Symmetries

Symmetry	Conservation Law
Translation in time	Energy
Translation in space	Momentum
Rotation	Angular momentum
Gauge transformation	Charge

Emmy Noether

Extremely powerful idea. Most of modern particle physics based upon the aesthetic concepts of symmetries

Emmy Noether

Not going to go into mathematical details of group theory. Perhaps some of you have already taken a group theory course here?

We move instead to discussing angular momentum, which will end up providing rich structure to bound states in particle physics

Reminder from QM that even fundamental point particles (such as electrons, muons and W bosons) can carry intrinsic spin, and that we cannot measure all 3 components of **L** simultaneously

Angular momentum - a reminder

We cannot measure L_x, L_y and L_z simultaneously, but we can measure L^2 and also L_z (by convention - nothing special about z direction!) simultaneously. They can take only discrete values

$$\mathbf{L^2}Q = l(l+1)Q$$

$$l = 0, 1, 2, 3...$$

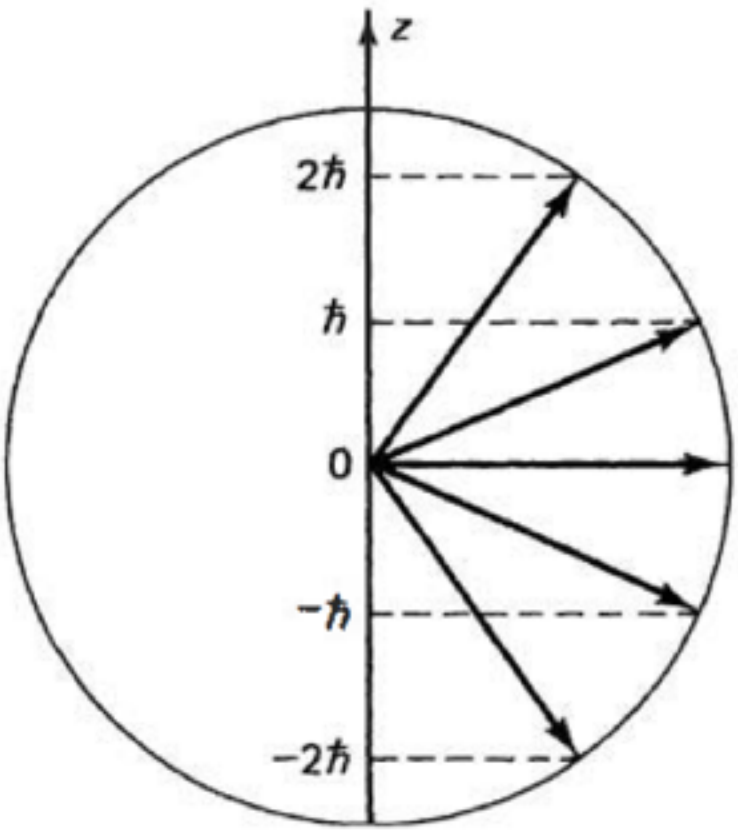
$$L_zQ = m_lQ$$

$$m_l = -l, -l+1, -l+2, ...0, l-2, l-1, l$$

$2l+1$ values for m_L

Reminder: hbar = 1 for us!

From Griffiths



l=2, so $|\mathbf{L}|^2$ = (2)(2+1)

|L| = sqrt(6)

L_z can be -2,-1,0,1,2

For spin

$$S_z Q = m_s Q$$

$$m_s = -s, -s+1, -s+2, ...0, s-2, s-1, s$$

$$S^2 Q = (\mathbf{S}\cdot\mathbf{S})Q = s(s+1)Q$$

$$s = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, ...$$

Different than orbital angular momentum (s can take half-integer values)

Other difference between s and l

Particle	Spin
electron, muon, tau, neutrinos	1/2
quarks	1/2
$W^{\pm}$, Z, γ, gluon	1
Higgs boson	0
π, K	0
proton, neutron	1/2
J/Ψ, ρ	1
Δ, Ω^{-}	3/2

s is an intrinsic quantity associated with the particle - we usually call is the **spin**. **Half-integer spin = fermion, integer spin = boson**

Reminder of ket notation

$|l \; m_l \;\; >$

$|s \; m_s \;\; >$

We use the ket notation to define j and j_z

$$|j_1 \; m_{j1} \;\; > \; + \; |j_2 \; m_{j2} \;\; > \; = \; |j \; m_j \;\; >$$

What is $|j \; m_j >$? How to add two kets? How does angular momentum add?

m_j is the easy one: $m_j = m_{j1} + m_{j2}$

Reminder of ket notation

$$|j_1\ m_{j1}\ \ > \ + |j_2\ m_{j2}\ \ > = |j\ m_j\ \ >$$

m_j is the easy one:

$m_j = m_{j1} + m_{j2}$

$j = |j_1 - j_2|, |j_1 - j_2|+1, |j_1 - j_2|+2,$

$|j_1 + j_2|-2, |j_1 + j_2|-1, |j_1 + j_2|$

$j_1=5$, $j_2=2$, j can be 3,4,5,6 or 7

What are the probabilities for the different values?

Clebsch-Gordan coefficients (we don't derive them here)

From Particle Data Group

36. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND *d* FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for −8/15 read $-\sqrt{8/15}$.

Notation:

		J	J	...
		M	M	...
m_1	m_2	Coefficients		
m_1	m_2			
.	.			

1/2×1/2

M	m_1	m_2	J=1	J=0
+1	+1/2	+1/2	1	
0	+1/2	−1/2	1/2	1/2
0	−1/2	+1/2	1/2	−1/2
−1	−1/2	−1/2	1	

1×1/2

M	m_1	m_2	J=3/2	J=1/2
+3/2	+1	+1/2	1	
+1/2	+1	−1/2	1/3	2/3
+1/2	0	+1/2	2/3	−1/3
−1/2	0	−1/2	2/3	1/3
−1/2	−1	+1/2	1/3	−2/3
−3/2	−1	−1/2	1	

2×1

M	m_1	m_2	J=3	J=2	J=1
+3	+2	+1	1		
+2	+2	0	1/3	2/3	
+2	+1	+1	2/3	−1/3	
+1	+2	−1	1/15	1/3	3/5
+1	+1	0	8/15	1/6	−3/10
+1	0	+1	2/5	−1/2	1/10
0	+1	−1	1/5	1/2	3/10
0	0	0	3/5	0	−2/5
0	−1	+1	1/5	−1/2	3/10
−1	0	−1	2/5	1/2	1/10
−1	−1	0	8/15	−1/6	−3/10
−1	−2	+1	1/15	−1/3	3/5
−2	−1	−1	2/3	1/3	
−2	−2	0	1/3	−2/3	
−3	−2	−1	1		

1×1

M	m_1	m_2	J=2	J=1	J=0
+2	+1	+1	1		
+1	+1	0	1/2	1/2	
+1	0	+1	1/2	−1/2	
0	+1	−1	1/6	1/2	1/3
0	0	0	2/3	0	−1/3
0	−1	+1	1/6	−1/2	1/3
−1	0	−1	1/2	1/2	
−1	−1	0	1/2	−1/2	
−2	−1	−1	1		

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

2×1/2

M	m_1	m_2	J=5/2	J=3/2
+5/2	+2	+1/2	1	
+3/2	+2	−1/2	1/5	4/5
+3/2	+1	+1/2	4/5	−1/5
+1/2	+1	−1/2	2/5	3/5
+1/2	0	+1/2	3/5	−2/5
−1/2	0	−1/2	3/5	2/5
−1/2	−1	+1/2	2/5	−3/5
−3/2	−1	−1/2	4/5	1/5
−3/2	−2	+1/2	1/5	−4/5
−5/2	−2	−1/2	1	

3/2×1/2

M	m_1	m_2	J=2	J=1
+2	+3/2	+1/2	1	
+1	+3/2	−1/2	1/4	3/4
+1	+1/2	+1/2	3/4	−1/4
0	+1/2	−1/2	1/2	1/2
0	−1/2	+1/2	1/2	−1/2
−1	−1/2	−1/2	3/4	1/4
−1	−3/2	+1/2	1/4	−3/4
−2	−3/2	−1/2	1	

3/2×1

M	m_1	m_2	J=5/2	J=3/2	J=1/2
+5/2	+3/2	+1	1		
+3/2	+3/2	0	2/5	3/5	
+3/2	+1/2	+1	3/5	−2/5	
+1/2	+3/2	−1	1/10	2/5	1/2
+1/2	+1/2	0	3/5	1/15	−1/3
+1/2	−1/2	+1	3/10	−8/15	1/6
−1/2	+1/2	−1	3/10	8/15	1/6
−1/2	−1/2	0	3/5	−1/15	−1/3
−1/2	−3/2	+1	1/10	−2/5	1/2
−3/2	−1/2	−1	3/5	2/5	
−3/2	−3/2	0	2/5	−3/5	
−5/2	−3/2	−1	1		

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

Ex 4.1 in Griffiths

What are the possible values of the spin of a meson if the quark and anti-quark have orbital angular momentum = 0?

Ex 4.1 in Griffiths

Spin=0: pseudoscalar mesons (pion, kaons, etas)
(Scalar meaning spin 0)
Spin = 1: vector mesons (rhos, K*, phi, omega)
(Vector meaning spin 1)

Ex 4.1-2 in Griffiths

What do we get if we add the angular momenta of 3 quarks, to form a baryon? What can the spin be? Does orbital angular momentum change anything?

Ex 4.3 in Griffiths

An electron in a hydrogen atom is in the |2 -1> state, and spin state |½ ½>. If we measure J^2 what values might we get, and with what probability? The first part should be easy

For the second part...

From Particle Data Group

36. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

m_1	m_2	J / M	J / M	...
m_1	m_2	Coefficients		
.	.			

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

1/2×1/2

m_1	m_2	J,M		
+1/2	+1/2	1, +1: 1		
		1, 0	0, 0	
+1/2	−1/2	1/2	1/2	
−1/2	+1/2	1/2	−1/2	
−1/2	−1/2	1, −1: 1		

1×1/2

m_1	m_2			
+1	+1/2	3/2, +3/2: 1		
		3/2, +1/2	1/2, +1/2	
+1	−1/2	1/3	2/3	
0	+1/2	2/3	−1/3	
		3/2, −1/2	1/2, −1/2	
0	−1/2	2/3	1/3	
−1	+1/2	1/3	−2/3	
−1	−1/2	3/2, −3/2: 1		

2×1

m_1	m_2			
+2	+1	3, +3: 1		
		3, +2	2, +2	
+2	0	1/3	2/3	
+1	+1	2/3	−1/3	
		3, +1	2, +1	1, +1
+2	−1	1/15	1/3	3/5
+1	0	8/15	1/6	−3/10
0	+1	2/5	−1/2	1/10
		3, 0	2, 0	1, 0
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10
		3, −1	2, −1	1, −1
0	−1	2/5	1/2	1/10
−1	0	8/15	−1/6	−3/10
−2	+1	1/15	−1/3	3/5
		3, −2	2, −2	
−1	−1	2/3	1/3	
−2	0	1/3	−2/3	
−2	−1	3, −3: 1		

1×1

m_1	m_2			
+1	+1	2, +2: 1		
		2, +1	1, +1	
+1	0	1/2	1/2	
0	+1	1/2	−1/2	
		2, 0	1, 0	0, 0
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3
		2, −1	1, −1	
0	−1	1/2	1/2	
−1	0	1/2	−1/2	
−1	−1	2, −2: 1		

3/2×1

m_1	m_2			
+3/2	+1	5/2, +5/2: 1		
		5/2, +3/2	3/2, +3/2	
+3/2	0	2/5	3/5	
+1/2	+1	3/5	−2/5	
		5/2, +1/2	3/2, +1/2	1/2, +1/2
+3/2	−1	1/10	2/5	1/2
+1/2	0	3/5	1/15	−1/3
−1/2	+1	3/10	−8/15	1/6
		5/2, −1/2	3/2, −1/2	1/2, −1/2
+1/2	−1	3/10	8/15	1/6
−1/2	0	3/5	−1/15	−1/3
−3/2	+1	1/10	−2/5	1/2
		5/2, −3/2	3/2, −3/2	
−1/2	−1	3/5	2/5	
−3/2	0	2/5	−3/5	
−3/2	−1	5/2, −5/2: 1		

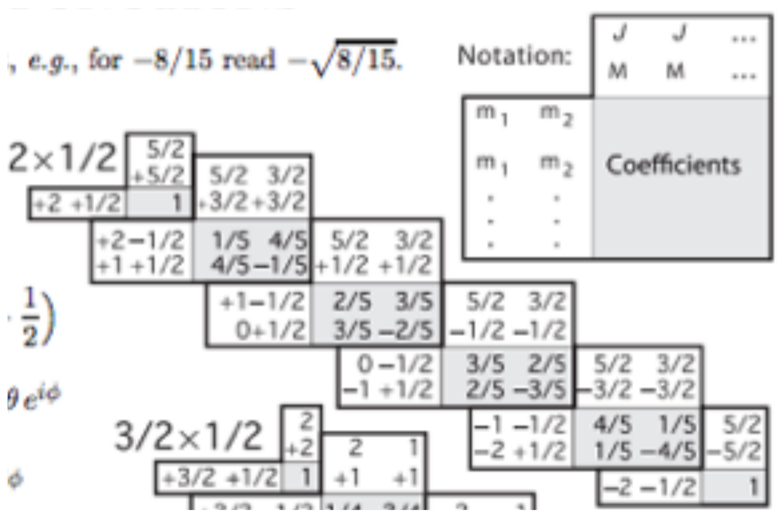
2×1/2

m_1	m_2		
+2	+1/2	5/2, +5/2: 1	
		5/2, +3/2	3/2, +3/2
+2	−1/2	1/5	4/5
+1	+1/2	4/5	−1/5
		5/2, +1/2	3/2, +1/2
+1	−1/2	2/5	3/5
0	+1/2	3/5	−2/5
		5/2, −1/2	3/2, −1/2
0	−1/2	3/5	2/5
−1	+1/2	2/5	−3/5
		5/2, −3/2	3/2, −3/2
−1	−1/2	4/5	1/5
−2	+1/2	1/5	−4/5
−2	−1/2	5/2, −5/2: 1	

3/2×1/2

m_1	m_2		
+3/2	+1/2	2, +2: 1	
		2, +1	1, +1
+3/2	−1/2	1/4	3/4
+1/2	+1/2	3/4	−1/4
		2, 0	1, 0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2
		2, −1	1, −1
−1/2	−1/2	3/4	1/4
−3/2	+1/2	1/4	−3/4
−3/2	−1/2	2, −2: 1	

For the second part...



Spin 1/2 objects

All fundamental matter particles (leptons and quarks) carry spin 1/2. So do neutrons and protons

Spin 1/2 objects can have spin up (m_s = +1/2) or spin down (m_s = -1/2). Represent them with spinors:

$$|j\ m_j^{up}\ >=|\frac{1}{2}\ \frac{1}{2}\ >=\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|j\ m_j^{down}\ >=|\frac{1}{2}\ -\frac{1}{2}\ >=\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Spinors

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\hat{S}_x = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\hat{S}_y = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\hat{S}_z = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Spin operators

Pauli spin matrices

Normalization

$$|\alpha|^2 + |\beta|^2 = ?$$

Flavor symmetries

M(proton) = 938.272046 MeV
M(neutron) = 939.565379 MeV

Same mass to 0.1%!

So perhaps they are different types of the "same" object (call it a nucleon)

Proton is "up" by convention

$$\text{Nucleon} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\text{Proton} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\text{Neutron} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Instead of spin we call this isospin (lousy name!)

Instead of spin we call this isospin

By analogy with regular spin, both p and n have isospin = 1/2

$$\text{Proton} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\text{Neutron} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

By analogy proton has $I_3 = +1/2$
By analogy neutron has $I_3 = -1/2$

(Could call it I_Z but we are not dealing with real, ordinary physical space)

Isospin

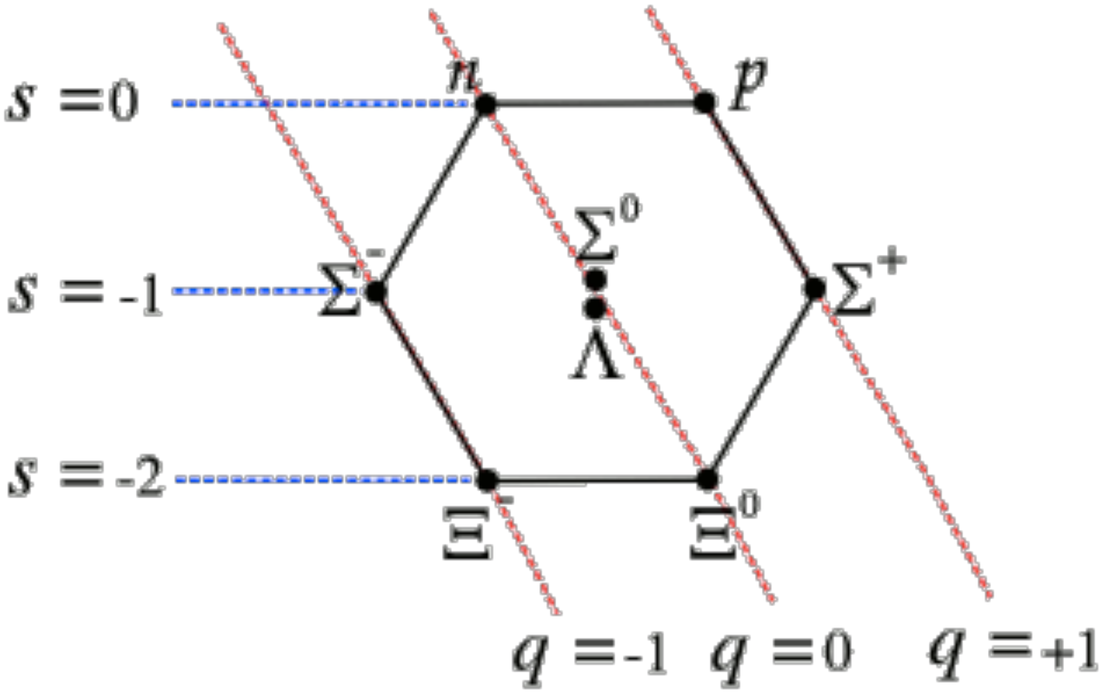
$$p = \left| \frac{1}{2} \ \frac{1}{2} \right>$$

$$n = \left| \frac{1}{2} \ \frac{-1}{2} \right>$$

Idea: QCD interactions are invariant under rotations in isospin space

Symmetry	Conservation Law
Translation in time	Energy
Translation in space	Momentum
Rotation	Angular momentum
Gauge transformation	Charge

Emmy Noether: This means that isospin is conserved in strong interactions



PDG:

Σ^+: 1189 MeV

Σ^0: 1193 MeV

Σ^-: 1197 MeV

Λ: 1116 MeV

Ξ^0: 1315 MeV

Ξ^-: 1322 MeV

$$p = |\ \frac{1}{2}\ \frac{1}{2}\ >$$

$$n = |\ \frac{1}{2}\ \frac{-1}{2}\ >$$

$$\Sigma^+ = |\ 1\ 1 >$$

$$\Sigma^0 = |\ 1\ 0\ >$$

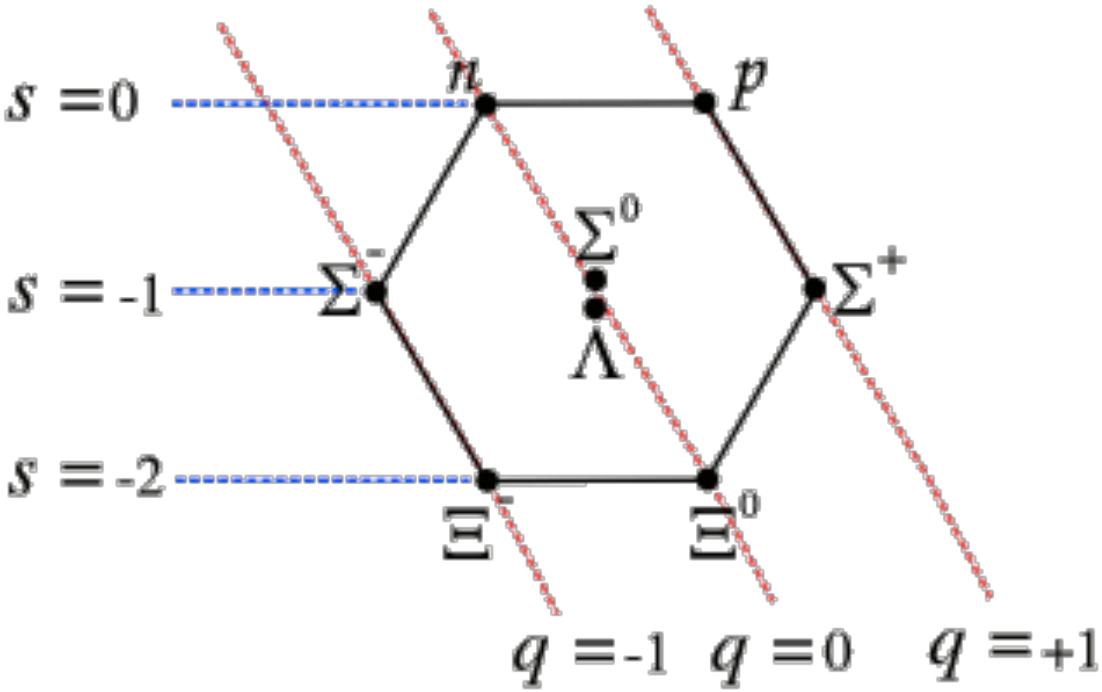
$$\Sigma^- = |\ 1\ -1\ >$$

$$\Lambda = |\ 0\ 0\ >$$

$$\Xi^0 = |\ \frac{1}{2}\ +\frac{1}{2} >$$

$$\Xi^- = |\ \frac{1}{2}\ -\frac{1}{2} >$$

Isospin counting



I3 goes from -I to I so multiplicity is 2I+1. By convention, highest charge gets highest I3

$$p = |\ \frac{1}{2}\ \frac{1}{2}\ >$$

$$n = |\ \frac{1}{2}\ \frac{-1}{2}\ >$$

$$\Sigma^+ = |\ 1\ 1 >$$

$$\Sigma^0 = |\ 1\ 0\ >$$

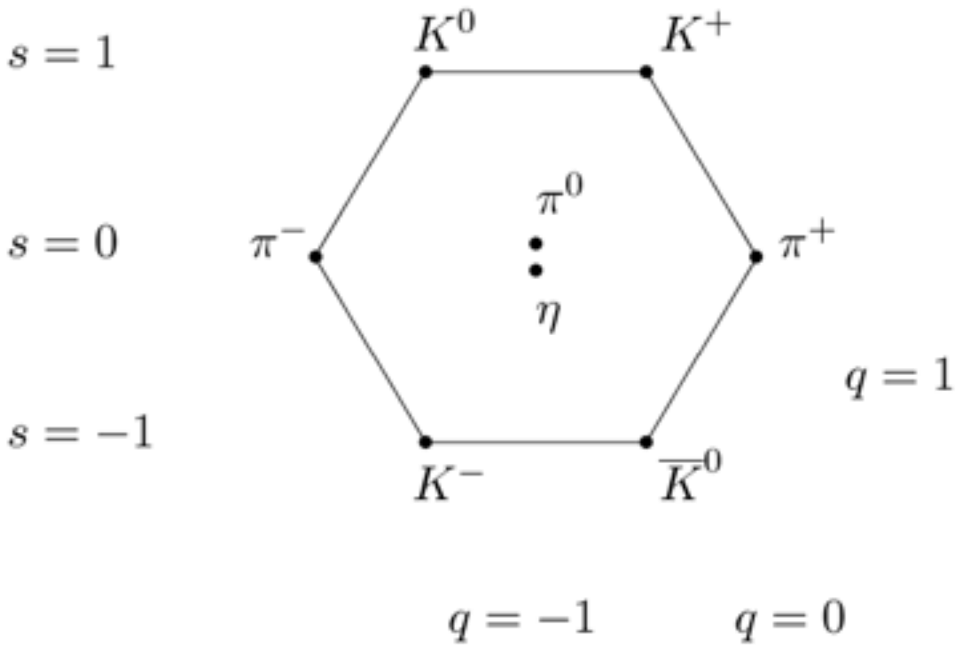
$$\Sigma^- = |\ 1\ -1\ >$$

$$\Lambda = |\ 0\ 0\ >$$

$$\Xi^0 = |\ \frac{1}{2}\ +\frac{1}{2} >$$

$$\Xi^- = |\ \frac{1}{2}\ -\frac{1}{2} >$$

Isospin for mesons



$$\pi^+ = |\,1\;1\;\,>$$

$$\pi^0 = |\,1\;0\;\,>$$

$$\pi^- = |\,1\;-1\;\,>$$

PDG:

K^0: 498 MeV

$K^\pm$: 494 MeV

π^0: 135 MeV

$\pi^\pm$: 140 MeV

η: 548 MeV

Isospin and forces

I_3 is conserved by electromagnetic forces, but I is not.

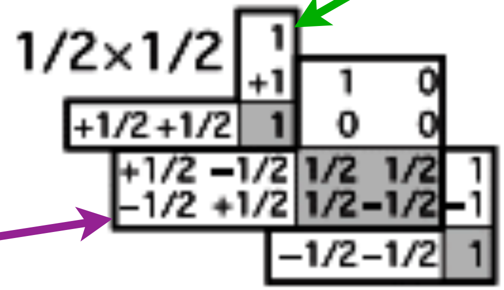
Weak interactions don't conserve isospin at all.

And of course, strong forces conserve it as we have discussed

Combining isospin

Combining two 1/2 x 1/2 particles

ex:
J=1



A pair of nucleons can have total isospin +1 or 0. What are the combinations of n and p?

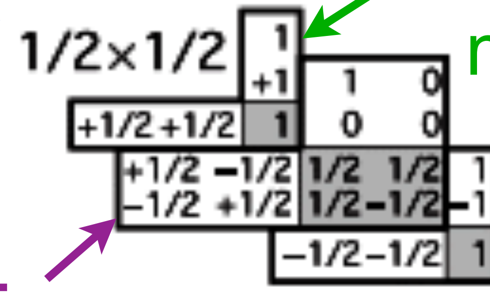
ex:
m_1,m_2=
-1/2,+/1-2

$$p = |\ \frac{1}{2}\ \frac{1}{2}\ >$$

$$n = |\ \frac{1}{2}\ \frac{-1}{2}\ >$$

Combining isospin

Combining two 1/2 x 1/2 particles



ex: J=1 m=1

ex: $m_1, m_2 =$ -1/2,+/1-2

$$|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{1}{2}> = |1\ 1>$$

$$|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{-1}{2}> = \sqrt{\frac{1}{2}}|1\ 0> + \sqrt{\frac{1}{2}}|0\ 0>$$

$$|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{1}{2}> = \sqrt{\frac{1}{2}}|1\ 0> - \sqrt{\frac{1}{2}}|0\ 0>$$

$$|\frac{-1}{2}\ \frac{1}{2}> + |\frac{-1}{2}\ \frac{1}{2}> = |1\ -1>$$

Putting it together

$$|\frac{1}{2}\ \frac{1}{2} > +|\frac{1}{2}\ \frac{1}{2} >= |1\ 1 >$$

$$|\frac{1}{2}\ \frac{1}{2} > +|\frac{1}{2}\ \frac{-1}{2} >= \sqrt{\frac{1}{2}}|1\ 0 > +\sqrt{\frac{1}{2}}|0\ 0 >$$

$$|\frac{1}{2}\ \frac{-1}{2} > +|\frac{1}{2}\ \frac{1}{2} >= \sqrt{\frac{1}{2}}|1\ 0 > -\sqrt{\frac{1}{2}}|0\ 0 >$$

$$|\frac{-1}{2}\ \frac{1}{2} > +|\frac{-1}{2}\ \frac{1}{2} >= |1\ -1 >$$

$\longrightarrow$

$$pp = |1\ 1 >$$

$$pn = \sqrt{\frac{1}{2}}(|1\ 0 > +|0\ 0 >)$$

$$np = \sqrt{\frac{1}{2}}(|1\ 0 > -|0\ 0 >)$$

$$nn = |1\ -1 >$$

Putting it together

$$pp = |1\ 1>$$

$$pn = \sqrt{\frac{1}{2}}(|1\ 0> + |0\ 0>)$$

$$np = \sqrt{\frac{1}{2}}(|1\ 0> - |0\ 0>)$$

$$nn = |1\ -1>$$

$$pp = |1\ 1>$$

$$\frac{1}{\sqrt{2}}(pn + np) = |1\ 0>$$

$$\frac{1}{\sqrt{2}}(pn - np) = |0\ 0>$$

$$nn = |1\ -1>$$

What is a deuteron

$$pp = |1\ 1>$$

$$\frac{1}{\sqrt{2}}(pn + np) = |1\ 0>$$

$$\frac{1}{\sqrt{2}}(pn - np) = |0\ 0>$$

$$nn = |1\ -1>$$

The deuteron is an ‘np’ bound state. There are no nn and no pp bound states. Since isospin is conserved by strong interactions, if the |1 0> state is a bound state, then so must be the |1 -1> and |1 1> states, which is not the case. So deuteron must be isosinglet |0 0>

Nucleon scattering

$p + p \rightarrow d + \pi^{+}$ |1 1>

$p + n \rightarrow d + \pi^{0}$ |1 0>

$n + n \rightarrow d + \pi^{-}$ |1 -1>

d is isosinglet (I=0) so it adds no isospin to objects on right

$$pp = |1\ 1>$$

$$pn = \sqrt{\frac{1}{2}}(|1\ 0> + |0\ 0>)$$

$$np = \sqrt{\frac{1}{2}}(|1\ 0> - |0\ 0>)$$

$$nn = |1\ -1>$$

$$\pi^{+} = |\ 1\ 1\ >$$

$$\pi^{0} = |\ 1\ 0\ >$$

$$\pi^{-} = |\ 1\ -1\ >$$

Nucleon scattering

$p + p \rightarrow d + \pi^+$ |1 1> → |1 1>

$p + n \rightarrow d + \pi^0$ 1/sqrt(2)[|1 0>+|0 0>] → |1 0>

$n + n \rightarrow d + \pi^-$ |1 -1> → |1 -1>

$$pp = |1\ 1>$$

$$pn = \sqrt{\frac{1}{2}}(|1\ 0> + |0\ 0>)$$

$$np = \sqrt{\frac{1}{2}}(|1\ 0> - |0\ 0>)$$

$$nn = |1\ -1>$$

$$\pi^+ = |\ 1\ 1\ >$$

$$\pi^0 = |\ 1\ 0\ >$$

$$\pi^- = |\ 1\ -1\ >$$

Nucleon scattering

$p + p \rightarrow d + \pi^+$ |1 1> → |1 1>

$p + n \rightarrow d + \pi^0$ 1/sqrt(2)[|1 0>+|0 0>] → |1 0>

$n + n \rightarrow d + \pi^-$ |1 -1> → |1 -1>

Assuming that these are purely QCD scattering, then middle reaction must have matrix element for this process 1/sqrt{2} of the others (since isospin is conserved), so will proceed at half the rate

Another example

$\pi^+ p \to \pi^+ p$

$\pi^- p \to \pi^- p$

$\pi^0 p \to \pi^0 p$

$\pi^+ n \to \pi^+ n$

$\pi^- n \to \pi^- n$

$\pi^0 n \to \pi^0 n$

$\pi^+ n \to \pi^0 p$

$\pi^0 n \to \pi^- p$

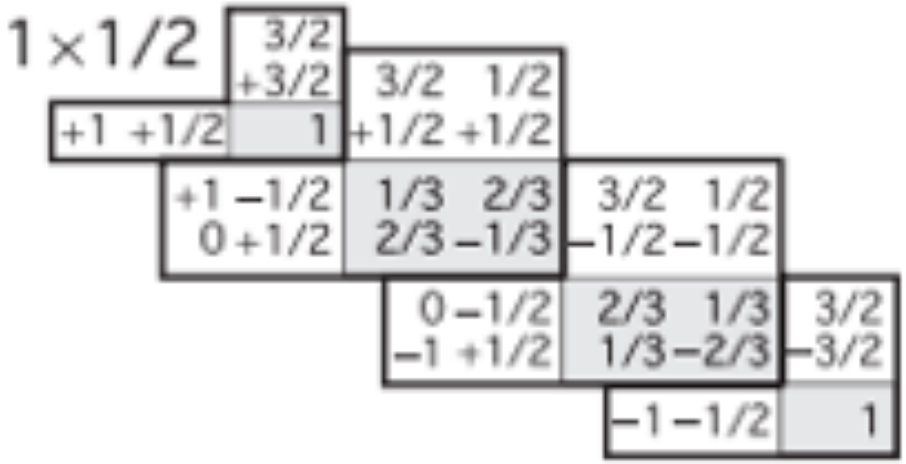
$\pi^0 p \to \pi^+ n$

$\pi^- p \to \pi^0 n$

Pion has I=1 and n/p have I=1/2 so total isospin can be 1/2 or 3/2

Let’s do this together

From Particle Data Group



Pion + nucleon isospin

$$\pi^+ + p : |1\ 1> |\frac{1}{2}\ \frac{1}{2}> = |\frac{3}{2}\ \frac{3}{2}>$$

$$\pi^0 + p : |1\ 0> |\frac{1}{2}\ \frac{1}{2}> = \sqrt{\frac{2}{3}}|\frac{3}{2}\ \frac{1}{2}> - \frac{1}{\sqrt{3}}|\frac{1}{2}\ \frac{1}{2}>$$

$$\pi^- + p : |1\ -1> |\frac{1}{2}\ \frac{1}{2}> = \sqrt{\frac{1}{3}}|\frac{3}{2}\ \frac{-1}{2}> - \sqrt{\frac{2}{3}}|\frac{1}{2}\ \frac{-1}{2}>$$

$$\pi^+ + n : |1\ 1> |\frac{1}{2}\ \frac{-1}{2}> = \sqrt{\frac{1}{3}}|\ \frac{3}{2}\frac{1}{2}> + \sqrt{\frac{2}{3}}|\frac{1}{2}\ \frac{1}{2}>$$

$$\pi^0 + n : |1\ 0> |\frac{1}{2}\ \frac{-1}{2}> = \sqrt{\frac{2}{3}}|\frac{3}{2}\ \frac{-1}{2}> + \frac{1}{\sqrt{3}}|\frac{1}{2}\ \frac{-1}{2}>$$

$$\pi^- + n : |1\ -1> |\frac{1}{2}\ \frac{-1}{2}> = |\frac{3}{2}\ \frac{-3}{2}>$$

Pion + nucleon scattering

$$\pi^+ p \rightarrow \pi^+ p : |\frac{3}{2}\ \frac{3}{2} > \rightarrow |\frac{3}{2}\ \frac{3}{2} >$$

$$\pi^- p \rightarrow \pi^- p : \sqrt{\frac{1}{3}}|\frac{3}{2}\ \frac{-1}{2} > -\sqrt{\frac{2}{3}}|\frac{1}{2}\ \frac{-1}{2} > \rightarrow \sqrt{\frac{1}{3}}|\frac{3}{2}\ \frac{-1}{2} > -\sqrt{\frac{2}{3}}|\frac{1}{2}\ \frac{-1}{2} >$$

$$\pi^0 p \rightarrow \pi^0 p : \sqrt{\frac{2}{3}}|\frac{3}{2}\ \frac{1}{2} > -\frac{1}{\sqrt{3}}|\frac{1}{2}\ \frac{1}{2} > \rightarrow \sqrt{\frac{2}{3}}|\frac{3}{2}\ \frac{1}{2} > -\frac{1}{\sqrt{3}}|\frac{1}{2}\ \frac{1}{2} >$$

$$\pi^+ n \rightarrow \pi^+ n : \sqrt{\frac{1}{3}}|\ \frac{3}{2}\frac{1}{2} > +\sqrt{\frac{2}{3}}|\frac{1}{2}\ \frac{1}{2} > \rightarrow \sqrt{\frac{1}{3}}|\ \frac{3}{2}\frac{1}{2} > +\sqrt{\frac{2}{3}}|\frac{1}{2}\ \frac{1}{2} >$$

$$\pi^- n \rightarrow \pi^- n : |\frac{3}{2}\ \frac{-3}{2} > \rightarrow |\frac{3}{2}\ \frac{-3}{2} >$$

$$\pi^0 n \rightarrow \pi^0 n : \sqrt{\frac{2}{3}}|\frac{3}{2}\ \frac{-1}{2} > +\frac{1}{\sqrt{3}}|\frac{1}{2}\ \frac{-1}{2} > \rightarrow \sqrt{\frac{2}{3}}|\frac{3}{2}\ \frac{-1}{2} > +\frac{1}{\sqrt{3}}|\frac{1}{2}\ \frac{-1}{2} >$$

$$\pi^+ n \rightarrow \pi^0 p : \sqrt{\frac{1}{3}}|\ \frac{3}{2}\frac{1}{2} > +\sqrt{\frac{2}{3}}|\frac{1}{2}\ \frac{1}{2} > \rightarrow \sqrt{\frac{2}{3}}|\frac{3}{2}\ \frac{1}{2} > -\frac{1}{\sqrt{3}}|\frac{1}{2}\ \frac{1}{2} >$$

$$\pi^0 n \rightarrow \pi^- p : \sqrt{\frac{2}{3}}|\frac{3}{2}\ \frac{-1}{2} > +\frac{1}{\sqrt{3}}|\frac{1}{2}\ \frac{-1}{2} > \rightarrow \sqrt{\frac{1}{3}}|\frac{3}{2}\ \frac{-1}{2} > -\sqrt{\frac{2}{3}}|\frac{1}{2}\ \frac{-1}{2} >$$

$$\pi^0 p \rightarrow \pi^+ n : \sqrt{\frac{2}{3}}|\frac{3}{2}\ \frac{1}{2} > -\frac{1}{\sqrt{3}}|\frac{1}{2}\ \frac{1}{2} > \rightarrow \sqrt{\frac{1}{3}}|\ \frac{3}{2}\frac{1}{2} > +\sqrt{\frac{2}{3}}|\frac{1}{2}\ \frac{1}{2} >$$

$$\pi^- p \rightarrow \pi^0 n : \sqrt{\frac{1}{3}}|\frac{3}{2}\ \frac{-1}{2} > -\sqrt{\frac{2}{3}}|\frac{1}{2}\ \frac{-1}{2} > \sqrt{\frac{2}{3}}|\frac{3}{2}\ \frac{-1}{2} > +\frac{1}{\sqrt{3}}|\frac{1}{2}\ \frac{-1}{2} >$$

Phew

Pion + nucleon scattering

$$\pi^+ p \to \pi^+ p : |\frac{3}{2}\ \frac{3}{2}> \to |\frac{3}{2}\ \frac{3}{2}> \quad \mathcal{M}_{3/2}$$

$$\pi^- p \to \pi^- p : \sqrt{\frac{1}{3}}|\frac{3}{2}\ \frac{-1}{2}> -\sqrt{\frac{2}{3}}|\frac{1}{2}\ \frac{-1}{2}> \to \sqrt{\frac{1}{3}}|\frac{3}{2}\ \frac{-1}{2}> -\sqrt{\frac{2}{3}}|\frac{1}{2}\ \frac{-1}{2}>$$

$$\pi^0 p \to \pi^0 p : \sqrt{\frac{2}{3}}|\frac{3}{2}\ \frac{1}{2}> -\frac{1}{\sqrt{3}}|\frac{1}{2}\ \frac{1}{2}> \to \sqrt{\frac{2}{3}}|\frac{3}{2}\ \frac{1}{2}> -\frac{1}{\sqrt{3}}|\frac{1}{2}\ \frac{1}{2}>$$

$$\pi^+ n \to \pi^+ n : \sqrt{\frac{1}{3}}|\ \frac{3}{2}\frac{1}{2}> +\sqrt{\frac{2}{3}}|\frac{1}{2}\ \frac{1}{2}> \to \sqrt{\frac{1}{3}}|\ \frac{3}{2}\frac{1}{2}> +\sqrt{\frac{2}{3}}|\frac{1}{2}\ \frac{1}{2}>$$

$$\sqrt{\frac{1}{3}}\sqrt{\frac{1}{3}}\mathcal{M}_{3/2} + (-\sqrt{\frac{2}{3}})(-\sqrt{\frac{2}{3}})\mathcal{M}_{1/2} = \frac{1}{3}\mathcal{M}_{3/2} + \frac{2}{3}\mathcal{M}_{1/2}$$

$$\pi^- n \to \pi^- n : |\frac{3}{2}\ \frac{-3}{2}> \to |\frac{3}{2}\ \frac{-3}{2}> \quad \mathcal{M}_{3/2}$$

$$\pi^0 n \to \pi^0 n : \sqrt{\frac{2}{3}}|\frac{3}{2}\ \frac{-1}{2}> +\frac{1}{\sqrt{3}}|\frac{1}{2}\ \frac{-1}{2}> \to \sqrt{\frac{2}{3}}|\frac{3}{2}\ \frac{-1}{2}> +\frac{1}{\sqrt{3}}|\frac{1}{2}\ \frac{-1}{2}>$$

$$\pi^+ n \to \pi^0 p : \sqrt{\frac{1}{3}}|\ \frac{3}{2}\frac{1}{2}> +\sqrt{\frac{2}{3}}|\frac{1}{2}\ \frac{1}{2}> \to \sqrt{\frac{2}{3}}|\frac{3}{2}\ \frac{1}{2}> -\frac{1}{\sqrt{3}}|\frac{1}{2}\ \frac{1}{2}>$$

$$\pi^0 n \to \pi^- p : \sqrt{\frac{2}{3}}|\frac{3}{2}\ \frac{-1}{2}> +\frac{1}{\sqrt{3}}|\frac{1}{2}\ \frac{-1}{2}> \to \sqrt{\frac{1}{3}}|\frac{3}{2}\ \frac{-1}{2}> -\sqrt{\frac{2}{3}}|\frac{1}{2}\ \frac{-1}{2}>$$

$$\pi^0 p \to \pi^+ n : \sqrt{\frac{2}{3}}|\frac{3}{2}\ \frac{1}{2}> -\frac{1}{\sqrt{3}}|\frac{1}{2}\ \frac{1}{2}> \to \sqrt{\frac{1}{3}}|\ \frac{3}{2}\frac{1}{2}> +\sqrt{\frac{2}{3}}|\frac{1}{2}\ \frac{1}{2}>$$

$$\pi^- p \to \pi^0 n : \sqrt{\frac{1}{3}}|\frac{3}{2}\ \frac{-1}{2}> -\sqrt{\frac{2}{3}}|\frac{1}{2}\ \frac{-1}{2}> \ \sqrt{\frac{2}{3}}|\frac{3}{2}\ \frac{-1}{2}> +\frac{1}{\sqrt{3}}|\frac{1}{2}\ \frac{-1}{2}>$$

Let's work out the rest together

From Griffiths

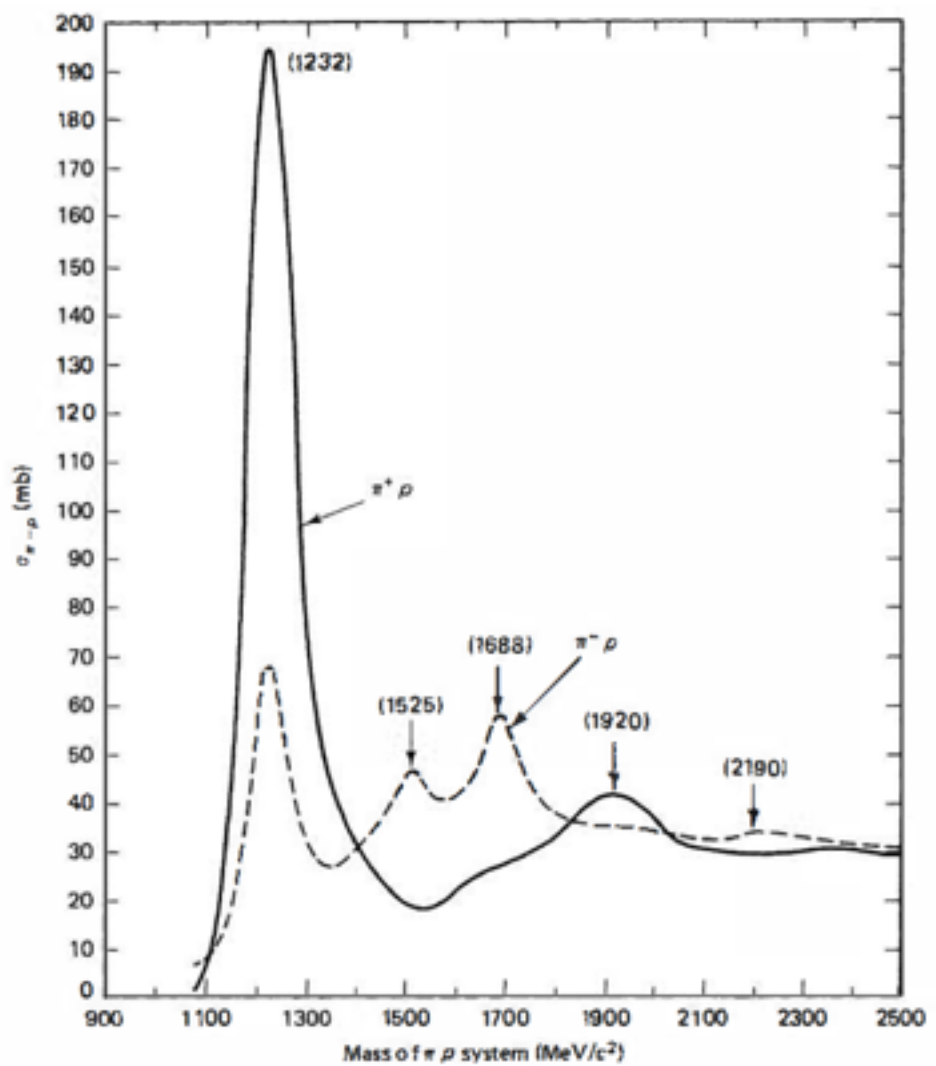


Fig. 4.6 Total cross sections for π^+p (solid line) and π^-p (dashed line) scattering. (*Source:* Gasiorowicz, S. (1966) *Elementary Particle Physics*, John Wiley & Sons, New York, p. 294. Reprinted by permission of John Wiley and Sons, Inc.)

Nice bump at 1232 MeV! This is the Δ^{++} resonance. But it has I=3/2, so at this around this mass we know something about the relative nature of matrix elements

On isospin

If isospin is conserved, why don't neutron and proton have identical mass? Maybe it's electric charge? Well, why do Σ^+ and Σ^- have different masses? Isospin is a very good symmetry because the quarks have a very small mass. But the mass != 0. Strange quark mass is similar to u and d masses, but not quite as small. And charm quark is significantly more massive.

Σ_c^{++} (uuc): 2454 MeV
Σ_c^{+} (udc): 2453 MeV
Σ_c^{0} (ddc): 2454 MeV

Σ^+ (uus): 1189 MeV
Σ^0 (uds): 1193 MeV
Σ^-(dds): 1197 MeV

Quark masses (from Griffiths)

Table 4.4 Quark masses (MeV/c^2)

Quark flavor	Bare mass	Effective mass
u	2	336
d	5	340
s	95	486
c	1300	1550
b	4200	4730
t	174000	177 000

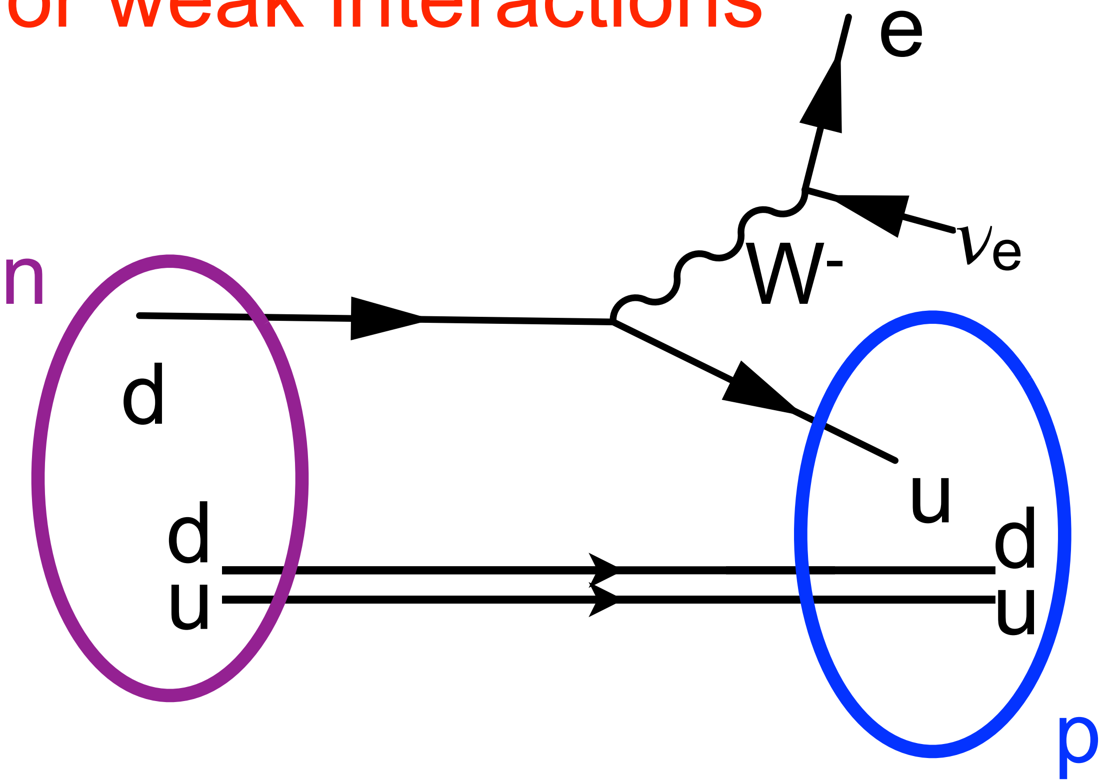
Note: These numbers may be a bit old but are not far from the modern values

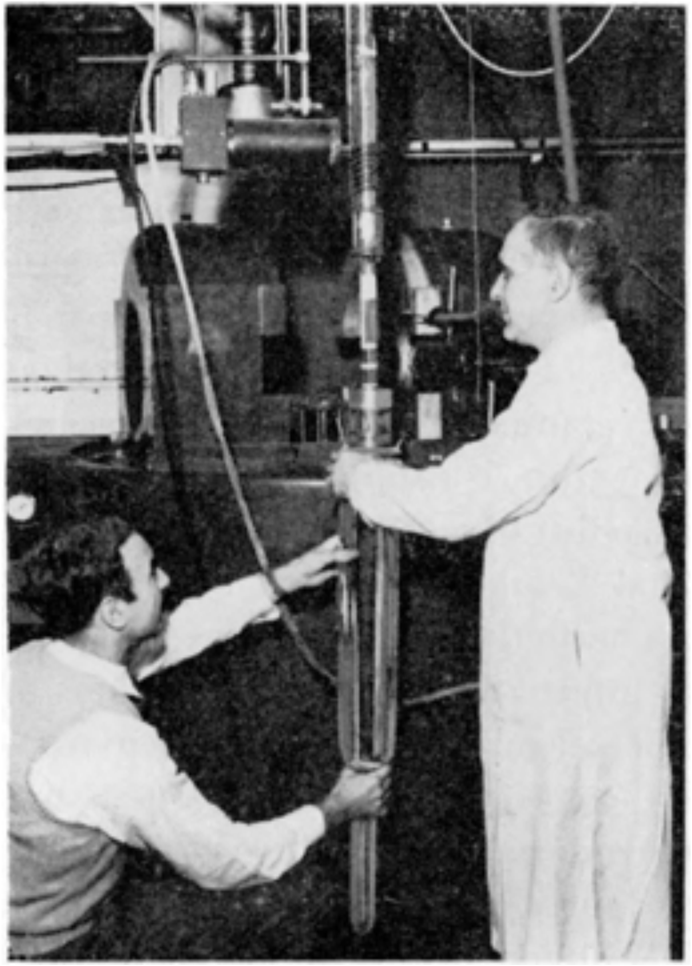
On to parity

$$^{60}\text{Co} \rightarrow ^{60}\text{Ni} + e^- + \overline{\nu}_e$$

Wu experiment at NIST to test for parity violation in Cobalt-60 radioactive decays

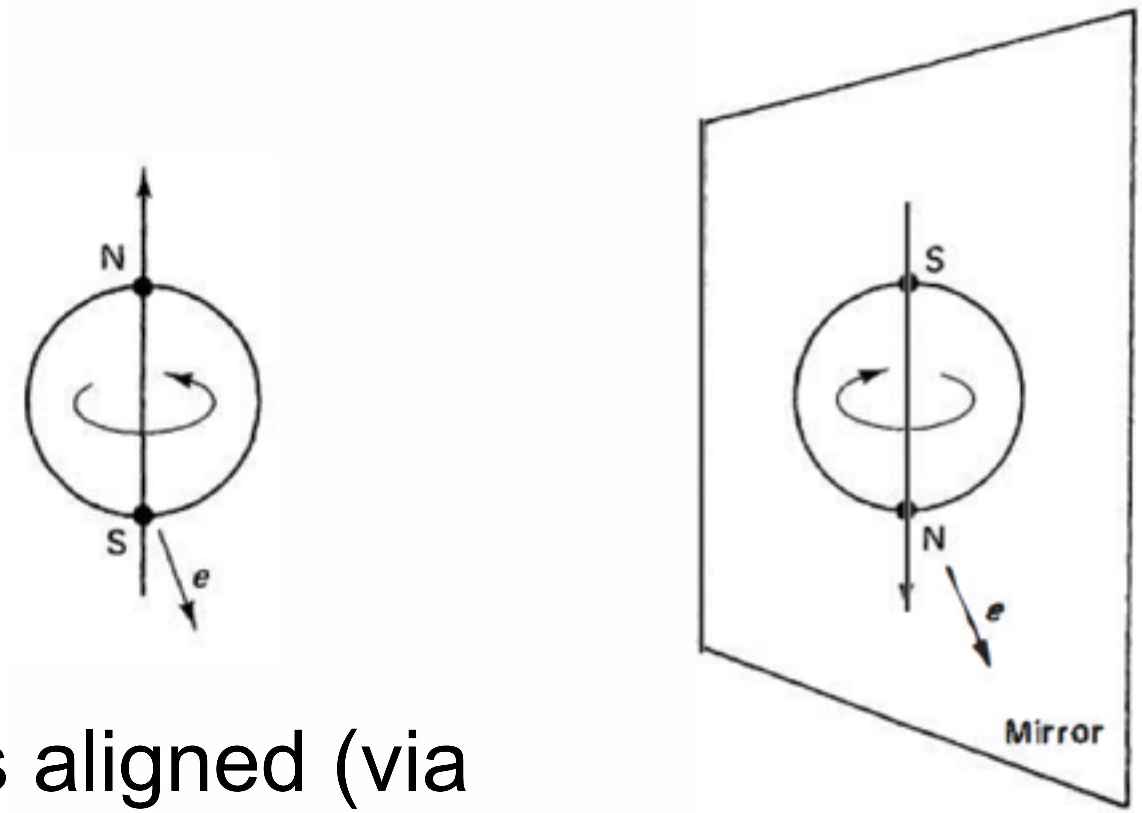
Lee and Yang (1956): Parity conserved in strong and EM processes, but no tests yet for weak interactions





Maximal parity violation

Image from Griffiths



Things look different in a hypothetical mirror!

Spins aligned (via B field) to point in z direction.
Find that electrons overwhelmingly prefer to come out towards south pole!

Image from Griffiths

Helicity = +1   Helicity = -1

Choose z axis as axis of motion of an object. Helicity = m_s/s. For spin-1/2 particles, $m_s = \pm 1/2$ so helicity = ±1. Only makes sense if another reference frame cannot overtake the particle and change the z axis direction! Neutrinos are ~massless so this is OK. What is found?

Image from Griffiths

Helicity = +1 Helicity = -1

Helicity (neutrino) = -1 (left-handed)
Helicity (anti-neutrino) = +1 (right-handed)

All neutrinos are left-handed
All anti-neutrinos are right-handed

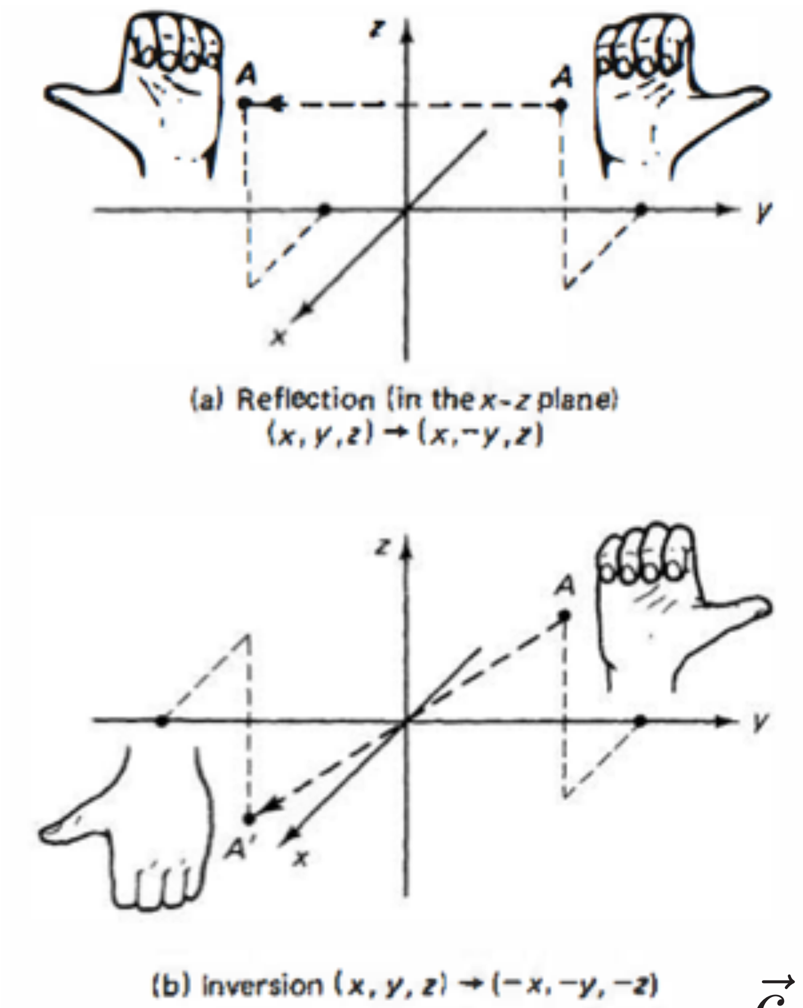
How to measure neutrino handedness

Image from Griffiths

Fig. 4.10 Decay of π^- at rest.

Pion has spin 0. In rest frame, muon and anti-neutrino are back-to-back and spins must be oppositely aligned. Measure muon helicity here as always right-handed, implies anti-neutrino is always right-handed

Image from Griffiths



Parity operator P applies inversion

$$P(\vec{a}) = -\vec{a}, P(\vec{b}) = -\vec{b}$$

$$\vec{c} = \vec{a} \times \vec{b} \rightarrow P(\vec{c}) = -\vec{a} \times -\vec{b} = \vec{c}$$

a,b are vectors (P changes sign), c is a pseudovector, also called an axial vector, and does not change sign under P

$$P(\vec{a}) = -\vec{a}, P(\vec{b}) = -\vec{b}$$

$$\vec{c} = \vec{a} \times \vec{b} \rightarrow P(\vec{c}) = -\vec{a} \times -\vec{b} = \vec{c}$$

Thinking about parity

a,b and q are vectors
c is then a pseudovector, as we saw

d is then a scalar (P doesn’t change sign)
r is then a pseudoscalar (changes sign under P)

$$P(\vec{a}) = -\vec{a}, P(\vec{b}) = -\vec{b}, P(\vec{c}) = P(\vec{a} \times \vec{b}) = \vec{c}$$

$$d = \vec{a} \cdot \vec{b}, r = \vec{c} \cdot \vec{q}$$

$$P(d) = P(\vec{a} \cdot \vec{b}) = -\vec{a} \cdot -\vec{b} = d$$

$$P(r) = P(\vec{c} \cdot \vec{q}) = P(\vec{a} \times \vec{b} \cdot \vec{q}) = P(\vec{a}) \times P(\vec{b}) \cdot P(\vec{q})$$

$$P(r) = -\vec{a} \times -\vec{b} \cdot -\vec{q} = -r$$

Parity operator

P^2 is the identify operator, so P has eigenvalues +/- 1. By convention, quarks have parity = +1, anti-quarks have parity = -1. (Each factor of orbital angular momentum gives an additional parity factor of -1). Photon parity = -1

Parity is a multiplicative quantum number.

Baryon $P = (-1)^L$

Anti-baryon $P = (-1)^{L+1}$

Pseudoscalar, vector meson P = -1

Pseudovector, scalar meson P = +1

Why did Lee and Yang propose parity violation?

Theta and tau (two strange mesons) seem to have same mass, but can't be the same particle since they have different parity. This is not allowed unless weak decays violate parity! These are one and the same particle, now known as charged kaon (K^+)

$$\theta^+ \to \pi^+ + \pi^0 \quad [P = (-1)^2 = +1]$$

$$\tau^+ \to \pi^+ + \pi^0 + \pi^0 \quad [P = (-1)^3 = -1]$$

$$\tau^+ \to \pi^+ + \pi^+ + \pi^- \quad [P = (-1)^3 = -1]$$

Charge conjugation

Charge operator changes the sign of all internal quantum numbers (electron number, baryon number, electric charge, etc). NOT a symmetry of weak force

C^2 = Identity, but only particles that are their own anti-particles are eigenstates of C. For spin-1/2 particle and anti-particle, $C=(-1)^{L+s}$

Photon C =-1

Explains why neutral pions do not decay to three photons

$$\pi^0 \to \gamma\gamma$$

so $C(\pi^0) = +1$

CP symmetry

$$C|\nu_{e,L}> = |\overline{\nu}_{e,L}>$$

Charge conjugation not a symmetry of weak interaction: left-handed anti-neutrinos do not exist

$$P|\nu_{e,L}> = |\nu_{e,R}>$$

Parity not a symmetry of weak interaction: right-handed neutrinos also do not exist

$$CP|\nu_{e,L}> = |\overline{\nu}_{e,R}>$$

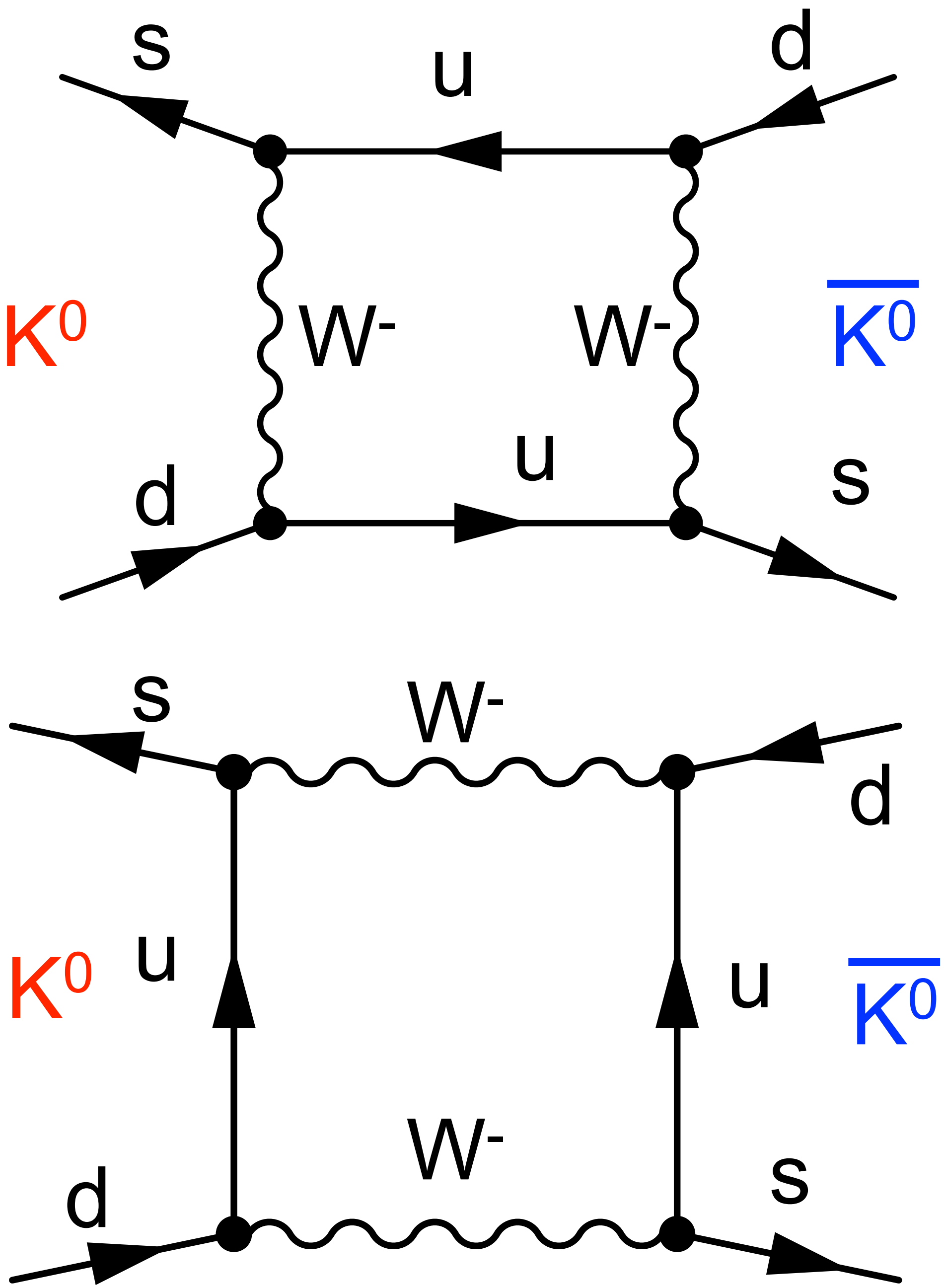
What about the combination? CP symmetry

CP symmetry

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

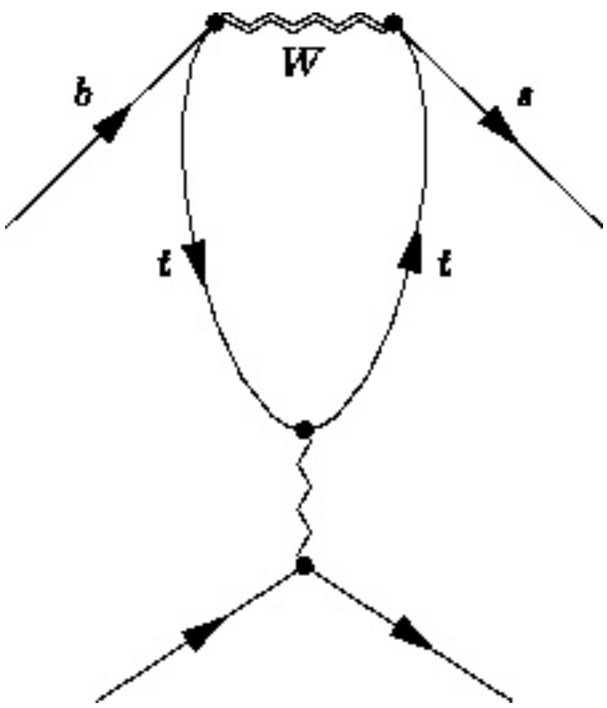
What do we get if we apply CP operators to both sides of this decay?

Neutral kaons



We call these “box” diagrams (hopefully for obvious if uncreative reasons). They are a way for neutral kaons to turn into their own antiparticles

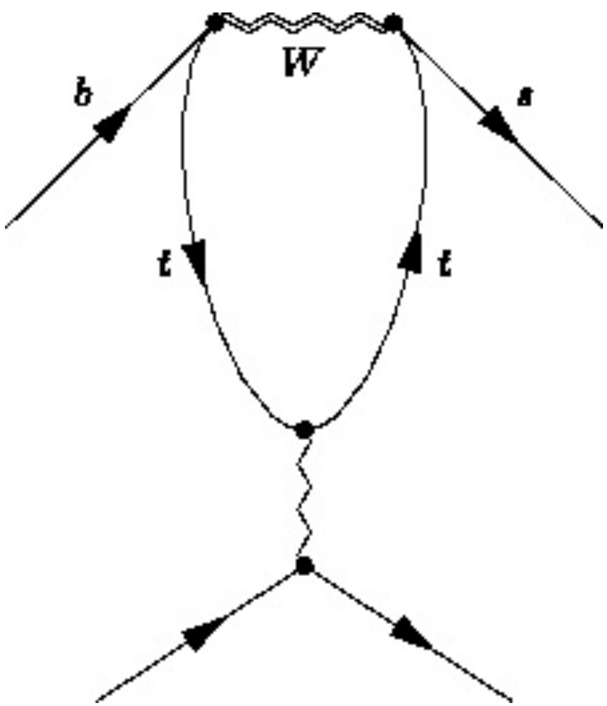
Fun aside on diagrams



What would you call this sort of diagram? It's a bit odd, but you have two quarks coming in and two quarks coming out (plays an important role in flavor physics). Any guesses?

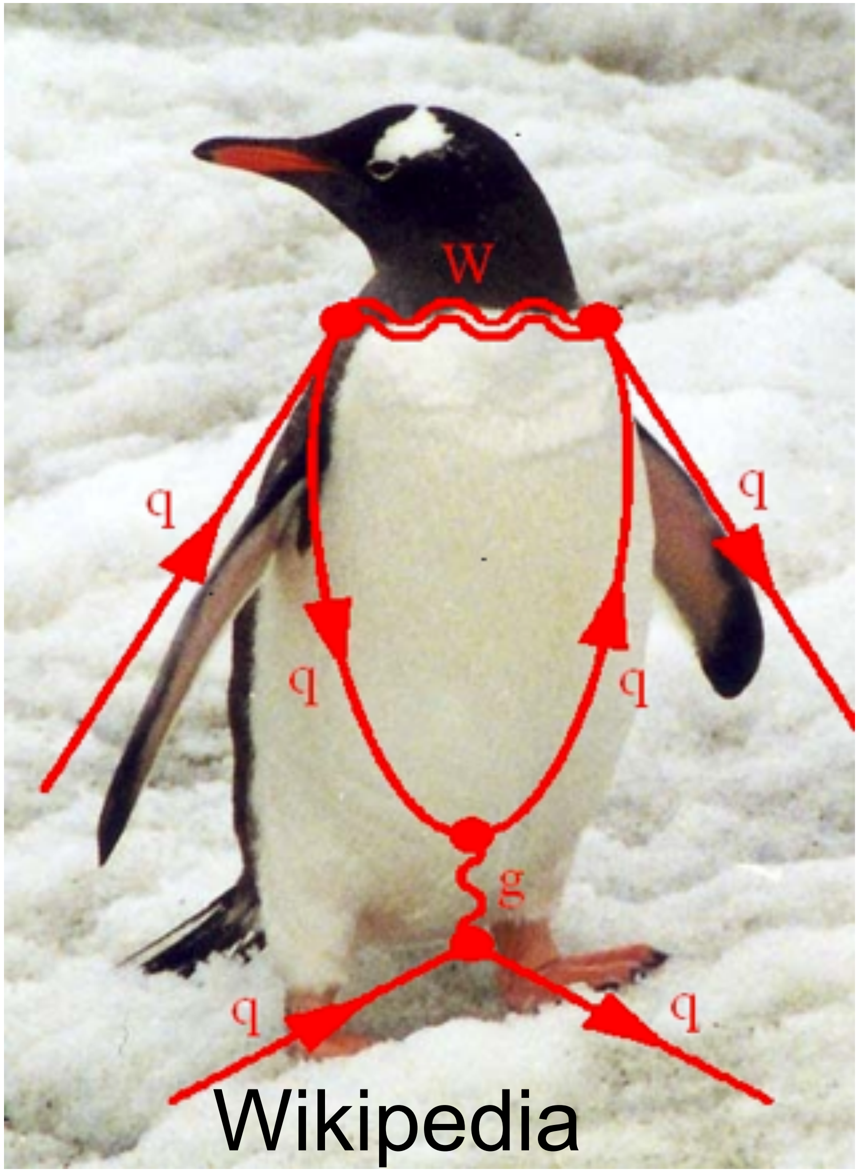
Fun aside on diagrams

http://asymptotia.com/2007/05/14/penguin-opportunity/



What would you call this sort of diagram? It's a bit odd, but you have two quarks coming in and two quarks coming out (plays an important role in flavor physics). Any guesses?

Fun aside on diagrams



A penguin diagram! Of course. I won't even ask you to guess how the name appeared, but will just quote the explanation :)

Penguin diagrams

http://arxiv.org/pdf/hep-ph/9510397v1.pdf

Could not have made this up if I tried. Disclaimer: I **do not** suggest that you can be a better physicist or get a better grade this way

By the way, about penguins. From time to time students ask about how this word could possibly penetrate high energy physics. This is a funny story, indeed. The first paper where the graphs that are now called penguins were considered in the weak decays appeared in JETP Letters in 1975, and there they did not look like penguins at all. Later on they were made look line penguins and called penguins by John Ellis. Here is his story as he recollects it himself. “Mary K. [Gaillard], Dimitri [Nanopoulos] and I first got interested in what are now called penguin diagrams while we were studying CP violation in the Standard Model in 1976... The penguin name came in 1977, as follows. In the spring of 1977, Mike Chanowitz, Mary K and I wrote a paper on GUTs predicting the b quark mass before it was found. When it was found a few weeks later, Mary K, Dimitri, Serge Rudaz and I immediately started working on its phenomenology. That summer, there was a student at CERN, Melissa Franklin who is now an experimentalist at Harvard. One evening, she, I and Serge went to a pub, and she and I started a game of darts. We made a bet that if I lost I had to put the word penguin into my next paper. She actually left the darts game before the end, and was replaced by Serge, who beat me. Nevertheless, I felt obligated to carry out the conditions of the bet. For some time, it was not clear to me how to get the word into this b quark paper that we were writing at the time. Then, one evening, after working at CERN, I stopped on my way back to my apartment to visit some friends living in Meyrin where I smoked some illegal substance. Later, when I got back to my apartment and continued working on our paper, I had a sudden flash that the famous diagrams look like penguins. So we put the name into our paper, and the rest, as they say, is history.”

From the PDG

LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$	• $\phi(1680)$	$0^-(1^{--})$
• π^0	$1^-(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$
• η	$0^+(0^{-+})$	• $\rho(1700)$	$1^+(1^{--})$
• $f_0(500)$	$0^+(0^{++})$	$a_2(1700)$	$1^-(2^{++})$
• $\rho(770)$	$1^+(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$
• $\omega(782)$	$0^-(1^{--})$	$\eta(1760)$	$0^+(0^{-+})$
• $\eta'(958)$	$0^+(0^{-+})$	• $\pi(1800)$	$1^-(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$	$f_2(1810)$	$0^+(2^{++})$
• $a_0(980)$	$1^-(0^{++})$	$X(1835)$	$?^?(?^{-+})$
• $\phi(1020)$	$0^-(1^{--})$	$X(1840)$	$?^?(?^{??})$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$
• $a_1(1260)$	$1^-(1^{++})$	• $\pi_2(1880)$	$1^-(2^{-+})$
• $f_2(1270)$	$0^+(2^{++})$	$\rho(1900)$	$1^+(1^{--})$
• $f_1(1285)$	$0^+(1^{++})$	$f_2(1910)$	$0^+(2^{++})$
• $\eta(1295)$	$0^+(0^{-+})$	• $f_2(1950)$	$0^+(2^{++})$
• $\pi(1300)$	$1^-(0^{-+})$	$\rho_3(1990)$	$1^+(3^{--})$
• $a_2(1320)$	$1^-(2^{++})$	• $f_2(2010)$	$0^+(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$	$f_0(2020)$	$0^+(0^{++})$
$h_1(1380)$	$?^-(1^{+-})$	• $a_4(2040)$	$1^-(4^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$	• $f_4(2050)$	$0^+(4^{++})$

STRANGE ($S = \pm1,\ C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K^0_S	$1/2(0^-)$
• K^0_L	$1/2(0^-)$
$K^*_0(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K^*_0(1430)$	$1/2(0^+)$
• $K^*_2(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K^*_3(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K^*_0(1950)$	$1/2(0^+)$

CHARMED, STRANGE ($C = S = \pm1$)

	$I(J^P)$
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D^*_{s0}(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}(2573)$	$0(?^?)$
• $D^*_{s1}(2700)^\pm$	$0(1^-)$
$D^*_{sJ}(2860)^\pm$	$0(?^?)$
$D_{sJ}(3040)^\pm$	$0(?^?)$

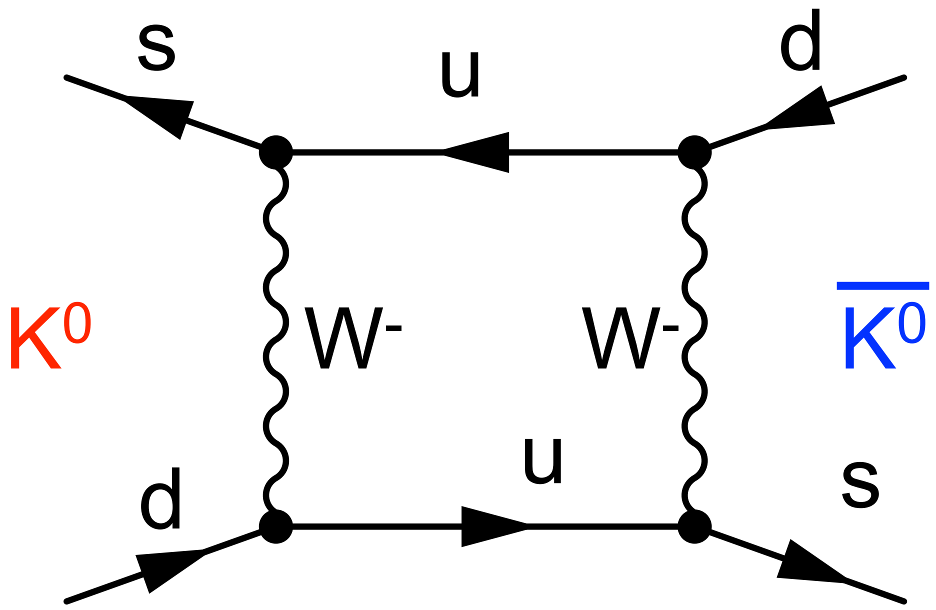
BOTTOM ($B = \pm1$)

• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B^0_s/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)^+$	$1/2(1^+)$
• $B_1(5721)^0$	$1/2(1^+)$

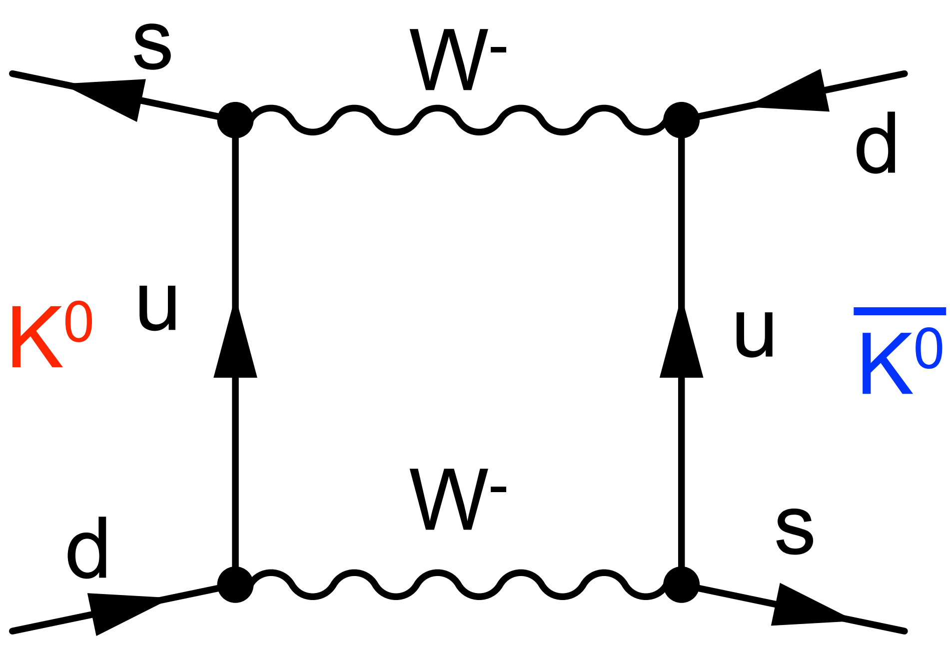
$c\bar{c}$

	$I^G(J^{PC})$
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $h_c(1P)$	$?^?(1^{+-})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
$X(3823)$	$?^?(?^{?-})$
• $X(3872)$	$0^+(1^{++})$
• $X(3900)^\pm$	$?(1^+)$
$X(3900)^0$	$?(?^?)$
• $\chi_{c0}(3915)$	$0^+(0^{++})$
• $\chi_{c2}(2P)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
$X(4020)^\pm$	$?(?^?)$
• $\psi(4040)$	$0^-(1^{--})$
$X(4050)^\pm$	$?(?^?)$
$X(4140)$	$0^+(?^{?+})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$

Back to more ... geometric animals



P = -1



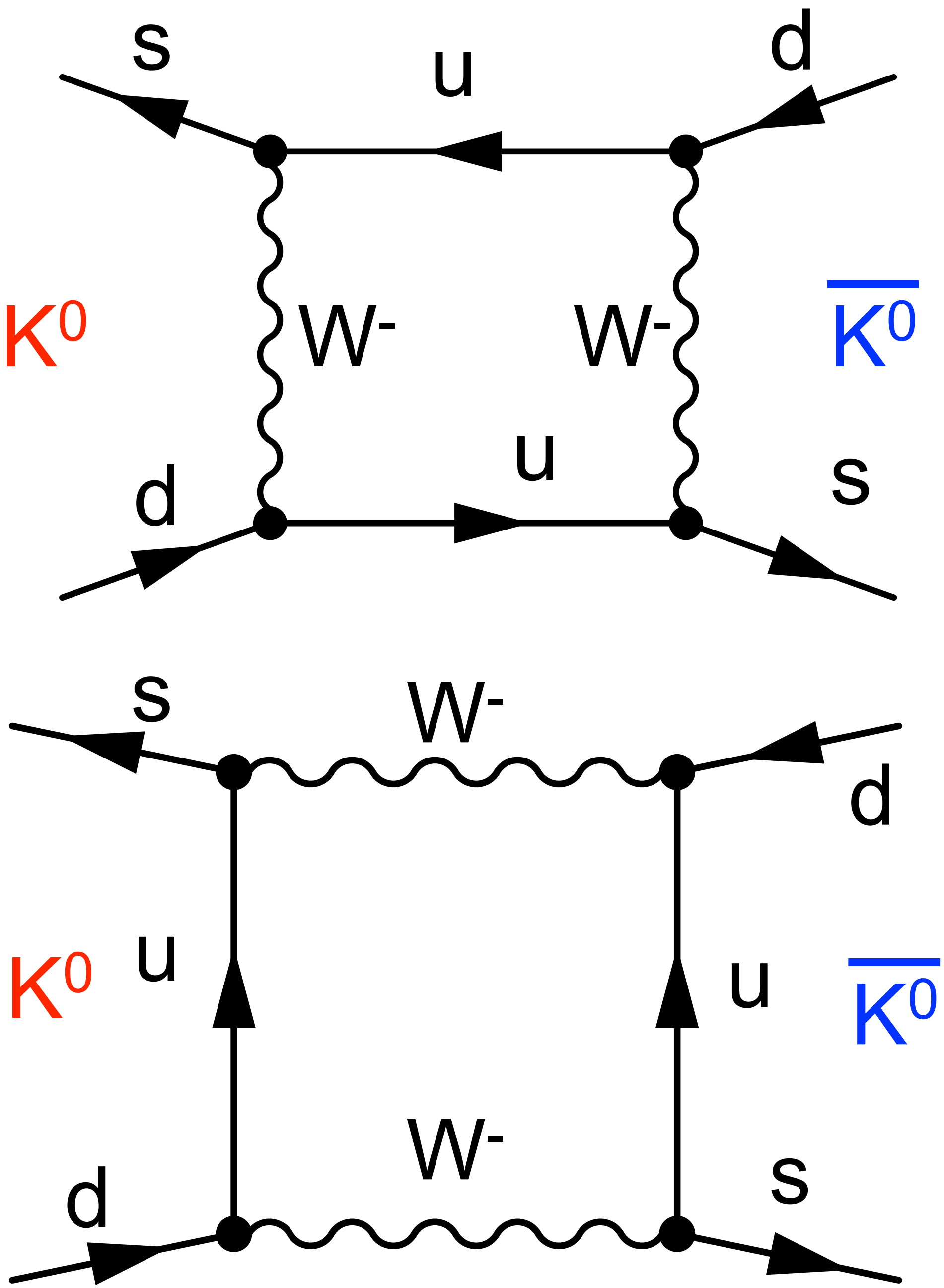
$$C|K^0 >= |\overline{K}^0 >$$

$$C|\overline{K}^0 >= |K^0 >$$

$$P|K^0 >= -|\overline{K}^0 >$$

$$P|\overline{K}^0 >= -|K^0 >$$

CP for neutral kaons



$$C|K^0> = |\overline{K}^0>$$

$$C|\overline{K}^0> = |K^0>$$

$$P|K^0> = -|\overline{K}^0>$$

$$P|\overline{K}^0> = -|K^0>$$

So...

CP for neutral kaons

$$CP|K^0> = -|\overline{K}^0>$$

$$CP|\overline{K}^0> = -|K^0>$$

$$CP|K_1> = |K_1>, CP|K_2> = -|K_2>$$

$$|K_1> = \frac{1}{\sqrt{2}}\left(|K^0> - |\overline{K}^0>\right)$$

$$|K_2> = \frac{1}{\sqrt{2}}\left(|K^0> + |\overline{K}^0>\right)$$

K_1 and K_2 are the eigenstates of CP, not the kaon and anti-kaon! If weak interactions conserve CP, then they will have different decays

CP for neutral kaons

$$CP|K^0> = -|\overline{K}^0>$$
$$CP|\overline{K}^0> = -|K^0>$$
$$CP|K_1> = |K_1>, CP|K_2> = -|K_2>$$
$$|K_1> = \frac{1}{\sqrt{2}}\left(|K^0> - |\overline{K}^0>\right)$$
$$|K_2> = \frac{1}{\sqrt{2}}\left(|K^0> + |\overline{K}^0>\right)$$

Neutral kayons decay most of the time to two-pion and three-pion final states

What is CP of the pions that kaons can decay to? Let's start with two-pion decay:

$$K^0/\overline{K}^0 \rightarrow \pi^+\pi^-$$
$$K^0/\overline{K}^0 \rightarrow \pi^0\pi^0$$
$$\pi^0 = \frac{1}{\sqrt{2}}(u\overline{u} - d\overline{d})$$
$$\pi^+ = ud$$
$$\pi^- = \overline{u}\overline{d}$$

CP for neutral kaons

What is CP of the pions that kaons can decay to? Let's start with two-pion decay:

$$\pi^+ = ud$$

$$\pi^- = \overline{u}\overline{d}$$

$$C|\pi^+ >= |\pi^- >$$

$$P|\pi^+ >= -|\pi^+ >$$

$$C|\pi^- >= |\pi^+ >$$

$$P|\pi^- >= -|\pi^- >$$

$$CP|\pi^-\pi^+ >= +|\pi^-\pi^+ >$$

$$K^0/\overline{K}^0 \rightarrow \pi^+\pi^-$$

$$K^0/\overline{K}^0 \rightarrow \pi^0\pi^0$$

$$\pi^0 = \frac{1}{\sqrt{2}}(u\overline{u} - d\overline{d})$$

$$C|\pi^0 >= |\pi^0 >$$

$$P|\pi^0 >= -|\pi^0 >$$

$$CP|\pi^0\pi^0 >= +|\pi^0\pi^0 >$$

So two pion decay is CP even

CP for neutral kaons

What about three-pion decay?

$$K^0/\overline{K}^0 \rightarrow \pi^+\pi^-\pi^0$$

$$K^0/\overline{K}^0 \rightarrow \pi^0\pi^0\pi^0$$

$$C|\pi^+\pi^-\pi^0> = +1|\pi^+\pi^-\pi^0>$$

$$C|\pi^0\pi^0\pi^0> = +1|\pi^0\pi^0\pi^0>$$

$$P|\pi^+\pi^-\pi^0> = -1|\pi^+\pi^-\pi^0>$$

$$P|\pi^0\pi^0\pi^0> = -1|\pi^0\pi^0\pi^0>$$

$$CP|\pi^+\pi^-\pi^0> = -1|\pi^+\pi^-\pi^0>$$

$$CP|\pi^0\pi^0\pi^0> = -1|\pi^0\pi^0\pi^0>$$

So three pion decay is CP odd

What does this mean?

$$CP|K^0> = -|\overline{K}^0>$$

$$CP|\overline{K}^0> = -|K^0>$$

Over long distance, only K longs remain!

$$CP|K_1> = |K_1>, CP|K_2> = -|K_2>$$

$$|K_1> = \frac{1}{\sqrt{2}}\left(|K^0> - |\overline{K}^0>\right)$$

$K_1 = K_S$ = "K short", lifetime ~ 9×10^{-11} s

$$|K_2> = \frac{1}{\sqrt{2}}\left(|K^0> + |\overline{K}^0>\right)$$

$K_2 = K_L$ = "K long" lifetime ~ 5×10^{-8} s

Assuming no CP violation in weak force, one eigenstate of kaons decays to two pions, the other to three pions. Three pion decay is closer to threshold, so it should be slower: K_2 should have longer lifetime

A common theme here

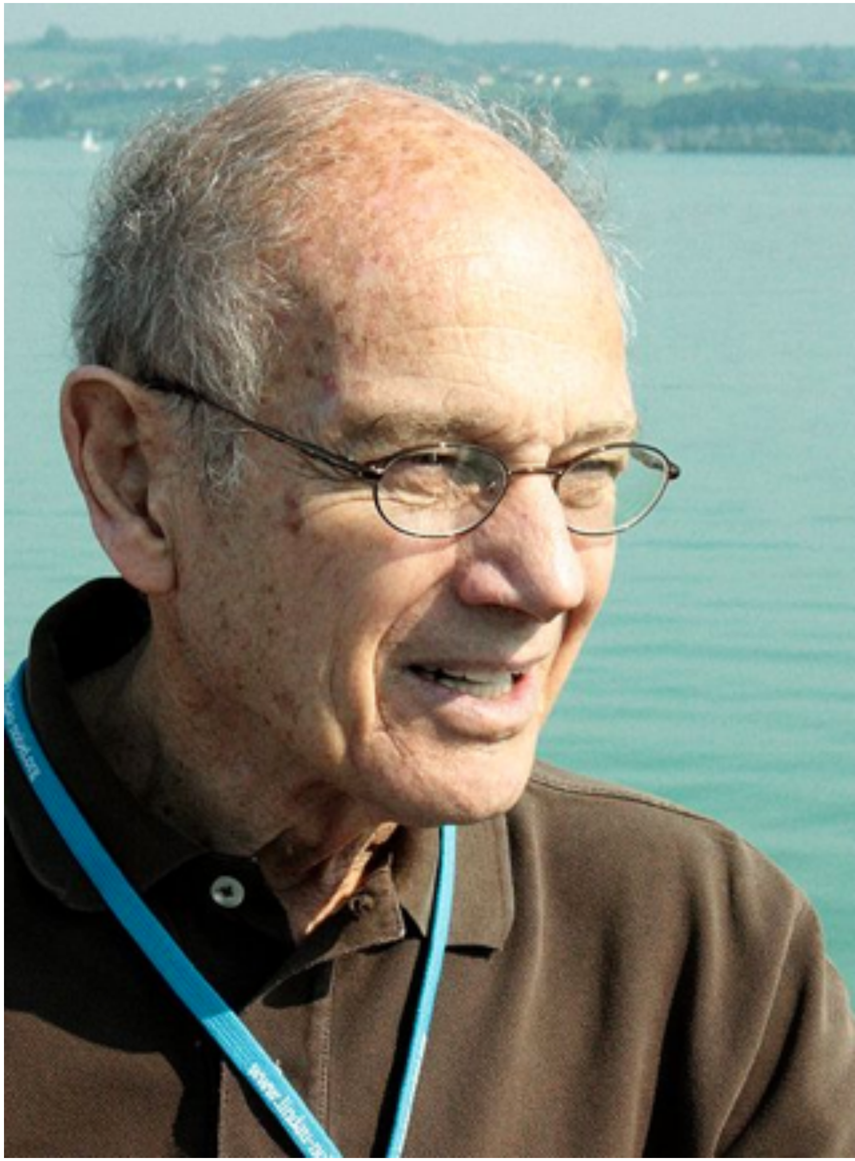
Need to consider different types of eigenstates. We observe quark/mass eigenstates d/s/b, but weak force couples to modified flavor versions d'/s'/b'

Kaons produced in eigenstates of strangeness, but oscillate back and forth, and decay as eigenstates of CP (mostly!)

Weak interactions couple to flavor eigenstates for neutrinos, but mass eigenstates are different!

It turns out that this isn't the full story

James Cronin and Val Fitch showed that weak interactions DO violate CP symmetry. Some K_L decay to two pions! Just not very often (~2 out of every thousand decays to pions). Matter and anti-matter are not the same! Led to prediction of third generation of quarks



James Cronin

It turns out that even this isn't the full story

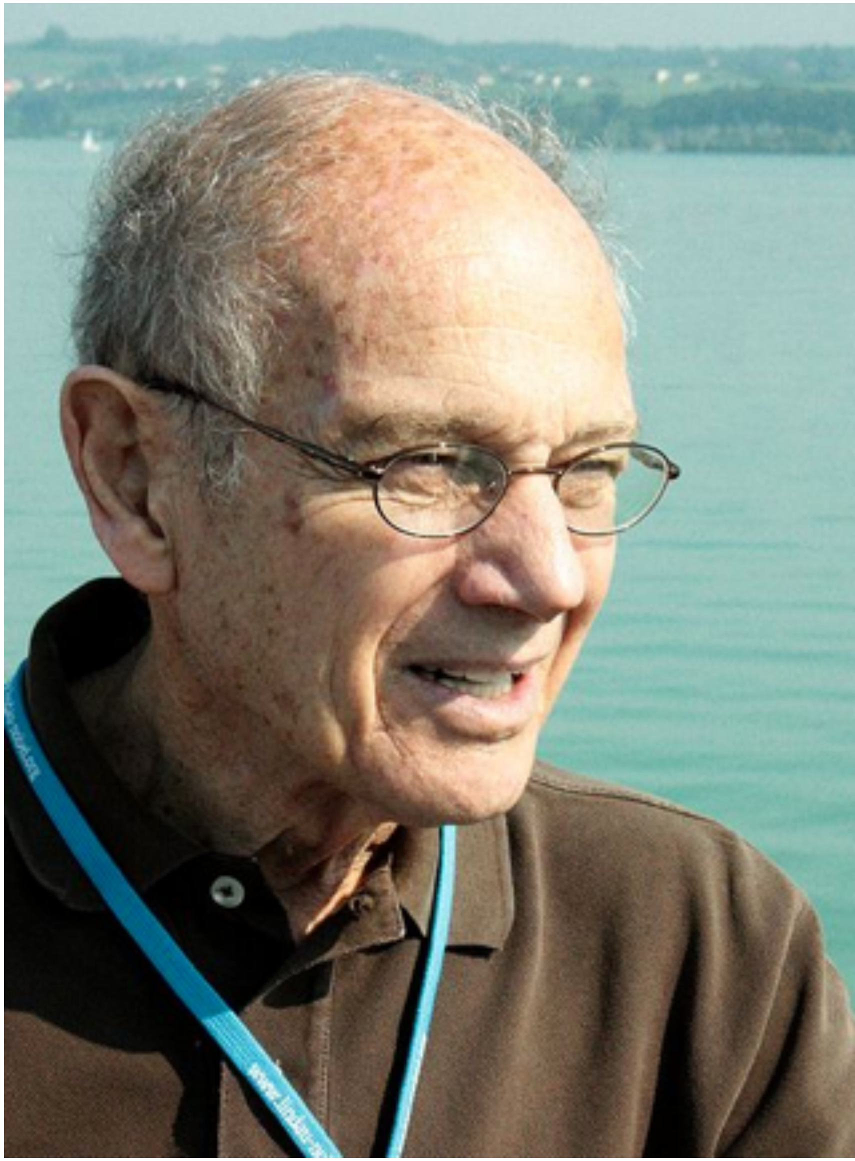
K_L prefers to decay to

$$\pi^+ + e^- + \overline{\nu}_e$$

instead of

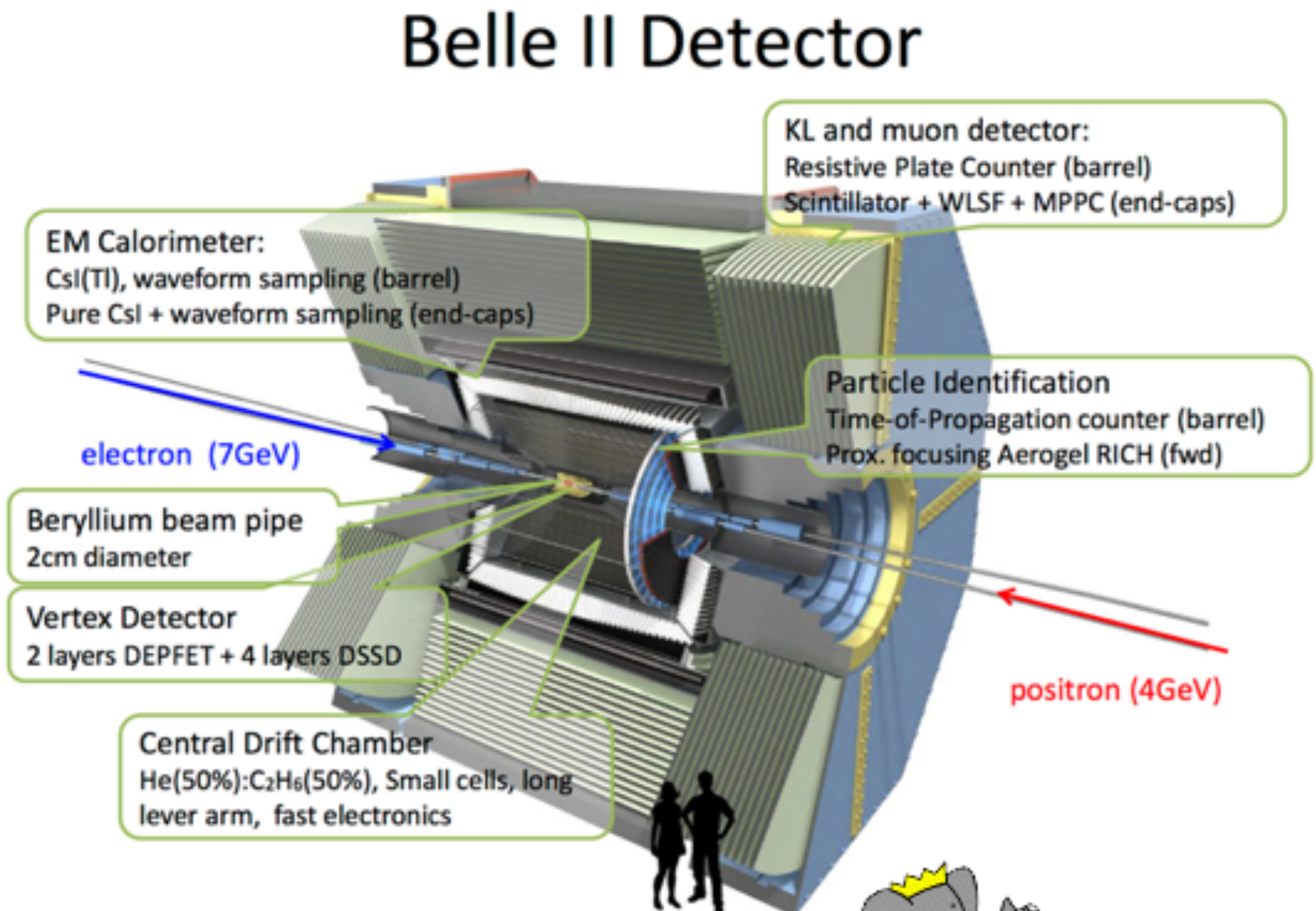
$$\pi^- + e^+ + \nu_e$$

by a few parts in a thousand! Matter and anti-matter are not really the same thing (well, we knew that already)



James Cronin

Getting the full story



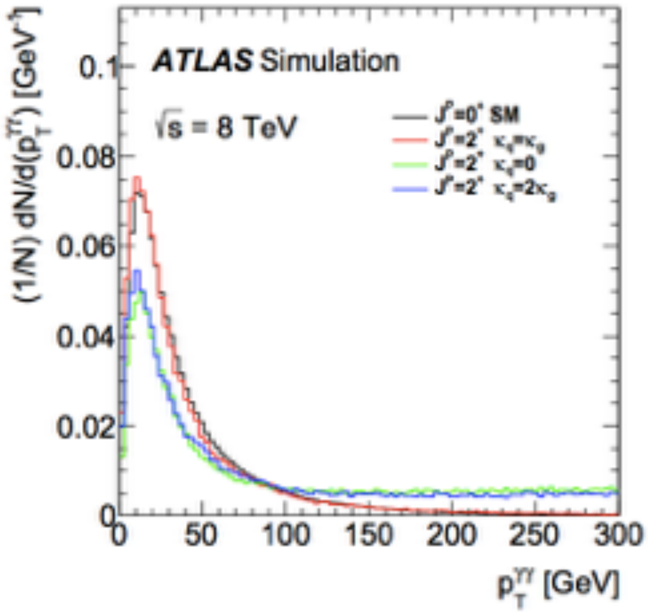
Data-taking stared ~2 years ago

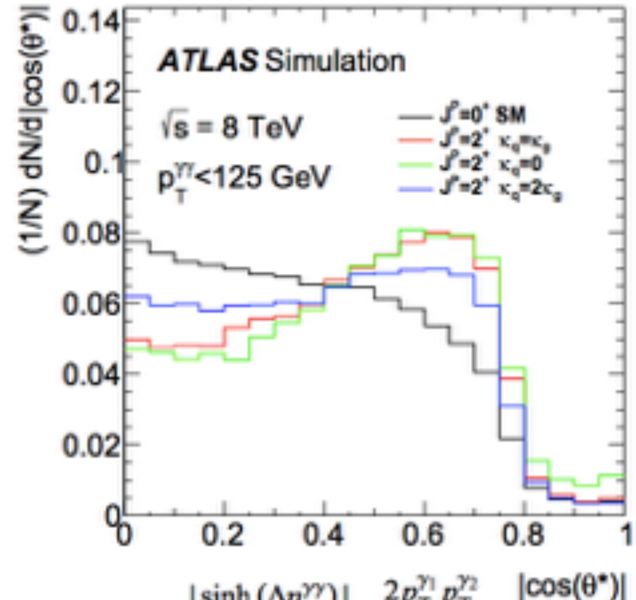
BABAR

arXiv: 1506.05669

For the SM Higgs boson, $J^{CP} = 0^{++}$

Look at a the center-of-mass frame of the diphoton system (Collins-Soper frame)





$$|\cos\theta^*| = \frac{|\sinh(\Delta\eta^{\gamma\gamma})|}{\sqrt{1+\left(p_T^{\gamma\gamma}/m_{\gamma\gamma}\right)^2}} \frac{2p_T^{\gamma_1} p_T^{\gamma_2}}{m_{\gamma\gamma}^2},$$

Tested Hypothesis	$H \to \gamma\gamma$				
	$p^{\text{alt}}_{\text{exp},\mu=1}$	$p^{\text{alt}}_{\text{exp},\mu=\hat{\mu}}$	$p^{\text{SM}}_{\text{obs}}$	$p^{\text{alt}}_{\text{obs}}$	Obs. CL_s (%)
$2^+(\kappa_q = \kappa_g)$	0.13	$7.5 \cdot 10^{-2}$	0.13	0.34	39
$2^+(\kappa_q = 0;\ p_T < 300$ GeV)	$4.3 \cdot 10^{-4}$	$< 3.1 \cdot 10^{-5}$	0.16	$2.9 \cdot 10^{-4}$	$3.5 \cdot 10^{-2}$
$2^+(\kappa_q = 0;\ p_T < 125$ GeV)	$9.4 \cdot 10^{-2}$	$5.6 \cdot 10^{-2}$	0.23	0.20	26
$2^+(\kappa_q = 2\kappa_g;\ p_T < 300$ GeV)	$9.1 \cdot 10^{-4}$	$< 3.1 \cdot 10^{-5}$	0.16	$8.6 \cdot 10^{-4}$	0.10
$2^+(\kappa_q = 2\kappa_g;\ p_T < 125$ GeV)	0.27	0.24	0.20	0.54	68