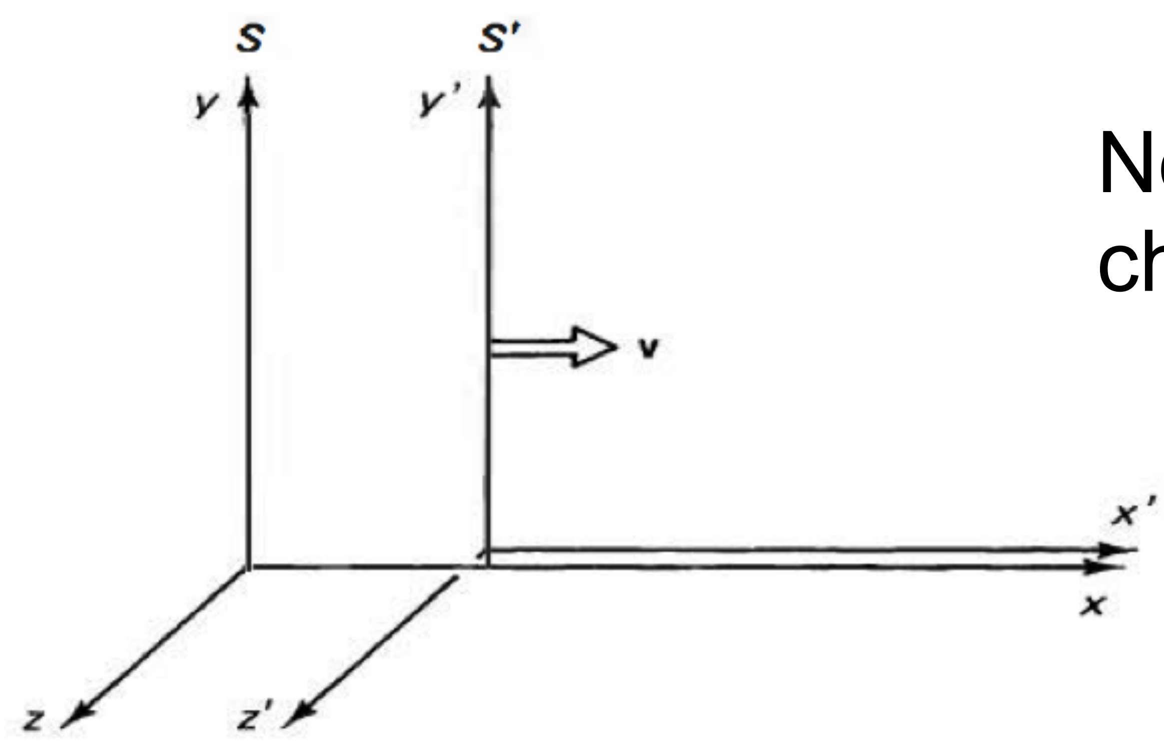
Time for some homework

Griffiths problems 1.19, 2.1, 2.2, 2.5, 2.6, 2.7, 2.8

Just a brief review at this stage before we delve in

Relativistic kinematics



Note simplicity from choice of units!

$$x' = \gamma\left(x - vt\right)$$

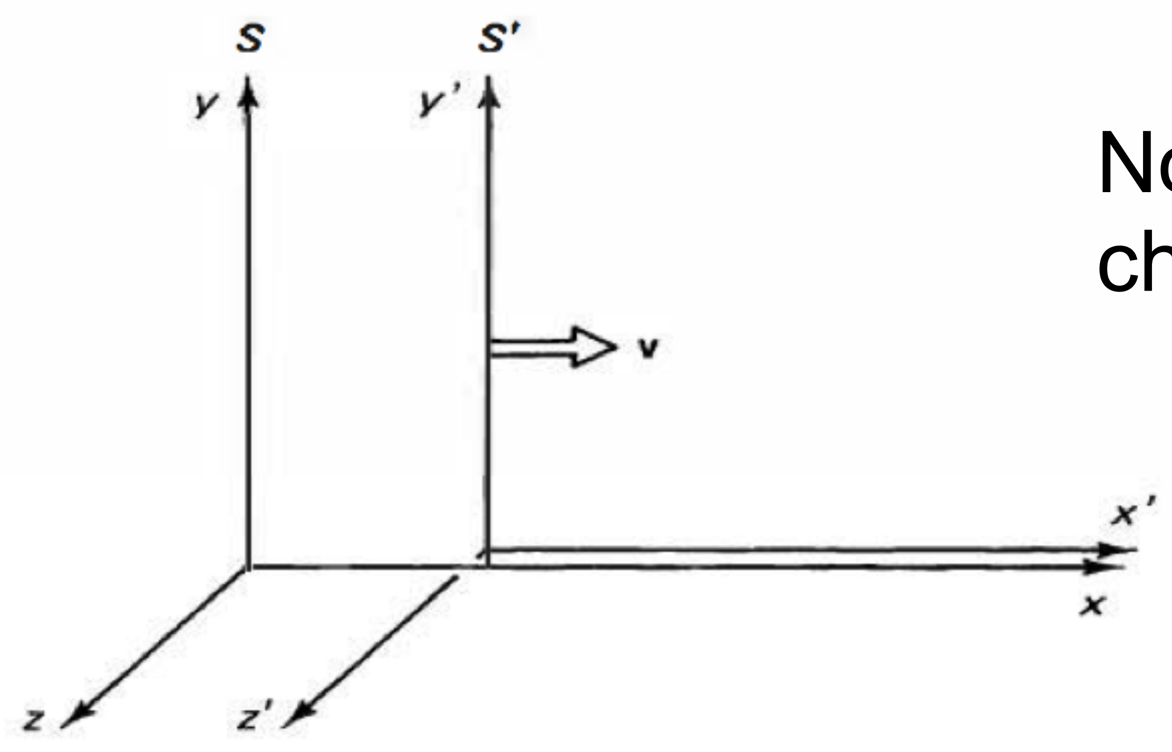
$$\gamma' = \gamma$$

$$z' = z, y' = y$$

$$t' = \gamma\left(t - vx\right)$$

$$\gamma = \left(1 - v^2\right)^{-1/2}$$

Inverse transformation



Note simplicity from choice of units!

$$x = \gamma \left(x' + vt' \right)$$

$$\gamma = \gamma'$$

$$z = z', y = y'$$

$$t = \gamma \left(t' + vx' \right)$$

$$\gamma = \left(1 - v^2\right)^{-1/2}$$

Consequences

$$t'_A = t'_B + \gamma v \left(x_B - x_A\right)$$

Cannot define simultaneity in all reference frames

$$L = \frac{L'}{\gamma}$$

Lorentz Contraction (objects shrink)

$$t = \gamma t'$$

Time dilation (moving clocks run slow)

$$u = \frac{u' + v}{1 + u'v}$$

Velocity addition is not as simple as before

What happens if u' = speed of light (which is ... what in our units?)

On to four-vectors

Are folks familiar with four-vector notation?

Get used to them :)

Space-time 4-vectors

$$x^0 = t$$
$$x^1 = x$$
$$x^2 = y$$
$$x^3 = z$$

All we've done here is label the 0th component of space-time as time, and then the normal (x,y,z)

Note that x in the 4-vector is standard notation, but it is **not** the position x

Lorentz transformations with four-vectors

$$x^{0'} = \gamma\left(x^0 - vx^1\right)$$

$$x^{1'} = \gamma\left(x^1 - vx^0\right)$$

$$x^{2'} = x^2$$

$$x^{3'} = x^3$$

Introduces a nice symmetry to the transformations, which leads us to re-writing it as...

$$x^{\mu'} = \sum_{\nu=0}^{3} \Lambda^{\mu}_{\nu} x^{\nu}$$

Where Λ is a 4x4 matrix. Let's get the coordinates of the matrix on the board...

Hopefully we got this correct :)

$$\Lambda = \begin{bmatrix} \gamma & -\gamma v & 0 & 0 \\ -\gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The symmetry really stands out here

$$x^{\mu'} = \sum_{\nu=0}^{3} \Lambda^{\mu}_{\nu} x^{\nu} = \Lambda^{\mu}_{\nu} x^{\nu}$$

Summation notation will be used a LOT

The second form has the sum implicit, and is shorthand to save smart (read: lazy) physicists from typing and writing

Define this invariant I. How does it transform? What is I' ?

$$I = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2$$

$$I^{'} = (x^{0'})^2 - (x^{1'})^2 - (x^{2'})^2 - (x^{3'})^2$$

$$I^{'} = (x^{0'})^2 - (x^{1'})^2 - (x^2)^2 - (x^3)^2$$

$$I^{'} = \gamma^2 \left(x^0 - vx^1\right)^2 - \gamma^2 (x^1 - vx^0)^2 - (x^2)^2 - (x^3)^2$$

$$I^{'} = \gamma^2 \left((x^0)^2 - (x^1)^2 + v^2 (x^1)^2 - v^2 (x^0)^2 - 2vx^0x^1 + 2vx^0x^1\right) - (x^2)^2 - (x^3)^2$$

$$I^{'} = \gamma^2 \left((x^0)^2 \left(1 - v^2\right) - (x^1)^2 \left(1 - v^2\right)\right) - (x^2)^2 - (x^3)^2$$

Now use $\gamma^2 = \dfrac{1}{1 - v^2}$

$$I = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2$$

$$I' = \gamma^2 \left((x^0)^2 \left(1 - v^2\right) - (x^1)^2 \left(1 - v^2\right)\right) - (x^2)^2 - (x^3)^2 = I$$

Now we see why we called I an invariant.

Define:

$$g = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Rewrite:

$$I = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} g_{\mu\nu} x^\mu x^\nu = g_{\mu\nu} x^\mu x^\nu$$

Covariant vs contravariant

covariant
vector (down)

contravariant
vector (up)

$$x_\mu = g_{\mu\nu} x^\nu$$

$$I = x_\mu x^\mu = x_\nu x^\nu$$

$x_0 \equiv x^0$
$x_1 \equiv -x^1$
$x_2 \equiv -x^2$
$x_3 \equiv -x^3$

The names are not quite as important as remembering that they are different (and that it’s easy to convert from one to another)

Generalizing this

$$a^\mu b_\mu = a_\mu b^\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$

Four-vector version of the dot production (which is also an invariant but only for three spacial dimensions)

Generalizing this

$$a^\mu b_\mu = a_\mu b^\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$

Four-vector version of the dot production (which is also an invariant but only for three spacial dimensions)

$$a^\mu b_\mu = a^0 b^0 - \vec{a} \cdot \vec{b}$$

$$a^2 = a^\mu a_\mu = a^0 a^0 - \vec{a}^2$$

More about the length of a 4-vector

$$a^2 = a^\mu a_\mu = a^0 a^0 - \vec{a}^2$$

$a^2 > 0$ time-like

$a^2 = 0$ light-like

$a^2 < 0$ space-like (like normal spatial 3-vectors)

Let’s look at conservation of momentum, classically

Conservation of momentum:

$$m_A v_A + m_B v_B = m_C v_C + m_D v_D$$

Now imagine a frame moving with constant velocity V with respect to this frame

$$v_A^{'} = v_A - V$$

$$v_B^{'} = v_B - V$$

$$v_C^{'} = v_C - V$$

$$v_D^{'} = v_D - V$$

Let's look at conservation of momentum

Now imagine a frame moving with constant velocity V with respect to this frame

$$m_A v_A' + m_B v_B' = m_C v_C' + m_D v_D'$$

$$v_A' = v_A - V$$
$$v_B' = v_B - V$$
$$v_C' = v_C - V$$
$$v_D' = v_D - V$$

$$m_A v_A + m_B v_B - V(m_A + m_B) = m_C v_C + m_D v_D - V(m_C + m_D)$$

And we know $m_A v_A + m_B v_B = m_C v_C + m_D v_D$

$$V(m_A + m_B) = V(m_C + m_D)$$

$$\boxed{m_A + m_B = m_C + m_D}$$

Classical conservation of momentum

Classical conservation of momentum (in all inertial reference frames) implies conservation of mass!

$$\boxed{m_A + m_B = m_C + m_D}$$

Conservation of momentum in SR

Conservation of momentum:

$$m_A v_A + m_B v_B = m_C v_C + m_D v_D$$

Now imagine a frame moving with constant velocity V with respect to this frame. Note the use of new velocity addition formula!

$$v_A' = \frac{v_A - V}{1 - v_A V}$$

$$v_B' = \frac{v_B - V}{1 - v_B V}$$

$$v_C' = \frac{v_C - V}{1 - v_C V}$$

$$v_D' = \frac{v_D - V}{1 - v_D V}$$

Conservation of momentum in SR

$$m_A v_A + m_B v_B = m_C v_C + m_D v_D$$
$$m_A v_A^{'} + m_B v_B^{'} = m_C v_C^{'} + m_D v_D^{'}$$

$$v_A^{'} = \frac{v_A - V}{1 - v_A V}$$
$$v_B^{'} = \frac{v_B - V}{1 - v_B V}$$
$$v_C^{'} = \frac{v_C - V}{1 - v_C V}$$
$$v_D^{'} = \frac{v_D - V}{1 - v_D V}$$

$$m_A \frac{v_A - V}{1 - v_A V} + m_B \frac{v_B - V}{1 - v_B V} = m_C \frac{v_C - V}{1 - v_C V} + m_D \frac{v_D - V}{1 - v_D V}$$

Velocities no longer nicely cancel, so we're stuck in the relativistic case! No longer have proof of mass conservation

Let's rethink conservation of momentum

Nominal definition of momentum $p = m\frac{dx}{dt}$

Not surprising that we run into trouble - both numerator and denominator are not Lorentz invariant! Let's make an ad hoc try and replace time with proper time

τ = t/γ

$$p = m\frac{dx}{d\tau} = m\frac{dx}{d(t/\gamma)} = \gamma m\frac{dx}{dt}$$

Conservation momentum with new definition

First frame

$$\frac{m_A v_A}{\sqrt{1-v_A^2}} + \frac{m_B v_B}{\sqrt{1-v_B^2}} = \frac{m_C v_C}{\sqrt{1-v_C^2}} + \frac{m_D v_D}{\sqrt{1-v_D^2}}$$

New frame

$$\frac{m_A v_A^{'}}{\sqrt{1-(v_A^{'})^2}} + \frac{m_B v_B^{'}}{\sqrt{1-(v_B^{'})^2}} = \frac{m_C v_C^{'}}{\sqrt{1-(v_C^{'})^2}} + \frac{m_D v_D^{'}}{\sqrt{1-(v_D^{'})^2}}$$

$$v_A^{'} = \frac{v_A - V}{1 - v_A V}$$

$$v_B^{'} = \frac{v_B - V}{1 - v_B V}$$

$$v_C^{'} = \frac{v_C - V}{1 - v_C V}$$

$$v_D^{'} = \frac{v_D - V}{1 - v_D V}$$

Some algebra in new frame

$$\frac{m_A v_A^{'}}{\sqrt{1-(v_A^{'})^2}} + \frac{m_B v_B^{'}}{\sqrt{1-(v_B^{'})^2}} = \frac{m_C v_C^{'}}{\sqrt{1-(v_C^{'})^2}} + \frac{m_D v_D^{'}}{\sqrt{1-(v_D^{'})^2}}$$

$$v_A^{'} = \frac{v_A - V}{1 - v_A V}$$

$$v_B^{'} = \frac{v_B - V}{1 - v_B V}$$

$$v_C^{'} = \frac{v_C - V}{1 - v_C V}$$

$$v_D^{'} = \frac{v_D - V}{1 - v_D V}$$

Let's look at first term:

$$\frac{m_A\left(\frac{v_A - V}{1-v_A V}\right)}{\sqrt{1-\left(\frac{v_A-V}{1-v_A V}\right)^2}} =$$

$$\frac{m_A\left(\frac{v_A - V}{1-v_A V}\right)}{\sqrt{\frac{(1-v_A V)^2-(v_A-V)^2}{(1-v_A V)^2}}} =$$

$$\frac{m_A(v_A - V)}{\sqrt{(1-v_A V)^2-(v_A-V)^2}} =$$

$$\frac{m_A(v_A - V)}{\sqrt{1+v_A^2 V^2 - 2v_A V - v_A^2 - V^2 + 2v_A V}} =$$

$$\frac{m_A(v_A - V)}{\sqrt{1+v_A^2 V^2 - v_A^2 - V^2}}$$

Some algebra in new frame

$$\frac{m_A v_A'}{\sqrt{1-(v_A')^2}} + \frac{m_B v_B'}{\sqrt{1-(v_B')^2}} = \frac{m_C v_C'}{\sqrt{1-(v_C')^2}} + \frac{m_D v_D'}{\sqrt{1-(v_D')^2}}$$

$$v_A' = \frac{v_A - V}{1 - v_A V}$$

$$v_B' = \frac{v_B - V}{1 - v_B V}$$

$$v_C' = \frac{v_C - V}{1 - v_C V}$$

$$v_D' = \frac{v_D - V}{1 - v_D V}$$

Let's look at first term:

$$\frac{m_A\left(\frac{v_A - V}{1 - v_A V}\right)}{\sqrt{1-\left(\frac{v_A - V}{1 - v_A V}\right)^2}} =$$

$$\frac{m_A(v_A - V)}{\sqrt{1 + v_A^2 V^2 - v_A^2 - V^2}} =$$

$$\frac{m_A v_A}{\sqrt{(1-v_A^2)(1-V^2)}} - \frac{m_A V}{\sqrt{(1-v_A^2)(1-V^2)}} =$$

$$\frac{m_A v_A}{\sqrt{1-v_A^2}}\frac{1}{\sqrt{1-V^2}} - \frac{m_A}{\sqrt{1-v_A^2}}\frac{V}{\sqrt{1-V^2}} =$$

Some algebra in new frame

$$\frac{m_A v_A^{'}}{\sqrt{1-(v_A^{'})^2}} + \frac{m_B v_B^{'}}{\sqrt{1-(v_B^{'})^2}} = \frac{m_C v_C^{'}}{\sqrt{1-(v_C^{'})^2}} + \frac{m_D v_D^{'}}{\sqrt{1-(v_D^{'})^2}}$$

First term = $\frac{m_A v_A}{\sqrt{1-v_A^2}} \frac{1}{\sqrt{1-V^2}} - \frac{m_A}{\sqrt{1-v_A^2}} \frac{V}{\sqrt{1-V^2}}$

So...

$$\frac{m_A v_A}{\sqrt{1-v_A^2}} \frac{1}{\sqrt{1-V^2}} - \frac{m_A}{\sqrt{1-v_A^2}} \frac{V}{\sqrt{1-V^2}} +$$
$$\frac{m_B v_B}{\sqrt{1-v_B^2}} \frac{1}{\sqrt{1-V^2}} - \frac{m_B}{\sqrt{1-v_B^2}} \frac{V}{\sqrt{1-V^2}} =$$
$$\frac{m_C v_C}{\sqrt{1-v_C^2}} \frac{1}{\sqrt{1-V^2}} - \frac{m_C}{\sqrt{1-v_C^2}} \frac{V}{\sqrt{1-V^2}} +$$
$$\frac{m_D v_D}{\sqrt{1-v_D^2}} \frac{1}{\sqrt{1-V^2}} - \frac{m_D}{\sqrt{1-v_D^2}} \frac{V}{\sqrt{1-V^2}}$$

But from original frame

$$\frac{m_A v_A}{\sqrt{1-v_A^2}} + \frac{m_B v_B}{\sqrt{1-v_B^2}} =$$
$$\frac{m_C v_C}{\sqrt{1-v_C^2}} + \frac{m_D v_D}{\sqrt{1-v_D^2}}$$

Thus we get

$$\frac{m_A}{\sqrt{1-v_A^2}}\frac{V}{\sqrt{1-V^2}}+\frac{m_B}{\sqrt{1-v_B^2}}\frac{V}{\sqrt{1-V^2}}=$$
$$\frac{m_C}{\sqrt{1-v_C^2}}\frac{V}{\sqrt{1-V^2}}+\frac{m_D}{\sqrt{1-v_D^2}}\frac{V}{\sqrt{1-V^2}}$$

Cancel the common term...

$$\frac{m_A}{\sqrt{1-v_A^2}}+\frac{m_B}{\sqrt{1-v_B^2}}=$$
$$\frac{m_C}{\sqrt{1-v_C^2}}+\frac{m_D}{\sqrt{1-v_D^2}}$$

This quantity must be conserved then! But what is it?

Thus we get

$$\frac{m}{\sqrt{1-v^2}} = m\left(1-v^2\right)^{-1/2} \sim m(1+v^2/2)$$

So this is a statement of the conservation of energy! At small velocities, energy is equal to the rest mass plus the kinematic energy (note that when v = 0 the energy is not zero, but m!)

Defining a four-vector

$$\frac{m}{\sqrt{1-v^2}} = m\left(1-v^2\right)^{-1/2} \sim m(1+v^2/2)$$

So this is a statement of the conservation of energy! At small velocities, energy is equal to the rest mass plus the kinematic energy

$$p^\mu = (E, p_x, p_y, p_z)$$
$$p^\mu p_\mu = E^2 - \mathbf{p}^2 = m^2$$

This way, p^2 is invariant, as it must be

For m = 0 (ex for: photons), |p|=E

Conveniently in our units

$$p^\mu = (E, p_x, p_y, p_z)$$
$$p^\mu p_\mu = E^2 - \mathbf{p}^2 = m^2$$

$$E = \gamma m$$

$$\vec{p} = \gamma m \vec{v}$$

$$\rightarrow \boxed{(\vec{p}/E) = \vec{v}}$$

As expected, for light,
|v| = 1 and |p| = E

We just saw that energy and momentum (correctly defined) are conserved. For process A+B→C+D:

$$E_A + E_B = E_C + E_D$$

$$\vec{p}_A + \vec{p}_B = \vec{p}_C + \vec{p}_D$$

Conveniently written as: $p^\mu_A + p^\mu_B = p^\mu_C + p^\mu_D$

Mass is clearly not conserved: $W^+ \rightarrow e^+ + \nu_e$

What about a particle decay?

$$A \rightarrow B + C$$

$$A_\mu A^\mu = m_A^2 \rightarrow (B+C)_\mu (B+C)^\mu = (E_B + E_C)^2 - (\mathbf{p_B} + \mathbf{p_C})^2$$

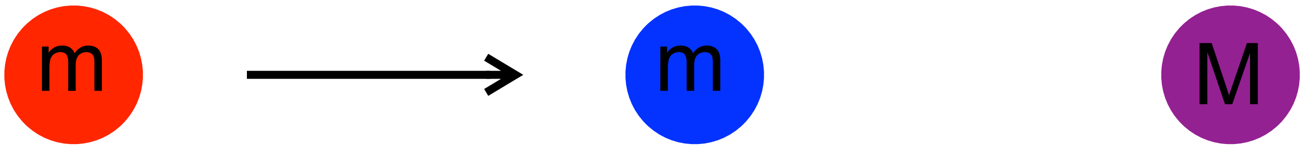
Lets us estimate the mass of a parent particle from its decay products. Conversation of energy also tells us:

$$m_A = \sqrt{m_B^2 + \mathbf{p}_B^2} + \sqrt{m_C^2 + \mathbf{p}_C^2}$$

Particle decay products cannot be more massive than mass of original particle! (Remember - proton stability...)

One fun example

Initial



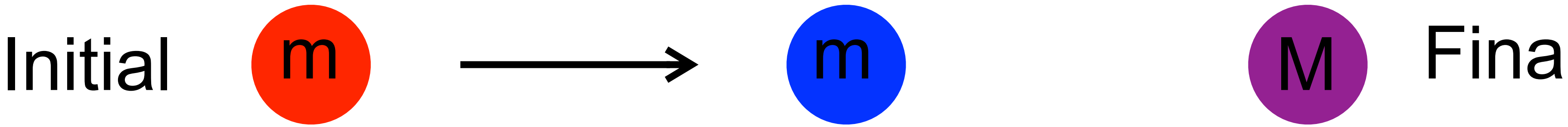
Final

$p_1=(2m,\mathbf{p})$ $p_2=(m,0)$ $p'=(E,\mathbf{p'})$

Particle with total energy = 2m collides with an identical particle at rest. They stick together. What is the mass of the resulting lump? What is its velocity?

Conservation of E: 2m+m = 3m = E

One fun example



p_1=(E_1=2m,**p**) p_2=(m,0) p'=(3m,**p'**)

$$E_1^2 - \mathbf{p}^2 = m^2$$

$$(2m^2) - m^2 = \mathbf{p}^2$$

$$\mathbf{p}^2 = 3m^2$$

$$|\mathbf{p}| = \sqrt{3}m$$

$$|\mathbf{p}| = |\mathbf{p}'| = \sqrt{3}m$$

One fun example

Initial m $\longrightarrow$ m M Final

p1=(2m,**p**) p2=(m,0) $p' = (3m, \sqrt{3}m)$

$$E_1^2 - \mathbf{p}^2 = m^2$$

$$(2m)^2 - m^2 = \mathbf{p}^2$$

$$\mathbf{p}^2 = 3m^2$$

$$|\mathbf{p}| = \sqrt{3}m$$

$$|\mathbf{p}| = |\mathbf{p}'| = \sqrt{3}m$$

One fun example

Initial $m \longrightarrow m$ M Final

$p_1=(2m,\mathbf{p})$ $p_2=(m,0)$ $p^{'} = (3m, \sqrt{3}m)$

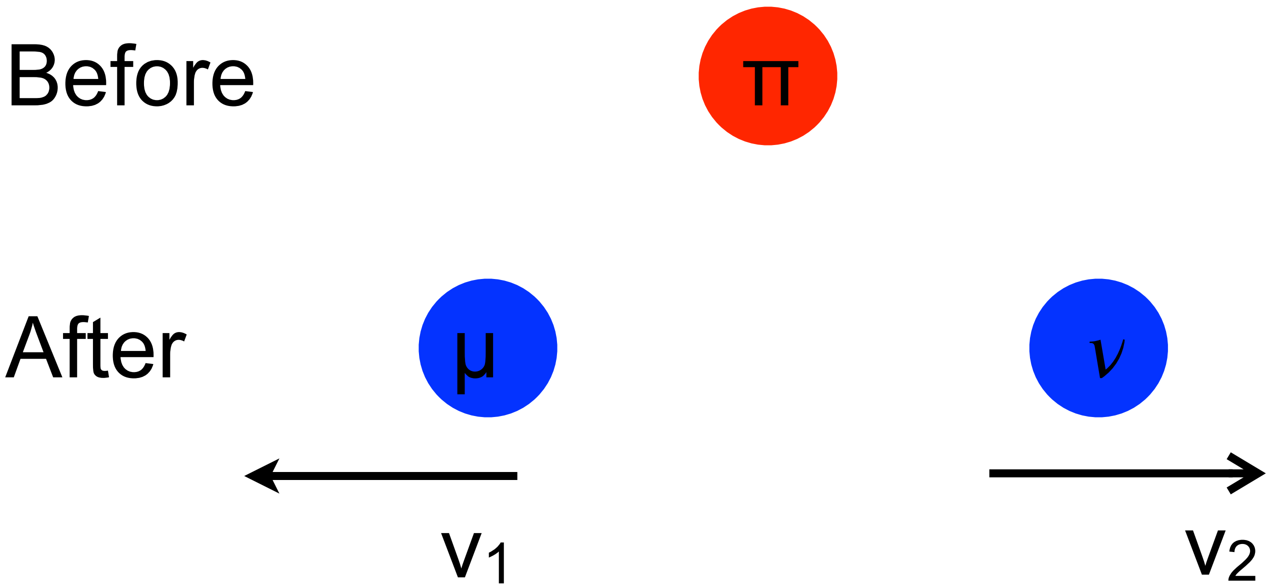
$$M^2 = E^2 - (\mathbf{p}^{'})^2 = 9m^2 - 3m^2$$

$$M = \sqrt{6}m$$

Total mass not conserved

$$|v| = \frac{|\mathbf{p}|}{E} = \frac{\sqrt{3}m}{3m} = \frac{1}{\sqrt{3}}$$

A pion at rest decays into a muon and a neutrino. What is the speed of the muon?



Let's follow Griffiths Ex 3.3

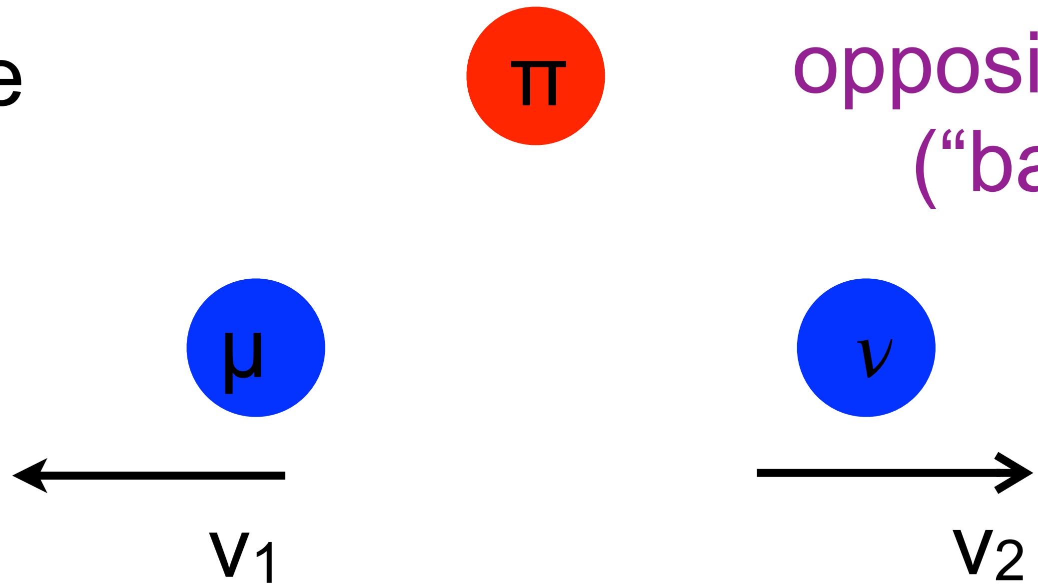
Conservation of momentum: $\mathbf{p}_\mu = -\mathbf{p}_\nu$

Conservation of energy: $E_\mu + E_\nu = E_\pi$

So muon and neutrino travel in opposite directions ("back to back")

Before

After



Let's follow Griffiths Ex 3.3

Apply conservation of energy:

$$E_\mu + E_\nu = E_\pi$$

$$\sqrt{m_\mu^2 + \vec{p}_\mu^2} + \sqrt{m_\nu^2 + \vec{p}_\nu^2} = \sqrt{m_\pi^2 + \vec{p}_\pi^2}$$

$$\sqrt{m_\mu^2 + \vec{p}_\mu^2} + |\vec{p}_\nu| = m_\pi$$

Pion was at rest

Neutrino mass ~0

Let's follow Griffiths Ex 3.3

Apply conservation of energy:

$$\sqrt{m_\mu^2 + \vec{p}_\mu^{\,2}} + |\vec{p}_\nu| = m_\pi$$

$$\sqrt{m_\mu^2 + \vec{p}_\mu^{\,2}} = m_\pi - |\vec{p}_\nu|$$

$$m_\mu^2 + \vec{p}_\mu^{\,2} = m_\pi^2 + |\vec{p}_\nu|^2 - 2m_\pi|\vec{p}_\nu|$$

Equal

Equal

$$m_\mu^2 = m_\pi^2 - 2m_\pi|\vec{p}_\nu|$$

$$m_\mu^2 = m_\pi^2 - 2m_\pi|\vec{p}_\mu|$$

$$|\vec{p}_\mu| = \frac{m_\pi^2 - m_\mu^2}{2m_\pi}$$

Let's follow Griffiths Ex 3.3

$$|\vec{p}_\mu| = \frac{m_\pi^2 - m_\mu^2}{2m_\pi}$$

$$E_\mu^2 = \vec{p}_\mu^{\,2} + m_\mu^2 = \frac{\left(m_\pi^2 - m_\mu^2\right)^2}{4m_\pi^2} + m_\mu^2$$

|v|=|p|/E

$$E_\mu^2 = \frac{m_\pi^4 + m_\mu^4 - 2m_\pi^2 m_\mu^2 + 4m_\pi^2 m_\mu^2}{4m_\pi^2}$$

$$\boxed{|v_\mu| = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2}}$$

$$E_\mu^2 = \frac{m_\pi^4 + m_\mu^4 + 2m_\pi^2 m_\mu^2}{4m_\pi^2}$$

$$E_\mu^2 = \frac{\left(m_\pi^2 + m_\mu^2\right)^2}{4m_\pi^2}$$

$$E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi}$$

Let's follow Griffiths Ex 3.3

Pion mass ~135 MeV
Muon mass ~106 MeV

$$\boxed{|v_\mu| = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2}}$$

v~0.25
(25% of the speed of light)

Let's follow Griffiths Ex 3.3 (continued)

Let's try using four-vector notation (remember that $p^2=m^2$ but be careful that p here is the four-vector p)

Let's follow Griffiths Ex 3.3 (continued)

$$p_\pi = p_\mu + p_\nu$$

$$p_\nu = p_\pi - p_\mu$$

$$p_\nu^2 = (p_\pi - p_\mu)^2 = m_\nu = 0$$

$$0 = p_\pi^2 + p_\mu^2 - 2p_\pi p_\mu$$

$$0 = m_\pi^2 + m_\mu^2 - 2p_\pi p_\mu$$

What is $p_\pi p_\mu$? The pion momentum is zero, so only first component (energy) contributes, and so $p_\pi p_\mu = E_\pi E_\mu$

Also, since pion was at rest, $E_\pi = m_\pi$ so $p_\pi p_\mu = m_\pi E_\mu$

$$0 = m_\pi^2 + m_\mu^2 - 2m_\pi E_\mu$$

$$\boxed{E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi}}$$

Let's follow Griffiths Ex 3.3 (continued)

$$p_\pi = p_\mu + p_\nu$$

$$p_\mu = p_\pi - p_\nu$$

$$p_\mu^2 = (p_\pi - p_\nu)^2 = m_\mu^2$$

$$p_\mu^2 = p_\pi^2 + p_\nu^2 - 2p_\pi p_\nu$$

$$m_\mu^2 = m_\pi^2 - 2m_\pi |p_\mu|$$

$$\boxed{|p_\mu| = \frac{m_\pi^2 - m_\mu^2}{2m_\pi}}$$

What is $p_\pi p_\nu$? The pion momentum is zero, so only first component (energy) contributes, and so $p_\pi p_\nu = E_\pi E_\nu$

Since pion was at rest, $E_\pi = m_\pi$. What is E_ν? It is the magnitude of the neutrino momentum, but that is the same as the magnitude of the muon momentum

Griffiths 3.4

$pp \rightarrow ppp\bar{p}$

What is the threshold energy for this process?

At threshold, all four final state objects are at rest (if they were not at rest we could reduce the CoM collision energy and reduce their momentum)

Invariant:

$$(p_1 + p_2)^2 = (E + m)^2 - \mathbf{p}^2$$

Initial

$p_1=(E,\mathbf{p})$ $p_2=(m,0)$

Griffiths 3.4

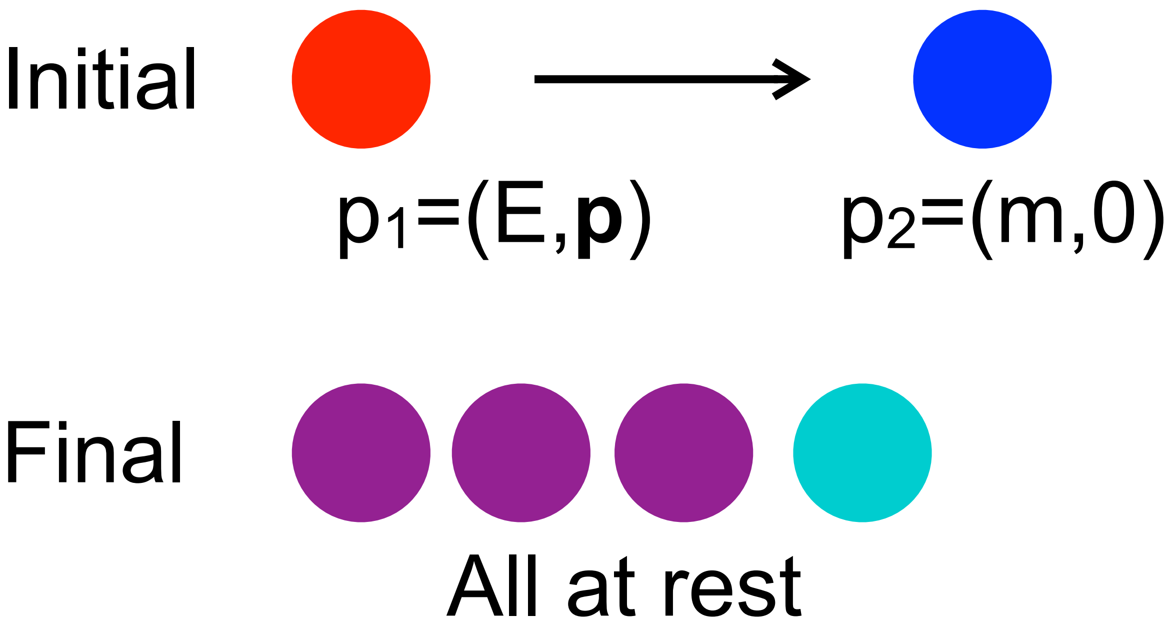
Invariant:

$$(p_1 + p_2)^2 = (E + m)^2 - \mathbf{p}^2$$

$$(p_3 + p_4 + p_5 + p_6)^2 = (4m)^2 = (p_1 + p_2)^2$$

$$16m^2 = (E + m)^2 - \mathbf{p}^2$$

$$\mathbf{p}^2 = E^2 - m^2$$



Griffiths 3.4

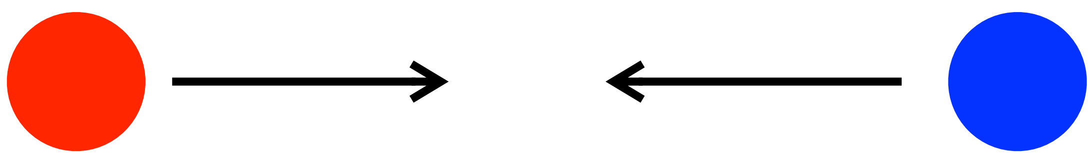
$$16m^2 = (E+m)^2 - \mathbf{p}^2 \qquad \mathbf{p}^2 = E^2 - m^2$$

$$16m^2 = E^2 + 2mE + m^2 - E^2 + m^2$$

$$14m^2 = 2mE \rightarrow \boxed{E = 7m}$$

What if we didn't fire at a stationary target?

Initial



$p_1=(E,\mathbf{p})$ $\qquad$ $p_2=(E,-\mathbf{p})$

$$(p_1 + p_2)^2 = (2E)^2 - (\mathbf{p} + -\mathbf{p})^2 = 4E^2$$

$$16m^2 = 4E^2 \rightarrow E = 2m$$

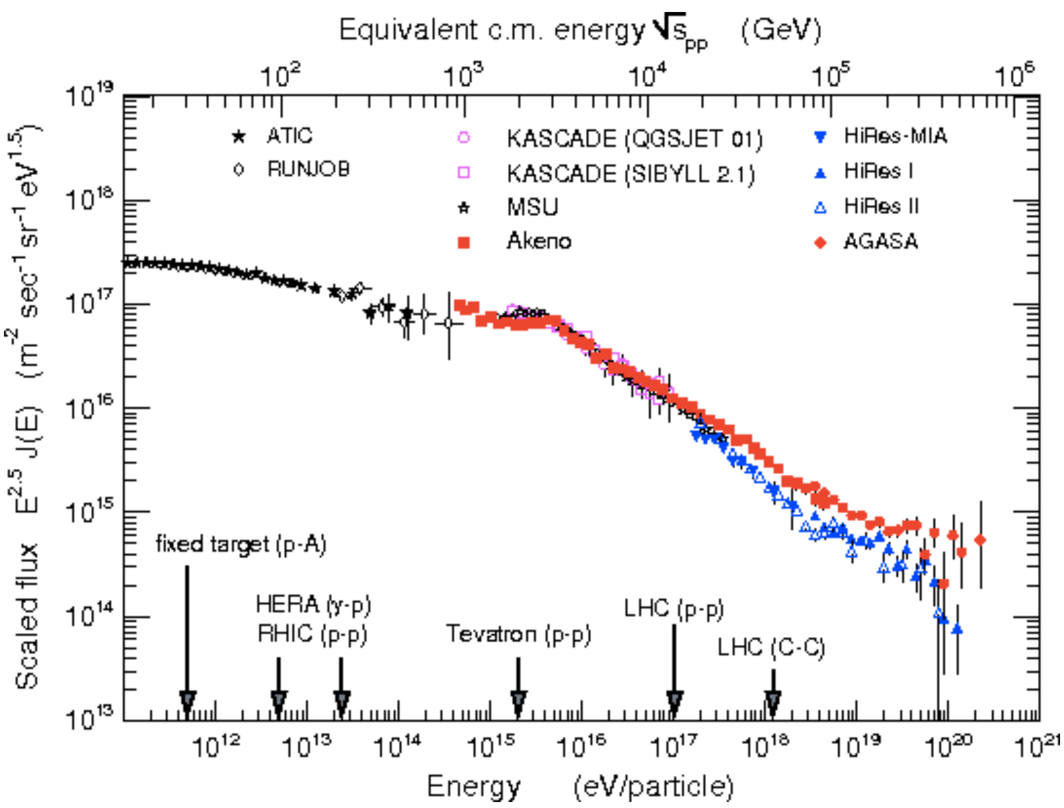
Huge advantage to colliding beam machines

In this example, have to give protons a lot more kinetic energy for the fixed target version! Of course, the accelerator for the beams is much more complex :)

GZK cutoff

Greisen–Zatsepin–Kuzmin: Over long enough distances, high-energy cosmic rays coming from the Universe should interact with the cosmic microwave background

http://www2.astro.psu.edu/users/nnp/cr.html



GZK cutoff

$$\gamma + p \rightarrow p\pi^0$$

$$p_\gamma + p_p = p_{p'} + p_{\pi^0}$$

$$(p_\gamma + p_p)^2 = (p_{p'} + p_{\pi^0})^2$$

$$p_\gamma^2 + p_p^2 + 2p_\gamma \cdot p_p = p_{p'}^2 + p_{\pi^0}^2 + 2p_{\pi^0} \cdot p_{p'}$$

$$m_p^2 + 2p_\gamma \cdot p_p = m_p^2 + m_{\pi^0}^2 + 2p_{\pi^0} \cdot p_{p'}$$

$$2p_\gamma \cdot p_p = m_{\pi^0}^2 + 2p_{\pi^0} \cdot p_{p'}$$

At threshold, pion and proton in final state are at rest and their momenta are zero (and energies equal to their masses)

$$2p_\gamma \cdot p_p = m_{\pi^0}^2 + 2m_{\pi^0} m_p$$

At threshold, have a head-on collision. Assume highly relativistic particles and we get...

GZK cutoff

$$2p_\gamma \cdot p_p = m_{\pi^0}^2 + 2m_{\pi^0} m_p$$

Head-on collision with energies >> m_p

$$p_\gamma = (E_\gamma, E_\gamma, 0, 0)$$
$$p_p = (E_p, -E_p, 0, 0)$$
$$2p_p \cdot p_\gamma = 2E_pE_\gamma - 2(-E_p)E_\gamma = 4E_pE_\gamma$$

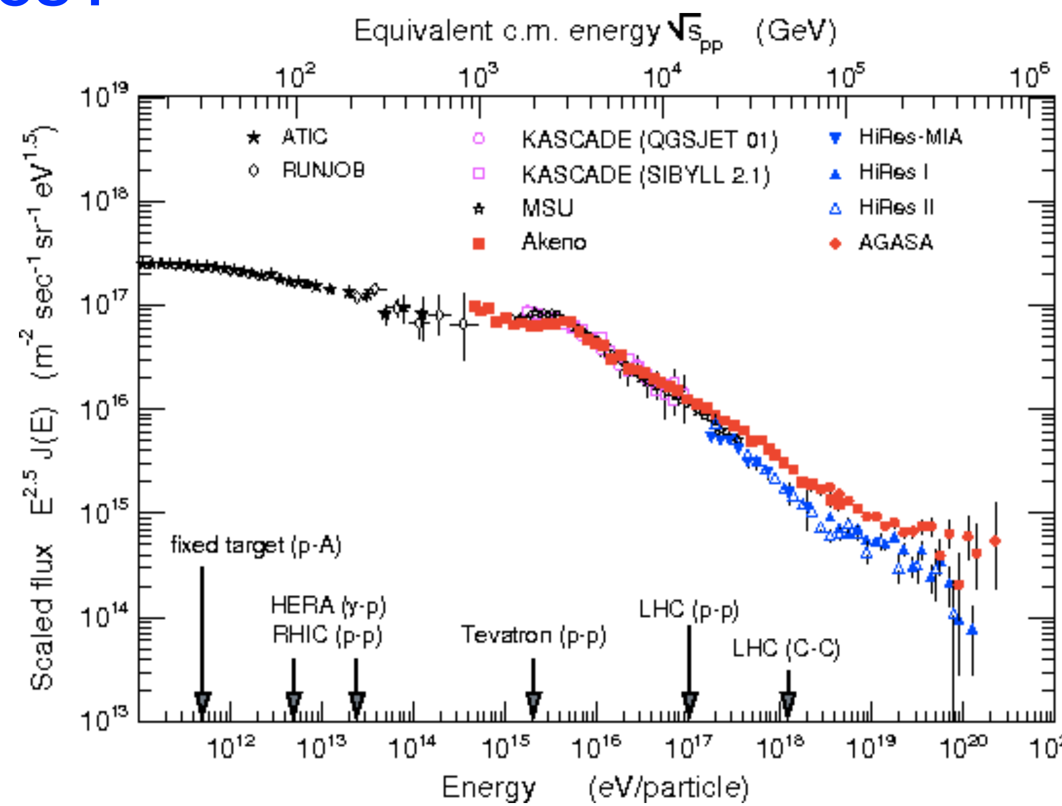
Want to solve for unknown cosmic rate proton energy. Photon energy is
CMB (6×10^{-4} ev). Plug in and we get ~2×10^{20} ev

Similarly have $\gamma + p \rightarrow n\pi^+$

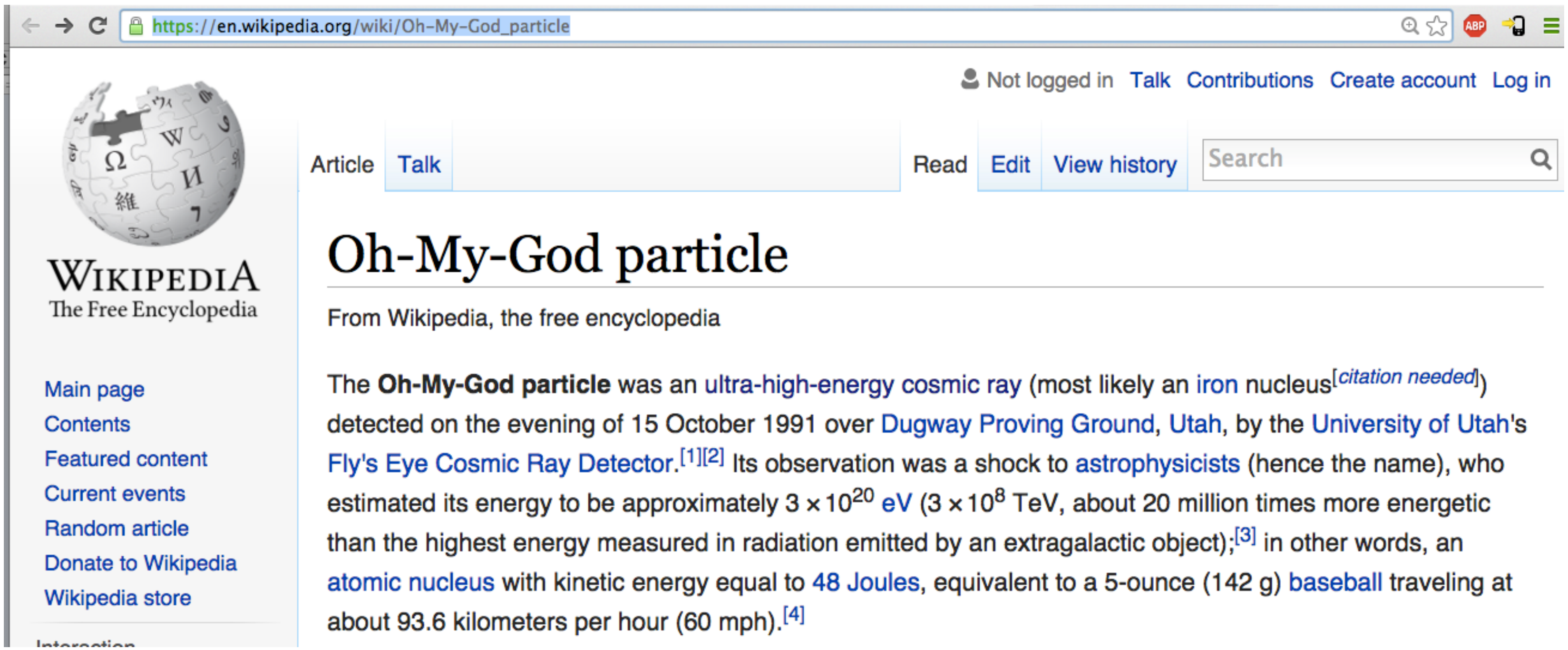
GZK cutoff

Of course, we used an “average” CMB photon, and ignored a full calculation of the kinematics (which goes through delta resonances) and assumed protons. Nevertheless, distribution doesn’t go to zero! Where are these sources coming from? Nearby sources? Or new physics?

http://www2.astro.psu.edu/users/nnp/cr.html



Fun thing to google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Oh-My-God particle

From Wikipedia, the free encyclopedia

The **Oh-My-God particle** was an ultra-high-energy cosmic ray (most likely an iron nucleus[*citation needed*]) detected on the evening of 15 October 1991 over Dugway Proving Ground, Utah, by the University of Utah's Fly's Eye Cosmic Ray Detector.[1][2] Its observation was a shock to astrophysicists (hence the name), who estimated its energy to be approximately 3×10^{20} eV (3×10^{8} TeV, about 20 million times more energetic than the highest energy measured in radiation emitted by an extragalactic object);[3] in other words, an atomic nucleus with kinetic energy equal to 48 Joules, equivalent to a 5-ounce (142 g) baseball traveling at about 93.6 kilometers per hour (60 mph).[4]