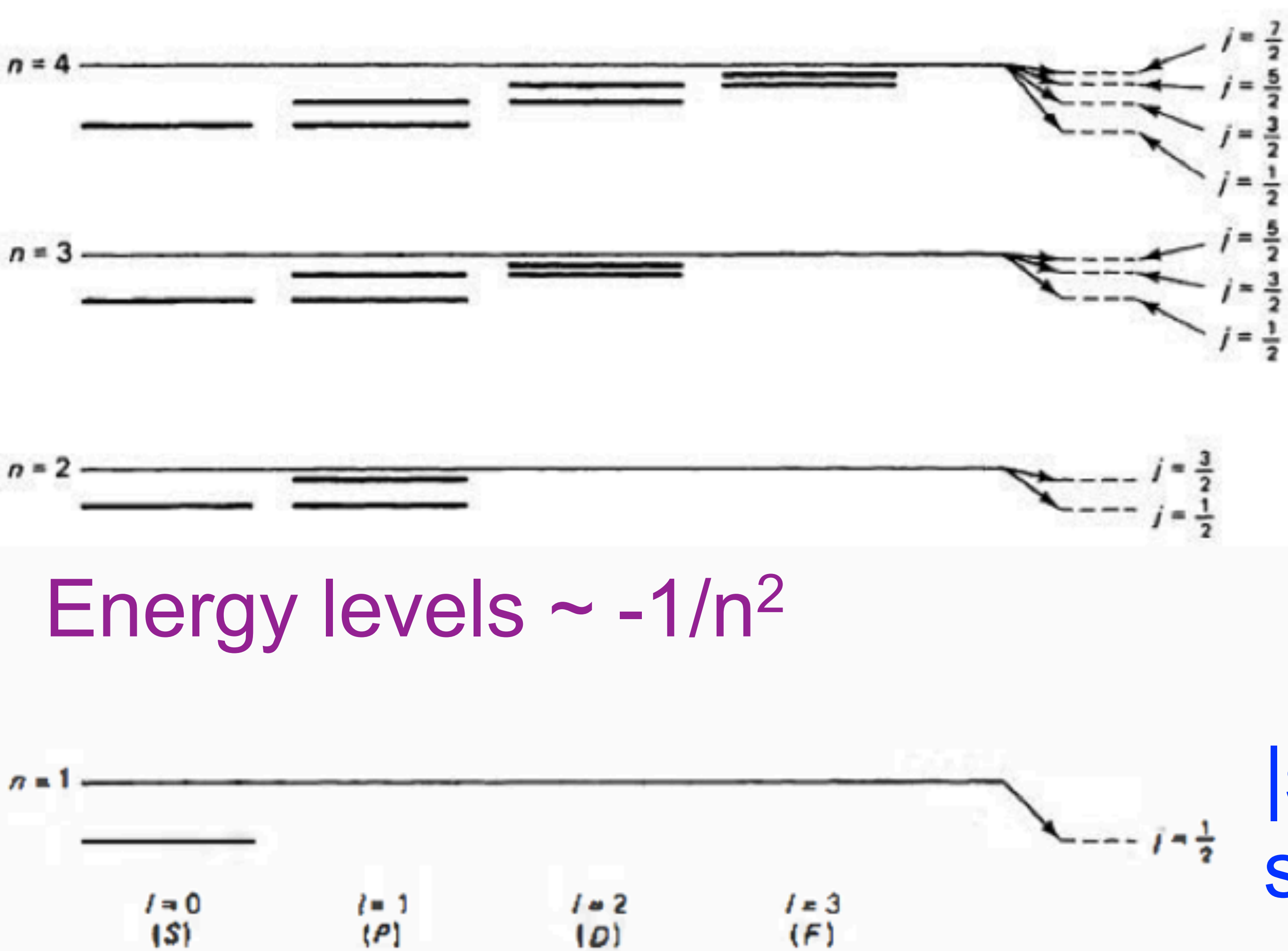
Let's move to Bound States

When we discuss bound states of two objects in central-force potential, kinetic energy and potential energy are ~the same. How does this compare to the rest energy of the objects?

Hydrogen ionization energy: 13.6 eV vs 0.5 MeV rest mass

Masses of b and c quarks are ~relatively large, so we can also consider them non-relativistically (which makes them much easier). NOT true for uds quarks

Briefest of overviews of hydrogen atom



Fine structure from relativistic corrections and spin-orbit coupling

Energy levels ~ $-1/n^2$

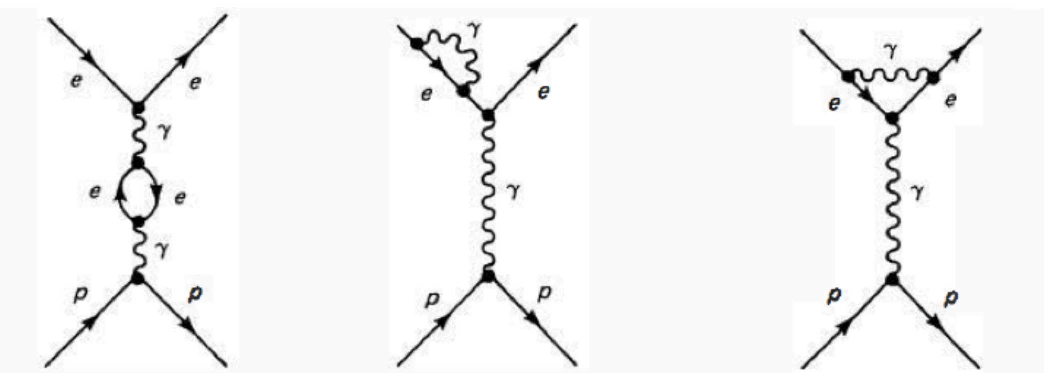
$|s(e)| = 1/2$
so $j = L \pm 1/2$

Fig. 5.2 Fine structure in hydrogen. The nth Bohr level (fine line) splits into n sublevels (dashed lines), characterized by $j = \frac{1}{2}, \frac{3}{2}, \ldots, (n - \frac{1}{2})$. Except for the last of these, two different values of l contribute to each level: $l = j - \frac{1}{2}$ and $l = j + \frac{1}{2}$. Spectroscopists' nomenclature – S for $l = 0$, P for $l = 1$, D for $l = 2$, F for $l = 3$ – is indicated. All levels are shifted downward, as shown (the diagram is not to scale, however).

Griffiths

Lamb shift

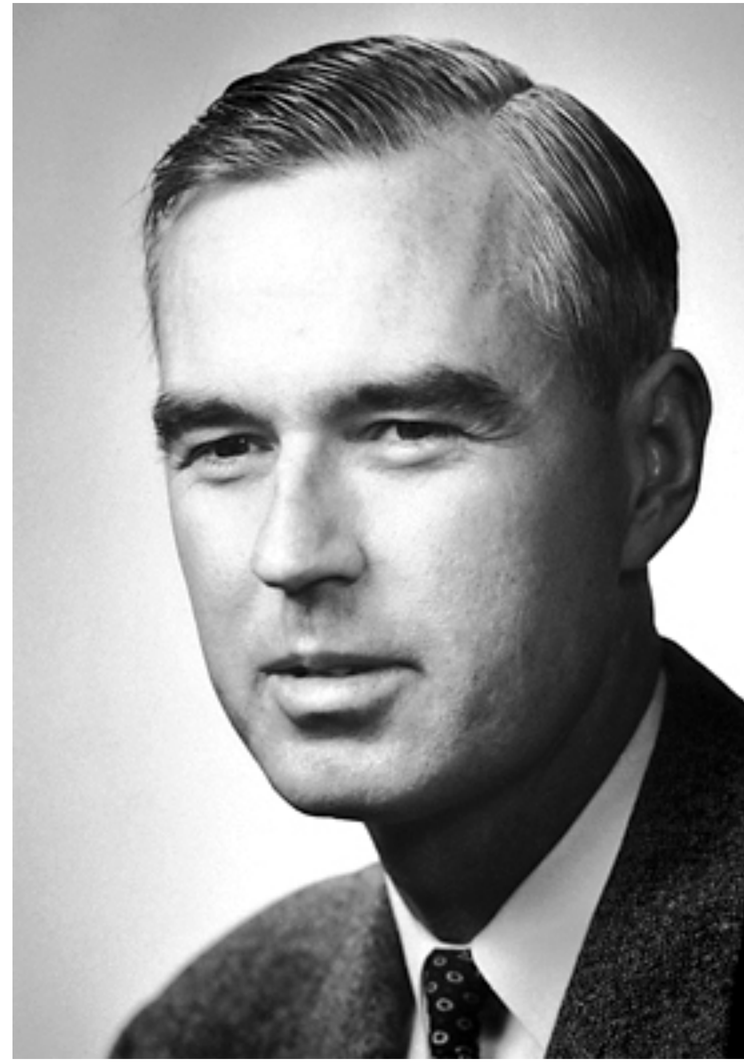
Lamb shift: Led to development of quantum electro-dynamics! QED corrections to the electron-proton interaction break degeneracy of two levels with same n, j but different L (so $2S_{1/2}$ and $2P_{1/2}$ are not fully degenerate)



Vacuum polarization | Electron mass renormalization | Anomalous magnetic moment

Fig. 5.3 Some loop diagrams contributing to the Lamb shift.

Griffiths



Willis Lamb

Finally, hyperfine splitting

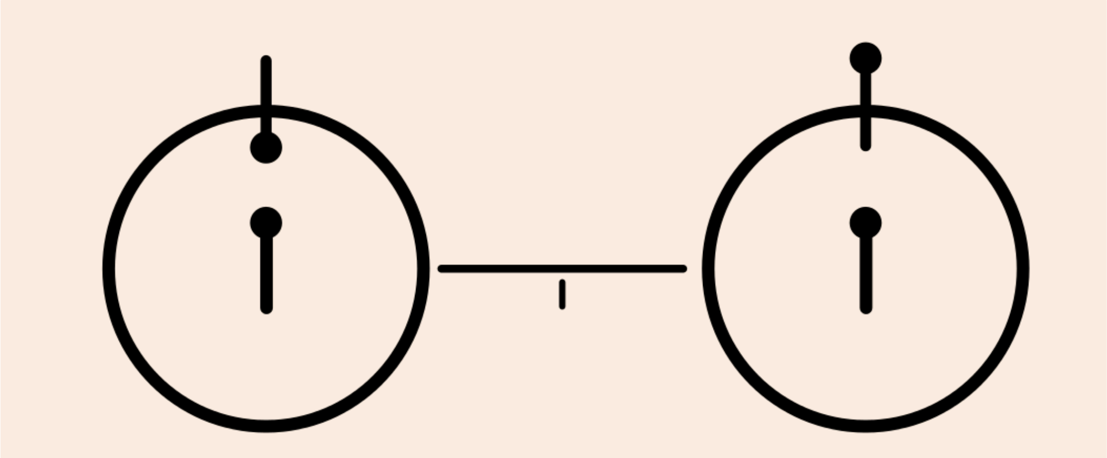
Spin-orbit coupling is principally due to spin of electron interacting with ‘B field’ from nucleus (fine structure). Much smaller is spin of nucleus interacting with ‘B field’ from electron. Goes as $(m_e/m_p)^4$ hence hyperfine (and not fine) splitting.

For n=1, the difference in energy states of proton (e and p spins aligned vs anti-aligned, which is lower) is 5.9 μeV = 1420 MHz = 21 cm. Famous 21 cm line (penetrates dust!)

On the 21 cm line

Lifetime of 21 cm is millions of years! Thankfully, enough hydrogen can provide this transition. Long lifetime = narrow width, so this is excellent for spectroscopy (Doppler shifts)

Used extensively in radio-astronomy, studying the early Universe, galaxy formation, measuring distances to objects, cosmology



Pioneer Plaque: 21 cm line defines distance and unit of time

On to quarkonium

Differences between quarkonium and hydrogen/ positronium: Don't really know the potential (strong force!) Also, interaction between quarks is large. Doesn't work for two light quarks, either

Instead of considering different states as energy levels of an atom, consider different bound states as different particles, each with a different mass. Start with mesons (much easier than baryons)

Q---Q'

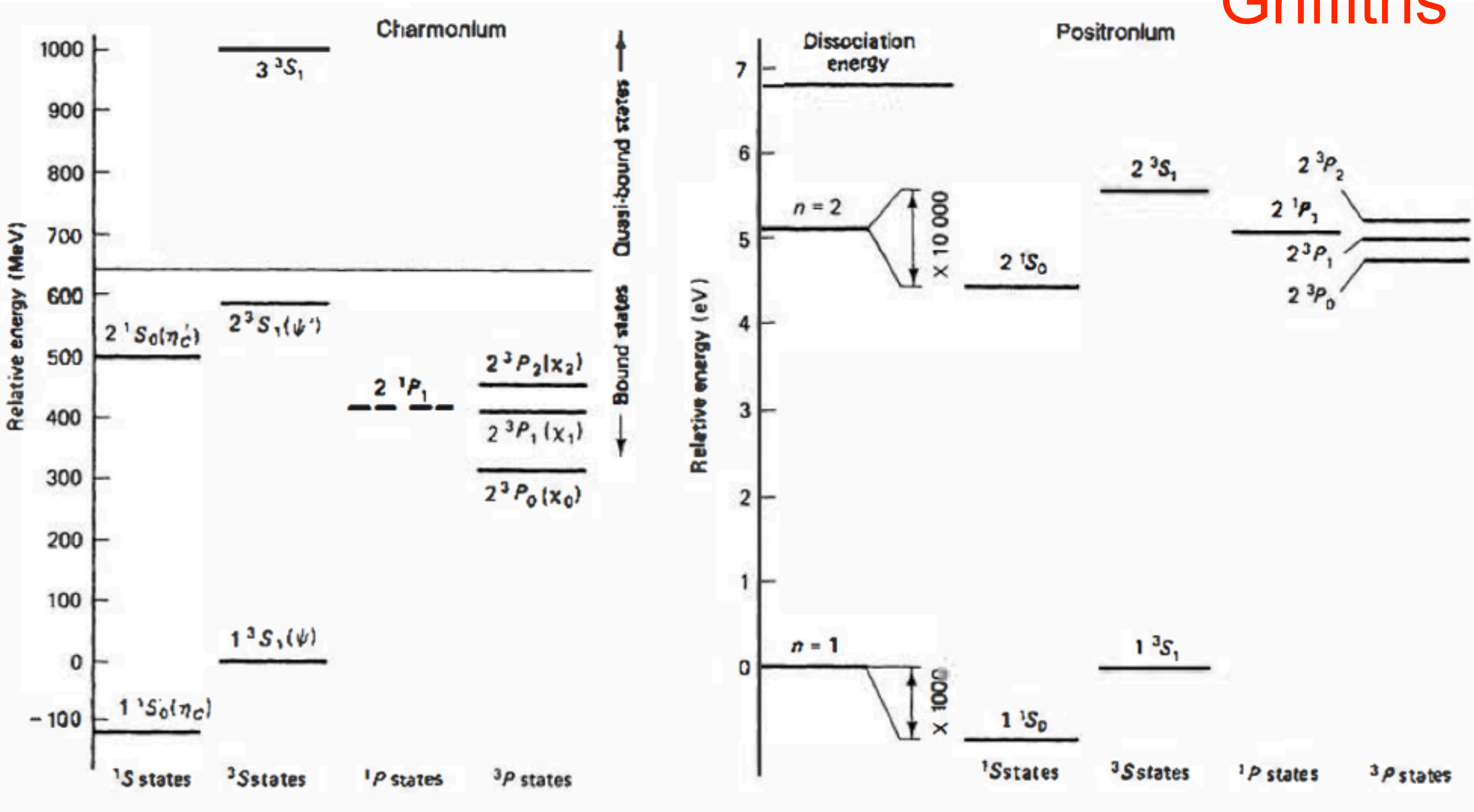
What sorts of potentials can we use?

At short distances, we know that QCD is not a strong force. Reasonable to start with 1/r potential.

At large distances, we know that force grows exponentially. Try V~kr (others could also work). Of course, k can be a function of r too!

Charmonium

Griffiths

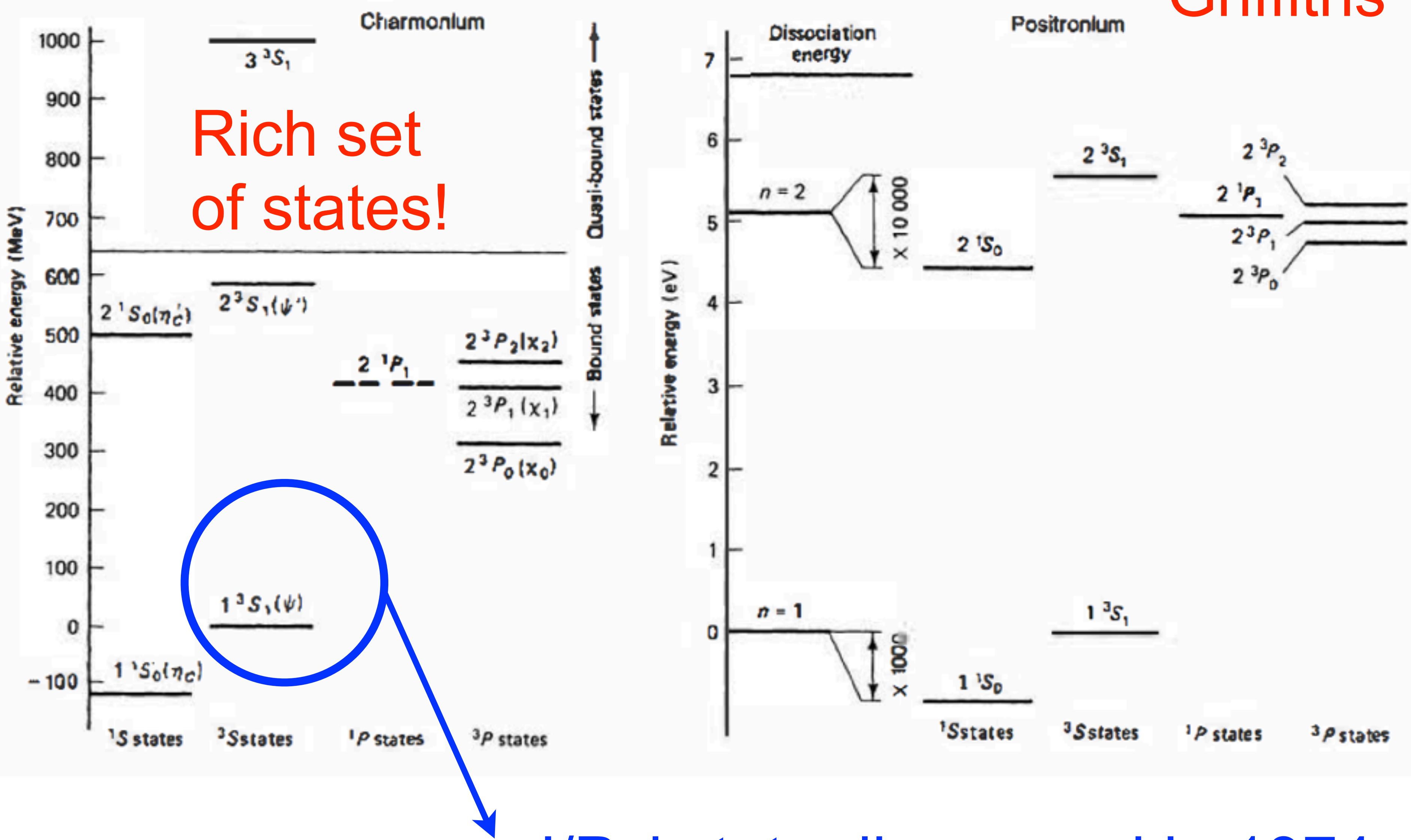


Labeled (confusingly, to me!) as $n^{(2s+1)}L_J$ (L=S,P,D,... for 0,1,2.... and s=0 or 1 for anti-aligned or aligned spins), with J=L+s

Charmonium

Griffiths

Rich set of states!



J/Psi state discovered in 1974

Why was J/Psi discovered first?

η

$I^G(J^{PC}) = 0^+(0^{-+})$

Charged modes

Γ_8	charged modes	(28.06 ± 0.34) %	S=1.2
Γ_9	$\pi^+\pi^-\pi^0$	(22.73 ± 0.28) %	S=1.2
Γ_{10}	$\pi^+\pi^-\gamma$	(4.60 ± 0.16) %	S=2.1
Γ_{11}	$e^+e^-\gamma$	$(6.8\pm0.8)\times10^{-3}$	S=1.7
Γ_{12}	$\mu^+\mu^-\gamma$	$(3.1\pm0.4)\times10^{-4}$	
Γ_{13}	e^+e^-	$< 7.7\times10^{-5}$	CL=90%
Γ_{14}	$\mu^+\mu^-$	$(5.8\pm0.8)\times10^{-6}$	
Γ_{15}	$e^+e^-e^+e^-$	$< 6.9\times10^{-5}$	CL=90%
Γ_{16}	$\pi^+\pi^-e^+e^-$	$(4.2\pm1.2)\times10^{-4}$	
Γ_{17}	$\pi^+\pi^-2\gamma$	$< 2.0\times10^{-3}$	
Γ_{18}	$\pi^+\pi^-\pi^0\gamma$	$< 5\times10^{-4}$	CL=90%
Γ_{19}	$\pi^0\mu^+\mu^-\gamma$	$< 3\times10^{-6}$	CL=90%

From PDG

Due to C-parity!

$J/\psi(1S)$

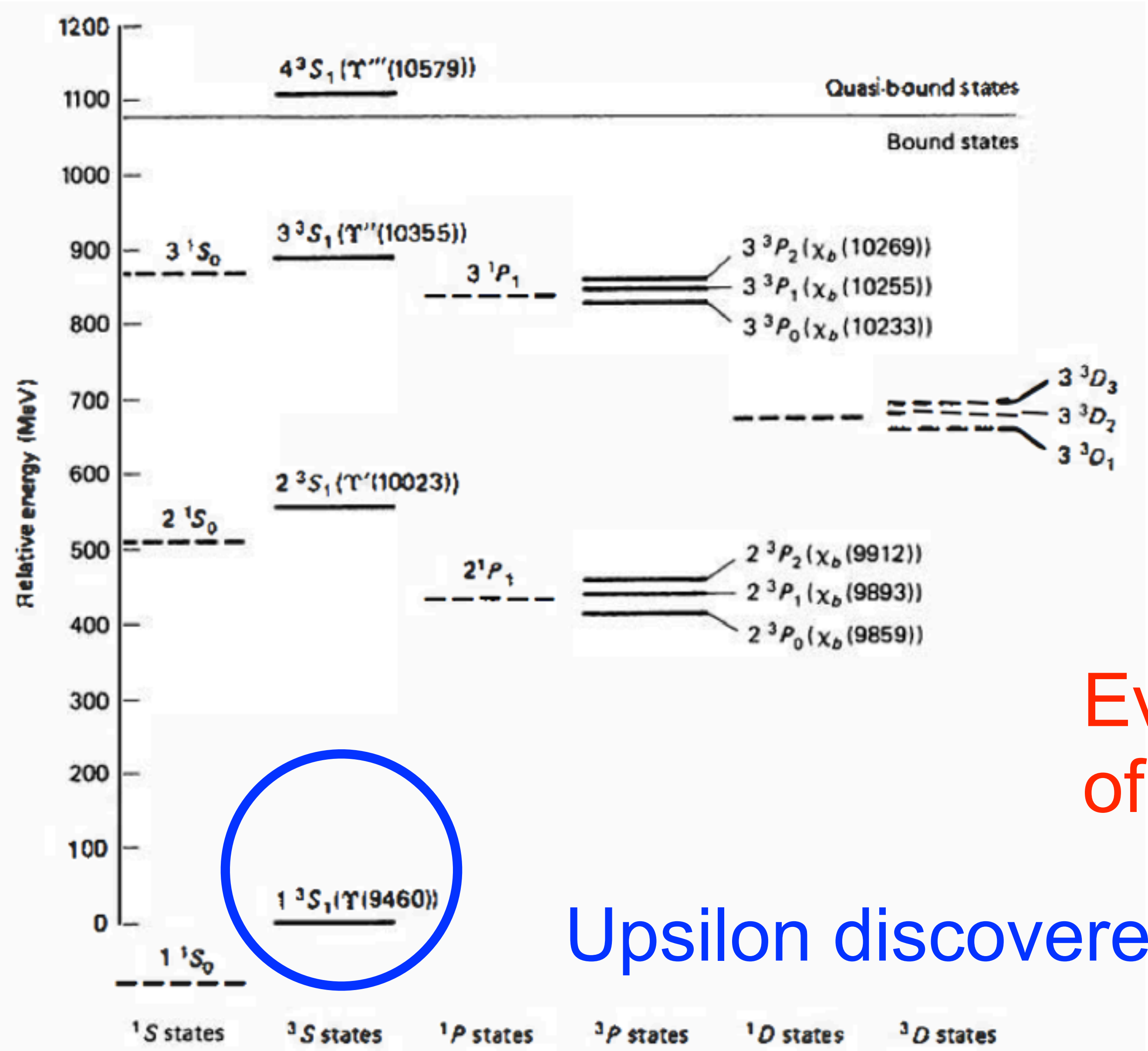
$I^G(J^{PC}) = 0^-(1^{--})$

$J/\psi(1S)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	hadrons	(87.7 ± 0.5) %	
Γ_2	virtual $\gamma\to$ hadrons	(13.50 ± 0.30) %	
Γ_3	ggg	(64.1 ± 1.0) %	
Γ_4	γgg	(8.8 ± 0.5) %	
Γ_5	e^+e^-	(5.94 ± 0.06) %	
Γ_6	$\mu^+\mu^-$	(5.93 ± 0.06) %	

Bottomonium

Griffiths



Even richer set of states!

Upsilon discovered in 1976

How to account for spin in hadron masses?

Recall magnetic moment formula: $\mu = -\frac{e}{m}\mathbf{S}$

Spin-spin interactions in hadrons have two components:

$$\mu_1 \cdot \mathbf{S_2} = -\frac{e}{m_1}\mathbf{S_1} \cdot \mathbf{S_2}$$

$$\mu_2 \cdot \mathbf{S_1} = -\frac{e}{m_2}\mathbf{S_2} \cdot \mathbf{S_1} = -\frac{e}{m_2}\mathbf{S_1} \cdot \mathbf{S_2}$$

Sum is then

$$-e\frac{m_1 + m_2}{m_1 m_2}\left(\mathbf{S_1} \cdot \mathbf{S_2}\right) = A(m_1 + m_2)\frac{1}{m_1 m_2}\left(\mathbf{S_1} \cdot \mathbf{S_2}\right)$$

What is the spin term?

$$\left(\mathbf{S_1} \cdot \mathbf{S_2}\right), \mathbf{S} = \mathbf{S_1} + \mathbf{S_2}$$

$$\mathbf{S}^2 = \mathbf{S_1}^2 + \mathbf{S_2}^2 + 2\left(\mathbf{S_1} \cdot \mathbf{S_2}\right)$$

$$\left(\mathbf{S_1} \cdot \mathbf{S_2}\right) = \frac{1}{2}\left(\mathbf{S}^2 - \mathbf{S_1}^2 - \mathbf{S_2}^2\right)$$

S_1 and $S_2 = \pm 1/2$

$\mathbf{S_1}^2 = \mathbf{S_2}^2 = (1/2)(1/2+1) = 3/4$

$\mathbf{S}^2 = (1)(1+1) = 2$ (spins aligned) or

$\mathbf{S}^2 = (0)(0+1) = 0$ (spins anti-aligned)

So: $\mathbf{S}_1 \cdot \mathbf{S}_2 = 1/4$ (spins aligned)

or: $\mathbf{S}_1 \cdot \mathbf{S}_2 = -3/4$ (spins anti-aligned)

How to account for spin in hadron masses?

$$M(m_1 - - - m_2) = m_1 + m_2 + A(m_1 + m_2)\frac{1}{m_1 m_2}(\mathbf{S_1} \cdot \mathbf{S_2})$$

Mass M of meson composed of quarks with mass m_1 and m_2 then generically looks like this

$\mathbf{S}_1 \cdot \mathbf{S}_2 = 1/4$ (spins aligned)
or: $\mathbf{S}_1 \cdot \mathbf{S}_2 = -3/4$ (spins anti-aligned)

Can try something simpler, and assume A is a constant (it surely is not, but maybe that is a reasonable approximation)

Fits for masses (from Griffiths)

Table 5.3 Pseudoscalar and vector meson masses. (MeV/c^2)

Meson	Calculated	Observed
π	139	138
K	487	496
η	561	548
ρ	775	776
ω	775	783
K^*	892	894
ϕ	1031	1020

Very nice agreement! But need to be careful...

For example: $$\eta = \frac{u\bar{u} + d\bar{d} - 2s\bar{s}}{\sqrt{6}}$$

On to baryons

A lot more complicated - have three quarks, and thus three spins to add together. Most importantly, mesons are always composed of a quark and an anti-quark, ie never contain two of the same particle. In baryons, however (example: proton = uud), this no longer has to be true.

Regardless, though, baryons have half-integer spin (three quarks with s=+/- 1/2 can combine to give s= +/- 1/2 or +/- 3/2 only)

How to add three spins

To add three spins together, we first start by adding two of them together. Back to those C-G tables from the PDG ...

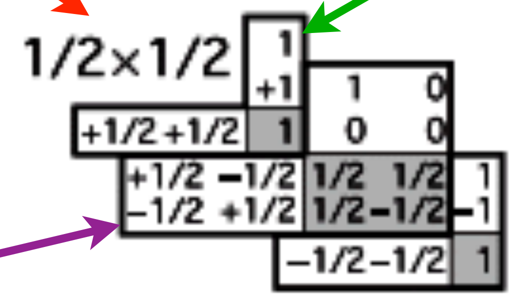
Combining two 1/2 x 1/2 particles

$$|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{1}{2}> = |1\ 1>$$

$$|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{-1}{2}> = \sqrt{\frac{1}{2}}|1\ 0> + \sqrt{\frac{1}{2}}|0\ 0>$$

$$|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{1}{2}> = \sqrt{\frac{1}{2}}|1\ 0> - \sqrt{\frac{1}{2}}|0\ 0>$$

$$|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{-1}{2}> = |1\ -1>$$



ex: J=1

ex: m_1,m_2= -1/2,+/1-2

Let's rearrange

$$|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{1}{2}> = |1\ 1>$$

$$|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{-1}{2}> = \sqrt{\frac{1}{2}}|1\ 0> + \sqrt{\frac{1}{2}}|0\ 0>$$

$$|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{1}{2}> = \sqrt{\frac{1}{2}}|1\ 0> - \sqrt{\frac{1}{2}}|0\ 0>$$

$$|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{-1}{2}> = |1\ -1>$$

These are the easy ones

$$|1\ 1> = |\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{1}{2}>$$

$$|1\ -1> = |\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{-1}{2}>$$

Let's rearrange (can also use tables for this)

$$|\frac{1}{2}\ \frac{1}{2} > + |\frac{1}{2}\ \frac{1}{2} >= |1\ 1 >$$

$$|\frac{1}{2}\ \frac{1}{2} > + |\frac{1}{2}\ \frac{-1}{2} >= \sqrt{\frac{1}{2}}|1\ 0 > + \sqrt{\frac{1}{2}}|0\ 0 >$$

Add together here

$$|\frac{1}{2}\ \frac{-1}{2} > + |\frac{1}{2}\ \frac{1}{2} >= \sqrt{\frac{1}{2}}|1\ 0 > - \sqrt{\frac{1}{2}}|0\ 0 >$$

$$|\frac{1}{2}\ \frac{-1}{2} > + |\frac{1}{2}\ \frac{-1}{2} >= |1\ -1 >$$

$$\sqrt{2}|1\ 0 >= \left(|\frac{1}{2}\ \frac{1}{2} > + |\frac{1}{2}\ \frac{-1}{2} >\right) + \left(|\frac{1}{2}\ \frac{-1}{2} > + |\frac{1}{2}\ \frac{1}{2} >\right)$$

$$|1\ 0 >= \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{1}{2} > + |\frac{1}{2}\ \frac{-1}{2} >\right) + \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{-1}{2} > + |\frac{1}{2}\ \frac{1}{2} >\right)$$

Let's rearrange (can also use tables for this)

$$|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{1}{2}> = |1\ 1>$$

$$|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{-1}{2}> = \sqrt{\frac{1}{2}}|1\ 0> + \sqrt{\frac{1}{2}}|0\ 0>$$

Subtract these

$$|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{1}{2}> = \sqrt{\frac{1}{2}}|1\ 0> - \sqrt{\frac{1}{2}}|0\ 0>$$

$$|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{-1}{2}> = |1\ -1>$$

$$\sqrt{2}|0\ 0> = \left(|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{-1}{2}>\right) - \left(|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|0\ 0> = \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{1}{2}> + |\frac{1}{2}\ \frac{-1}{2}>\right) - \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{-1}{2}> + |\frac{1}{2}\ \frac{1}{2}>\right)$$

Putting it together

$$|0\ 0>=\frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{1}{2}>+|\frac{1}{2}\ \frac{-1}{2}>\right)-\frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{-1}{2}>+|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|1\ 0>=\frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{1}{2}>+|\frac{1}{2}\ \frac{-1}{2}>\right)+\frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{-1}{2}>+|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|1\ 1>=|\frac{1}{2}\ \frac{1}{2}>+|\frac{1}{2}\ \frac{1}{2}>$$

$$|1\ -1>=|\frac{1}{2}\ \frac{-1}{2}>+|\frac{1}{2}\ \frac{-1}{2}>$$

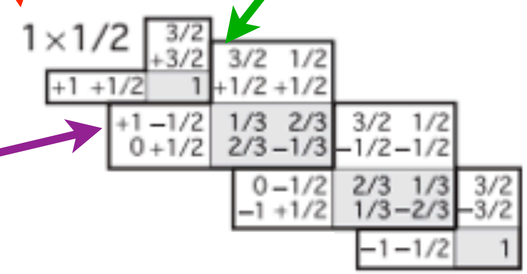
When we add the third quark we will have to add spin 1/2 with either spin 0 or spin 1

Now we add the third one

Combining spin 1 x 1/2 particles

ex:
J=3/2
m=1/2

We want the “inverse” of what we have been reading off. Can also use the tables for that!



ex:
$m_1, m_2 =$
+1,-1/2

Now we add the third one

Combining spin 1 x 1/2 particles

$$|\frac{3}{2}\ \frac{3}{2}>=|1\ 1>+|\frac{1}{2}\ \frac{1}{2}>$$

$$|\frac{3}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{3}}\left(|1\ 1>+|\frac{1}{2}\ \frac{-1}{2}>\right)+\sqrt{\frac{2}{3}}\left(|1\ 0>+|\frac{1}{2}\ \frac{1}{2}>\right)$$

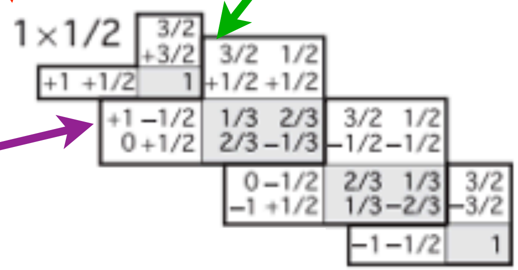
$$|\frac{1}{2}\ \frac{1}{2}>=\sqrt{\frac{2}{3}}\left(|1\ 1>+|\frac{1}{2}\ \frac{-1}{2}>\right)-\sqrt{\frac{1}{3}}\left(|1\ 0>+|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{3}{2}\ \frac{-1}{2}>=\sqrt{\frac{2}{3}}\left(|1\ 0>+|\frac{1}{2}\ \frac{-1}{2}>\right)+\sqrt{\frac{1}{3}}\left(|1\ -1>+|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\sqrt{\frac{1}{3}}\left(|1\ 0>+|\frac{1}{2}\ \frac{-1}{2}>\right)-\sqrt{\frac{2}{3}}\left(|1\ -1>+|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{3}{2}\ \frac{-3}{2}>=|1\ -1>+|\frac{1}{2}\ \frac{-1}{2}>$$

ex:
J=3/2
m=1/2



ex:
$m_1, m_2=$
+1,-1/2

Now we add the third one

Combining spin 0 x 1/2 particles is trivial

$$|\frac{1}{2}\ \frac{1}{2}>=|0\ 0>+|\frac{1}{2}\ \frac{1}{2}>$$

$$|\frac{1}{2}\ \frac{-1}{2}>=|0\ 0>+|\frac{1}{2}\ \frac{-1}{2}>$$

So in total

$$|0\ 0>= \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{1}{2}> +|\frac{1}{2}\ \frac{-1}{2}>\right) - \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{-1}{2}> +|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|1\ 0>= \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{1}{2}> +|\frac{1}{2}\ \frac{-1}{2}>\right) + \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{-1}{2}> +|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|1\ 1>= |\frac{1}{2}\ \frac{1}{2}> +|\frac{1}{2}\ \frac{1}{2}>$$

$$|1\ -1>= |\frac{1}{2}\ \frac{-1}{2}> +|\frac{1}{2}\ \frac{-1}{2}>$$

From first combination

$$|\frac{3}{2}\ \frac{3}{2}>= |1\ 1> +|\frac{1}{2}\ \frac{1}{2}>$$

$$|\frac{3}{2}\ \frac{1}{2}>= \frac{1}{\sqrt{3}}\left(|1\ 1> +|\frac{1}{2}\ \frac{-1}{2}>\right) + \sqrt{\frac{2}{3}}\left(|1\ 0> +|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{1}{2}\ \frac{1}{2}>= \sqrt{\frac{2}{3}}\left(|1\ 1> +|\frac{1}{2}\ \frac{-1}{2}>\right) - \sqrt{\frac{1}{3}}\left(|1\ 0> +|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{3}{2}\ \frac{-1}{2}>= \sqrt{\frac{2}{3}}\left(|1\ 0> +|\frac{1}{2}\ \frac{-1}{2}>\right) + \sqrt{\frac{1}{3}}\left(|1\ -1> +|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{1}{2}\ \frac{-1}{2}>= \sqrt{\frac{1}{3}}\left(|1\ 0> +|\frac{1}{2}\ \frac{-1}{2}>\right) - \sqrt{\frac{2}{3}}\left(|1\ -1> +|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{3}{2}\ \frac{-3}{2}>= |1\ -1> +|\frac{1}{2}\ \frac{-1}{2}>$$

$$|0\ 0> +|\frac{1}{2}\ \frac{1}{2}>= |\frac{1}{2}\ \frac{1}{2}>$$

$$|0\ 0> +|\frac{1}{2}\ \frac{-1}{2}>= |\frac{1}{2}\ \frac{-1}{2}>$$

Let's introduce some nicer notation

$$|0\ 0>= \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{1}{2}> +|\frac{1}{2}\ \frac{-1}{2}>\right) - \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{-1}{2}> +|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|1\ 0>= \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{1}{2}> +|\frac{1}{2}\ \frac{-1}{2}>\right) + \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\ \frac{-1}{2}> +|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|1\ 1>= |\frac{1}{2}\ \frac{1}{2}> +|\frac{1}{2}\ \frac{1}{2}>$$

$$|1\ -1>= |\frac{1}{2}\ \frac{-1}{2}> +|\frac{1}{2}\ \frac{-1}{2}>$$

$$|\frac{3}{2}\ \frac{3}{2}>= |1\ 1> +|\frac{1}{2}\ \frac{1}{2}>$$

$$|\frac{3}{2}\ \frac{1}{2}>= \frac{1}{\sqrt{3}}\left(|1\ 1> +|\frac{1}{2}\ \frac{-1}{2}>\right) + \sqrt{\frac{2}{3}}\left(|1\ 0> +|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{1}{2}\ \frac{1}{2}>= \sqrt{\frac{2}{3}}\left(|1\ 1> +|\frac{1}{2}\ \frac{-1}{2}>\right) - \sqrt{\frac{1}{3}}\left(|1\ 0> +|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{3}{2}\ \frac{-1}{2}>= \sqrt{\frac{2}{3}}\left(|1\ 0> +|\frac{1}{2}\ \frac{-1}{2}>\right) + \sqrt{\frac{1}{3}}\left(|1\ -1> +|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{1}{2}\ \frac{-1}{2}>= \sqrt{\frac{1}{3}}\left(|1\ 0> +|\frac{1}{2}\ \frac{-1}{2}>\right) - \sqrt{\frac{2}{3}}\left(|1\ -1> +|\frac{1}{2}\ \frac{1}{2}>\right)$$

$$|\frac{3}{2}\ \frac{-3}{2}>= |1\ -1> +|\frac{1}{2}\ \frac{-1}{2}>$$

From first combination

Don't forget: order matters!

$$|0\ 0> +|\frac{1}{2}\ \frac{1}{2}>= |\frac{1}{2}\ \frac{1}{2}>$$

$$|0\ 0> +|\frac{1}{2}\ \frac{-1}{2}>= |\frac{1}{2}\ \frac{-1}{2}>$$

$$|\frac{1}{2}\ \frac{1}{2}>= (\uparrow)$$

$$|\frac{1}{2}\ \frac{-1}{2}>= (\downarrow)$$

Using the notation

$$|\frac{3}{2}\ \frac{3}{2}>=|1\ 1>+(\uparrow)$$

$$|\frac{3}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{3}}(|1\ 1>+(\downarrow))+\sqrt{\frac{2}{3}}(|1\ 0>+(\uparrow))$$

$$|\frac{1}{2}\ \frac{1}{2}>=\sqrt{\frac{2}{3}}(|1\ 1>+(\downarrow))-\sqrt{\frac{1}{3}}(|1\ 0>+(\uparrow))$$

$$|\frac{3}{2}\ \frac{-1}{2}>=\sqrt{\frac{2}{3}}(|1\ 0>+(\downarrow))+\sqrt{\frac{1}{3}}(|1\ -1>+(\uparrow))$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\sqrt{\frac{1}{3}}(|1\ 0>+(\downarrow))-\sqrt{\frac{2}{3}}(|1\ -1>+(\uparrow))$$

$$|\frac{3}{2}\ \frac{-3}{2}>=|1\ -1>+(\downarrow)$$

$$|0\ 0>=\frac{1}{\sqrt{2}}(\uparrow\downarrow)-\frac{1}{\sqrt{2}}(\downarrow\uparrow)$$

$$|1\ 0>=\frac{1}{\sqrt{2}}(\uparrow\downarrow)+\frac{1}{\sqrt{2}}(\downarrow\uparrow)$$

$$|1\ 1>=\uparrow\uparrow$$

$$|1\ -1>=\downarrow\downarrow$$

$$|\frac{1}{2}\ \frac{1}{2}>=|0\ 0>+\uparrow$$

$$|\frac{1}{2}\ \frac{-1}{2}>=|0\ 0>+\downarrow$$

Putting it together

$$|\frac{3}{2}\ \frac{3}{2}>=\uparrow\uparrow\uparrow$$

$$|\frac{3}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow)+\sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{1}{2}\ \frac{1}{2}>=\sqrt{\frac{2}{3}}(\uparrow\uparrow\downarrow)-\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{3}{2}\ \frac{-1}{2}>=\sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)(\downarrow)\right)+\sqrt{\frac{1}{3}}(\downarrow\downarrow\uparrow)$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\downarrow\right)-\sqrt{\frac{2}{3}}(\downarrow\downarrow\uparrow)$$

$$|\frac{3}{2}\ \frac{-3}{2}>=\downarrow\downarrow\downarrow$$

Phew

$$|\frac{1}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\uparrow$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\downarrow$$

How to interpret

$$|\frac{3}{2}\ \frac{3}{2}>=\uparrow\uparrow\uparrow$$

$$|\frac{3}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow)+\sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{1}{2}\ \frac{1}{2}>=\sqrt{\frac{2}{3}}(\uparrow\uparrow\downarrow)-\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{3}{2}\ \frac{-1}{2}>=\sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)(\downarrow)\right)+\sqrt{\frac{1}{3}}(\downarrow\downarrow\uparrow)$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\downarrow\right)-\sqrt{\frac{2}{3}}(\downarrow\downarrow\uparrow)$$

$$|\frac{3}{2}\ \frac{-3}{2}>=\downarrow\downarrow\downarrow$$

$$|\frac{1}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\uparrow$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\downarrow$$

$$|\frac{3}{2}\ \frac{3}{2}>=\uparrow\uparrow\uparrow$$

$$|\frac{3}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow\ +\uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow)$$

$$|\frac{3}{2}\ \frac{-1}{2}>=\frac{1}{\sqrt{3}}(\uparrow\downarrow\downarrow\ +\downarrow\uparrow\downarrow+\downarrow\downarrow\uparrow)$$

$$|\frac{3}{2}\ \frac{-3}{2}>=\downarrow\downarrow\downarrow$$

Spin 3/2 states are easy to interpret: symmetric if we interchange any two quarks

How to interpret

$$|\frac{3}{2}\ \frac{3}{2}>=\uparrow\uparrow\uparrow$$

$$|\frac{3}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow)+\sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{1}{2}\ \frac{1}{2}>=\sqrt{\frac{2}{3}}(\uparrow\uparrow\downarrow)-\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{3}{2}\ \frac{-1}{2}>=\sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)(\downarrow)\right)+\sqrt{\frac{1}{3}}(\downarrow\downarrow\uparrow)$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\downarrow\right)-\sqrt{\frac{2}{3}}(\downarrow\downarrow\uparrow)$$

$$|\frac{3}{2}\ \frac{-3}{2}>=\downarrow\downarrow\downarrow$$

$$|\frac{1}{2}\ \frac{1}{2}>=\sqrt{\frac{2}{3}}(\uparrow\uparrow\downarrow)-\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\downarrow\right)-\sqrt{\frac{2}{3}}(\downarrow\downarrow\uparrow)$$

$$|\frac{1}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\uparrow$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\downarrow$$

Two of spin 1/2 states are asymmetric under interchange of first and second quarks

How to interpret

$$|\frac{3}{2}\ \frac{3}{2}>=\uparrow\uparrow\uparrow$$

$$|\frac{3}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow)+\sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{1}{2}\ \frac{1}{2}>=\sqrt{\frac{2}{3}}(\uparrow\uparrow\downarrow)-\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{3}{2}\ \frac{-1}{2}>=\sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)(\downarrow)\right)+\sqrt{\frac{1}{3}}(\downarrow\downarrow\uparrow)$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\downarrow\right)-\sqrt{\frac{2}{3}}(\downarrow\downarrow\uparrow)$$

$$|\frac{3}{2}\ \frac{-3}{2}>=\downarrow\downarrow\downarrow$$

$$|\frac{1}{2}\ \frac{1}{2}>=\sqrt{\frac{2}{3}}(\uparrow\uparrow\downarrow)-\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right)$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\downarrow\right)-\sqrt{\frac{2}{3}}(\downarrow\downarrow\uparrow)$$

$$|\frac{1}{2}\ \frac{1}{2}>=\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\uparrow$$

$$|\frac{1}{2}\ \frac{-1}{2}>=\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\downarrow$$

These last two spin 1/2 states are symmetric under interchange of first and second quarks

Back to baryons

We need our 3 quarks to satisfy fermi-dirac statistics (must be anti-symmetric under exchange of any two quarks)

For ground state (l=0), space wave function is symmetric. Left off with wave functions for spin, color and flavor. We will see that color wave function is necessarily anti-symmetric. That means that **flavor x spin** combination must be symmetric