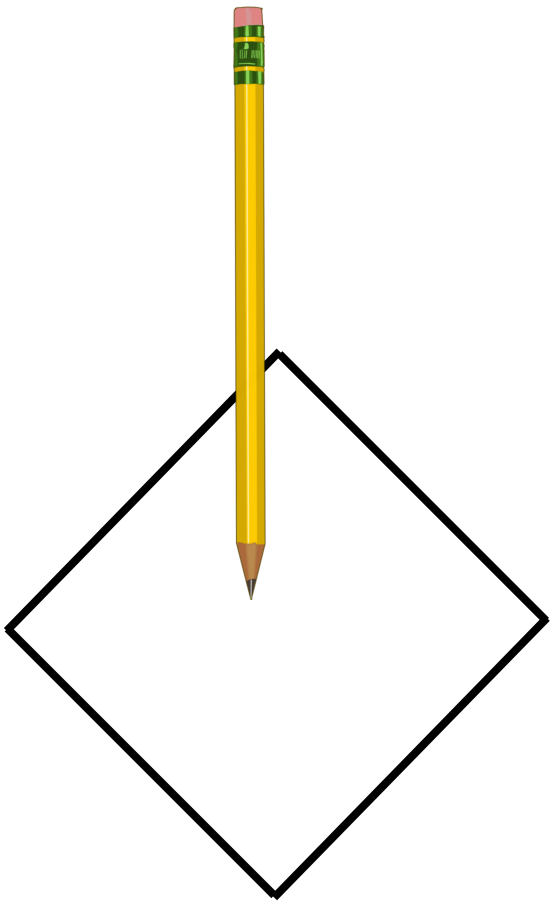
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - m^2 \eta^2 \pm mk\eta^3 - \frac{k^2}{4}\eta^4$$

Original Lagrangian is symmetric in Φ→-Φ




New Lagrangian is not symmetric in this way! Selection of **specific ground state** **hides** this symmetry! We have expanded this around the minimum (otherwise perturbation theory makes no sense!)

# Classical example of spontaneous symmetry breaking



Mechanical laws describing this pencil under gravity are symmetrical with respect to angle from the vertical

System choses to move to a ground state of lower energy, at the price of hiding that symmetry!

# Lagrangian with two scalar fields + symmetry breaking



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_1)(\partial^\mu \phi_1) + \frac{1}{2}m^2\phi_1^2 + \frac{1}{2}(\partial_\mu \phi_2)(\partial^\mu \phi_2) + \frac{1}{2}m^2\phi_2^2 - \frac{1}{4}k^2(\phi_1^2 + \phi_2^2)^2$$

$$U = -\frac{1}{2}m^2(\phi_1^2 + \phi_2^2) + \frac{1}{4}k^2(\phi_1^2 + \phi_2^2)^2$$

$$\frac{\partial U}{\partial \phi_1} = -m^2\phi_1 + \frac{k^2}{2}(\phi_1^2 + \phi_2^2)2\phi_1$$

$$\frac{\partial U}{\partial \phi_1} = -m^2\phi_1 + k^2(\phi_1^3 + \phi_1\phi_2^2) = 0$$

$$\frac{\partial U}{\partial \phi_2} = -m^2\phi_2 + k^2(\phi_2\phi_1^2 + \phi_2^3) = 0$$

# Lagrangian with two scalar fields + symmetry breaking



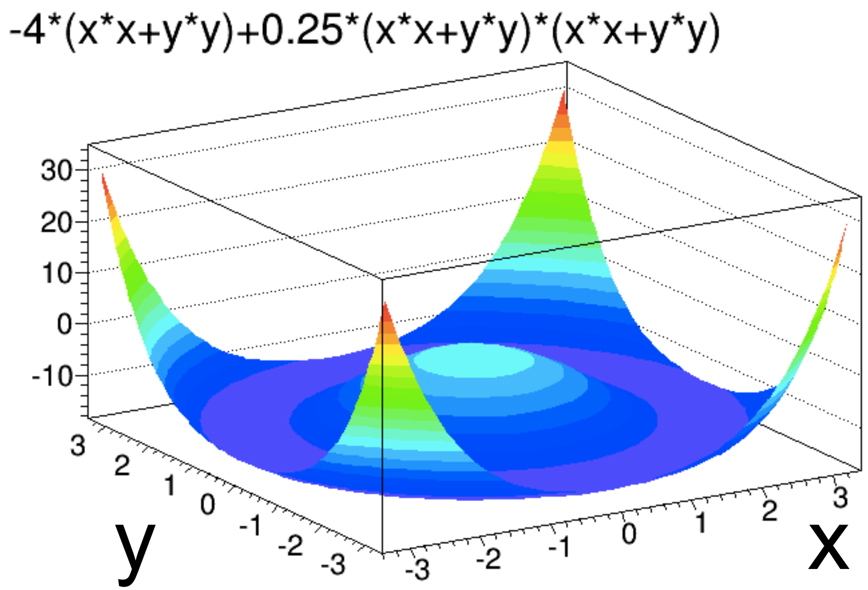
$$\frac{\partial U}{\partial \phi_1} = -m^2\phi_1 + k^2(\phi_1^3 + \phi_1\phi_2^2) = 0$$

$$\frac{\partial U}{\partial \phi_2} = -m^2\phi_2 + k^2(\phi_2\phi_1^2 + \phi_2^3) = 0$$

$$-m^2 + k^2(\phi_1^2 + \phi_2^2) = 0$$

$$\boxed{\phi_1^2 + \phi_2^2 = \frac{m^2}{k^2}}$$

A continous set of minima! But not at zero field



# Lagrangian with two scalar fields + symmetry breaking



$$\phi_1^2 + \phi_2^2 = \frac{m^2}{k^2}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_1)(\partial^\mu \phi_1) + \frac{1}{2}m^2\phi_1^2 + \frac{1}{2}(\partial_\mu \phi_2)(\partial^\mu \phi_2) + \frac{1}{2}m^2\phi_2^2 - \frac{1}{4}k^2(\phi_1^2 + \phi_2^2)^2$$

Let's pick $\Phi_1$ = m/k, $\Phi_2$ = 0, and expand around that

$$\eta = \phi_1 - m/k$$

$$\phi_1 = \eta + m/k$$

$$\theta = \phi_2$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) + \frac{1}{2}(\partial_\mu \theta)(\partial^\mu \theta) + \frac{1}{2}m^2(\eta^2 + \frac{m^2}{k^2} + 2\frac{m}{k}\eta + \theta^2) - \frac{1}{4}k^2(\eta^2 + \frac{m^2}{k^2} + 2\frac{m}{k}\eta + \theta^2)^2$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) + \frac{1}{2}(\partial_\mu \theta)(\partial^\mu \theta) + \frac{1}{2}m^2(\eta^2 + \frac{m^2}{k^2} + 2\frac{m}{k}\eta + \theta^2) -$$

$$\frac{k^2}{4}(\eta^4 + \frac{m^4}{k^4} + 4\frac{m^2}{k^2}\eta^2 + \theta^4 + 2\eta^2\frac{m^2}{k^2} + 4\eta^3\frac{m}{k} + 2\eta^2\theta^2 + 4\frac{m^3}{k^3}\eta + 2\frac{m^2}{k^2}\theta^2 + 4\frac{m}{k}\eta\theta^2)$$

# Lagrangian with two scalar fields + symmetry breaking



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) + \frac{1}{2}(\partial_\mu \theta)(\partial^\mu \theta) + \frac{1}{2}m^2(\eta^2 + \frac{m^2}{k^2} + 2\frac{m}{k}\eta + \theta^2) -$$
$$\frac{k^2}{4}(\eta^4 + \frac{m^4}{k^4} + 4\frac{m^2}{k^2}\eta^2 + \theta^4 + 2\eta^2\frac{m^2}{k^2} + 4\eta^3\frac{m}{k} + 2\eta^2\theta^2 + 4\frac{m^3}{k^3}\eta + 2\frac{m^2}{k^2}\theta^2 + 4\frac{m}{k}\eta\theta^2)$$

## Remove constant terms and combine

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) + \frac{1}{2}(\partial_\mu \theta)(\partial^\mu \theta) + \frac{1}{2}m^2(\eta^2 + 2\frac{m}{k}\eta + \theta^2) -$$
$$\frac{k^2}{4}(\eta^4 + \theta^4 + 6\frac{m^2}{k^2}\eta^2 + 4\eta^3\frac{m}{k} + 2\eta^2\theta^2 + 4\frac{m^3}{k^3}\eta + 2\frac{m^2}{k^2}\theta^2 + 4\frac{m}{k}\eta\theta^2)$$

## Combine more terms

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) + \frac{1}{2}(\partial_\mu \theta)(\partial^\mu \theta) - m^2\eta^2 - \frac{k^2}{4}\eta^4 - \frac{k^2}{4}\theta^4 - km\eta^3 - \frac{k^2}{2}\theta^2\eta^2 - mk\eta\theta^2$$

# Lagrangian with two scalar fields + symmetry breaking



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) + \frac{1}{2}(\partial_\mu \theta)(\partial^\mu \theta) - m^2\eta^2 - \frac{k^2}{4}\eta^4 - \frac{k^2}{4}\theta^4 - km\eta^3 - \frac{k^2}{2}\theta^2\eta^2 - mk\eta\theta^2$$

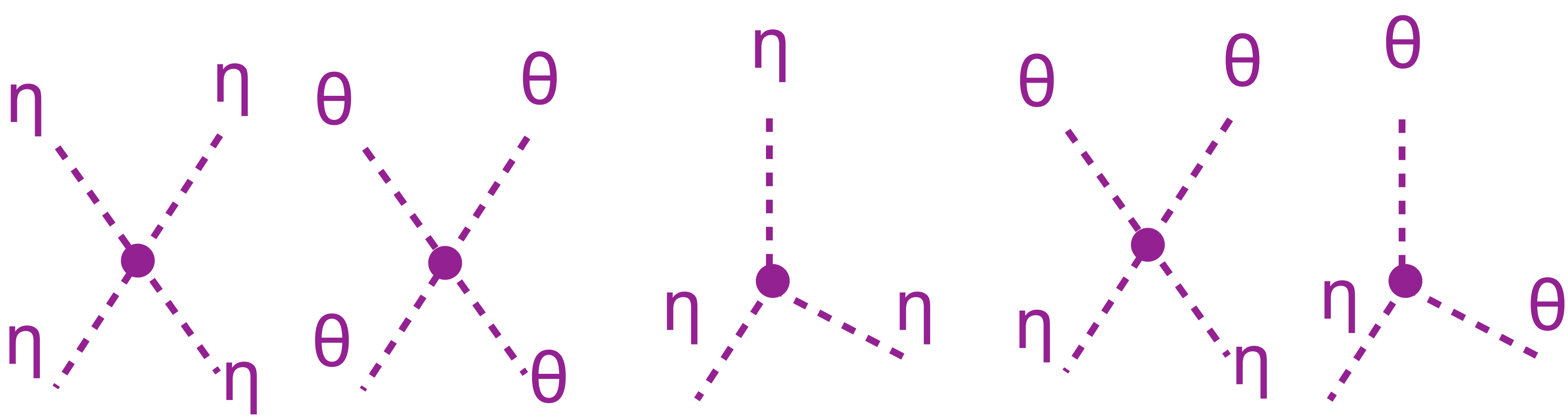
Let's rewrite this

$$\mathcal{L} = \left[\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - m^2\eta^2\right] + \left[\frac{1}{2}(\partial_\mu \theta)(\partial^\mu \theta)\right] - \left[\frac{k^2}{4}\eta^4 + \frac{k^2}{4}\theta^4 + km\eta^3 + \frac{k^2}{2}\theta^2\eta^2 + mk\eta\theta^2\right]$$

$$m_\eta = \sqrt{2}m$$
$$m_\theta = 0$$

One of the fields massless! Why?

And have new vertices:



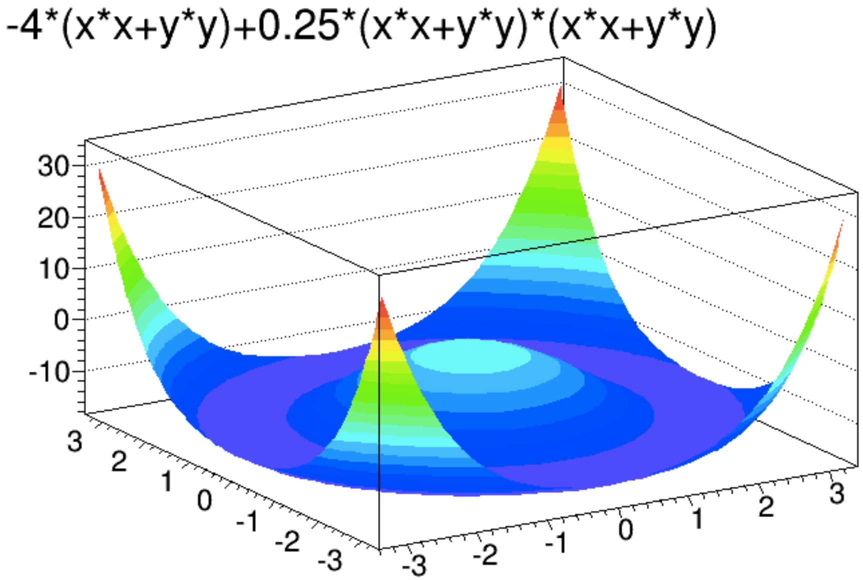
# Massless field

$$\mathcal{L} = \left[\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - m^2\eta^2\right] + \left[\frac{1}{2}(\partial_\mu \theta)(\partial^\mu \theta)\right] - \left[\frac{k^2}{4}\eta^4 + \frac{k^2}{4}\theta^4 + km\eta^3 + \frac{k^2}{2}\theta^2\eta^2 + mk\eta\theta^2\right]$$

$$m_\eta = \sqrt{2}m$$
$$m_\theta = 0$$

One of the fields massless! Why?

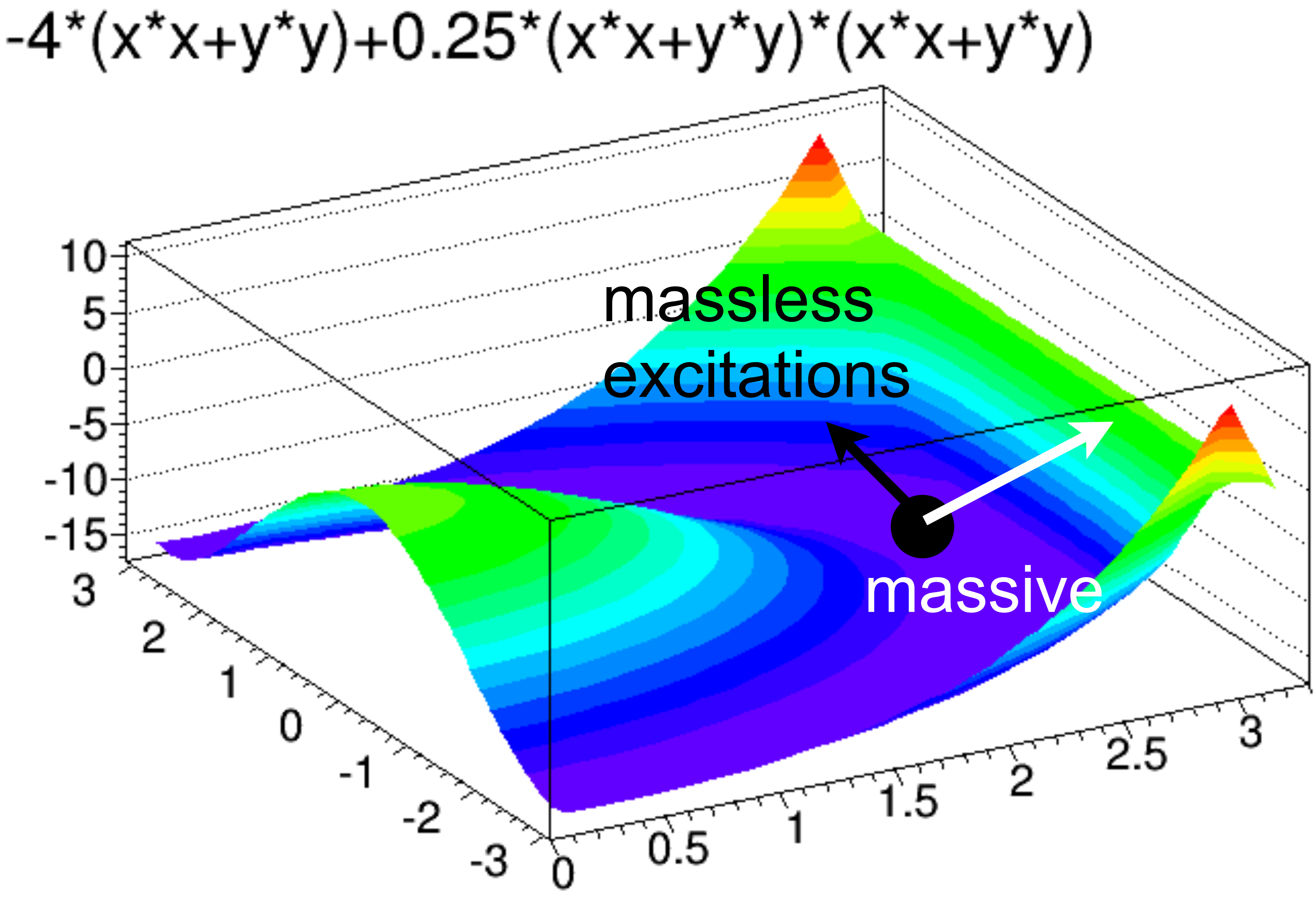
Continuous global symmetry! Goldstone's theorem - have you seen this?




Massless in direction where potential does not change, massive in orthogonal direction

# Massless and massive field





# Two scalar fields with spontaneous symmetry breaking



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_1)(\partial^\mu \phi_1) + \frac{1}{2}m^2\phi_1^2 + \frac{1}{2}(\partial_\mu \phi_2)(\partial^\mu \phi_2) + \frac{1}{2}m^2\phi_2^2 - \frac{1}{4}k^2(\phi_1^2 + \phi_2^2)^2$$

Rewrite this as a single complex field

$$\phi = \phi_1 + i\phi_2$$

$$\phi^*\phi = \phi_1^2 + \phi_2^2$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi) + \frac{1}{2}m^2(\phi^*\phi) - \frac{1}{4}k^2(\phi^*\phi)^2$$

# Now we want to apply local gauge transformations



$$\phi = \phi_1 + i\phi_2$$

$$\phi^*\phi = \phi_1^2 + \phi_2^2$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^*(\partial^\mu\phi) + \frac{1}{2}m^2(\phi^*\phi) - \frac{1}{4}k^2(\phi^*\phi)^2$$

Can get this: $\phi \rightarrow e^{i\theta(x)}\phi$

If we make this change: $\partial_\mu \rightarrow \mathcal{D}_\mu = \partial_\mu + iqA_\mu$

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu + iqA_\mu)\phi\right]^*\left[(\partial^\mu + iqA^\mu)\phi\right] + \frac{1}{2}m^2(\phi^*\phi) - \frac{1}{4}k^2(\phi^*\phi)^2 - \frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}$$

Interaction terms for new massless field

# Writing out our Lagrangian

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu - iqA_\mu)\phi^*\right]\left[(\partial^\mu + iqA^\mu)\phi\right] + \frac{1}{2}m^2(\phi^*\phi) - \frac{1}{4}k^2(\phi^*\phi)^2 - \frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}$$

Again expand about real minimum (has not changed due to adding gauge symmetry):

$$\eta = \phi_1 - m/k$$

$$\phi_1 = \eta + m/k$$

$$\psi = \phi_2$$

The F terms do not change via this transformation. And the $m^2$ and $k^2$ terms were calculated by us a few slides ago. So let's work out the nasty first two terms

# The ugly pieces



$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu - iqA_\mu)\phi^*\right]\left[(\partial^\mu + iqA^\mu)\phi\right] + \frac{1}{2}m^2(\phi^*\phi) - \frac{1}{4}k^2(\phi^*\phi)^2 - \frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}$$

$$\eta = \phi_1 - m/k$$

$$\phi_1 = \eta + m/k$$

$$\psi = \phi_2$$

$$\frac{1}{2}\left[(\partial_\mu - iqA_\mu)\phi^*\right]\left[(\partial^\mu + iqA^\mu)\phi\right]$$

$$\frac{1}{2}\left[(\partial_\mu - iqA_\mu)(\phi_1 - i\phi_2)\right]\left[(\partial^\mu + iqA^\mu)(\phi_1 + i\phi_2)\right]$$

Can already get apprehensive about the algebra!

# The ugly pieces



$$\frac{1}{2}\left[(\partial_\mu - iqA_\mu)(\phi_1 - i\phi_2)\right]\left[(\partial^\mu + iqA^\mu)(\phi_1 + i\phi_2)\right]$$

$$\frac{1}{2}\left[\partial_\mu\phi_1 - i\partial_\mu\phi_2 - iqA_\mu\phi_1 - qA_\mu\phi_2\right]\left[\partial^\mu\phi_1 + i\partial^\mu\phi_2 + iqA^\mu\phi_1 - qA^\mu\phi_2\right]$$

$$\frac{1}{2}[\partial_\mu\phi_1\partial^\mu\phi_1 + i\partial_\mu\phi_1\partial^\mu\phi_2 + iqA^\mu\phi_1\partial_\mu\phi_1 - qA^\mu\phi_2\partial_\mu\phi_1 +$$

$$-i\partial_\mu\phi_2\partial^\mu\phi_1 + \partial_\mu\phi_2\partial^\mu\phi_2 + qA^\mu\phi_1\partial_\mu\phi_2 + iq\phi_2A^\mu\partial_\mu\phi_2$$

$$-iqA_\mu\phi_1\partial^\mu\phi_1 + qA_\mu\phi_1\partial^\mu\phi_2 + q^2A_\mu A^\mu\phi_1^2 + iq^2A_\mu A^\mu\phi_1\phi_2 +$$

$$-qA_\mu\phi_2\partial^\mu\phi_1 - iqA_\mu\phi_2\partial^\mu\phi_2 - iq^2\phi_2A_\mu A^\mu\phi_1 + q^2A_\mu A^\mu\phi_2^2]$$

# Simplifying things



$$\frac{1}{2}[\partial_\mu\phi_1\partial^\mu\phi_1 + i\partial_\mu\phi_1\partial^\mu\phi_2 + iqA^\mu\phi_1\partial_\mu\phi_1 - qA^\mu\phi_2\partial_\mu\phi_1 +$$
$$-i\partial_\mu\phi_2\partial^\mu\phi_1 + \partial_\mu\phi_2\partial^\mu\phi_2 + qA^\mu\phi_1\partial_\mu\phi_2 + iq\phi_2A^\mu\partial_\mu\phi_2$$
$$-iqA_\mu\phi_1\partial^\mu\phi_1 + qA_\mu\phi_1\partial^\mu\phi_2 + q^2A_\mu A^\mu\phi_1^2 + iq^2A_\mu A^\mu\phi_1\phi_2 +$$
$$-qA_\mu\phi_2\partial^\mu\phi_1 - iqA_\mu\phi_2\partial^\mu\phi_2 - iq^2\phi_2A_\mu A^\mu\phi_1 + q^2A_\mu A^\mu\phi_2^2]$$

$$\frac{1}{2}[\partial_\mu\phi_1\partial^\mu\phi_1 - qA^\mu\phi_2\partial_\mu\phi_1 +$$
$$\partial_\mu\phi_2\partial^\mu\phi_2 + qA^\mu\phi_1\partial_\mu\phi_2 +$$
$$qA_\mu\phi_1\partial^\mu\phi_2 + q^2A_\mu A^\mu\phi_1^2 +$$
$$-qA_\mu\phi_2\partial^\mu\phi_1 + q^2A_\mu A^\mu\phi_2^2]$$

# Simplifying things



$$\frac{1}{2}[\partial_\mu\phi_1\partial^\mu\phi_1 - qA^\mu\phi_2\partial_\mu\phi_1 +$$
$$\partial_\mu\phi_2\partial^\mu\phi_2 + qA^\mu\phi_1\partial_\mu\phi_2 +$$
$$qA_\mu\phi_1\partial^\mu\phi_2 + q^2A_\mu A^\mu\phi_1^2 +$$
$$-qA_\mu\phi_2\partial^\mu\phi_1 + q^2A_\mu A^\mu\phi_2^2]$$

$$\frac{1}{2}[\partial_\mu\phi_1\partial^\mu\phi_1 + \partial_\mu\phi_2\partial^\mu\phi_2 + q^2A^2(\phi_1^2 + \phi_2^2) + 2qA^\mu(\phi_1\partial_\mu\phi_2 - \phi_2\partial_\mu\phi_1)]$$

Now let's apply this and remember that we don't care about derivatives of constants

$$\eta = \phi_1 - m/k$$
$$\phi_1 = \eta + m/k$$
$$\psi = \phi_2$$

# Putting it all together



$$\frac{1}{2}[\partial_\mu\phi_1\partial^\mu\phi_1 + \partial_\mu\phi_2\partial^\mu\phi_2 + q^2A^2(\phi_1^2+\phi_2^2) + 2qA^\mu(\phi_1\partial_\mu\phi_2 - \phi_2\partial_\mu\phi_1)]$$

Now let's apply this and remember that we don't care about derivatives of constants

$$\eta = \phi_1 - m/k$$
$$\phi_1 = \eta + m/k$$
$$\psi = \phi_2$$

$$\frac{1}{2}[\partial_\mu\eta\partial^\mu\eta + \partial_\mu\psi\partial^\mu\psi + q^2A^2(\psi^2+\eta^2+\frac{m^2}{k^2}+\eta\frac{2m}{k}) +$$
$$2qA^\mu(\eta\partial_\mu\psi + \frac{m}{k}\partial_\mu\psi - \psi\partial_\mu\eta)]$$

Now let's add in our missing pieces from the full Lagrangian

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu - iqA_\mu)\phi^*\right]\left[(\partial^\mu + iqA^\mu)\phi\right] + \frac{1}{2}m^2(\phi^*\phi) - \frac{1}{4}k^2(\phi^*\phi)^2 - \frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}$$

# Putting it all together

$$\left[\frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - m^2\eta^2\right] + \left[\frac{1}{2}(\partial_\mu\psi)(\partial^\mu\psi)\right] +$$

$$+\frac{q^2A^2}{2}(\psi^2 + \eta^2 + \frac{m^2}{k^2} + \eta\frac{2m}{k})+$$

$$qA^\mu(\eta\partial_\mu\psi + \frac{m}{k}\partial_\mu\psi - \psi\partial_\mu\eta)]+$$

$$-\left[\frac{k^2}{4}\eta^4 + \frac{k^2}{4}\psi^2 + km\eta^3 + \frac{k^2}{2}\eta^2\psi^2 + mk\eta\psi^2\right]+$$

$$-\frac{F^{\mu\nu}F_{\mu\nu}}{16\pi}$$

Let’s rewrite all of that

# Putting it all together



Massive field as before

Massless field as before

Gauge field description

Interactions between scalars and vectors

$$\left[\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - m^2\eta^2\right] + \left[\frac{1}{2}(\partial_\mu \psi)(\partial^\mu \psi)\right] +$$

$$+\frac{q^2A^2}{2}(\psi^2 + \eta^2 + \frac{m^2}{k^2} + \eta\frac{2m}{k})+$$

$$qA^\mu(\eta\partial_\mu\psi + \frac{m}{k}\partial_\mu\psi - \psi\partial_\mu\eta)]+$$

$$-\left[\frac{k^2}{4}\eta^4 + \frac{k^2}{4}\psi^4 + km\eta^3 + \frac{k^2}{2}\eta^2\psi^2 + mk\eta\psi^2\right]+$$

$$-\frac{F^{\mu\nu}F_{\mu\nu}}{16\pi}$$

# Most importantly



Mass of scalar field, scalar field self-interaction strength and interactions between scalars and vectors not all independent!

$$\left[\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - m^2\eta^2\right] + \left[\frac{1}{2}(\partial_\mu \psi)(\partial^\mu \psi)\right] +$$

$$+\frac{q^2A^2}{2}(\psi^2 + \eta^2 + \frac{m^2}{k^2} + \eta\frac{2m}{k})+$$

$$qA^\mu(\eta\partial_\mu\psi + \frac{m}{k}\partial_\mu\psi - \psi\partial_\mu\eta)]+$$

$$-\left[\frac{k^2}{4}\eta^4 + \frac{k^2}{4}\psi^4 + km\eta^3 + \frac{k^2}{2}\eta^2\psi^2 + mk\eta\psi^2\right]+$$

$$-\frac{F^{\mu\nu}F_{\mu\nu}}{16\pi}$$

Vector field now has a mass!!!!

Scalar field self-interactions

# Finally we get to the Higgs Lagrangian



$$\mathcal{L} = \frac{1}{2}(\mathcal{D}_\mu\phi)^\dagger(\mathcal{D}^\mu\phi) - V(\phi)$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

It's a complex scalar doublet - one field has electric charge (will be giving mass to the W bosons) and the other is neutral (giving mass to the Z boson)

# Minimum of potential



$$\phi^\dagger\phi = \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = \frac{v^2}{2} = -\frac{\mu^2}{2\lambda}$$

We know that in the ground state, after symmetry breaking, photon remains massless. So ground state should only contain electrically neutral piece

**Note original symmetry among all 4 directions. Now we are going to choose one direction, breaking the symmetry! (Other 3 directions are going to give mass to $W^+$, $W^-$ and Z) Compare with our Mexican hat previous examples**

$$\phi(vacuum) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\phi \rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

# Higgs potential



$$\phi^\dagger\phi = \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = \frac{v^2}{2} = -\frac{\mu^2}{2\lambda}$$

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

$$\phi(vacuum) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\phi \rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

$$V(\phi) = \frac{\mu^2}{2}\left(v^2 + h^2 + 2vh\right) + \frac{\lambda}{4}\left(v^4 + h^4 + 4v^2h^2 + 2v^2h^2 + 4v^3h + 4vh^3\right)$$

# Higgs potential



$$V(\phi) = \frac{\mu^2}{2}\left(v^2 + h^2 + 2vh\right) + \frac{\lambda}{4}\left(v^4 + h^4 + 4v^2h^2 + 2v^2h^2 + 4v^3h + 4vh^3\right)$$

Keeping terms only up to second order in h and ignoring constants

$$V(\phi) = \frac{\mu^2}{2}\left(h^2 + 2vh\right) + \frac{\lambda}{4}\left(6v^2h^2 + 4v^3h\right)$$

$$\frac{v^2}{2} = -\frac{\mu^2}{2\lambda}$$

$$V(\phi) = \frac{-\lambda v^2}{2}\left(h^2 + 2vh\right) + \frac{\lambda}{4}\left(6v^2h^2 + 4v^3h\right)$$

$$V(\phi) = \lambda v^2 h^2$$ Higgs boson mass term!

# On the vacuum

You might hear about the Higgs field's non-zero field, or "vacuum expectation value" (or "vev"). Find a very simple relation between W boson mass, weak coupling $g_W$ and v... calculate v = 246 GeV!

$$\phi(vacuum) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\phi \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

# After applying local gauge symmetry



Previous symmetry that we demanded was a U(1) local gauge symmetry. Now we ask for the Lagrangian to respect the symmetry of the electroweak theory, namely [$SU(2)_L \times U(1)_Y$]

Find that the $W^{\pm}$ and Z bosons obtain mass through the breaking of the symmetry. We just found that the vacuum has massive excitations (Higgs boson!)

# Full Higgs Lagrangian



$$\mathcal{L} = \frac{1}{2}(\mathcal{D}_\mu \phi)^\dagger (\mathcal{D}^\mu \phi) - V(\phi)$$

Want to pick the covariant derivative so that it also respects [SU(2)$_L$ x U(1)$_Y$] symmetry

$$\mathcal{D}_\mu = \partial_\mu + i g_W \vec{\tau} \cdot \vec{W}_\mu + i g' B_\mu$$

$$\frac{1}{2\sqrt{2}}(D_\mu \phi) = \frac{1}{2} \begin{pmatrix} i g_W W_{3\mu} + i g' B_\mu + \partial_\mu & i g_W W_{1\mu} + g_W W_{2\mu} \\ i g_W W_{1\mu} - g_W W_{2\mu} & -i g_W W_{3\mu} + i g' B_\mu + \partial_\mu \end{pmatrix} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

$$\frac{1}{2\sqrt{2}}(D_\mu \phi) = \frac{1}{2\sqrt{2}} \begin{bmatrix} (v+h)(i g_W W_{1\mu} + g_W W_{2\mu}) \\ (-i g_W W_{3\mu} + i g' B_\mu + \partial_\mu)(v+h) \end{bmatrix}$$

# Full Higgs Lagrangian



$$\frac{1}{2\sqrt{2}}(D_\mu\phi) = \frac{1}{2\sqrt{2}}\begin{bmatrix} (v+h)(ig_W W_{1\mu} + g_W W_{2\mu}) \\ (-ig_W W_{3\mu} + ig' B_\mu + \partial_\mu)(v+h) \end{bmatrix}$$

$$\frac{1}{2}(D_\mu\phi)^\dagger)(D^\mu\phi) =$$

$$\frac{1}{8}\left[(v+h)(-ig_W W_{1\mu} + g_W W_{2\mu}) \quad (ig_W W_{3\mu} - ig' B_\mu)(v+h)\right] \times$$

$$\begin{bmatrix} (v+h)(ig_W W_1^\mu + g_W W_2^\mu) \\ (-ig_W W_3^\mu + ig' B^\mu)(v+h) \end{bmatrix}$$

$$\frac{1}{2}(D_\mu\phi)^\dagger)(D^\mu\phi) =$$

$$\frac{1}{8}\left((v+h)^2(g_W^2 W_{1\mu}W_1^\mu + g_W^2 W_{2\mu}W_2^\mu) + (v+h)^2(g_W^2 W_3\mu W_3^\mu - g' g_W B_\mu W_3^\mu - g' g_W B^\mu W_{3\mu} + g'^2 B_\mu B^\mu\right)$$

**Recall that:**

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \pm iW_\mu^2)$$

# Full Higgs Lagrangian



$$\frac{1}{2}(D_\mu\phi)^\dagger)(D^\mu\phi) =$$

$$\frac{1}{8}\left((v+h)^2(g_W^2 W_{1\mu}W_1^\mu + g_W^2 W_{2\mu}W_2^\mu) + (v+h)^2(g_W^2 W_{3}\mu W_3^\mu - g' g_W B_\mu W_3^\mu - g' g_W B^\mu W_{3\mu} + g'^2 B_\mu B^\mu\right)$$

**Recall that:**

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \pm iW_\mu^2)$$

Mass of W1, W2 given by:

$$\frac{1}{2}m_{W1,2}^2 W_{\mu,1,2}W_{1,2}^\mu \text{ term: } \frac{1}{8}v^2 g_W^2 W_{\mu,1,2}W_{1,2}^\mu$$

$$m_W = \frac{1}{2}g_W v$$

# Full Higgs Lagrangian

$$\frac{1}{2}(D_\mu\phi)^\dagger)(D^\mu\phi) =$$

$$\frac{1}{8}\left((v+h)^2(g_W^2 W_{1\mu}W_1^\mu + g_W^2 W_{2\mu}W_2^\mu) + (v+h)^2(g_W^2 W_3\mu W_3^\mu - g' g_W B_\mu W_3^\mu - g' g_W B^\mu W_{3\mu} + g'^2 B_\mu B^\mu\right)$$

**Quadratic terms for spin-1 fields:**

**Mass matrix M**

$$\frac{v^2}{8}\left(W_{\mu,3}\ B_\mu\right)\begin{pmatrix} g_W^2 & -g_W g' \\ -g_W g' & g'^2 \end{pmatrix}\begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix}$$

$$\det(\mathbf{M} - \lambda I) = 0 \rightarrow (g_W^2 - \lambda)(g'^2 - \lambda) - g_W^2 g'^2 = 0$$

$$\lambda = 0, \lambda = g_W^2 + g'^2$$

# Eigenvalues for neutral gauge bosons



$$\frac{v^2}{8}(W_{\mu,3}\ B_\mu)\left(\begin{array}{cc} g_W^2 & -g_W g' \\ -g_W g' & g'^2 \end{array}\right)\left(\begin{array}{c} W_3^\mu \\ B^\mu \end{array}\right)$$

$$\lambda = 0, \lambda = g_W^2 + g'^2$$

**Mass terms diagonalized:**

$$\frac{v^2}{8}(A_\mu Z_\mu)\left(\begin{array}{cc} 0 & 0 \\ 0 & g'^2 + g_W^2 \end{array}\right)\left(\begin{array}{c} A^\mu \\ Z^\mu \end{array}\right)$$

$$m_A = 0, m_Z = \frac{1}{2}v\sqrt{g'^2 + g_W^2}$$

**Photon remains massless, but have a massive Z!**

# Eigenvectors for neutral gauge bosons



$$\frac{v^2}{8}(W_{\mu,3}\ B_\mu)\begin{pmatrix} g_W^2 & -g_W g' \\ -g_W g' & g'^2 \end{pmatrix}\begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix}$$

$$\lambda = 0, \lambda = g_W^2 + g'^2$$

**First eigenvector:**

$$\begin{pmatrix} g_W^2 & -g_W g' \\ -g_W g' & g'^2 \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix}$$

$$p g_W^2 - g_W g' q = 0 \to p = \frac{g'}{g_W} q \to A_\mu = g' W_\mu^3 + g_W B_\mu$$

**Normalize:**

$$A_\mu = \frac{g' W_\mu^3 + g_W B_\mu}{\sqrt{g_W^2 + g'^2}}$$

# Eigenvectors for neutral gauge bosons

$$\frac{v^2}{8}\left(W_{\mu,3}\ B_\mu\right)\begin{pmatrix} g_W^2 & -g_W g' \\ -g_W g' & g'^2 \end{pmatrix}\begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix}$$

$$\lambda = 0, \lambda = g_W^2 + g'^2$$

**Second eigenvector:**

$$\begin{pmatrix} g_W^2 - g_W^2 - g'^2 & -g_W g' \\ -g_W g' & g'^2 - g_W^2 - g'^2 \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix}$$

$$-pg'^2 - g_W g' q = 0 \rightarrow p = \frac{-g_W}{g'} q \rightarrow Z_\mu = g_W W_\mu^3 - g' B_\mu$$

**Normalize:**

$$Z_\mu = \frac{g_W W_\mu^3 - g' B_\mu}{\sqrt{g_W^2 + g'^2}}$$

# Neutral gauge bosons



$$A_\mu = \frac{g' W^3_\mu + g_W B_\mu}{\sqrt{g_W^2 + g'^2}} \qquad Z_\mu = \frac{g_W W^3_\mu - g' B_\mu}{\sqrt{g_W^2 + g'^2}}$$

$$\frac{g'}{g_W} = \tan\theta_W \to \sqrt{g_W^2 + g'^2} = \sqrt{g_W^2 + g_W^2 \tan^2\theta}$$

$$\sqrt{g_W^2 + g'^2} = \sqrt{g_W^2 (1 + \tan^2\theta)} = \frac{g_W}{\cos\theta}$$

$$A_\mu = \frac{g' W^3_\mu + g_W B_\mu}{\sqrt{g_W^2 + g'^2}} = \frac{\cos\theta (g' W^3_\mu + g_W B_\mu)}{g_W}$$

$$A_\mu = \cos\theta \left(\frac{g'}{g_W} W^3_\mu + B_\mu\right) = \cos\theta (\tan\theta W^3_\mu + B_\mu)$$

**Exactly what we proposed earlier!**

$$A_\mu = \sin\theta W^3_\mu + \cos\theta B_\mu$$

# Neutral gauge bosons



$$A_\mu = \frac{g' W_\mu^3 + g_W B_\mu}{\sqrt{g_W^2 + g'^2}} \qquad Z_\mu = \frac{g_W W_\mu^3 - g' B_\mu}{\sqrt{g_W^2 + g'^2}}$$

$$\frac{g'}{g_W} = \tan\theta_W \rightarrow \sqrt{g_W^2 + g'^2} = \sqrt{g_W^2 + g_W^2 \tan^2\theta}$$

$$\sqrt{g_W^2 + g'^2} = \sqrt{g_W^2 (1 + \tan^2\theta)} = \frac{g_W}{\cos\theta}$$

$$Z_\mu = \frac{g_W W_\mu^3 - g' B_\mu}{\sqrt{g_W^2 + g'^2}} = \frac{\cos\theta}{g_W}(g_W W_\mu^3 - g' B_\mu)$$

$$Z_\mu = \cos\theta W_\mu^3 - \frac{g'\cos\theta}{g_W} B_\mu = \cos\theta W_\mu^3 - \cos\theta\tan\theta B_\mu$$

**Exactly what we proposed earlier!**

$$Z_\mu = \cos\theta W_\mu^3 - \sin\theta B_\mu$$

# In addition



$$\sqrt{g_W^2 + g'^2} = \frac{g_W}{\cos\theta} \qquad m_A = 0, m_Z = \frac{1}{2} v \sqrt{g'^2 + g_W^2}$$

**So that** $$m_Z = \frac{v g_W}{2\cos\theta}$$ **From before** $$m_W = \frac{1}{2} g_W v$$

$$\frac{m_W}{m_Z} = \cos\theta_w$$

Plug in measured values and find $v = 246$ GeV

# Similarly

Find that mass terms for the fermions do not observe the [SU(2)$_L$ x U(1)$_Y$] symmetry. But interactions with the Higgs field allow them to obtain mass in the same way!

$$\mathcal{L}_{\text{fermion mass}} = -\lambda_f[\overline{\psi}_L \phi \psi_R + \overline{\psi}_R \overline{\phi} \psi_L]$$

$$\phi(vacuum) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\phi \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

# Fermion mass term for electrons (example)



$$\mathcal{L}_{\text{fermion mass}} = -\lambda_f [\overline{\psi}_L \phi \psi_R + \overline{\psi}_R \overline{\phi} \psi_L]$$

$$\mathcal{L}_{\text{fermion mass}} = -\lambda_f \left[ (\overline{\nu}, \overline{e})_L \begin{pmatrix} 0 \\ v + h \end{pmatrix} e_R + \overline{e}_R (0, v + h) \begin{pmatrix} \nu \\ e \end{pmatrix}_L \right]$$

Fermion-specific Higgs Yukawa coupling

$$\mathcal{L}_{\text{fermion mass}} = -\lambda_f \left[ \overline{e}_L (v + h) e_r + \overline{e}_R (v + h) e_L \right]$$

$$\mathcal{L}_{\text{fermion mass}} = -\lambda_f (v + h) \left[ \overline{e}_L e_R + \overline{e}_R e_L \right] = -\lambda_f (v + h) (\overline{e} e)$$

# Fermion mass term for electrons (example)



$$\mathcal{L}_{\text{fermion mass}} = -\lambda_f (v + h)\left[\bar{e}_L e_R + \bar{e}_R e_L\right] = -\lambda_f (v + h)(\bar{e}e)$$

$$\mathcal{L}_{\text{fermion mass}} = -\lambda_f v\bar{e}e + -\lambda_f h\bar{e}e$$

**Electron mass term**

**Electron-Higgs boson vertex, with interaction strength proportional to electron mass (very small)**

**Note something - where did the neutrino mass term go?**

# Some thoughts on Fermion masses

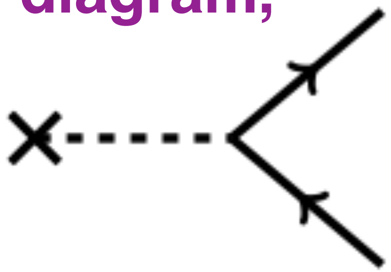


We have a new idea of mass in particle physics - how much the object has a well-defined helicity. This is measured by chirality

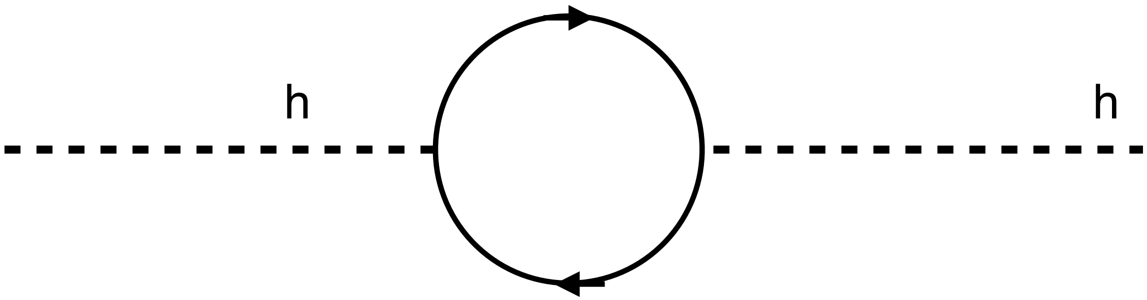
We've seen that the left-handed electron and right-handed electron are different objects (the first interacts with the W boson, the second does not)

Similarly, the left-handed positron and right-handed positron are different objects (the second interacts with the W boson, the first does not)

The Higgs field contains weak charge, and the non-zero value of the field allows a left-handed electron to convert to a right-handed electron. It's just like any other Feynman diagram, converting one object into another



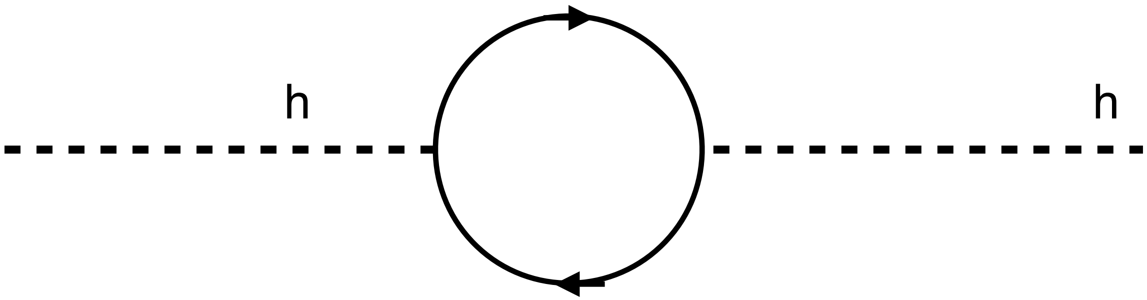
# The Hierarchy problem




Find that diagrams like this (with any objects running through the loop) contribute to the Higgs boson mass and self-energy. Naively, these can have energies up through the Planck scale ($10^{19}$ GeV). But the Higgs boson mass is only 125 GeV - why?!?! Some diagrams cancel with opposite sign, but getting a ~$10^2$ number from integrals involving $10^{19}$ is: 1) lucky, 2) coincidence, 3) pointing to some hidden, deep symmetry.

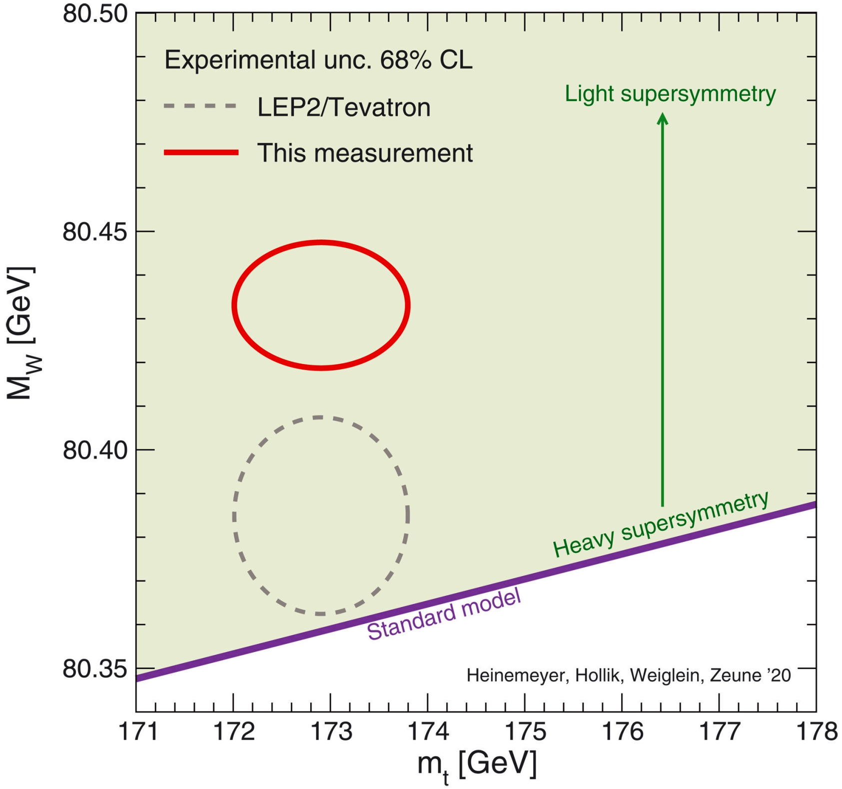
One strong motivation for supersymmetry (SUSY), among other theories. In SUSY, every SM boson has a SUSY boson counterpart (bosino), and every SM fermion has a SUSY fermion counterpart (sfermion). This symmetry is obviously broken, but these then have opposite signs in cancellations and can explain the small Higgs boson mass (and thus the small electroweak mass scale).

# Connections and predictions



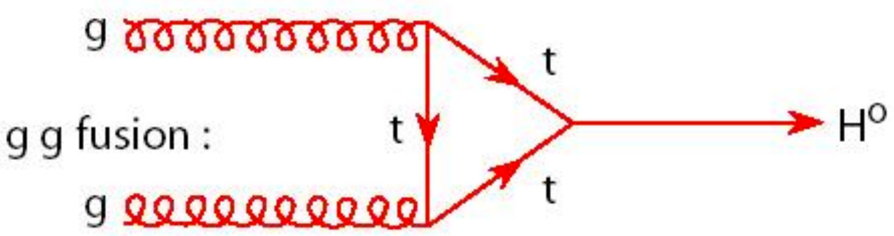

Given a Higgs boson mass, there is a strong SM prediction between the top quark mass and the W boson mass (both connect to the Higgs potential). CDF just published a new result, and it provides lots of tension - new physics somewhere?!?!

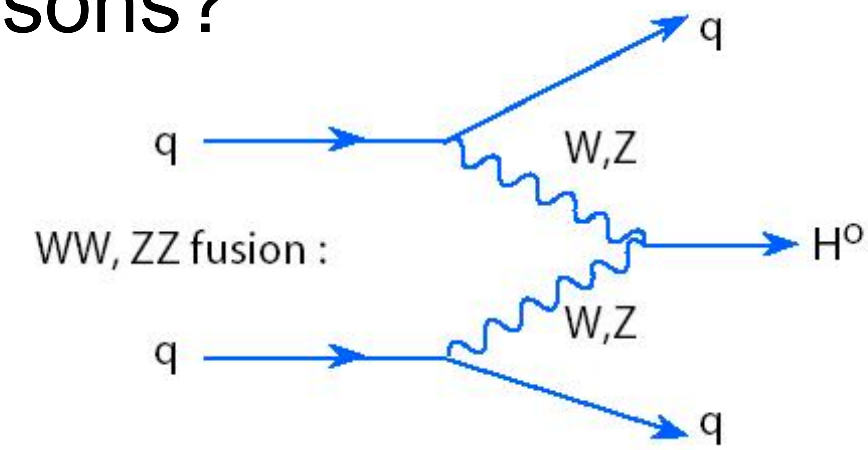
https://www.science.org/doi/10.1126/science.abk1781

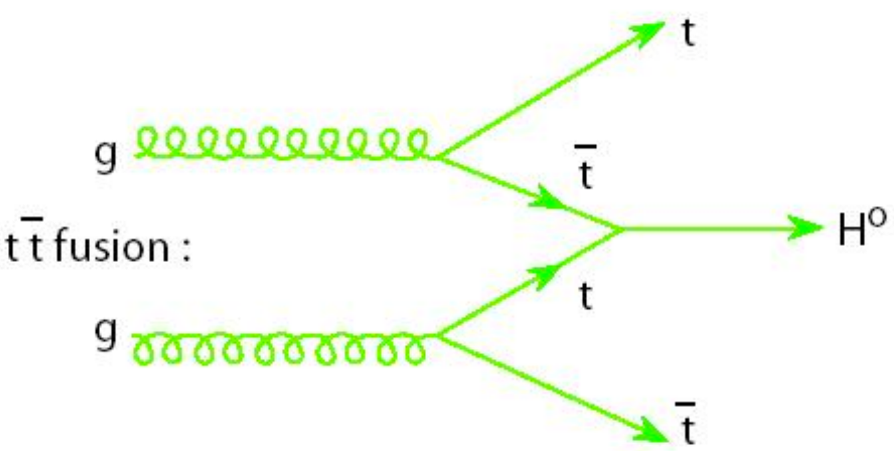


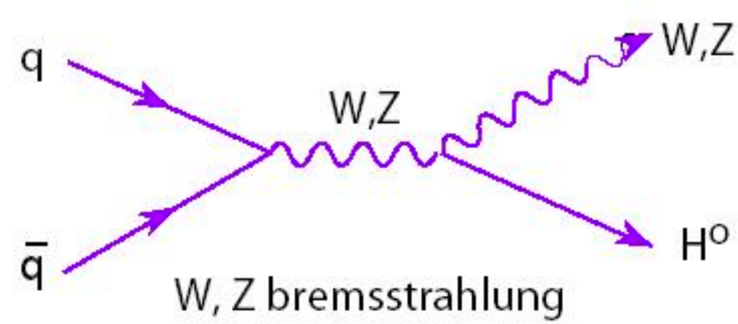
http://sites.uci.edu/energyobserver/2012/11/26/higgs-production-and-decay-channels/

## How does the LHC produce Higgs bosons?



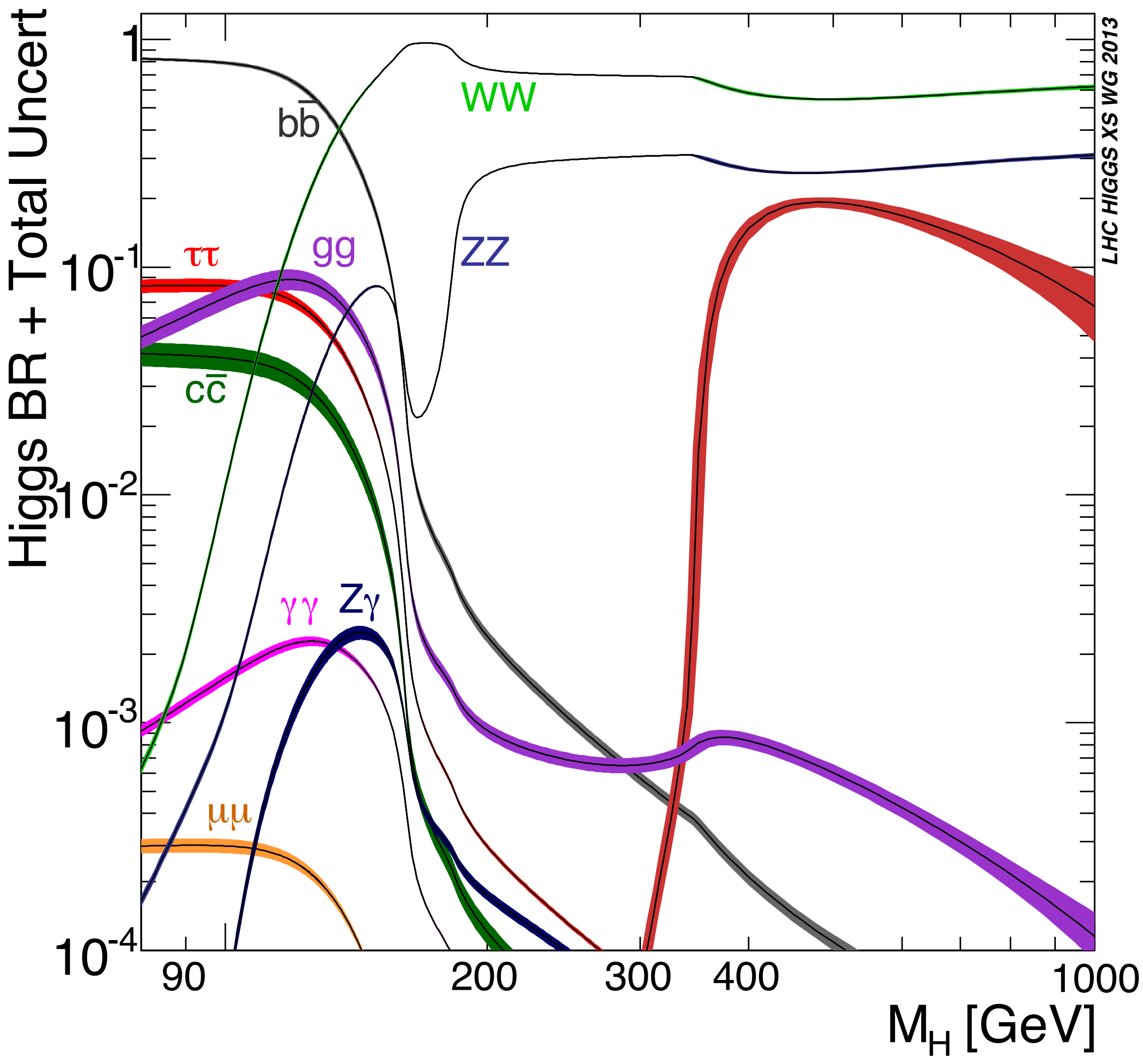



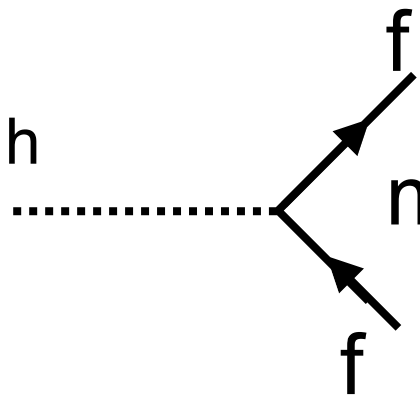
# Higgs boson decay?



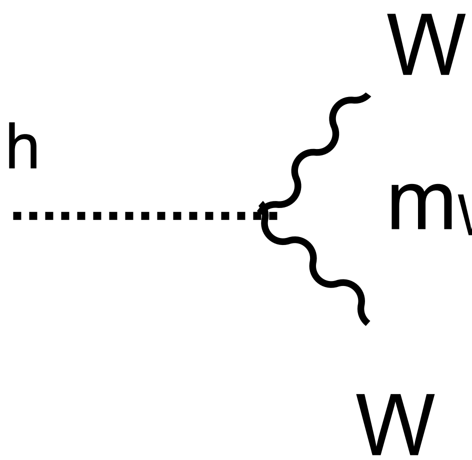



LHC Higgs XS Working Group

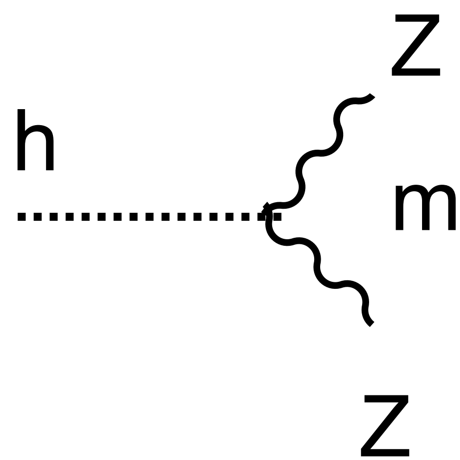
# Where do decay rates come from?




$m_f \, g_W / 2m_W$

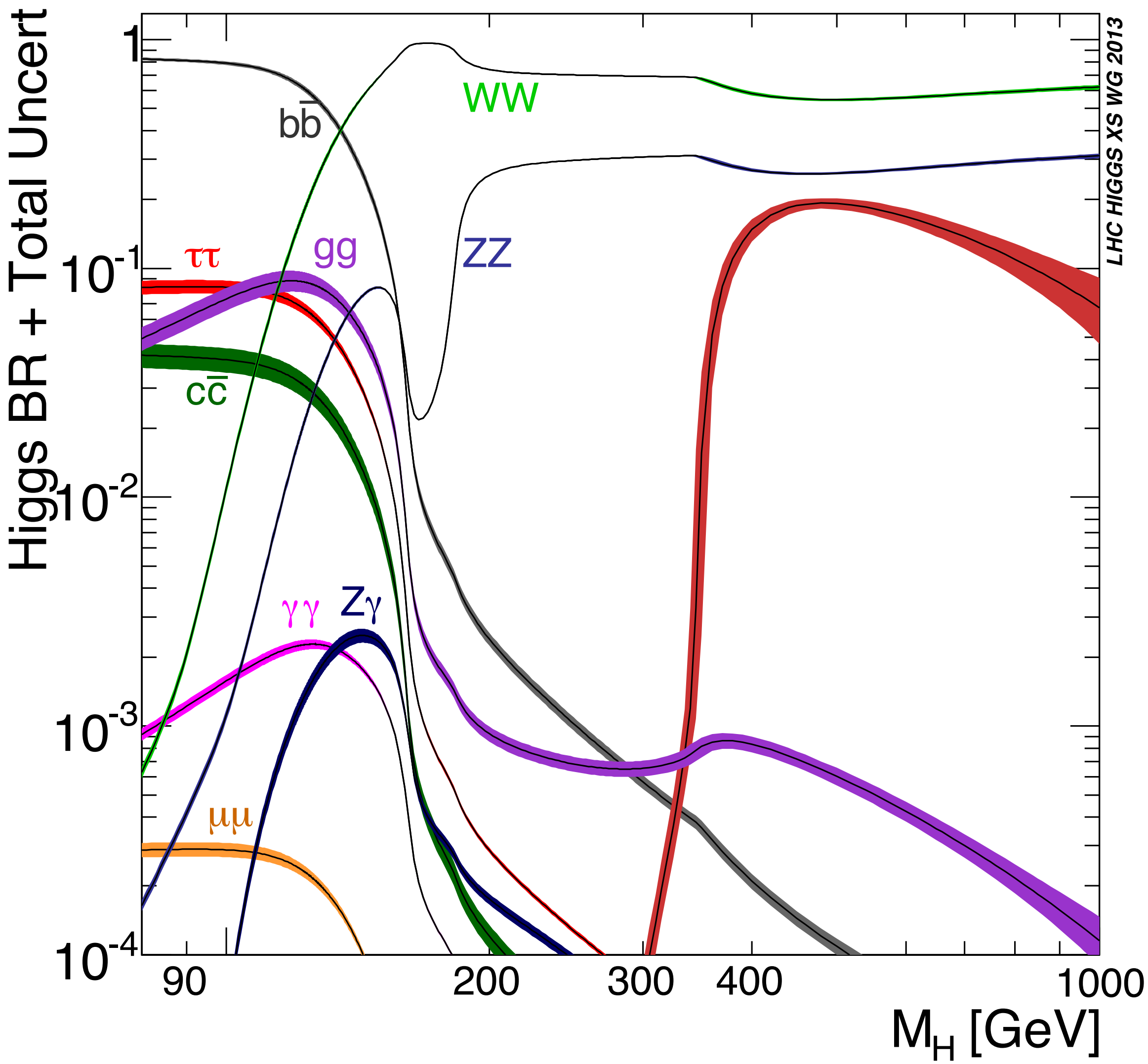



$m_W \, g_W$




$m_Z \, g_W / \cos\theta_W$

# Higgs boson decay?




$\Gamma(h \rightarrow bb)$:

$\Gamma(h \rightarrow cc)$:

$\Gamma(h \rightarrow \tau\tau)$~

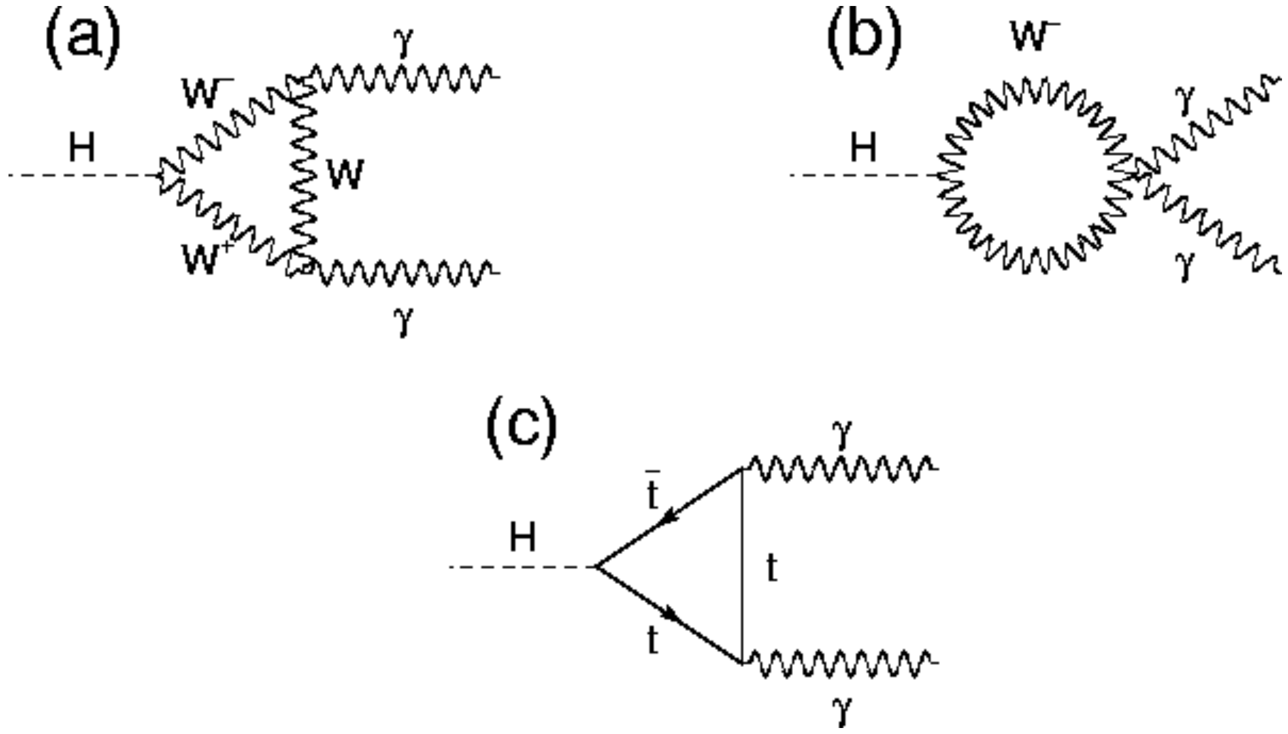
$3m_b^2$:$3m_c^2$:

$m_\tau^2$:

# How do Higgs bosons decay to gluons or photons, then?



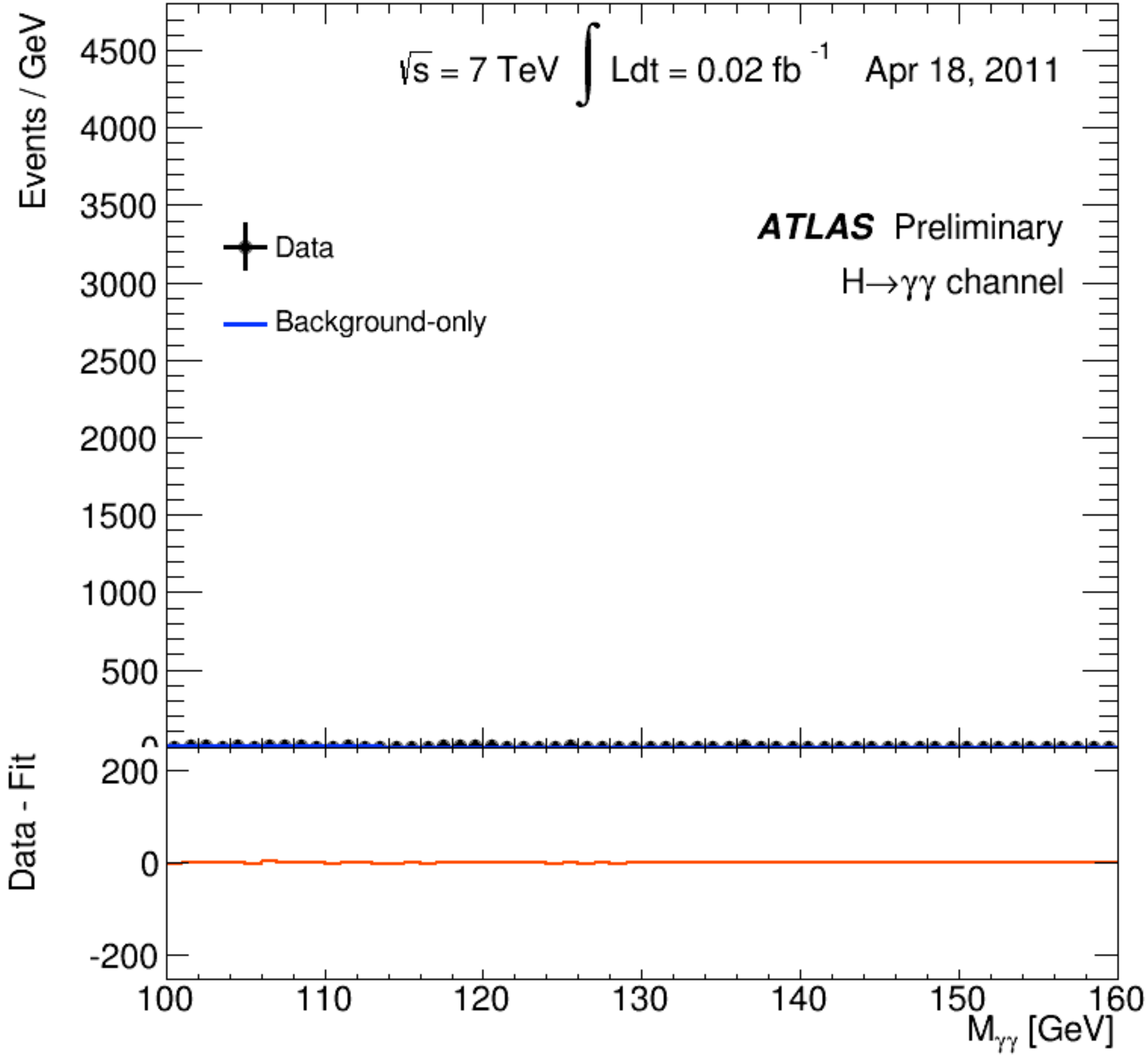
http://www.hep.lu.se/atlas/thesis/egede/thesis-node17.html




Indirect, induced! couplings Diagram (c) can also produce decays to gluons

# An animation that you've probably all seen already

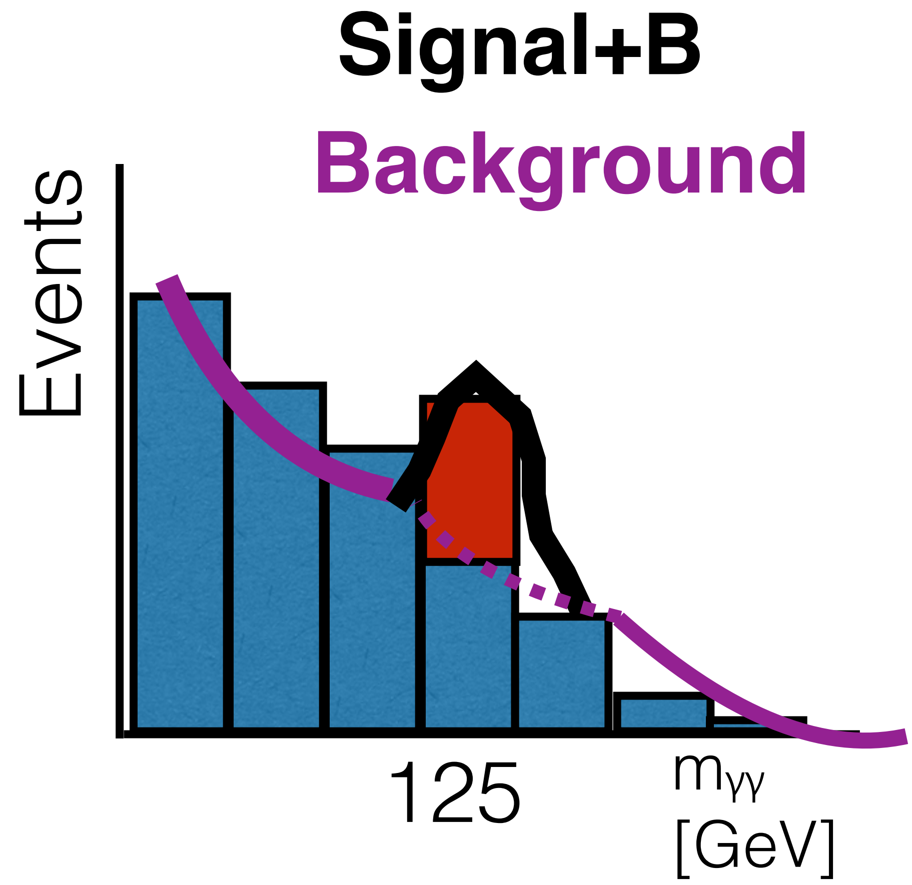




Bump-hunting in various bins of kinematic quantities

Take advantage of nearly 100% efficient diphoton triggers

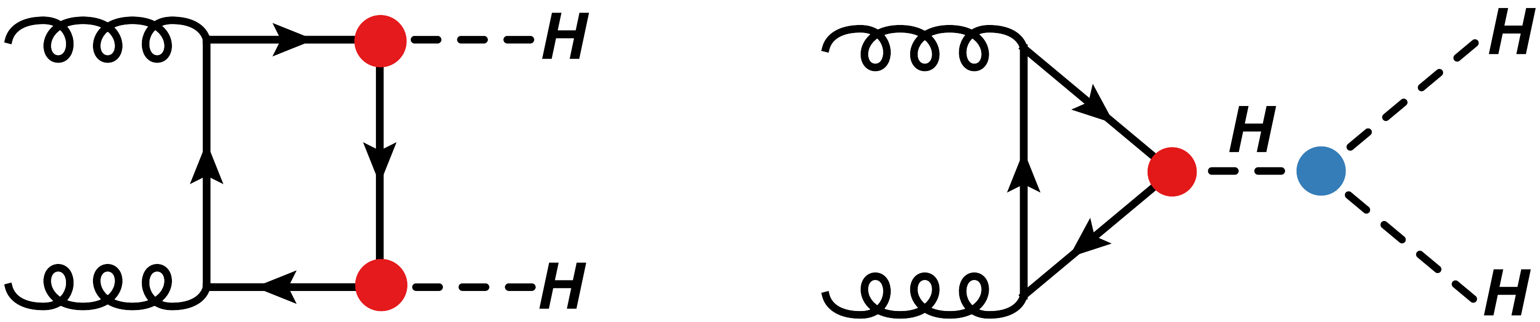
Require two tight, isolated, high-$p_T$ photons offline




**Disadvantage: Low BR, S/B still not great**

# Looking for pair production of Higgs bosons

arXiv:1807.04873

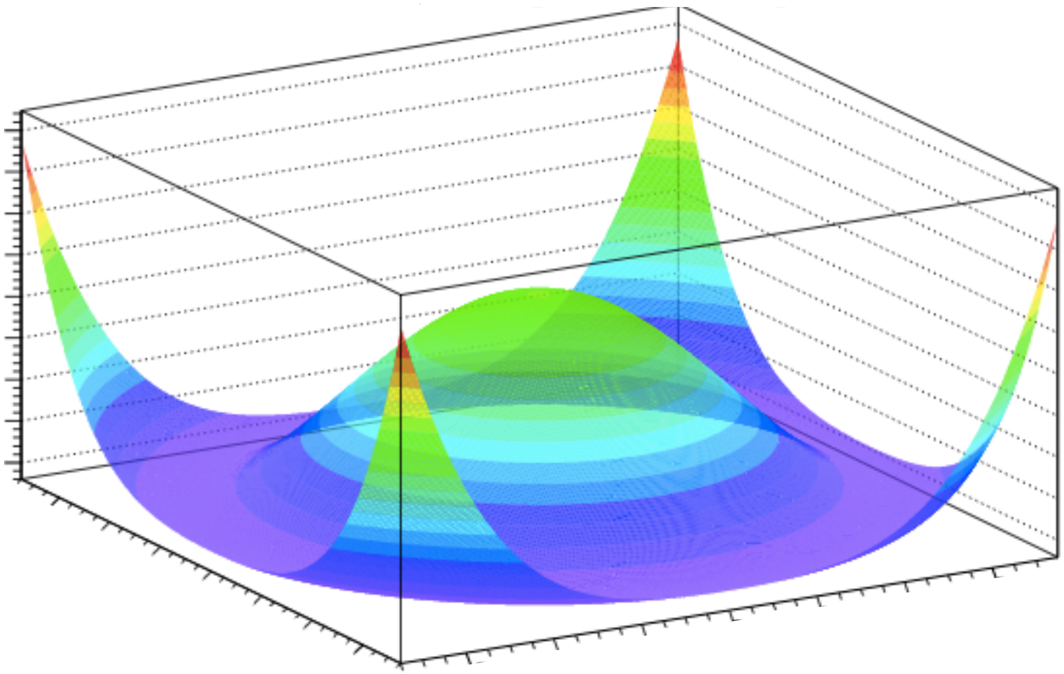


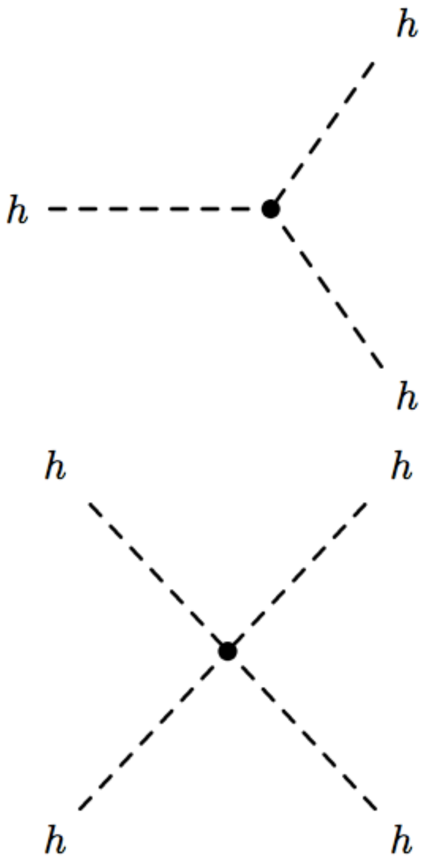

**Pair production of Higgs bosons directly probes the Higgs boson-self coupling, and thus EWSB and also the non-zero vacuum expectation value of the Higgs field! Problem: The above two diagrams contribute, and only one is of interest. And they interfere destructively**

# A long-term LHC goal

arXiv:1212.5581 (Baglio et al), among many

Observe the Higgs boson self-coupling, crucial to testing if the Higgs potential is the one predicted in the SM





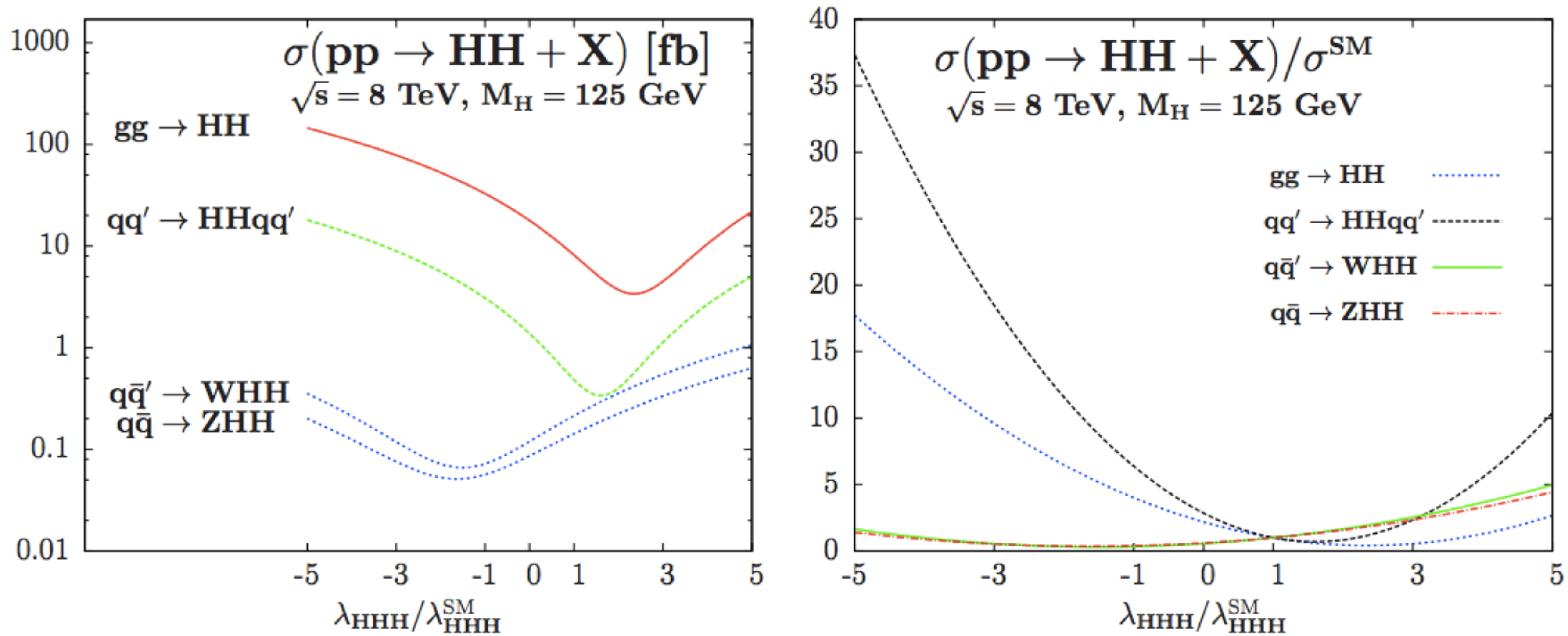

**Expand Lagrangian to higher orders…**

$$\mathcal{L}_V = -\lambda v^2 h^2 - \lambda v h^3 - \frac{\lambda}{4} h^4$$

**Need HL-LHC data sets to observe the process**

# What about extensions to the SM?

arXiv:1212.5581
(Baglio et al)

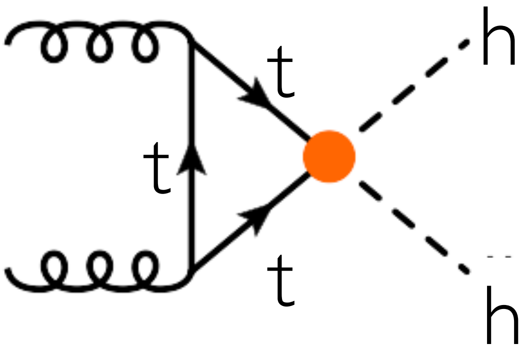


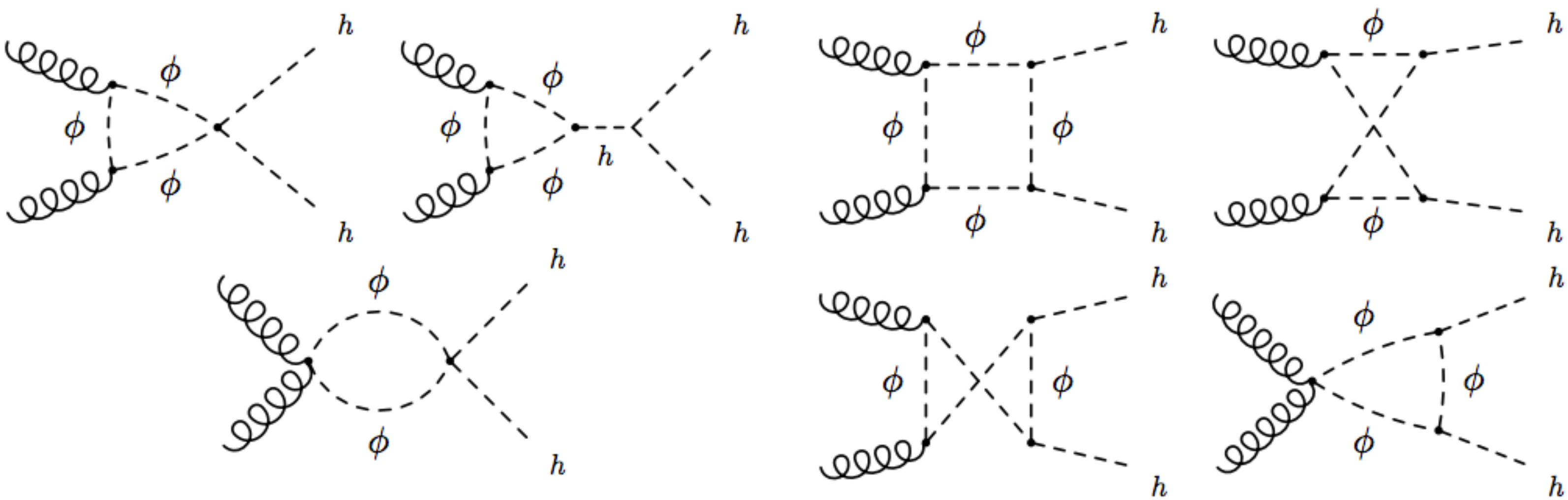

Altered self-coupling can significantly increase hh production rates

1205.5444 (Contino et al) 1207.4496 (Kribs and Martin), 1212.5581 (Baglio et al), 1512.00068 (Huang et al) among many



- Can enhance non-resonant hh production in many extensions to the SM
  - tthh interactions, light colored scalars, if Higgs boson self-coupling were altered, or if top quark had non-standard Yukawa coupling
    - Study EW phrase transition!



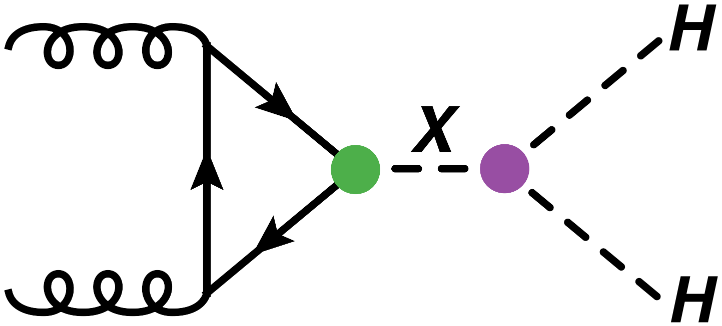



# Looking for pair production of Higgs bosons



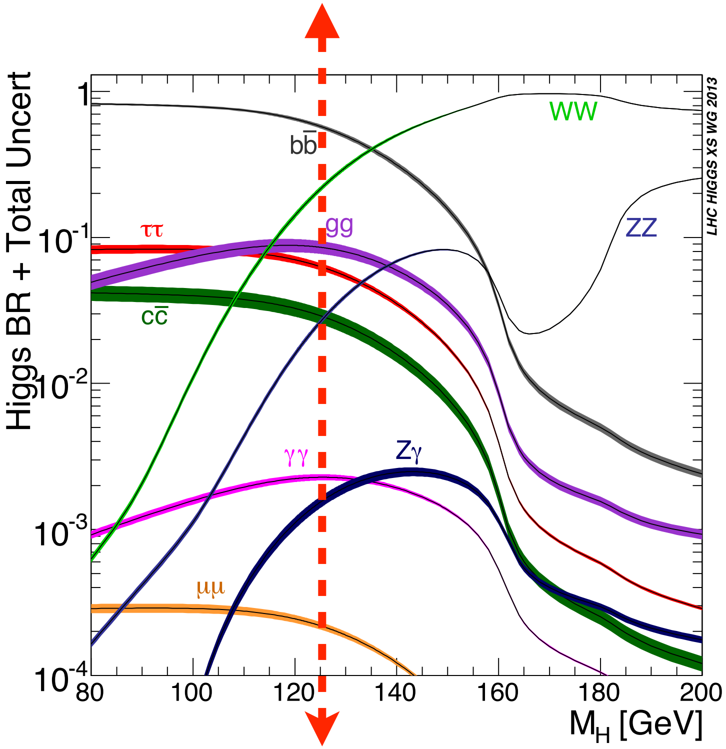
hep-ph/0009232 (Cheung), hep-ph/0503173 (Djouadi), 1210.8166 (Dolan et al), 1206.6949 (Tang), 1404.0996 (Kumar & Martin), among many

**Can enhance hh production resonantly as well:**
**Two Higgs doublet models, Randall-Sundrum gravitons, radions, stoponium, ...**



# Looking for hh→bbγγ. Why?

- At known $m_h$, h→bb has highest Higgs BR (0.57)
- h→γγ has high efficiency and good mass resolution
  - Can perform full mass reconstruction
- h→bb, h→$\tau\tau$ and h→WW have poor mass resolution vs γγ
- h→ZZ few events after require leptonic decays
- Sensitive to lower mass resonances and also the region testing hhh vertex

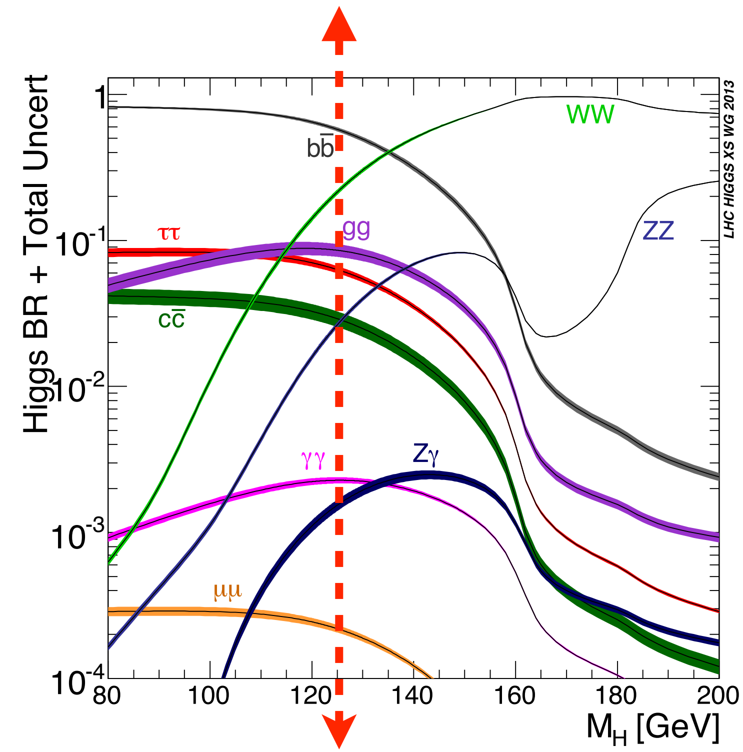



arXiv: 1307.1345

# Looking for hh→bbγγ

arXiv:1408.7084

- Start with common ATLAS h→γγ selection
  - Loose diphoton trigger nearly 100% efficient for offline cuts
  - $E_T > 0.35(0.25)m_{\gamma\gamma}$ for leading (subleading) photon
  - $|\eta| < 2.37$ excluding $1.37 < |\eta| < 1.56$




arXiv: 1307.1345

# Jets and b-tagging

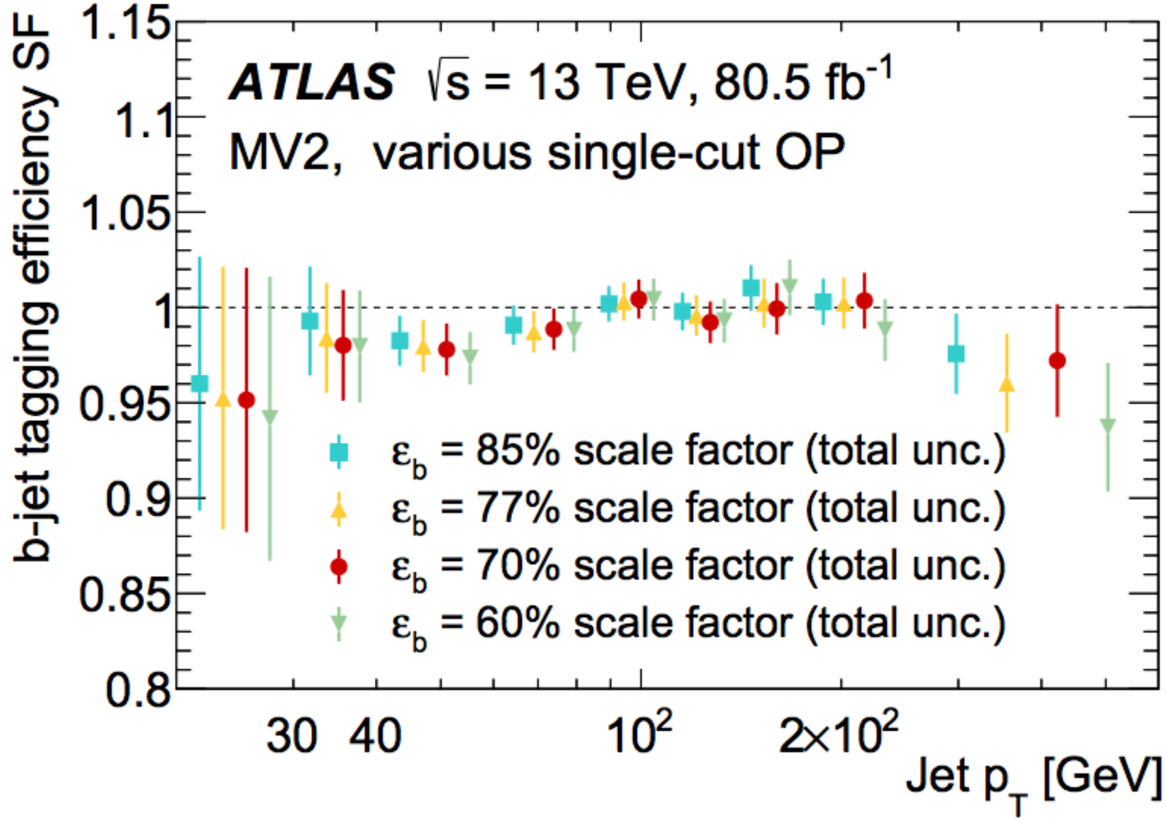
arXiv: 1907.05120

- Require two anti-$k_T$ R=0.4 jets with |η| < 2.5 and pT > 25 GeV
- Perform b-tagging using Boosted Decision Tree b-tagger at several efficiency working points
  - Calibrate b-tag scale factors using dilepton ttbar events

Table 1: Input variables used by the MV2 and the DL1 algorithms. The JetFitter *c*-tagging variables are only used by the DL1 algorithm.

| Input | Variable | Description |
|---|---|---|
| Kinematics | $p_T$ | Jet $p_T$ |
| | $\eta$ | Jet $\|\eta\|$ |
| IP2D/IP3D | $\log(P_b/P_{\mathrm{light}})$ | Likelihood ratio between the *b*-jet and light-flavour jet hypotheses |
| | $\log(P_b/P_c)$ | Likelihood ratio between the *b*- and *c*-jet hypotheses |
| | $\log(P_c/P_{\mathrm{light}})$ | Likelihood ratio between the *c*-jet and light-flavour jet hypotheses |
| SV1 | $m(\mathrm{SV})$ | Invariant mass of tracks at the secondary vertex assuming pion mass |
| | $f_E(\mathrm{SV})$ | Energy fraction of the tracks associated with the secondary vertex |
| | $N_{\mathrm{TrkAtVtx}}(\mathrm{SV})$ | Number of tracks used in the secondary vertex |
| | $N_{\mathrm{2TrkVtx}}(\mathrm{SV})$ | Number of two-track vertex candidates |
| | $L_{xy}(\mathrm{SV})$ | Transverse distance between the primary and secondary vertex |
| | $L_{xyz}(\mathrm{SV})$ | Distance between the primary and the secondary vertex |
| | $S_{xyz}(\mathrm{SV})$ | Distance between the primary and the secondary vertex divided by its uncertainty |
| | $\Delta R(\vec{p}_{\mathrm{jet}}, \vec{p}_{\mathrm{vtx}})(\mathrm{SV})$ | $\Delta R$ between the jet axis and the direction of the secondary vertex relative to the primary vertex. |
| JetFitter | $m(\mathrm{JF})$ | Invariant mass of tracks from displaced vertices |
| | $f_E(\mathrm{JF})$ | Energy fraction of the tracks associated with the displaced vertices |
| | $\Delta R(\vec{p}_{\mathrm{jet}}, \vec{p}_{\mathrm{vtx}})(\mathrm{JF})$ | $\Delta R$ between the jet axis and the vectorial sum of momenta of all tracks attached to displaced vertices |
| | $S_{xyz}(\mathrm{JF})$ | Significance of the average distance between PV and displaced vertices |
| | $N_{\mathrm{TrkAtVtx}}(\mathrm{JF})$ | Number of tracks from multi-prong displaced vertices |
| | $N_{\mathrm{2TrkVtx}}(\mathrm{JF})$ | Number of two-track vertex candidates (prior to decay chain fit) |
| | $N_{\mathrm{1\text{-}trk\ vertices}}(\mathrm{JF})$ | Number of single-prong displaced vertices |
| | $N_{\geq 2\text{-}\mathrm{trk\ vertices}}(\mathrm{JF})$ | Number of multi-prong displaced vertices |
| JetFitter *c*-tagging | $L_{xyz}(2^{\mathrm{nd}}/3^{\mathrm{rd}}\mathrm{vtx})(\mathrm{JF})$ | Distance of $2^{\mathrm{nd}}$ or $3^{\mathrm{rd}}$ vertex from PV |
| | $L_{xy}(2^{\mathrm{nd}}/3^{\mathrm{rd}}\mathrm{vtx})(\mathrm{JF})$ | Transverse displacement of the $2^{\mathrm{nd}}$ or $3^{\mathrm{rd}}$ vertex |
| | $m_{\mathrm{Trk}}(2^{\mathrm{nd}}/3^{\mathrm{rd}}\mathrm{vtx})(\mathrm{JF})$ | Invariant mass of tracks associated with $2^{\mathrm{nd}}$ or $3^{\mathrm{rd}}$ vertex |
| | $E_{\mathrm{Trk}}(2^{\mathrm{nd}}/3^{\mathrm{rd}}\mathrm{vtx})(\mathrm{JF})$ | Energy fraction of the tracks associated with $2^{\mathrm{nd}}$ or $3^{\mathrm{rd}}$ vertex |
| | $f_E(2^{\mathrm{nd}}/3^{\mathrm{rd}}\mathrm{vtx})(\mathrm{JF})$ | Fraction of charged jet energy in $2^{\mathrm{nd}}$ or $3^{\mathrm{rd}}$ vertex |
| | $N_{\mathrm{TrkAtVtx}}(2^{\mathrm{nd}}/3^{\mathrm{rd}}\mathrm{vtx})(\mathrm{JF})$ | Number of tracks associated with $2^{\mathrm{nd}}$ or $3^{\mathrm{rd}}$ vertex |
| | $Y_{\mathrm{trk}}^{\mathrm{min}}, Y_{\mathrm{trk}}^{\mathrm{max}}, Y_{\mathrm{trk}}^{\mathrm{avg}}(2^{\mathrm{nd}}/3^{\mathrm{rd}}\mathrm{vtx})(\mathrm{JF})$ | Min., max. and avg. track rapidity of tracks at $2^{\mathrm{nd}}$ or $3^{\mathrm{rd}}$ vertex |

- Moving to more advanced DL1 tagger in the next iteration of the analysis

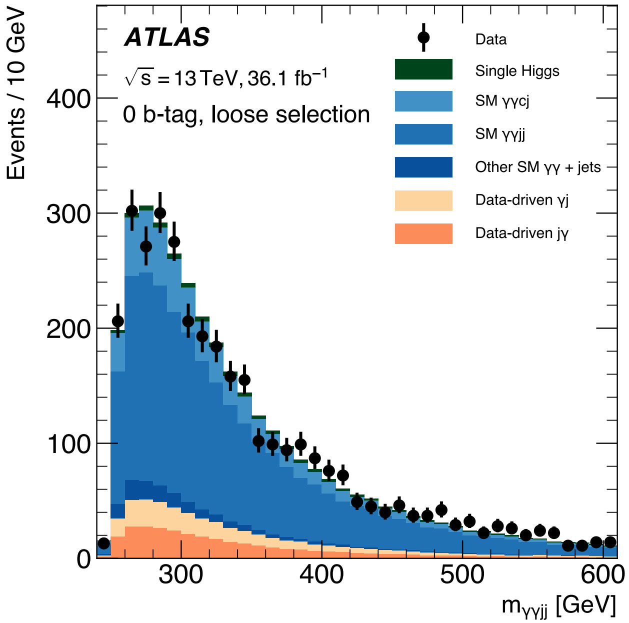



| $\epsilon_b$ | MV2 | | | | DL1 | | | |
|---|---|---|---|---|---|---|---|---|
| | Selection | Rejection | | | Selection | Rejection | | |
| | | $c$-jet | $\tau$-jet | Light-flavour jet | | $c$-jet | $\tau$-jet | Light-flavour jet |
| 60% | > 0.94 | 23 | 140 | 1200 | > 2.74 | 27 | 220 | 1300 |
| 70% | > 0.83 | 8.9 | 36 | 300 | > 2.02 | 9.4 | 43 | 390 |
| 77% | > 0.64 | 4.9 | 15 | 110 | > 1.45 | 4.9 | 14 | 130 |
| 85% | > 0.11 | 2.7 | 6.1 | 25 | > 0.46 | 2.6 | 3.9 | 29 |

# Jets and bb invariant mass



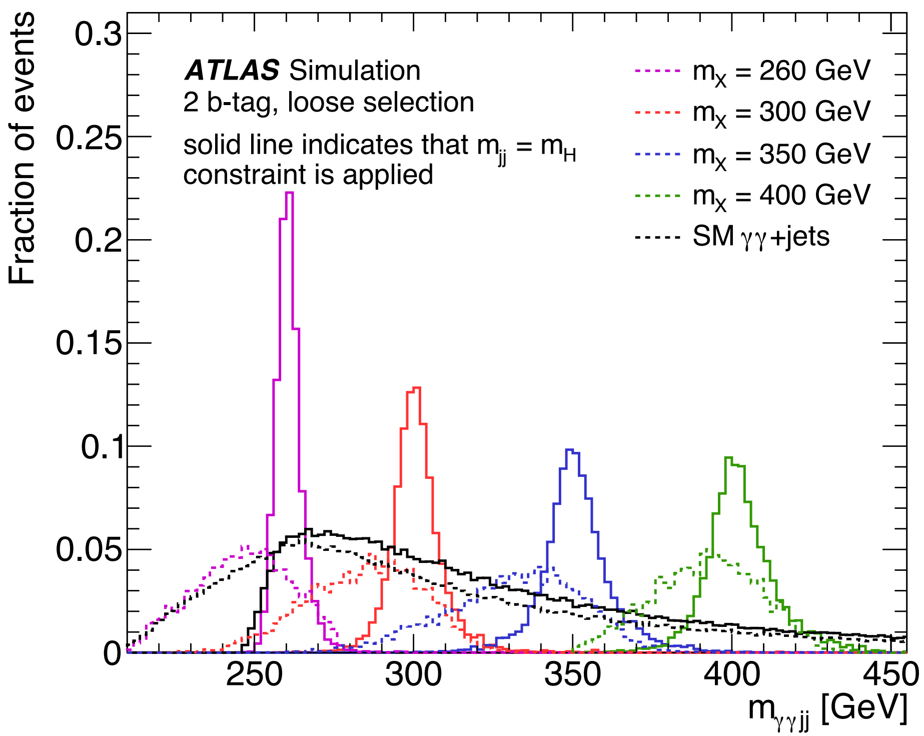
- Loose analysis: Leading jet pT > 40 GeV, subleading pT > 25 GeV after adding in 4-vectors of any muons with pT > 4 GeV with dR < 0.4 to jet
  - Require $80 < m_{bb} < 140$ [GeV]
    - Asymmetric cut optimized in simulation and largely due to energies losses from escaping neutrinos




**0 b-tag data as a good way to test our modeling of background events, including accounting for "fake" photons (done using double sideband method in ID and isolation)**

# Resonance analysis

- Also look for production of X decaying to a pair of higgs bosons
  - First select events with diphoton mass consistent with Higgs boson mass, then look for bumps in 4-body mass




**Use Higgs mass constraint to improve the resonance search**

# Number of events



| | 1-tag | | 2-tag | |
|---|---|---|---|---|
| | Loose selection | Tight selection | Loose selection | Tight selection |
| Continuum background | 117.5 ±4.7 | 15.7 ± 1.6 | 21.0 ± 2.0 | 3.74 ± 0.78 |
| SM single-Higgs-boson background | 5.51 ±0.10 | 2.20 ± 0.05 | 1.63 ± 0.04 | 0.56 ± 0.02 |
| Total background | 123.0 ±4.7 | 17.9 ± 1.6 | 22.6 ± 2.0 | 4.30 ± 0.79 |
| SM Higgs boson pair signal | 0.219±0.006 | 0.120± 0.004 | 0.305± 0.007 | 0.175± 0.005 |
| Data | 125 | 19 | 21 | 3 |

**Only counting events with diphoton mass near mH (125 GeV), dominated by continuum background! Note that in new versions of the analysis we use a BDT to separate signal and background, and this isn't necessarily true anymore**

# Systematic uncertainties

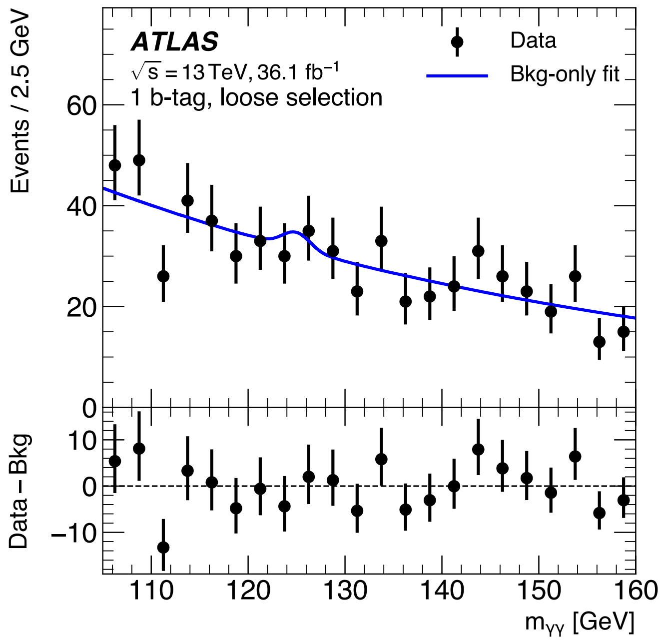


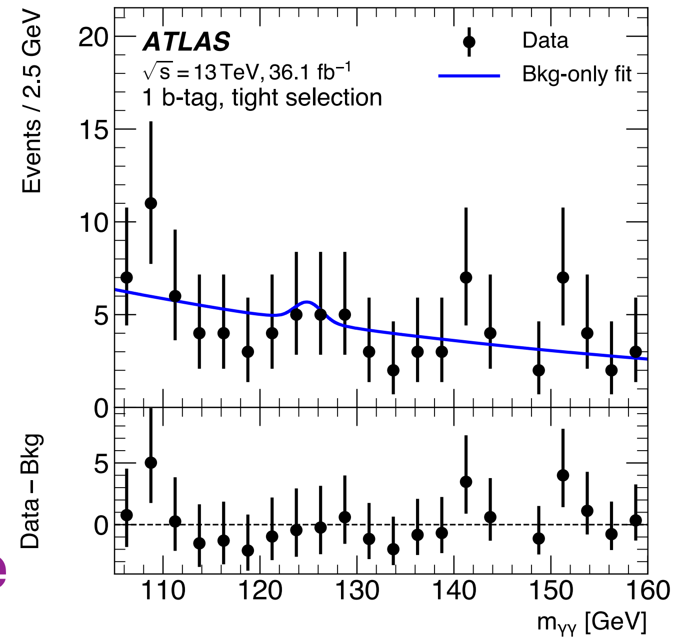
| Source of systematic uncertainty | | % effect relative to nominal in the 2-tag (1-tag) category | | | |
|---|---|---|---|---|---|
| | | Non-resonant analysis | | Resonant analysis: BSM $HH$ | |
| | | SM $HH$ signal | Single-$H$ bkg | Loose selection | Tight selection |
| Luminosity | | ±2.1 (±2.1) | ±2.1 (± 2.1) | ±2.1 (±2.1) | ±2.1 (±2.1) |
| Trigger | | ±0.4 (±0.4) | ±0.4 (± 0.4) | ±0.4 (±0.4) | ±0.4 (±0.4) |
| Pile-up modelling | | ±3.2 (±1.3) | ±2.0 (± 0.8) | ±4.0 (±4.2) | ±4.0 (±3.8) |
| Photon | identification | ±2.5 (±2.4) | ±1.7 (± 1.8) | ±2.6 (±2.6) | ±2.5 (±2.5) |
| | isolation | ±0.8 (±0.8) | ±0.8 (± 0.8) | ±0.8 (±0.8) | ±0.9 (±0.9) |
| | energy resolution | - | - | ±1.0 (±1.3) | ±1.8 (±1.2) |
| | energy scale | - | - | ±0.9 (±3.0) | ±0.9 (±2.4) |
| Jet | energy resolution | ±1.5 (±2.2) | ±2.9 (± 6.4) | ±7.5 (±8.5) | ±6.4 (±6.4) |
| | energy scale | ±2.9 (±2.7) | ±7.8 (± 5.6) | ±3.0 (±3.3) | ±2.3 (±3.4) |
| Flavour tagging | $b$-jets | ±2.4 (±2.5) | ±2.3 (± 1.4) | ±3.4 (±2.6) | ±2.5 (±2.6) |
| | $c$-jets | ±0.1 (±1.0) | ±1.8 (±11.6) | - | - |
| | light-jets | <0.1 (±5.0) | ±1.6 (± 2.2) | - | - |
| Theory | PDF+$\alpha_S$ | ±2.3 (±2.3) | ±3.1 (± 3.3) | n/a | n/a |
| | Scale | +4.3 (+4.3) | +4.9 (+ 5.3) | n/a | n/a |
| | | −6.0 (−6.0) | +7.0 (+ 8.0) | n/a | n/a |
| | EFT | ±5.0 (±5.0) | n/a | n/a | n/a |

**These are all small compared to statistical uncertainties**

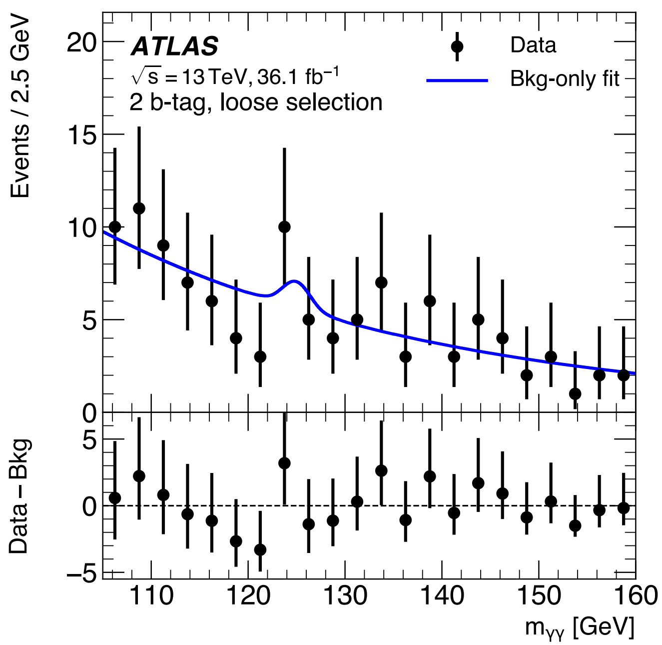
# Results!

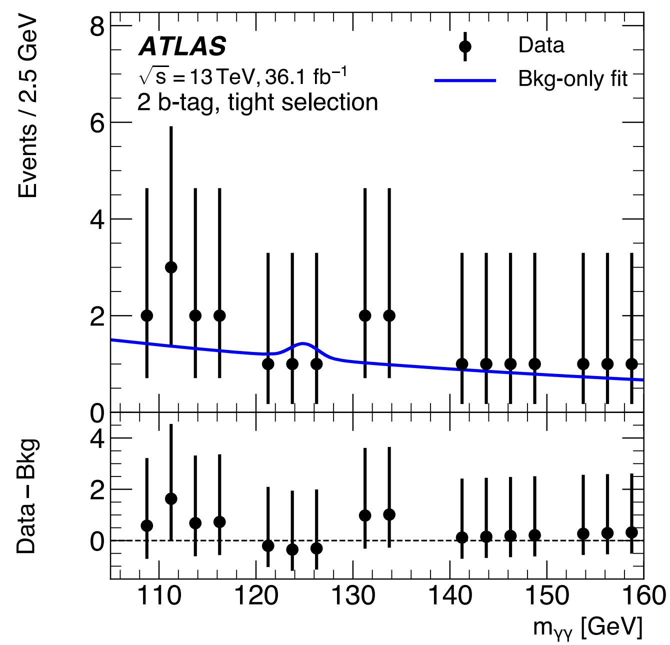




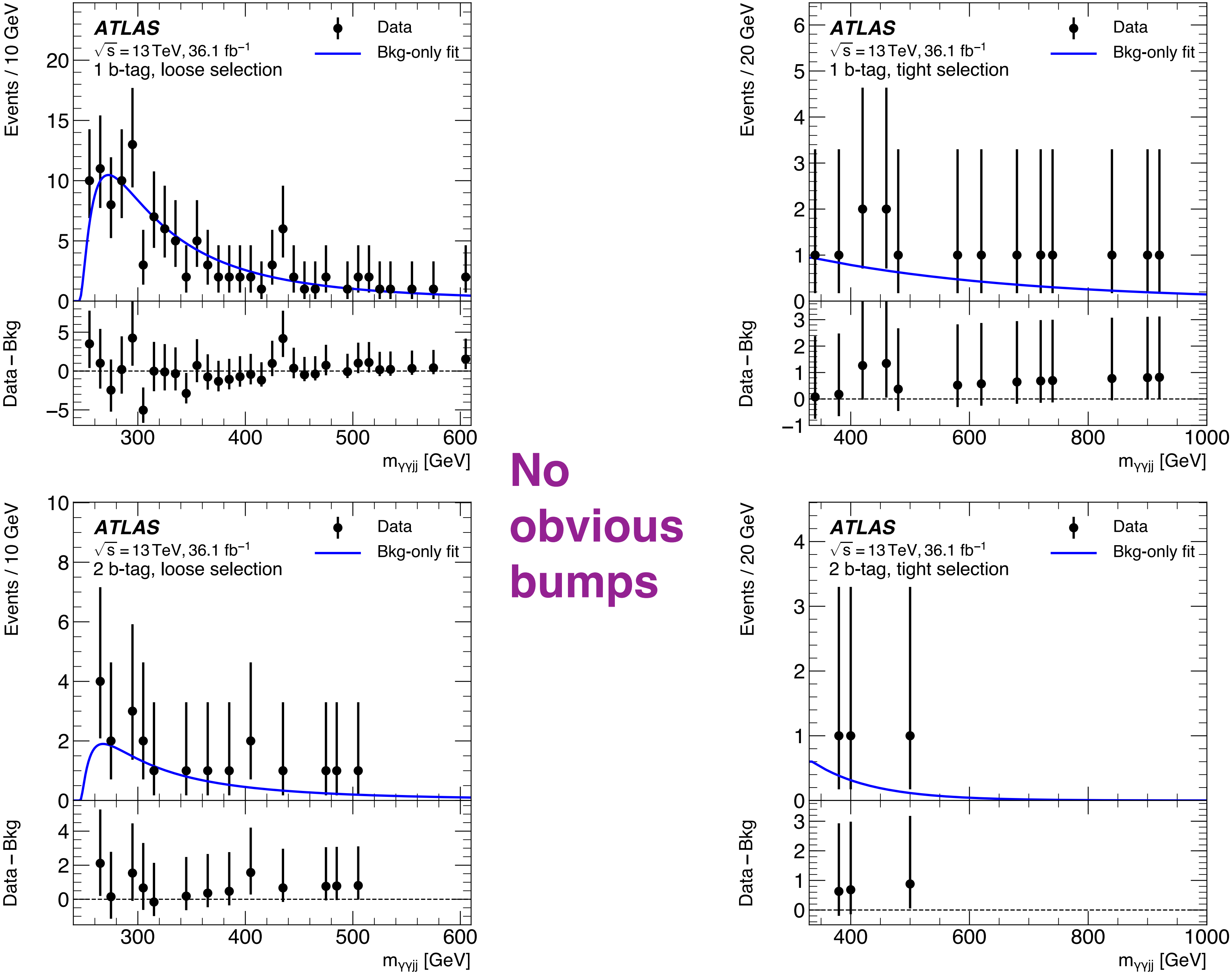




**See single Higgs bosons being fit**



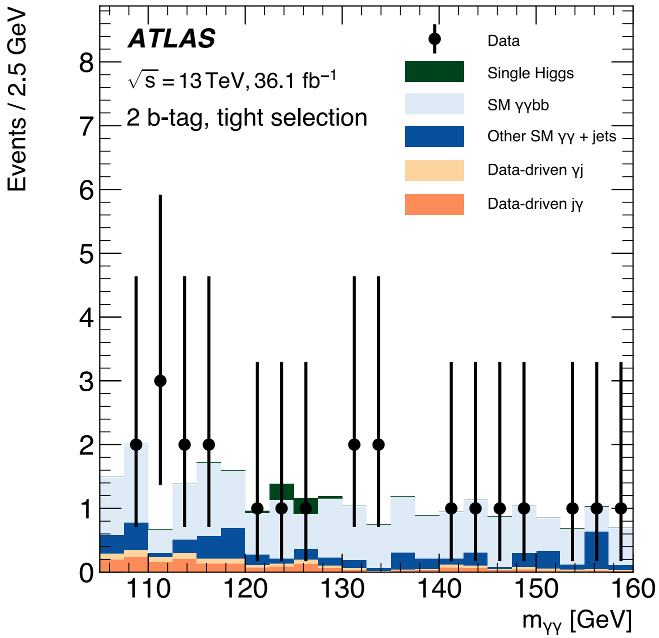
# Resonant analysis plots

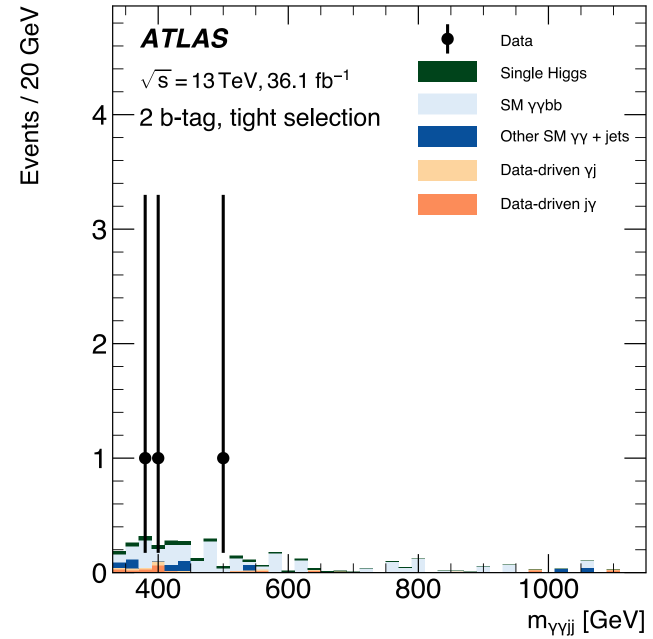



**No obvious bumps**

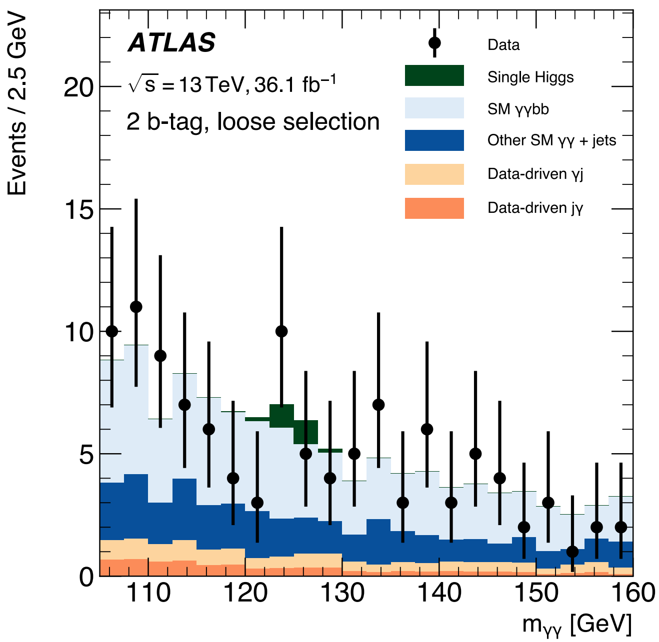
# More ways to look at the data

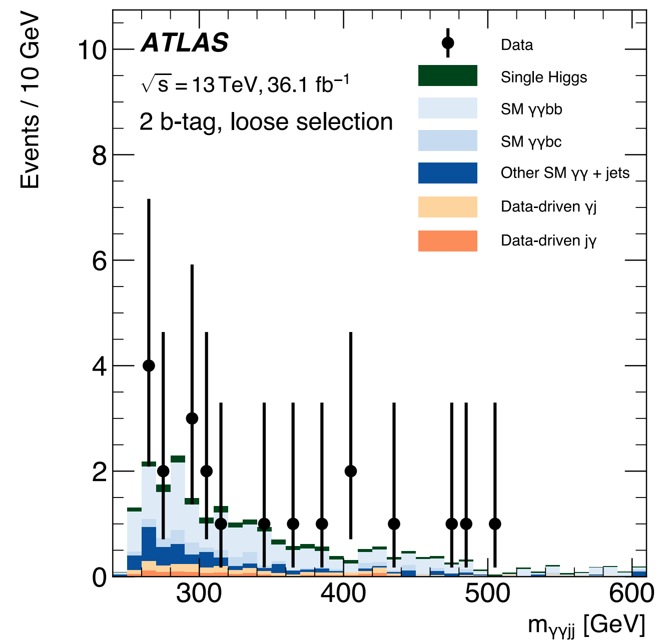




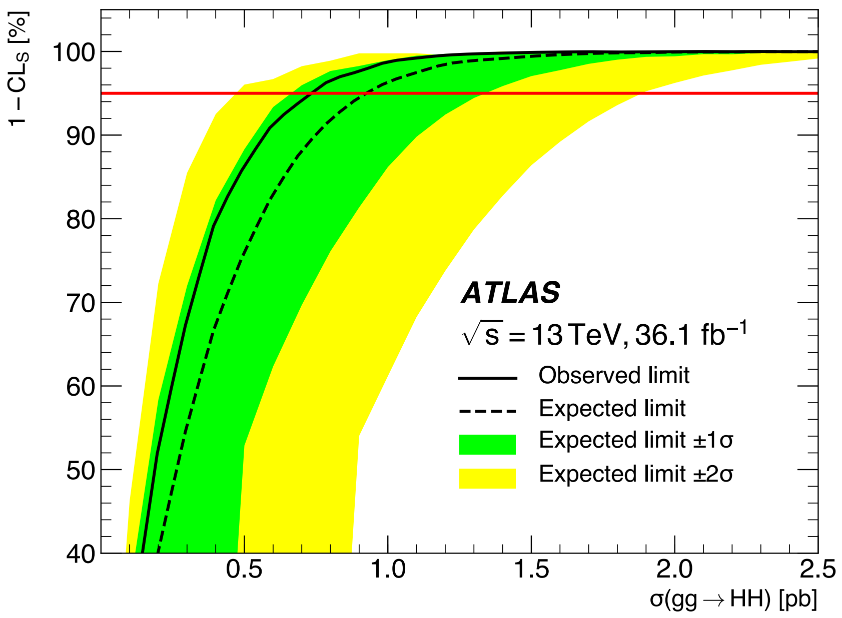



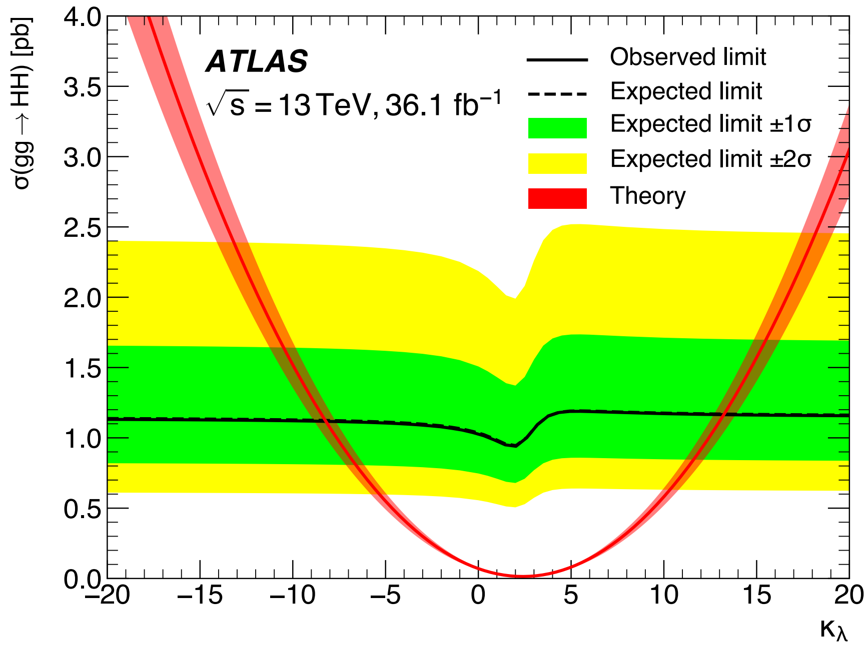

**We have lots more data in hand!**



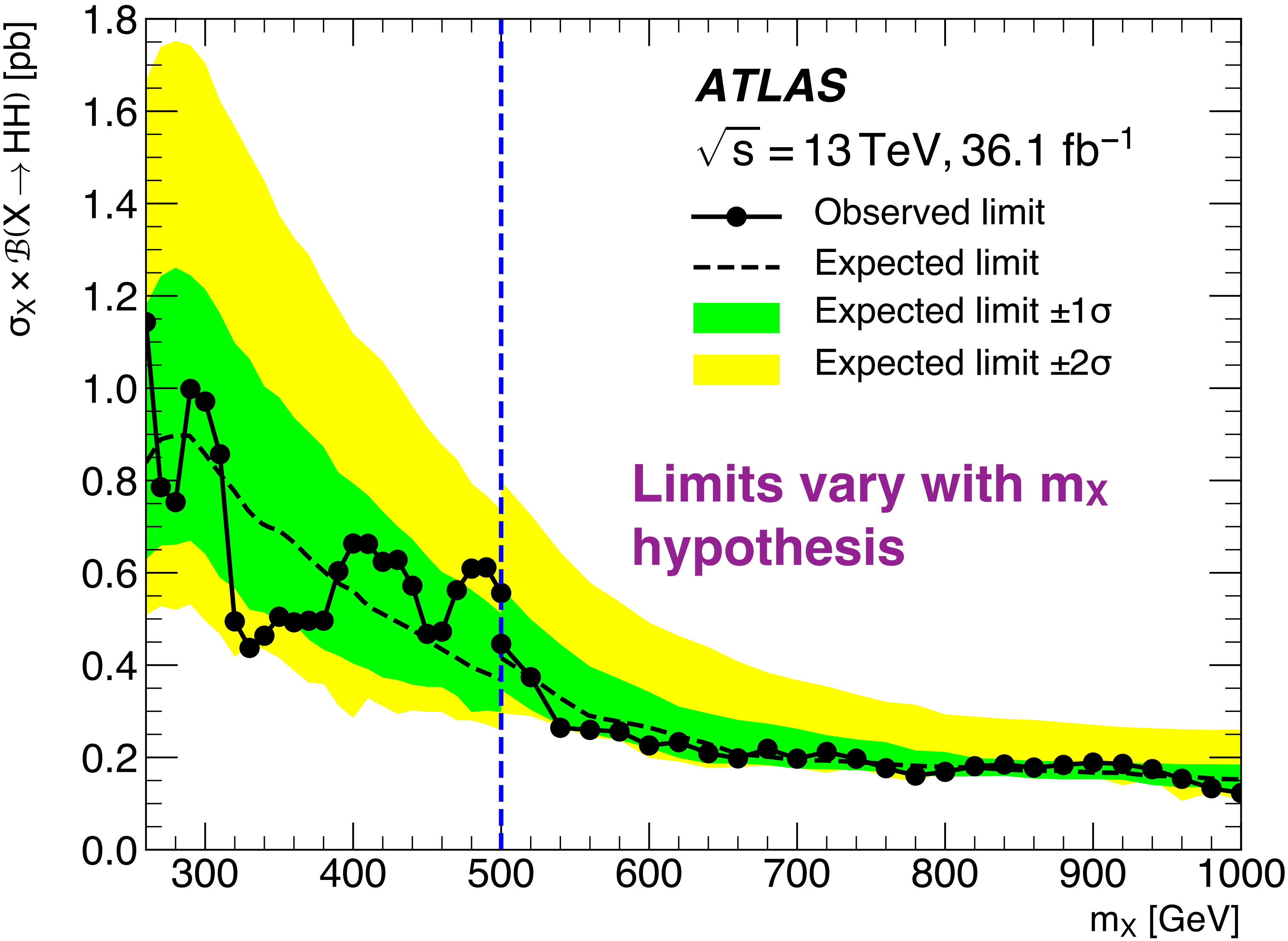
# Limits



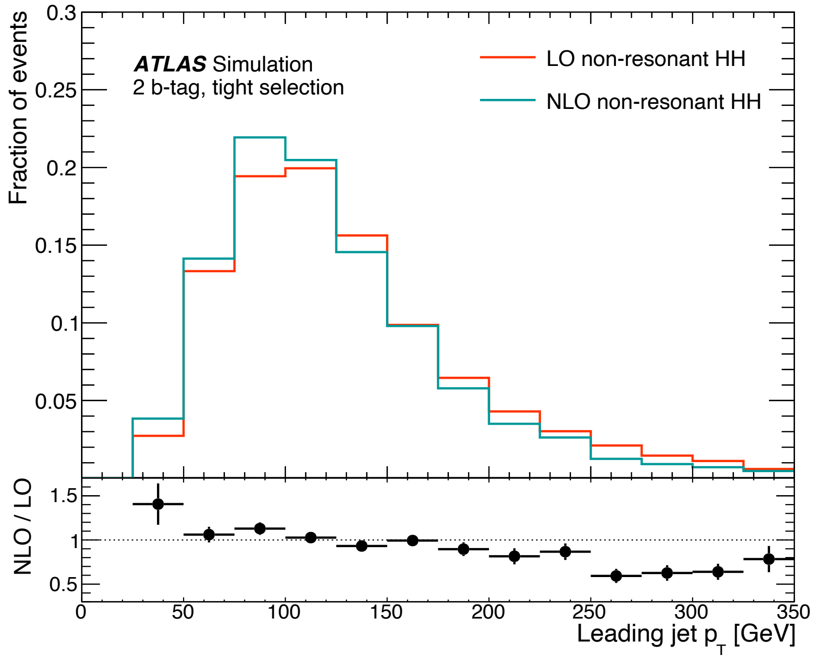



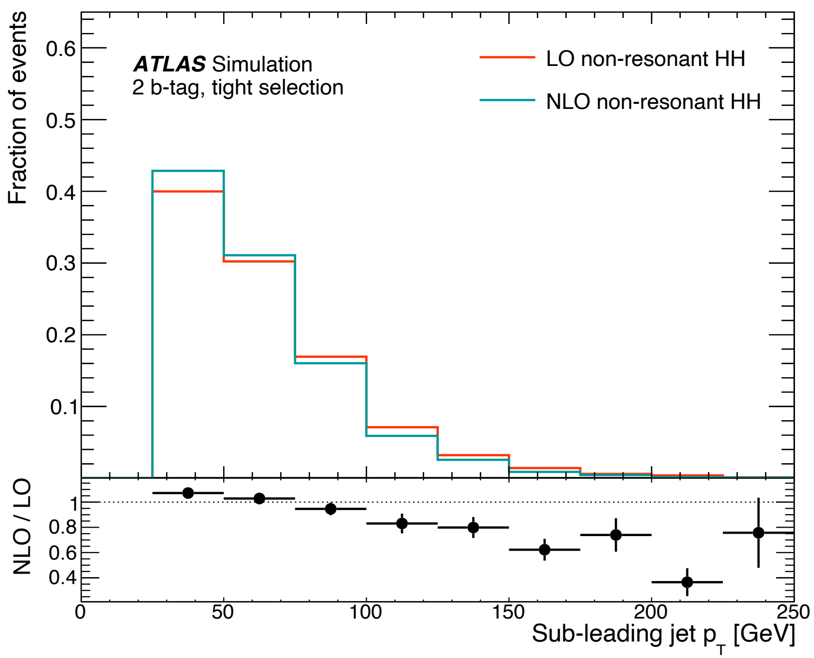

**Self-coupling is constrained at 95% CL to $-8.2 < \kappa\lambda < 13.2$; expected limits are $-8.3 < \kappa\lambda < 13.2$**

# Resonance limits



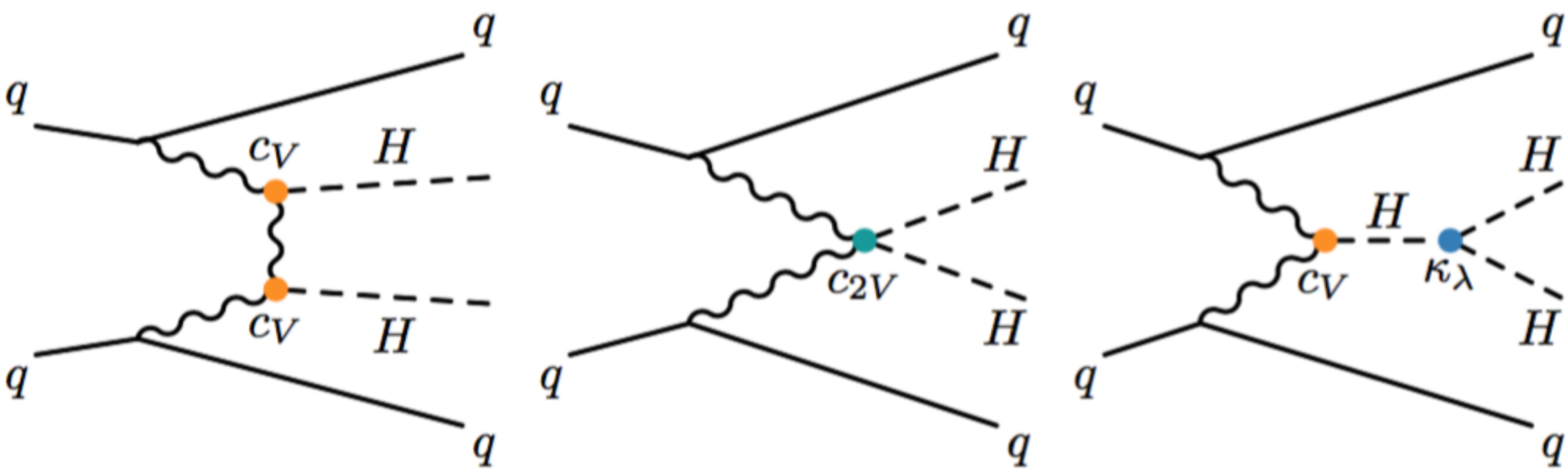
# On the Monte Carlo




**NLO corrections can be non-negligible!**

# For the future




- Do a better job of discriminating S and B (use a BDT)
- Think about improvements to photon ID (again, multivariate classifiers are helpful)
- More data!
- New channels (VBF)…

# VBF channel

arXiv:1611.03860

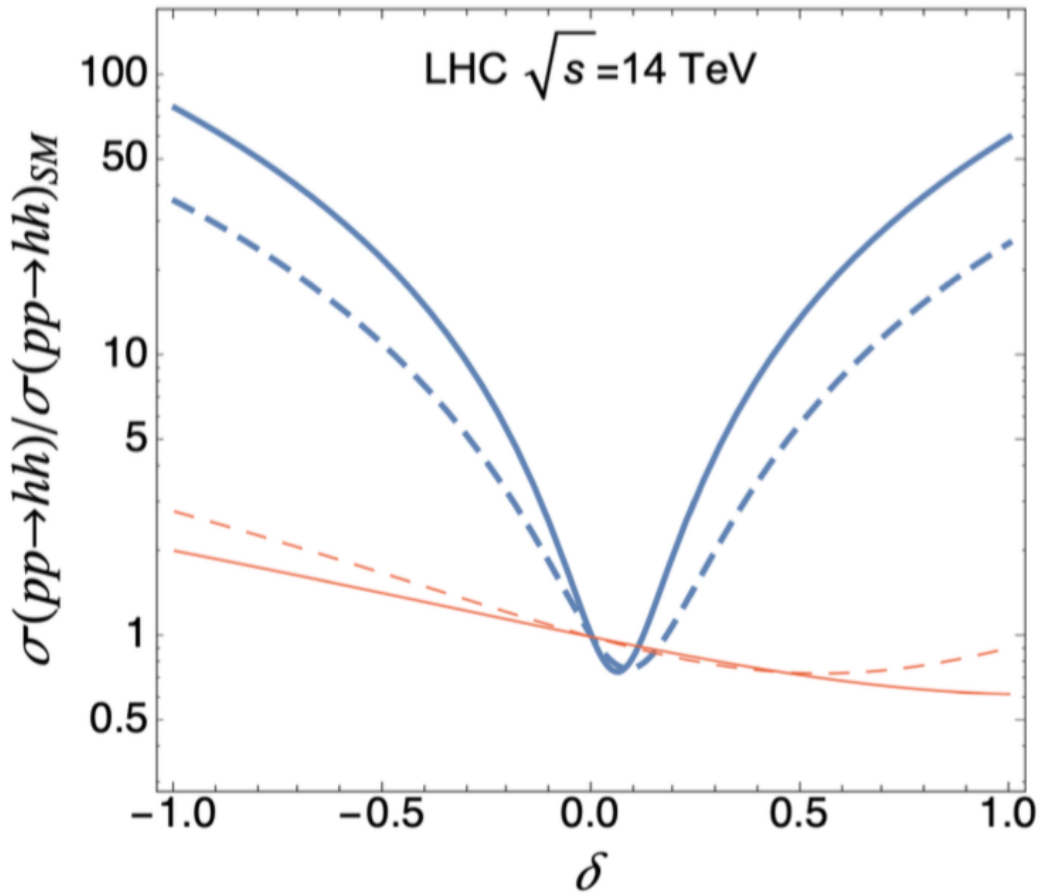
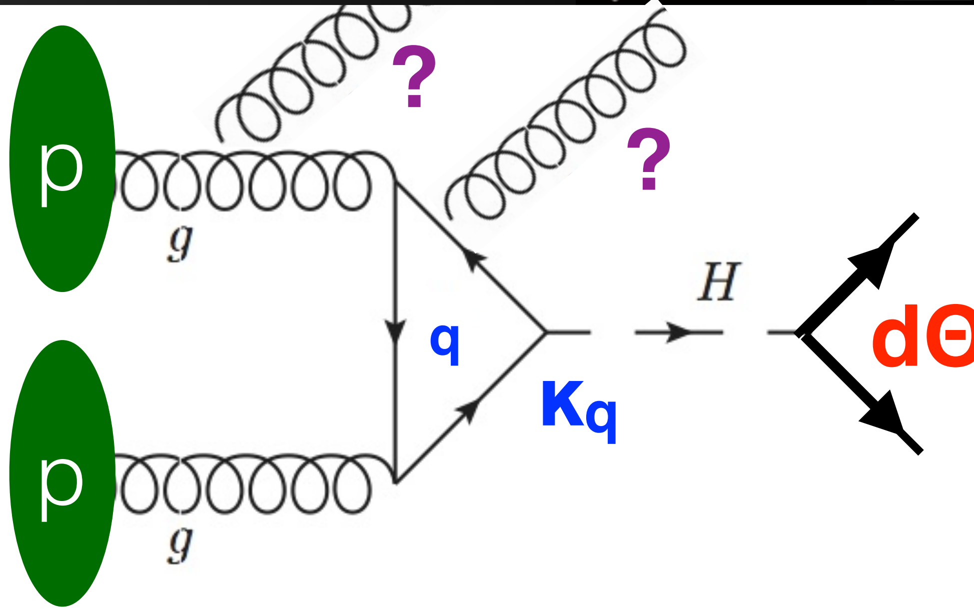



Figure 1.6: The VBF HH cross-section as a function of coupling deviation from prediction, in units of the SM value ($\delta = c - c_{SM}$ for coupling, $c$. I.e. $\delta = 0$ corresponds to the SM prediction, $\delta = 1$ is twice the SM prediction). The solid blue line shows deviations in the $c_{2V}$ vertex, $\delta_{c_{2V}}$, while the red line shows deviations for the $c_3$ vertex, $\delta_{\kappa_\lambda}$. The dashed lines represent the cross-section after simple analysis cuts presented in Reference [25].

**Huge potential for excess from small changes in the coupling (Tyler's thesis)**

Measure Higgs boson cross sections in bins of various kinematic quantities sensitive to Higgs boson modeling and BSM physics

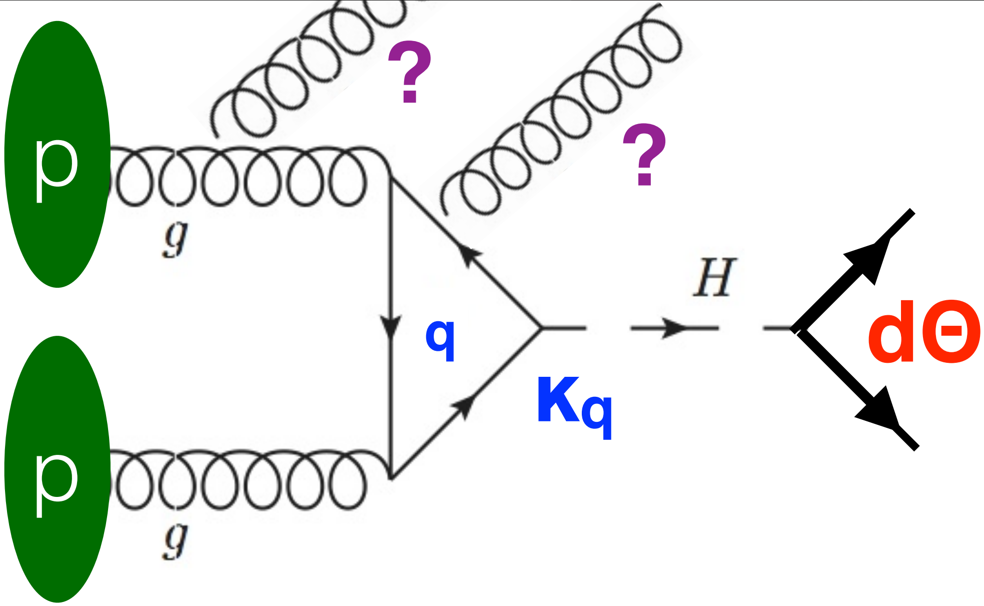



Focus on h→$\gamma\gamma$ final state, which is clean and offer advantages of bump-hunting, despite low branching ratios

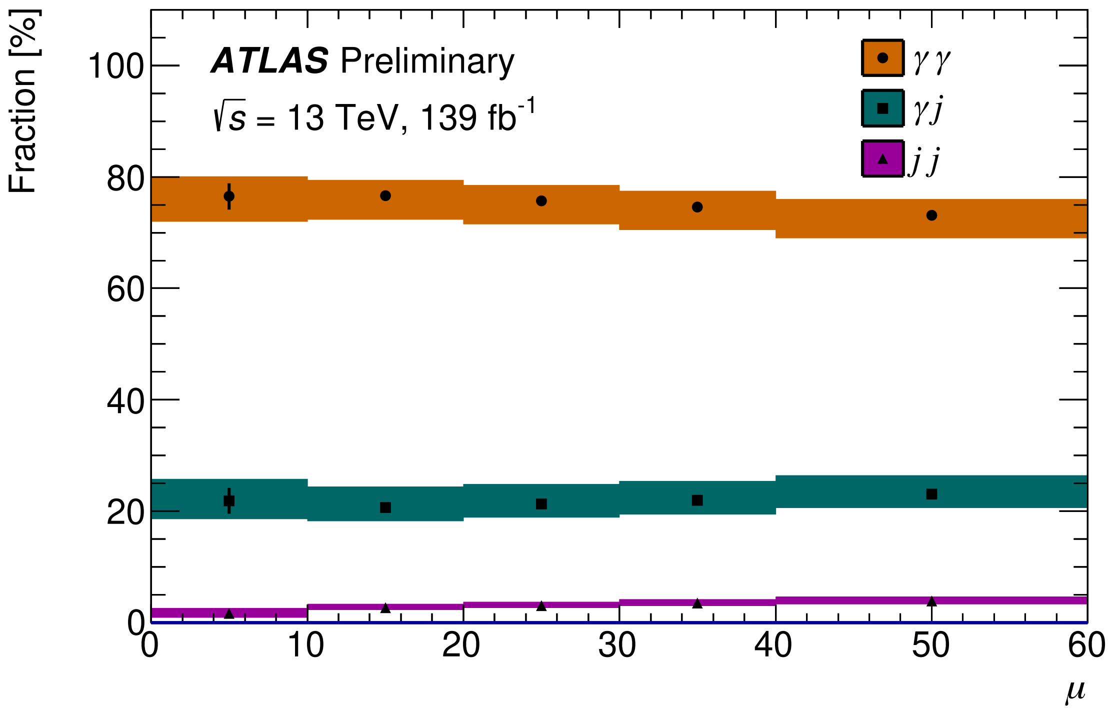
Measurements stat-limited in almost all bins. Theory uncertainties also important

Individual measurements within the fiducial volume, combinations extract to full phase space

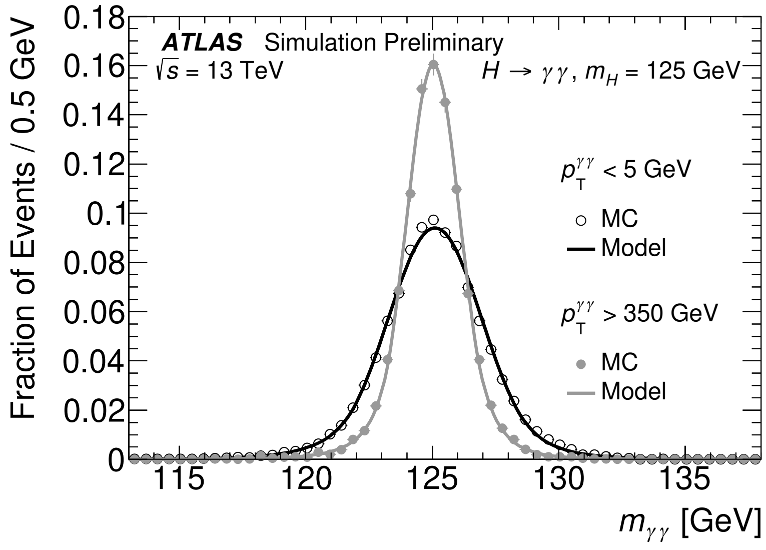
- $p_T$(h) probes QCD modeling of dominant ggF triangle, including b/c/t Yukawa coupling, and also to new heavy particles in loop
- y(h) sensitive to modeling Higgs production and PDFs inside proton
- $p_T$ and rapidity of jets sensitive to Higgs modeling and different production mechanics
- Angular variables sensitive to spin and CP of Higgs

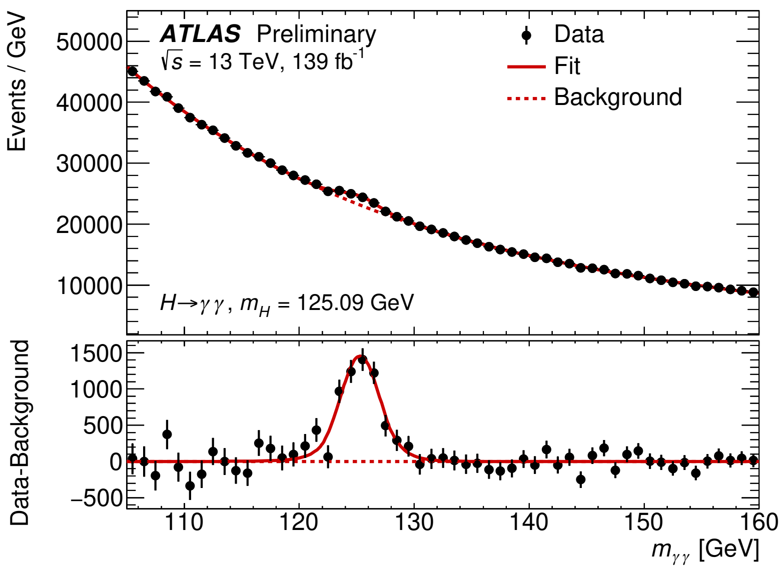



- Define and study and set limits on interactions in terms of an effective Lagrangian, which introduce CP-even and CP-odd interactions to change event rates and kinematics




- Sherpa $\gamma\gamma$ MC to study the irreducible background, fractions from double 2D sideband fit in photon ID and isolation
- Shapes of reducible background from control regions used to reweight nominal MC
- Evaluate "Spurious signal" (maximum of fitted signal yield in fits to background), then fit data with function with smallest number of fit parameters

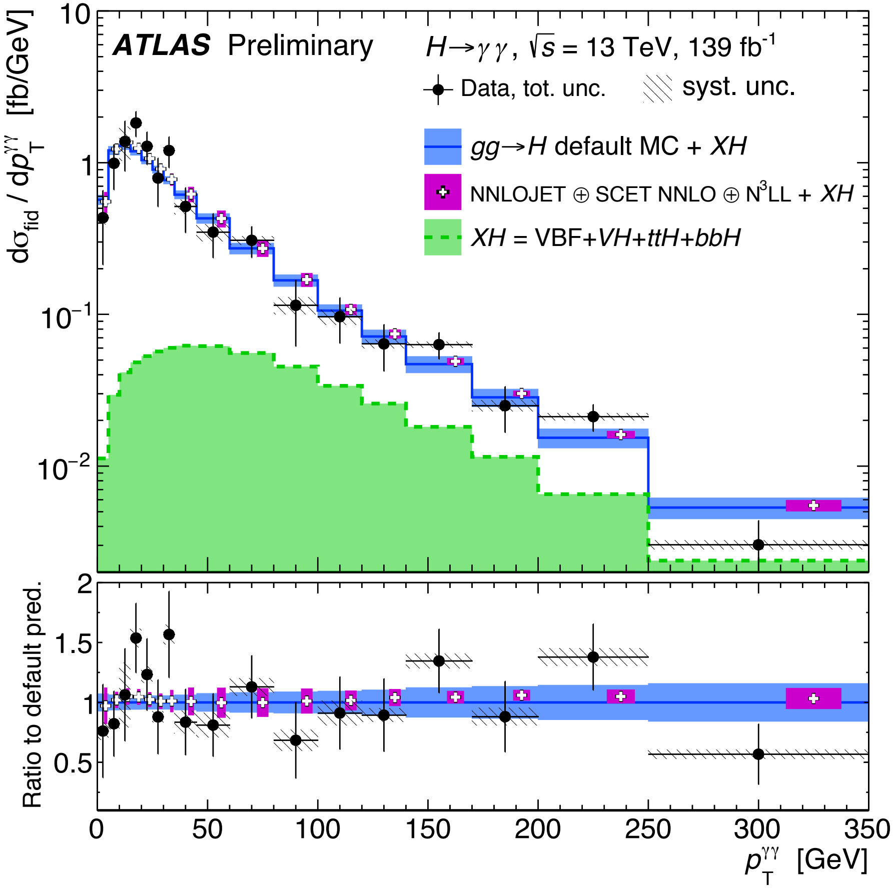


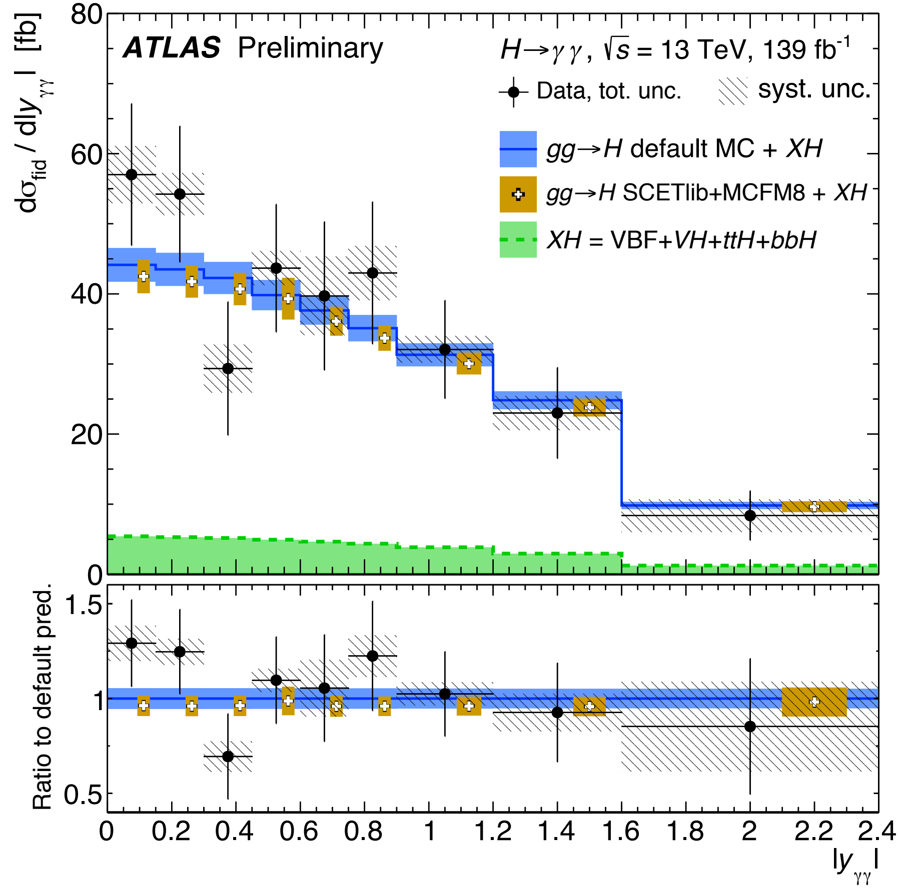




- Signal shape depends strongly on the momenta of the Higgs decay products. Can clearly see bump on top of large background in this big data set!
- Challenge is knowing that you have modeled the background correctly

# ATLAS h→$\gamma\gamma$ differential

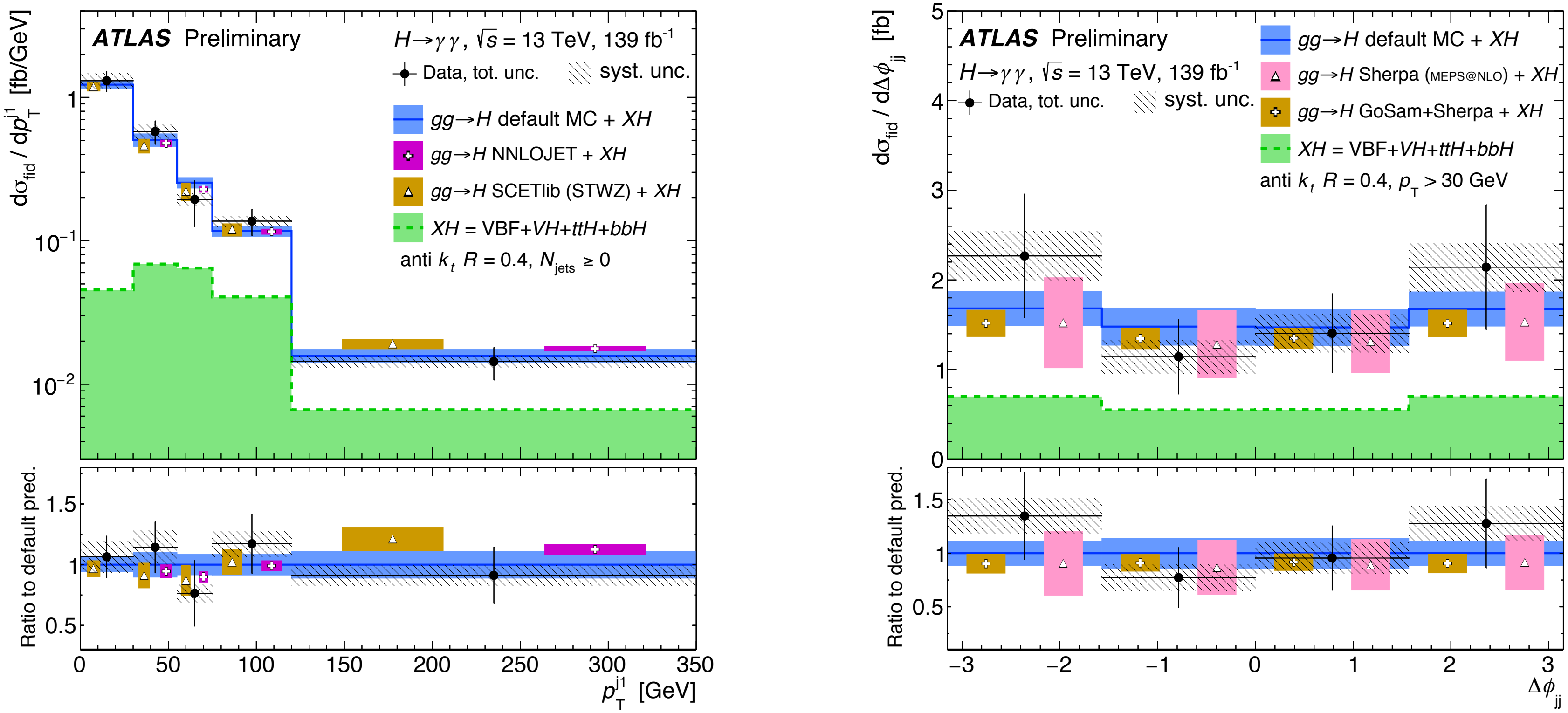





**Each of these bins has a diphoton mass fit!**

# ATLAS h→$\gamma\gamma$ differential

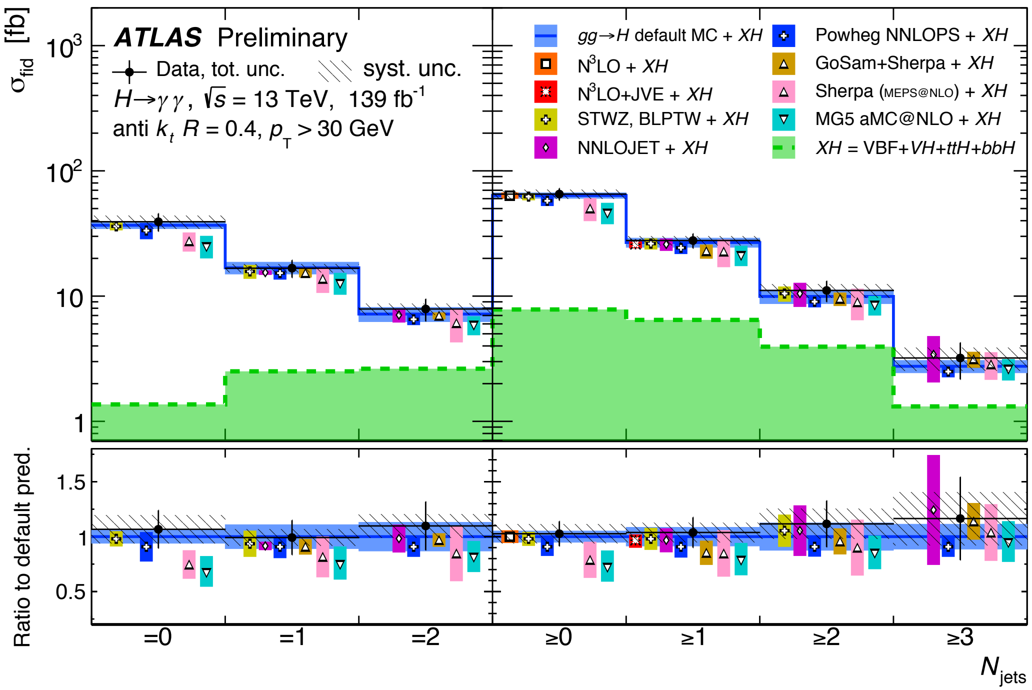



**No obvious discrepancies**

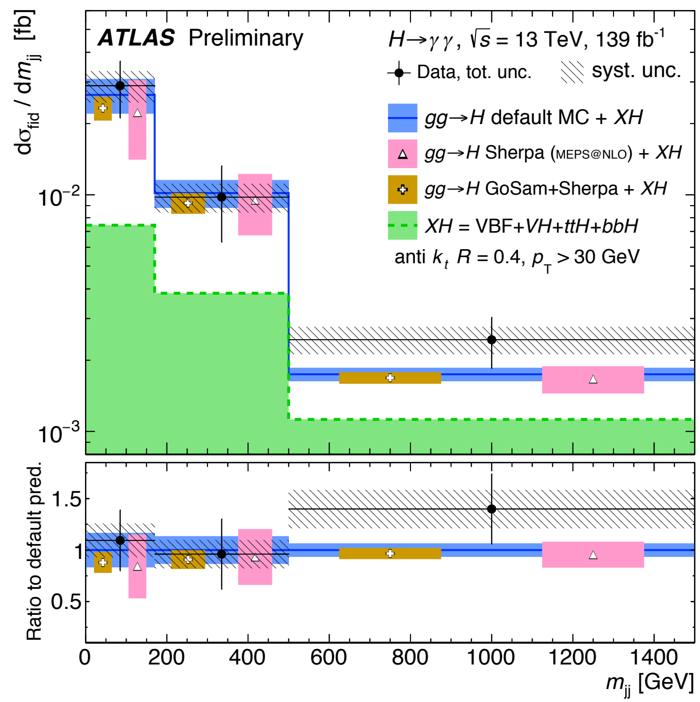
# ATLAS h→$\gamma\gamma$ differential



Northern Illinois University




**Probing Higgs boson + 3 or more jets!**

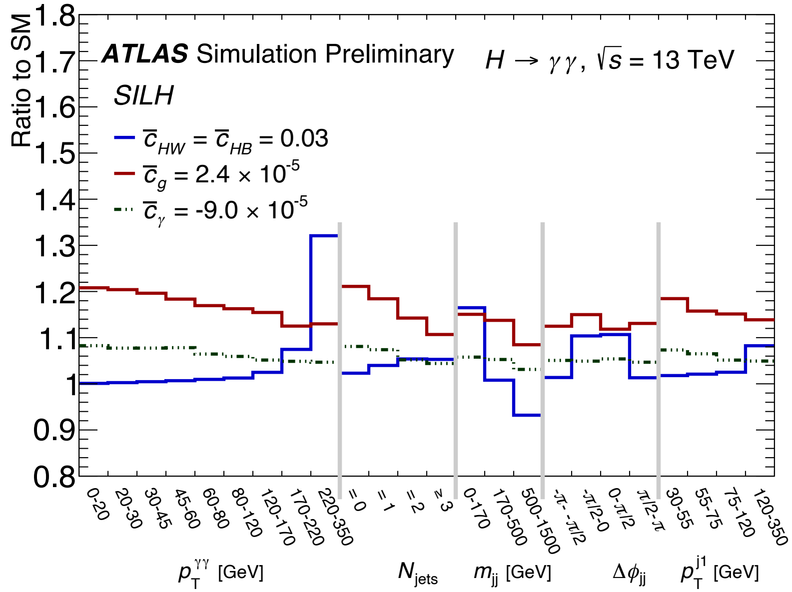


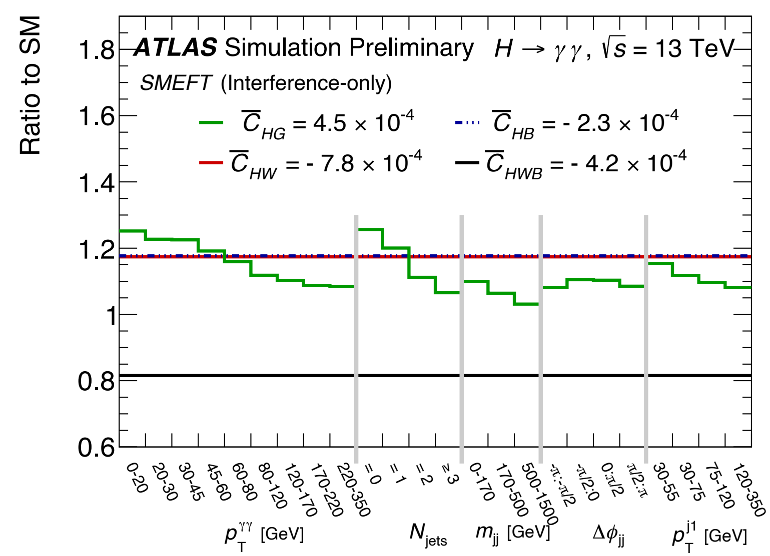
# ATLAS $h \to \gamma\gamma$ differential interpretation



- Study strength and structure of Higgs boson interaction using effective field theory approach
- All coefficients in effective Lagrangian are zero in the SM, non-zero values change rates and overall shapes

$$\mathcal{L}_{\text{eff}} = \bar{c}_g O_g + \bar{c}_{HW} O_{HW} + \bar{c}_{HB} O_{HB} + \tilde{c}_g \tilde{O}_g + \tilde{c}_{HW} \tilde{O}_{HW} + \tilde{c}_{HB} \tilde{O}_{HB}$$

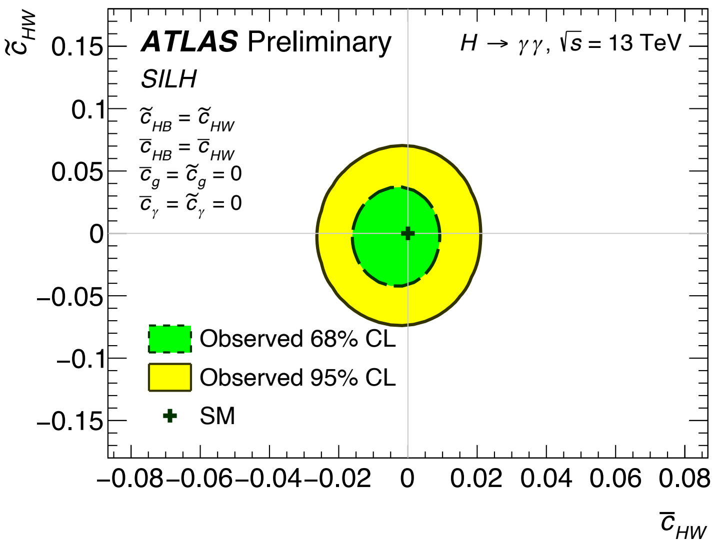


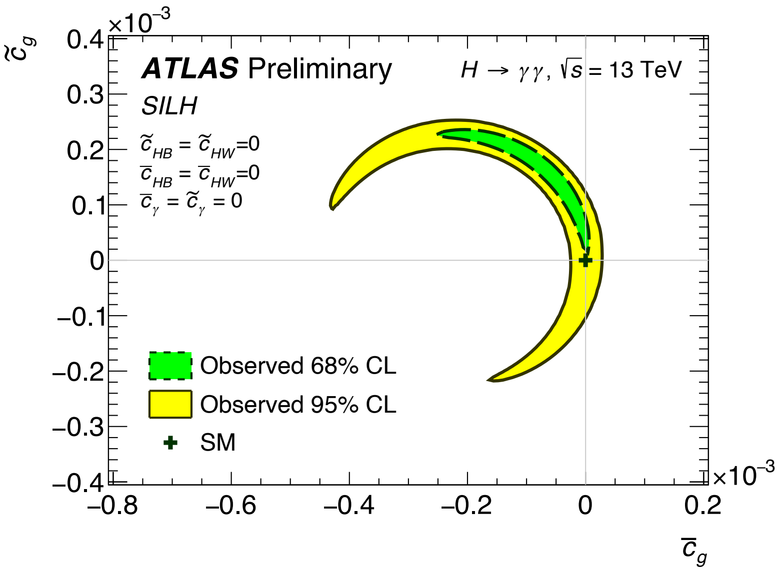



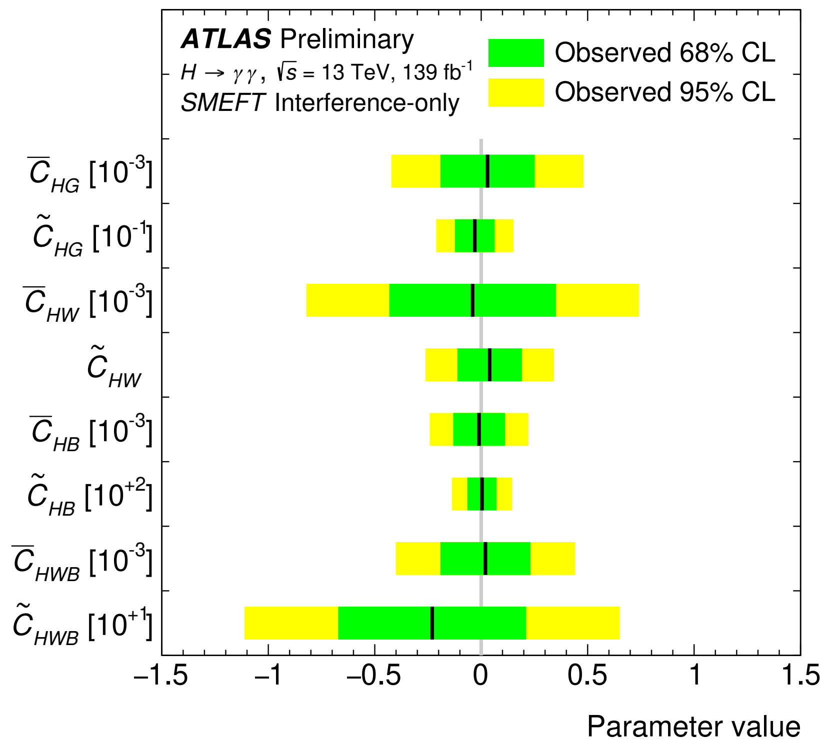
# ATLAS $h \to \gamma\gamma$ EFT results





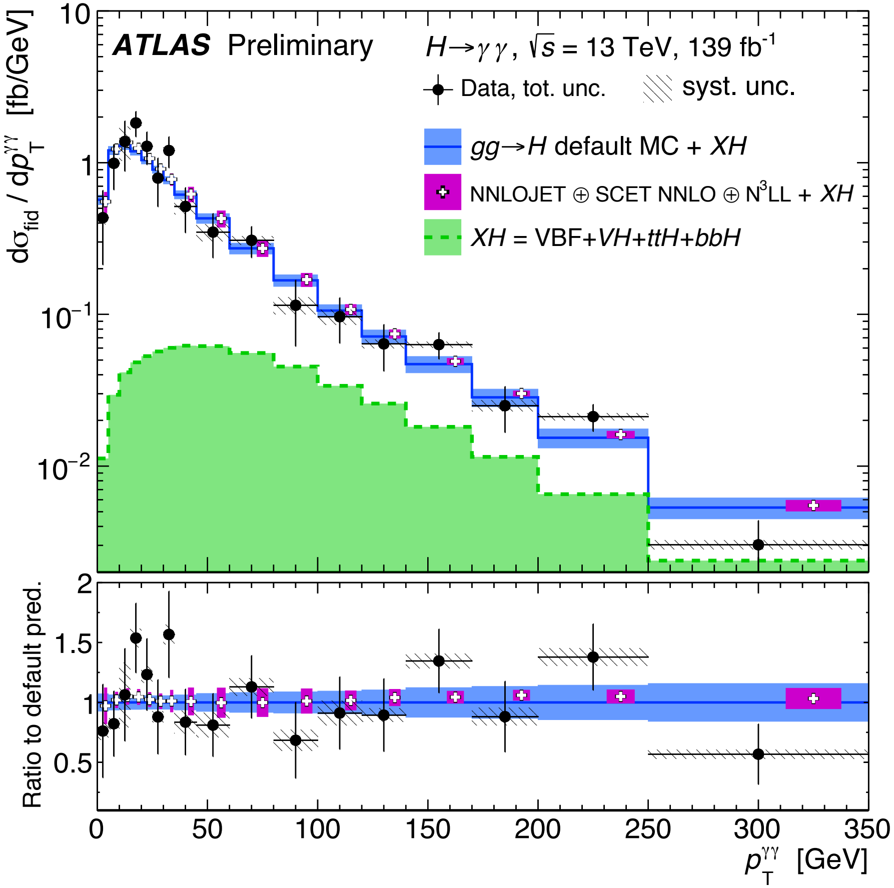


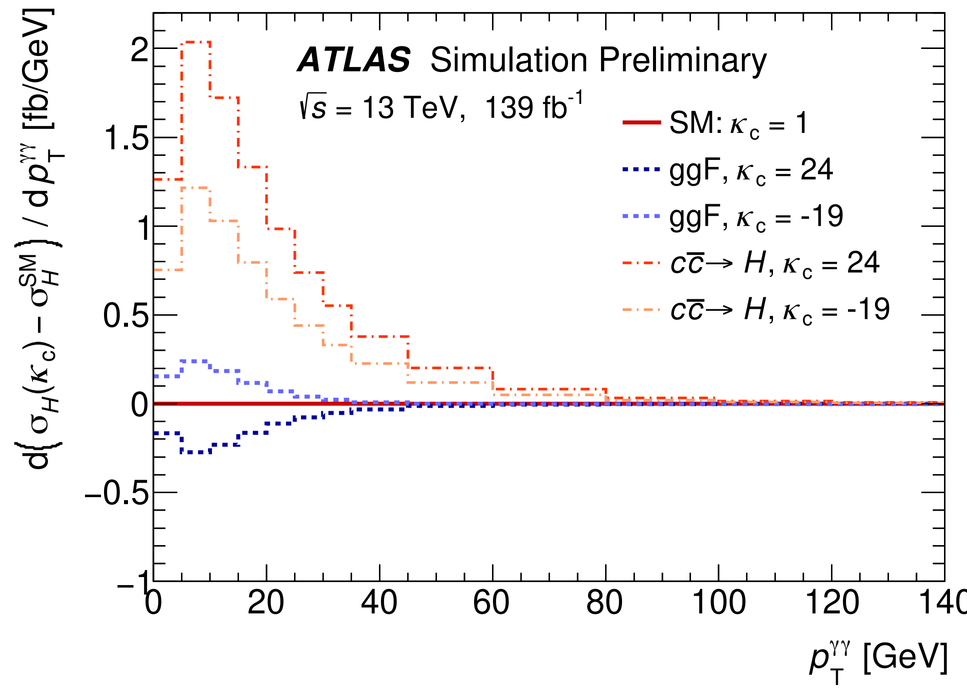



# Using Higgs boson $p_T$

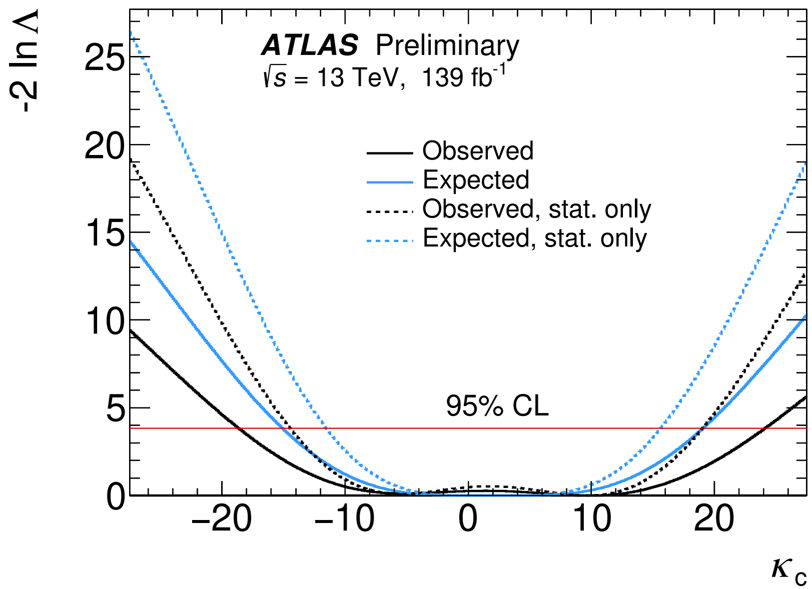


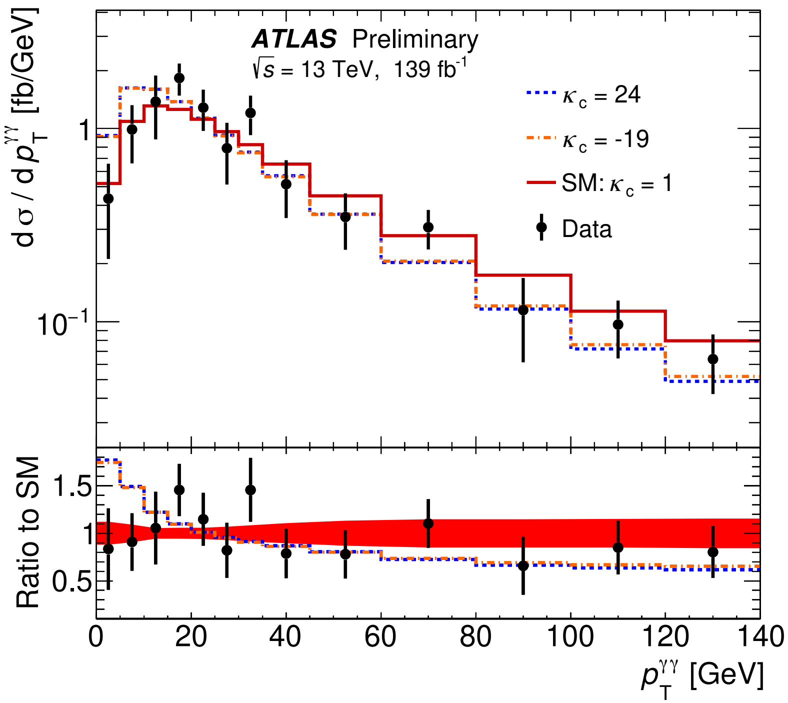



**Higgs boson $p_T$ distribution changes if Yukawa couplings change, including the charm Yukawa coupling, which is difficult to study otherwise**

# Charm Yukawa results 

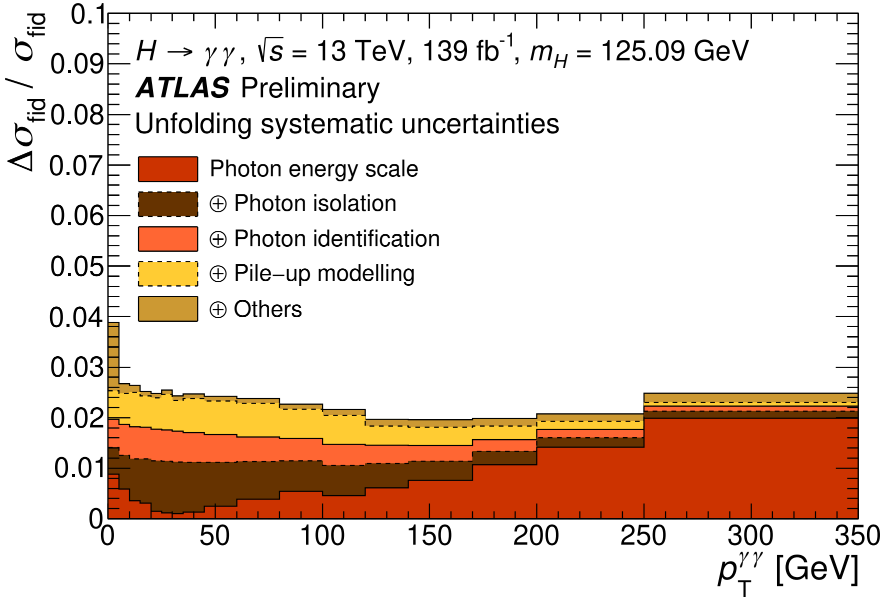


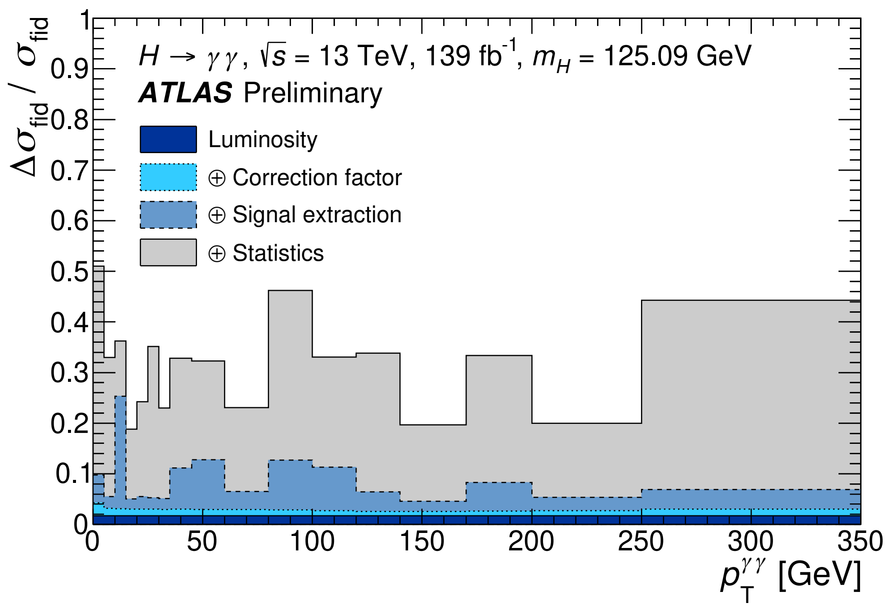

**Higgs boson $p_T$ distribution changes if Yukawa couplings change, including the charm Yukawa coupling, which is difficult to study otherwise**

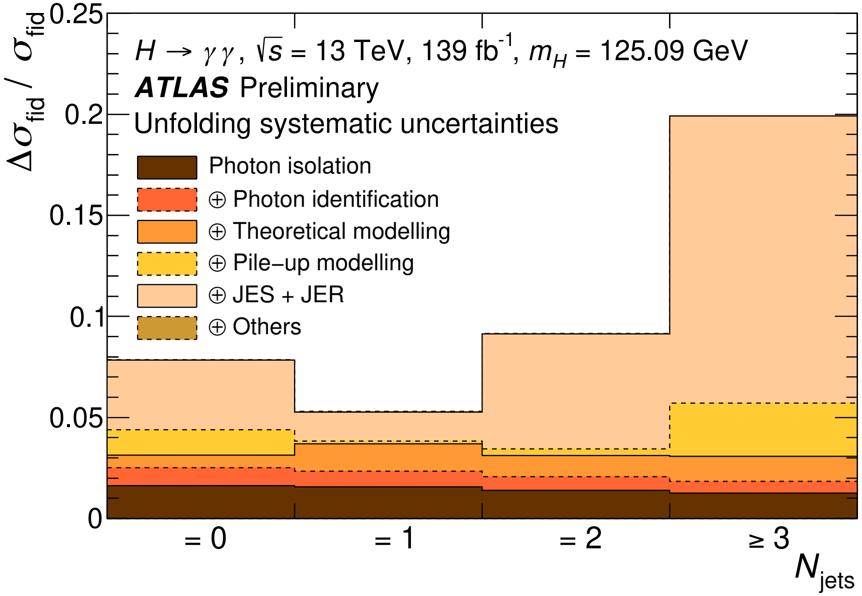
# ATLAS h→$\gamma\gamma$ differential syst uncertainties

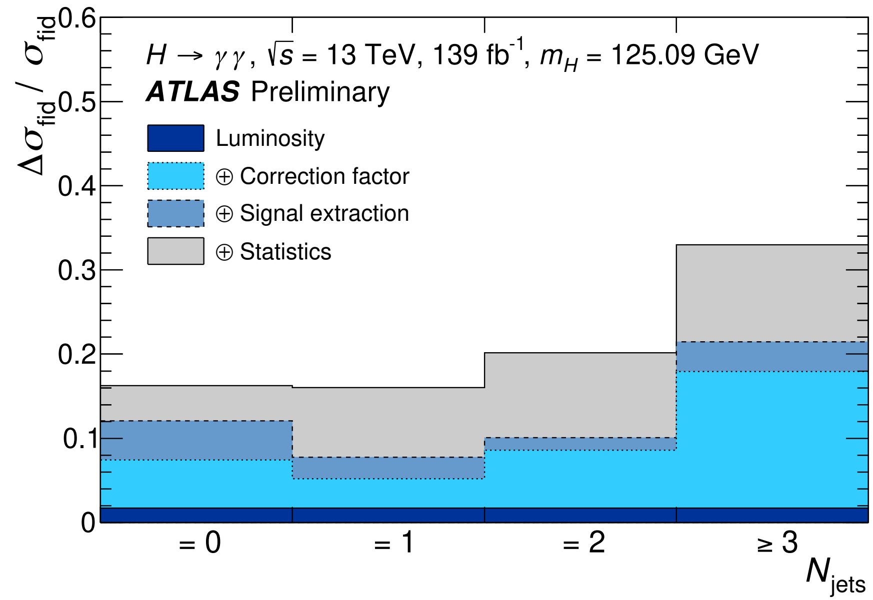





**Need to "unfold" the results, but systematic uncertainties are small compared to stat uncertainties (except for inclusive result)**





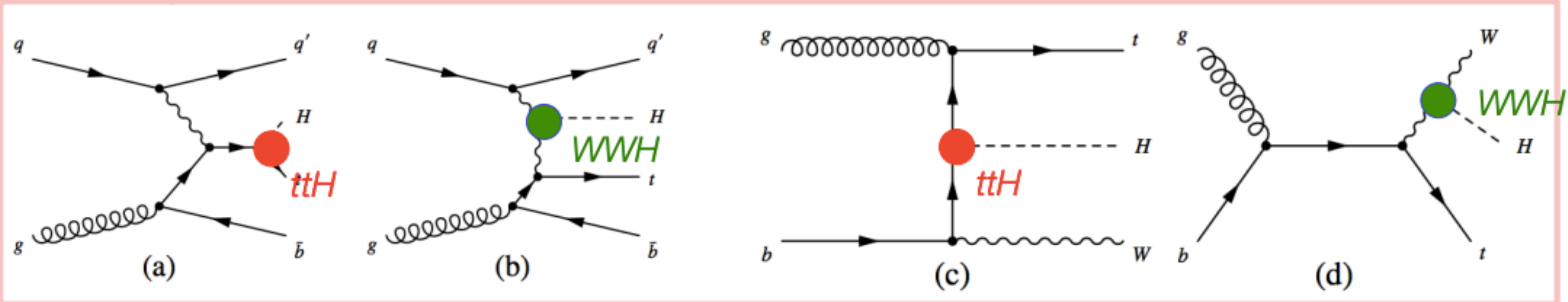




**Jet uncertainties are typically larger than photon uncertainties**

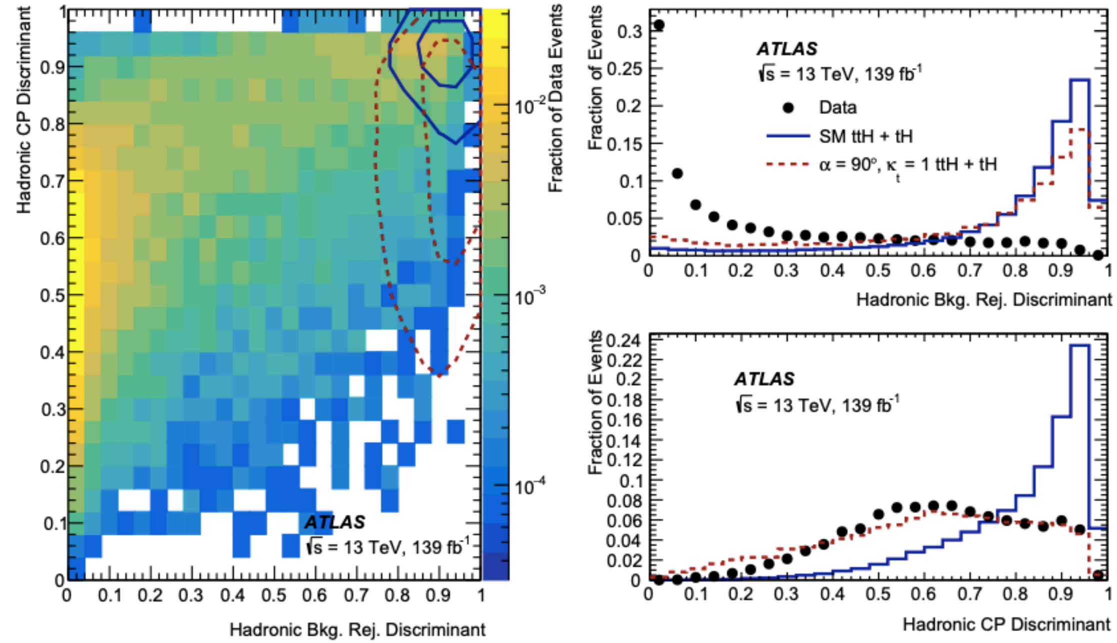
The EFT definition used in this Letter is provided by the Higgs Characterization model [20], which is implemented in the MadGraph5_aMC@NLO generator [21]. Within this model, the term in the effective Lagrangian that describes the top Yukawa coupling is:

$$\mathcal{L} = -\frac{m_t}{v}\left\{\bar{\psi}_t \kappa_t \left[\cos(\alpha) + i\sin(\alpha)\gamma_5\right]\psi_t\right\} H$$

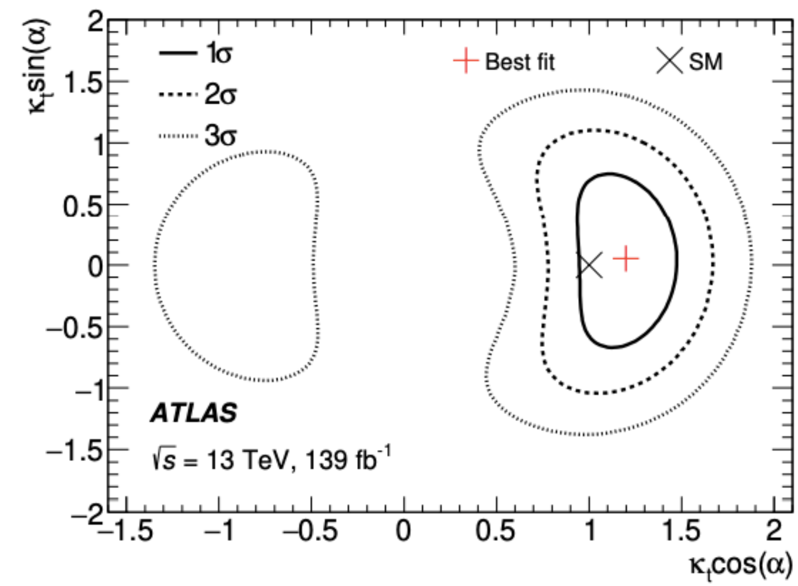
where $m_t$ is the top quark mass, $v$ is the Higgs vacuum expectation value, $\kappa_t$ (> 0) is the top Yukawa coupling parameter, and $\alpha$ is the $CP$-mixing angle. The SM corresponds to a $CP$-even coupling with $\alpha = 0$ and $\kappa_t = 1$ while a $CP$-odd coupling is realized when $\alpha = 90°$.




**Single top + Higgs is very sensitive to CP of the Higgs boson because of interference effects in the SM. But single top + Higgs gets an ~order of magnitude increase in rate for a CP-odd coupling!**




**Dominated by non-ttH backgrounds!**




**Exclude the opposite-sign top quark Yukawa coupling**

# Last homework

Filling out the time allotted to you but staying within your time budget is an important skill. Stand in front of the mirror and practice your talk at LEAST three times (separated out over at least 2 days) before your presentation. Also a last chance to ask me any questions