Computer assignment 1

Projectiles with quadratic drag?

Recall that we could not solve this analytically

$$m\ddot{\mathbf{r}} = m\mathbf{g} - cv^2\hat{\mathbf{v}}$$

$$\ddot{x} = -\frac{c}{m}v_x v$$

$$\ddot{x} = -\frac{c}{m}v_x\sqrt{v_x^2 + v_y^2}$$

$$\ddot{x} = -\frac{c}{m}\dot{x}\sqrt{v_x^2 + v_y^2}$$

Coupled equations!

$$\ddot{y} = -g - \frac{c}{m}\dot{y}\sqrt{v_x^2 + v_y^2}$$

So what can we do?

From Newton

$$v_x(t+\Delta t) = v_x(t) + a_x(t)\Delta t$$

$$x(t+\Delta t) = x(t) + \frac{[v_x(t) + v_x(t+\Delta t)]}{2}\Delta t$$

$$x(t+\Delta t) = x(t) + \frac{[v_x(t) + v_x(t) + a_x(t)\Delta t)]}{2}\Delta t$$

$$x(t+\Delta t) = x(t) + v_x(t)\Delta t + \frac{1}{2}a_x(t)(\Delta t)^2$$

From Newton

Above is more and more valid as Δt gets smaller and smaller! Can imagine other (better) approximations, but this is a good first step. Similar equations hold for y direction

In other words

We know the initial conditions at t=0 (initial x, y, v_x, v_y). From this, can calculate a_x, a_y using Newton's Laws

After this, we can find x, y, v_x, v_y at $t=\Delta t$ using approximations of last slide. From this, calculate new a_x, a_y

Then we can repeat to find x, y, v_x, v_y at $t=2\Delta t$, and then new a_x, a_y, etc

How best to do this?

Obviously need a computer to keep track of all of this. Could easily be programmed in python, C, C++, java, or (even better) using Mathematica, Matlab, ROOT, or Maple

If you are proficient in any of the above, please go ahead. I can probably help with C, C++, python or ROOT, and encourage any and all options.

But if not, we'll do something even simpler ...

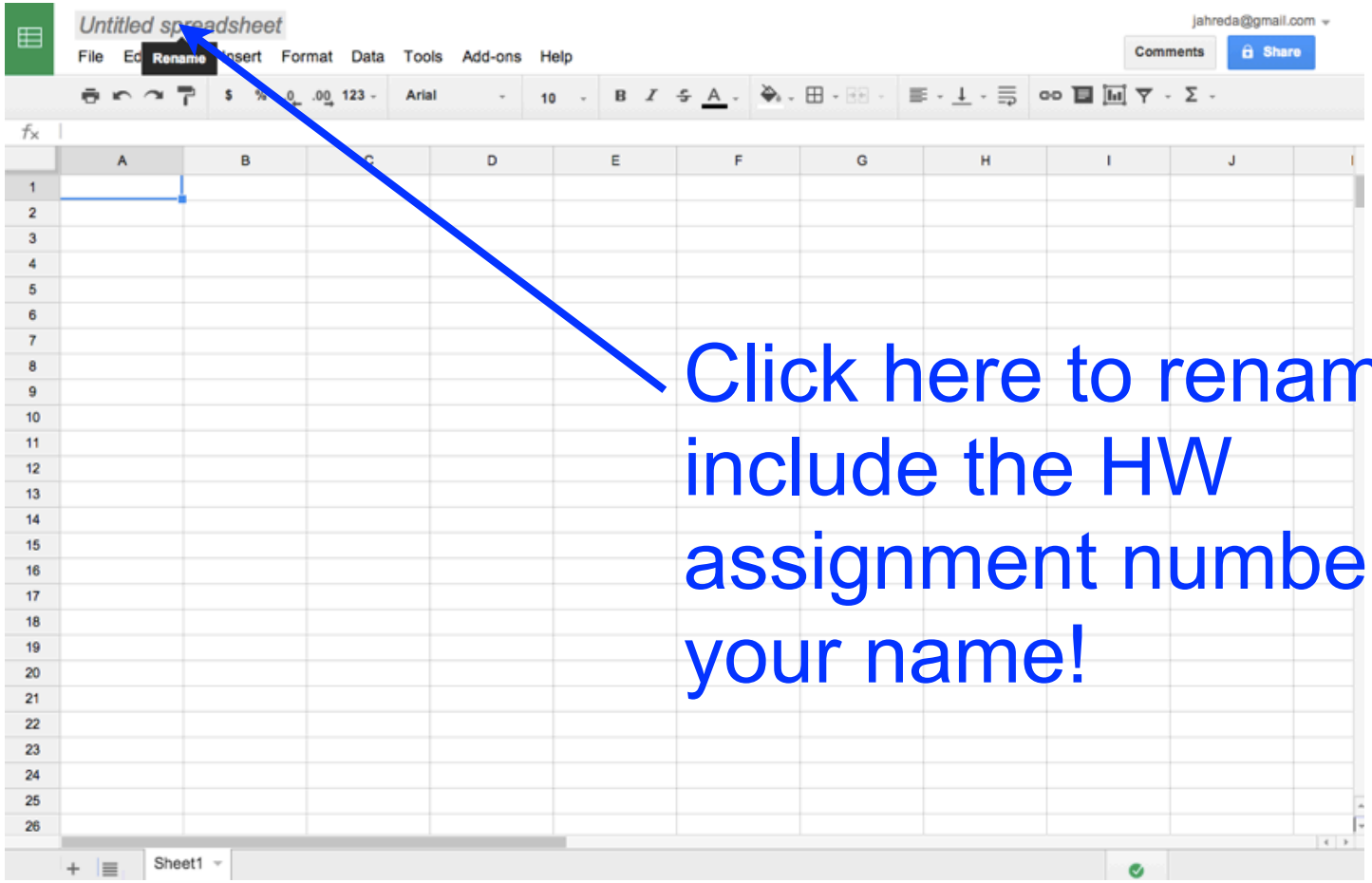
Spreadsheets

You can easily use Microsoft Excel, Apple Numbers or OpenOffice Calc to solve problems. All are allowed, of course.

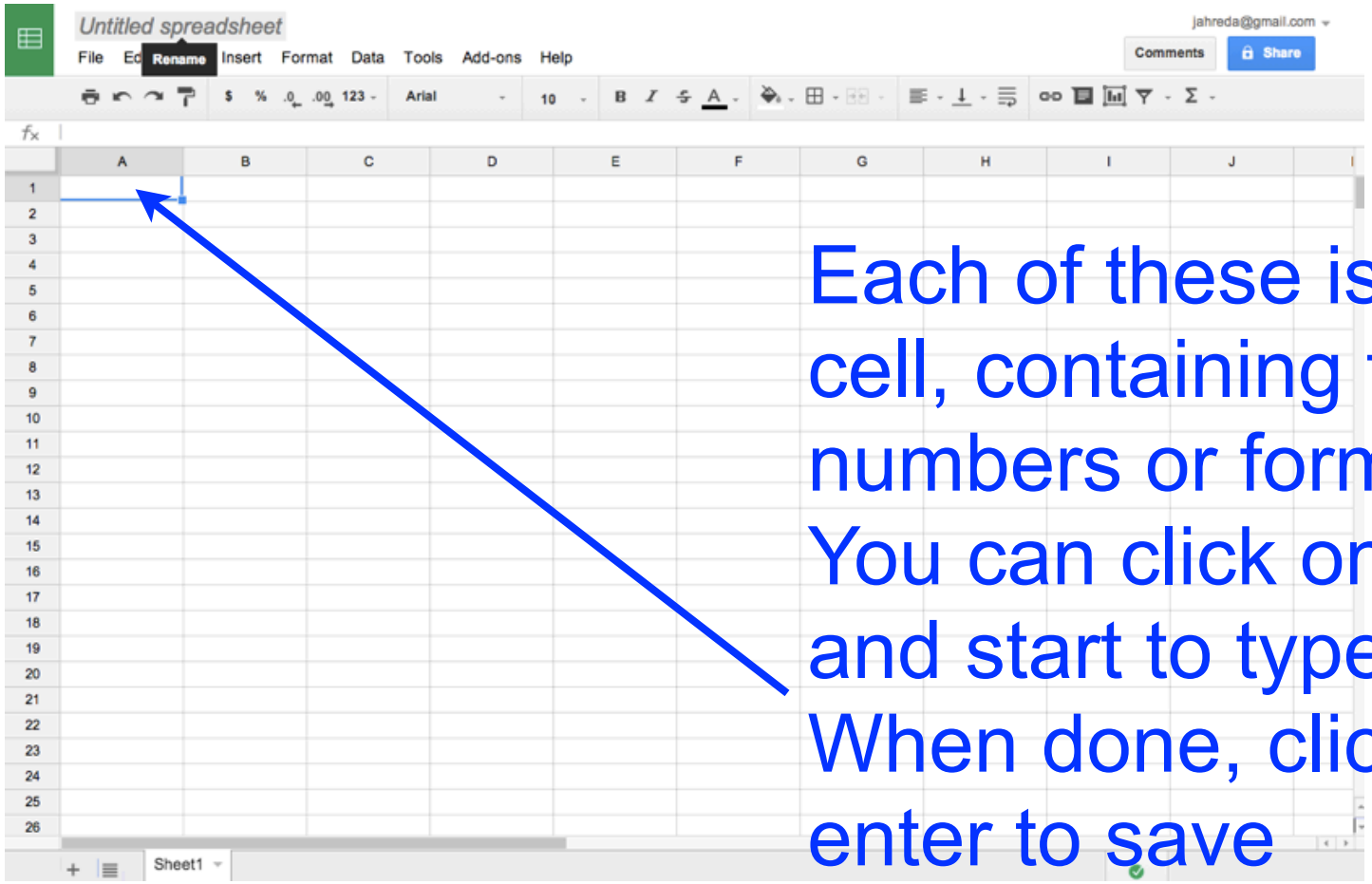
However, the easiest thing is likely to be the use of Google Spreadsheet, since you can save this, edit it and hand in your assignment all on the web

To start, create/sign into a google account, and then go to drive.google.com, then click create->spreadsheet

Google docs automatically get saved (you can access later, and share your HW with me, by going to drive.google.com later on)

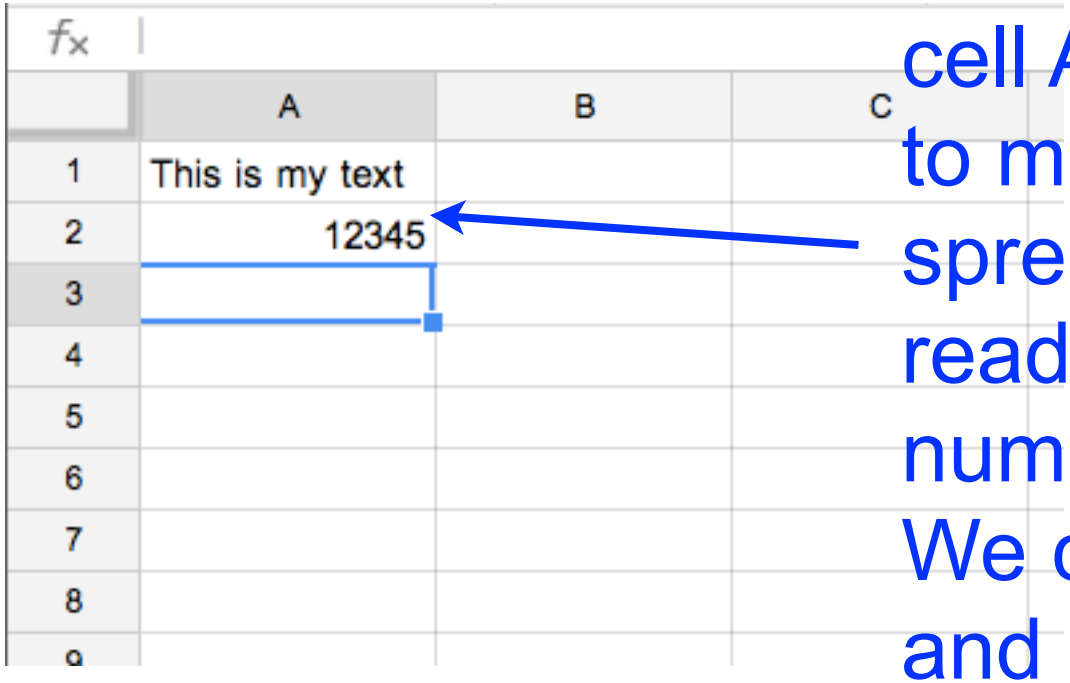


Spreadsheets



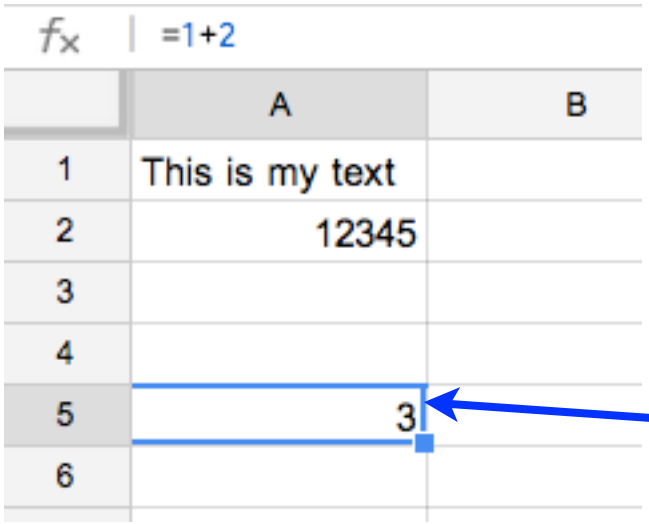
Each of these is a cell, containing text, numbers or formulas. You can click on it and start to type. When done, click enter to save

All data is stored in a spreadsheet in a cell, which is referenced by its row and column



Here we have text in cell A1 (text is useful to make the spreadsheet more readable), and a number in cell A2. We can copy, paste and cut the contents of cells as with any other text

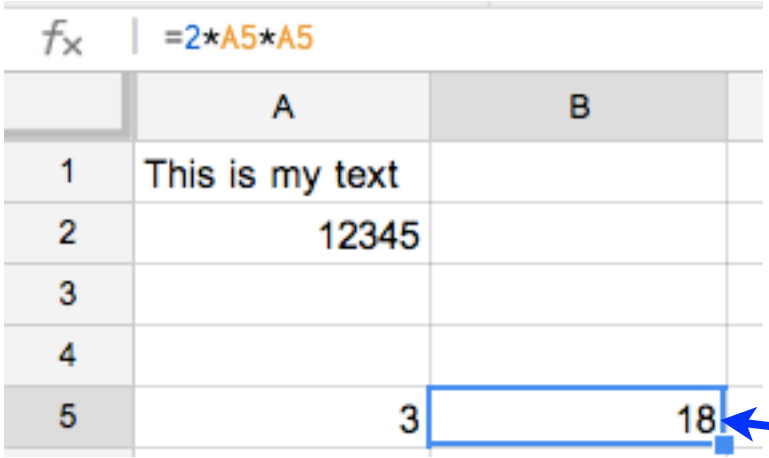
Formulas are entered by starting text entry off with an '='



We have entered '=1+2' into cell A5, and since this is a formula, it gets calculated, ie the cell shows 3

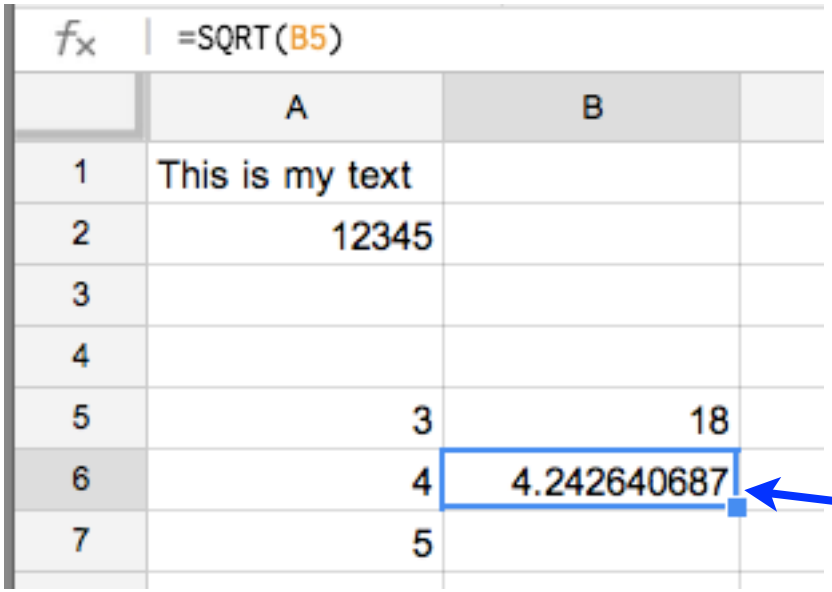
Referring to other cells

Formulas can also refer to other cells via CR, where C is the column (such as B) and R is the row (such as 5)



We have entered '=2*A5*A5' into cell B5, and since this is a formula, it gets calculated, ie the cell shows 2*3*3 = 18

There are a large variety of functions that you can use in formulas



We have entered '=SQRT(B5)' into cell B6, and since this is a formula, it gets calculated, ie the cell shows sqrt(18) = 4.24

Other useful functions: SIN,COS,POW,etc... Be careful to use radians with trigonometric functions

Copying a cell

A cell that is just text or a number can be copied and pasted, whether with Edit->Copy/Paste, or the usual CTRL-C, CTRL-V (or using apple on a mac)

But what happens when we copy a formula?

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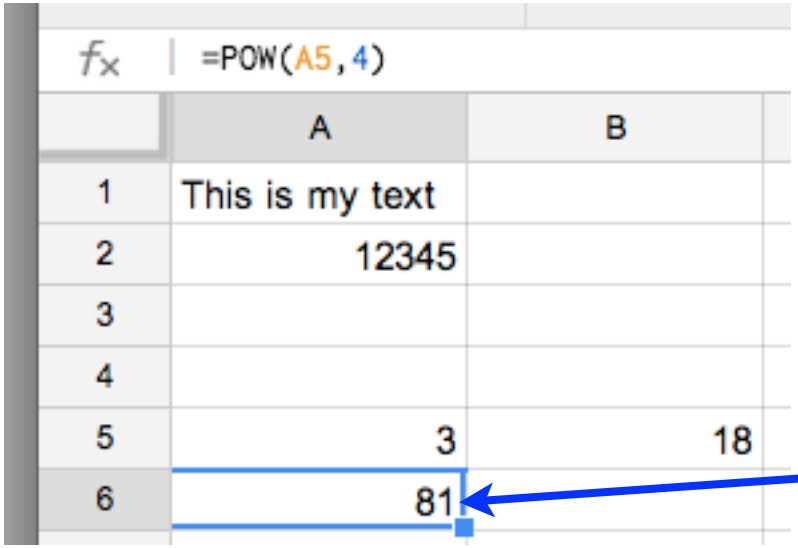
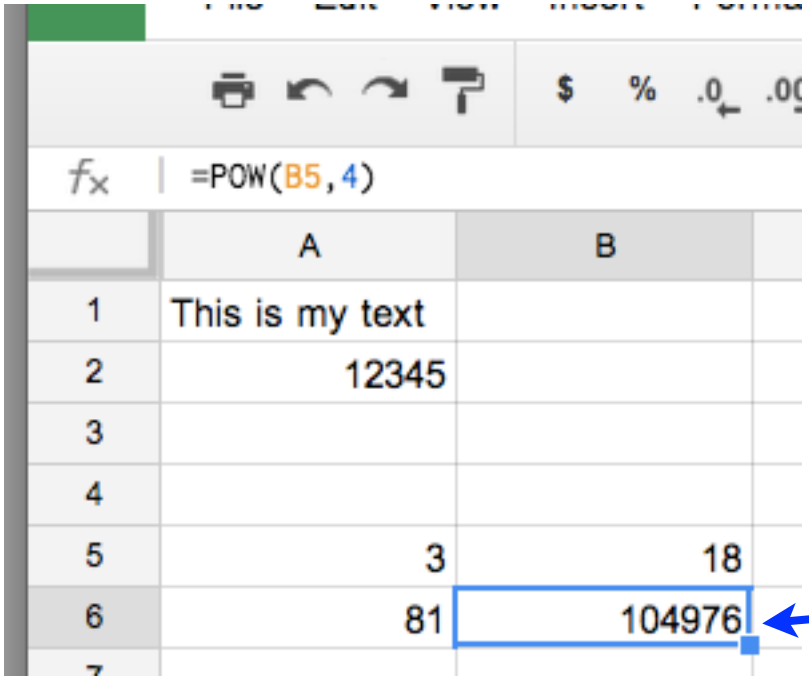


Here, A6 is A5 raised to the 4th power

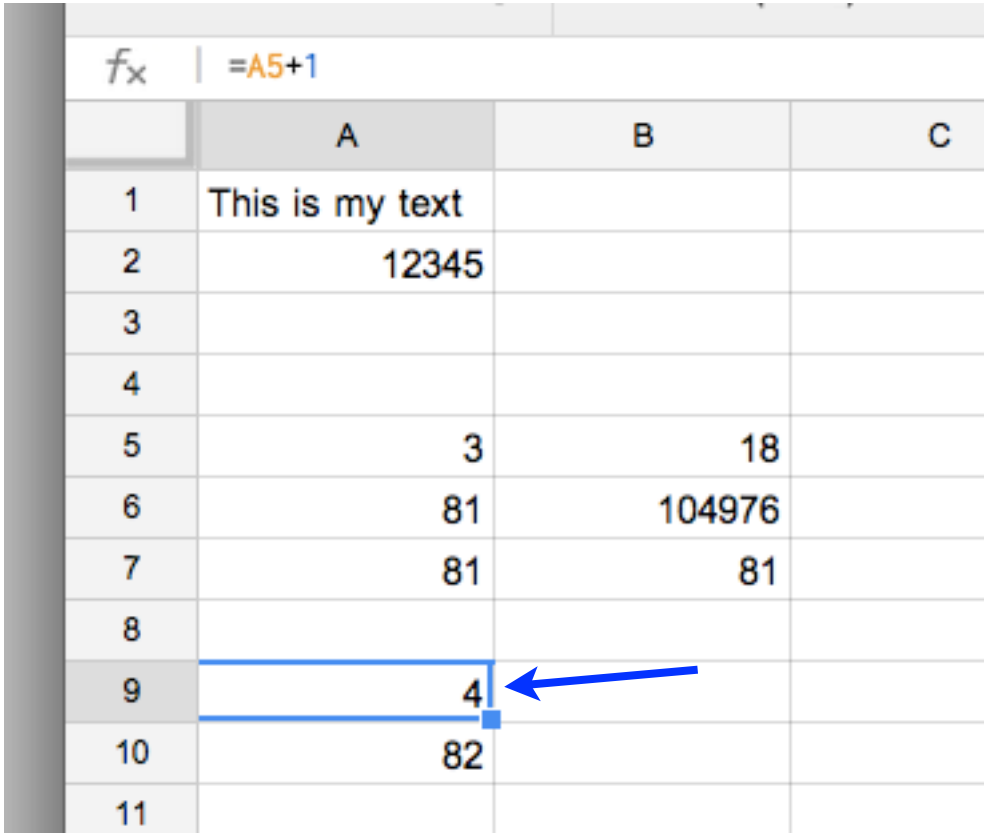
Copying a cell

Unless specially marked, a cell that is cut/copied and pasted elsewhere will have references shifted. This may seem weird, but is very useful



When I copied cell A6 to B6, I don't get A5 to the 4th power, but B5 to the 4th power!

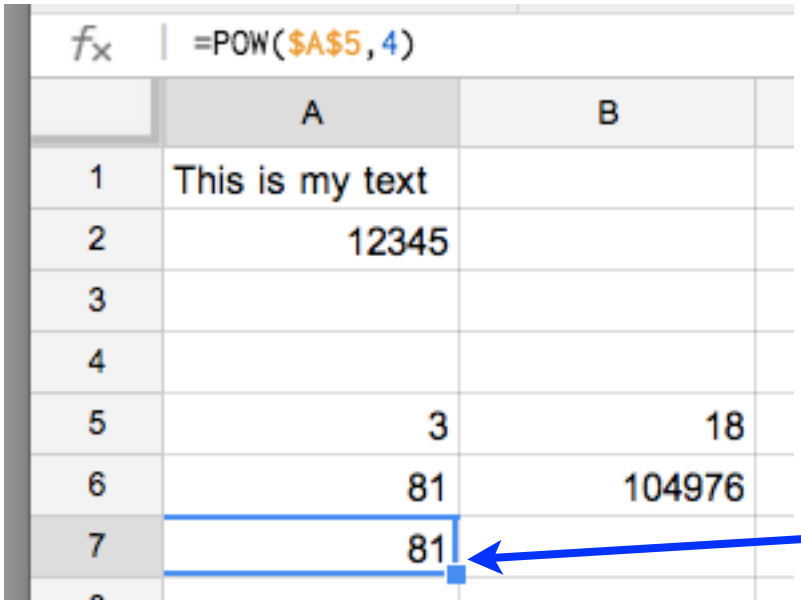
Shifting works both along rows as well as columns



A9 is set to 1+A5. When I copy A9 to A10, A10 beomes 1+A6 (it also gets shifted)!

Copying a cell without shifting references

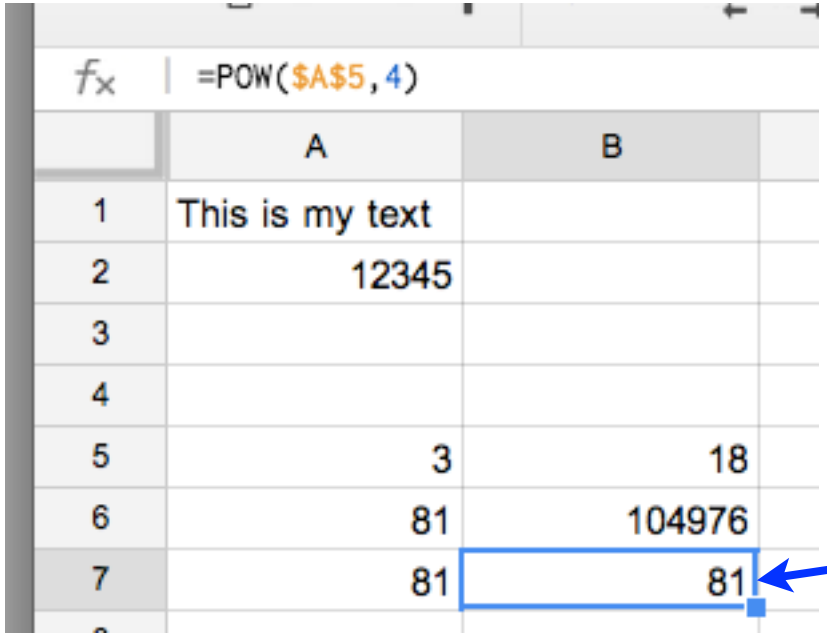
To mark a cell as something that should not be shifted, use a $ sign. This can be used for both the column and/or the row



Now A7 is $A$5 to the 4th power, which gives the same answer as A5 to the 4th power

Copying a cell without shifting references

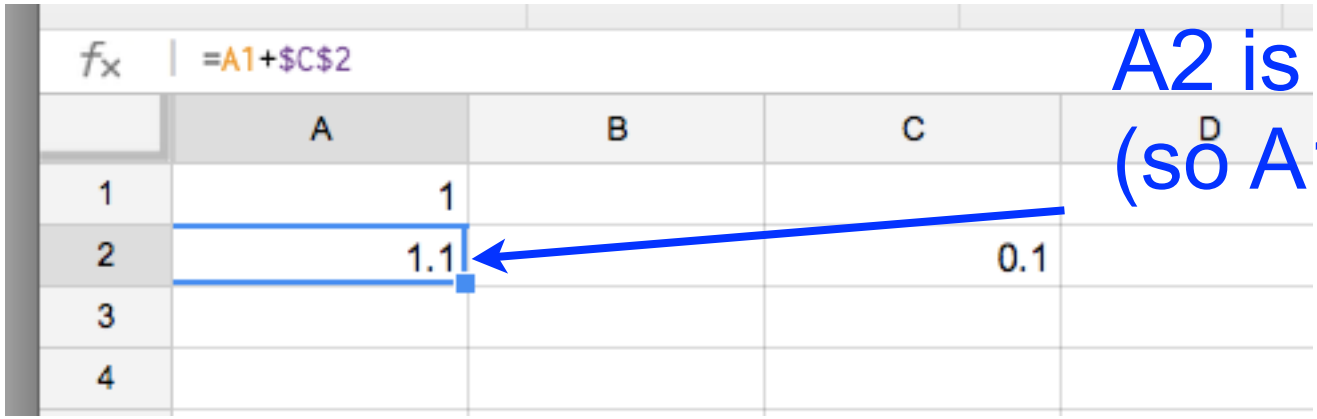
To mark a cell as something that should not be shifted, use a $ sign. This can be used for both the column and/or the row



But now when I copy A7 to B7, the reference does not get shifted

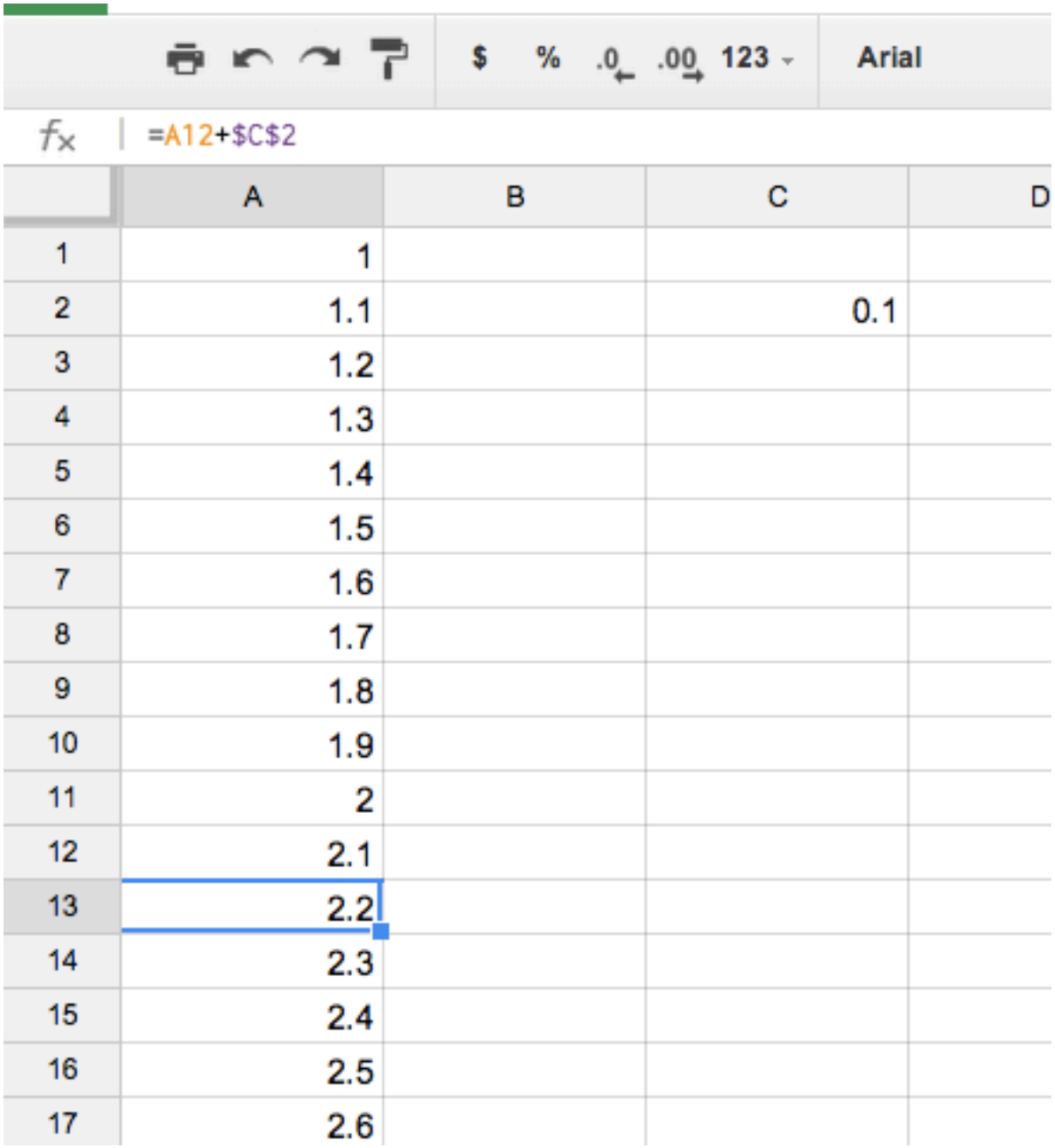
Why are references useful?

If we want to recalculate the same thing over and over again (for example, x, y, v_x, v_y) we don't want to enter the same information multiple times



A2 is just A1+$C$2 (so A1+0.1)

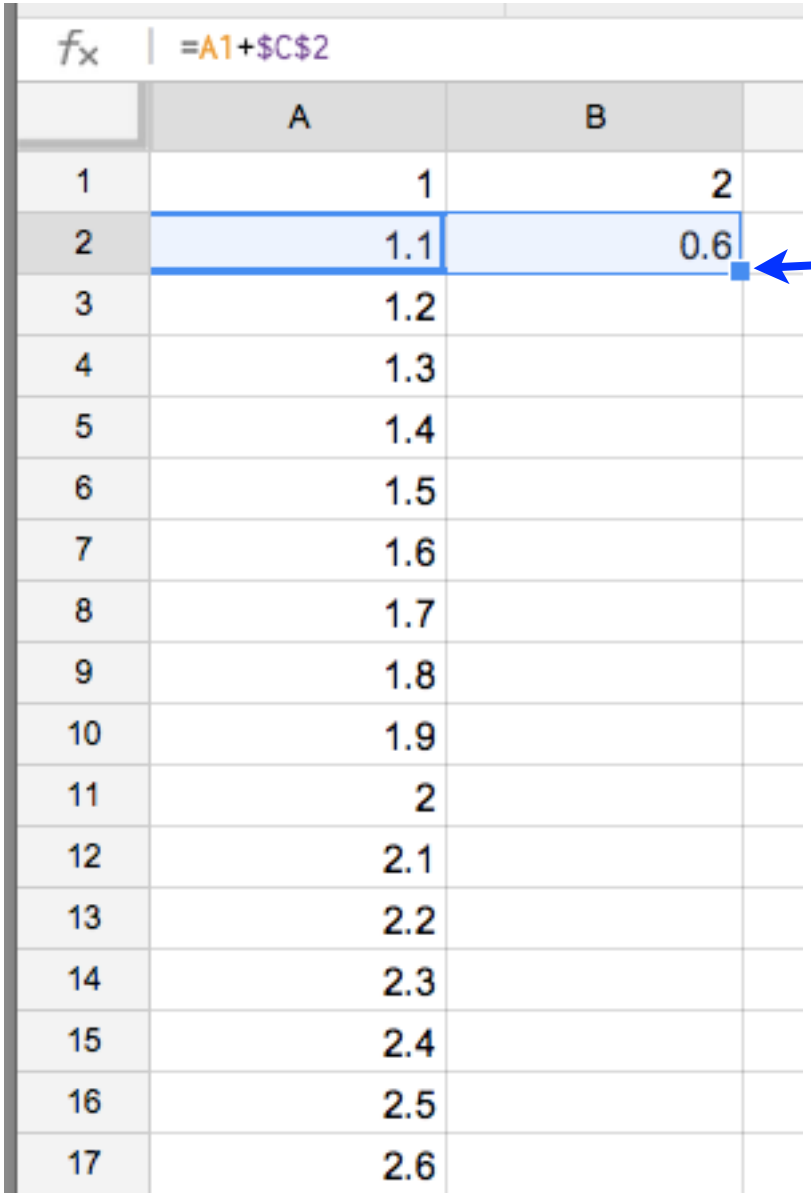
Dragging cells



	A	B	C	D
1	1			
2	1.1		0.1	
3	1.2			
4	1.3			
5	1.4			
6	1.5			
7	1.6			
8	1.7			
9	1.8			
10	1.9			
11	2			
12	2.1			
13	2.2			
14	2.3			
15	2.4			
16	2.5			
17	2.6			

Easy way to get a loop over a series of information/ calculation very quickly!

More on dragging cells



fx | =A1+$C$2

	A	B
1	1	2
2	1.1	0.6
3	1.2	
4	1.3	
5	1.4	
6	1.5	
7	1.6	
8	1.7	
9	1.8	
10	1.9	
11	2	
12	2.1	
13	2.2	
14	2.3	
15	2.4	
16	2.5	
17	2.6	

You can also select multiple rows/columns and drag them all at the same time.

Dragging can be done up, down, left and right

Your assignment - based on Taylor 2.43

1. In the case of no air resistance, solve analytically for the time to hit ground (t'), the range (R), and the maximum height (h)

2. Use a computer to solve the case of no air resistance for t', R and h, separately using Δt=0.02,0.05,0.2,0.4s. How do the answers compare to the analytic answers?
   - Taylor says to read the answer off of the plot, but I'd like you to find the answer among the cells of your spreadsheet

3. Use a computer to solve the case with air resistance, again for the four different Δt above. How do the answers change?
   - Again, not just from the plot, but the cells

4. Using the computer, plot y as a function of x with and without air resistance

How to get started

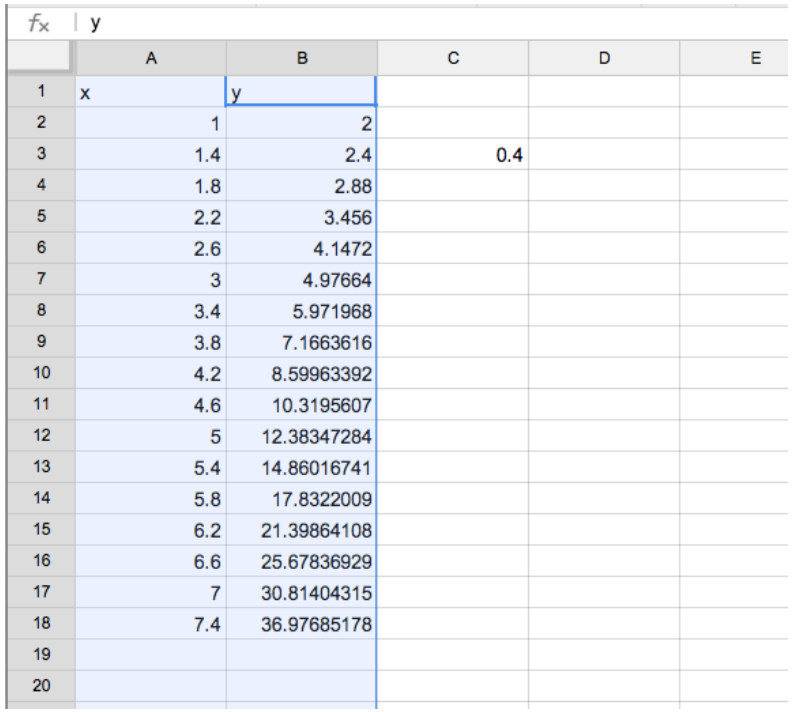
1. We want cells for all our constants - set them aside! We also want a row up top corresponding to all our quantities of interest: time (starting from 0), x, y, v_x and v_y, from which we can calculate a_x, a_y

2.Next we want to move to time $t+\Delta t$. And to recalculate all our other quantities at this new time

3. Then copy-paste/drag and repeat this over and over until we reach the necessary point (where basketball hits ground)

4. How do we know how to read the answer from our cells of numbers? We will never reach ground (ie y = 0), but rather a point where y becomes negative instead of positive. We can use this to interpolate/estimate where the actual value occurred

Making a plot



	A	B	C	D	E
1	x	y			
2	1	2			
3	1.4	2.4	0.4		
4	1.8	2.88			
5	2.2	3.456			
6	2.6	4.1472			
7	3	4.97664			
8	3.4	5.971968			
9	3.8	7.1663616			
10	4.2	8.59963392			
11	4.6	10.3195607			
12	5	12.38347284			
13	5.4	14.86016741			
14	5.8	17.8322009			
15	6.2	21.39864108			
16	6.6	25.67836929			
17	7	30.81404315			
18	7.4	36.97685178			
19					
20					

Our sample x and y data. First select both columns.

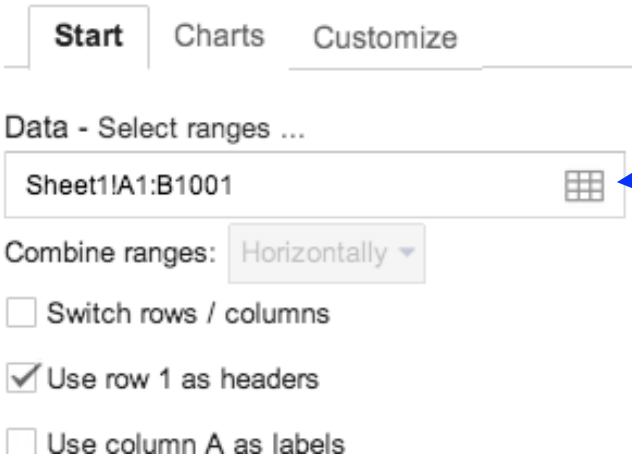
Trick 1: can click on A to select “all of A”)

Trick 2: can use ctrl/ apple/shift keys to select multiple columns/rows

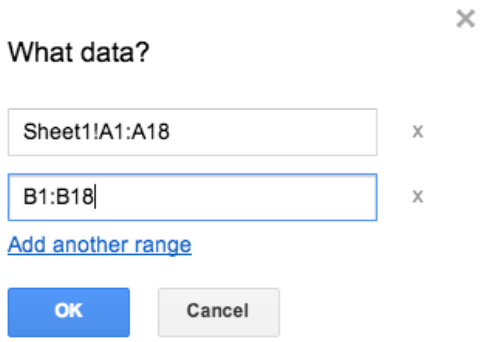
Then go to Insert->Chart

Making a plot

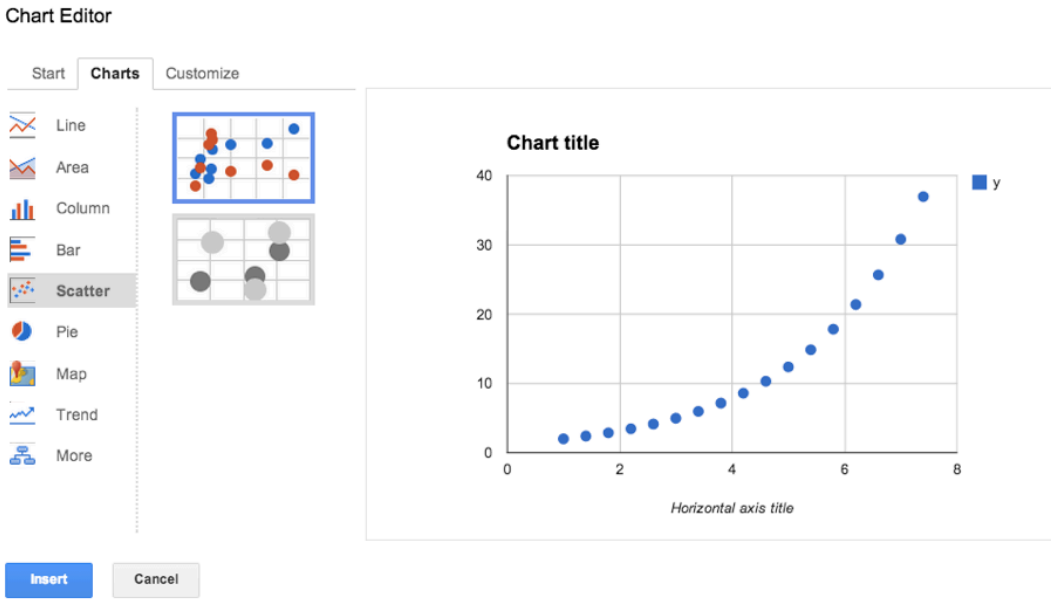
Click here if you want to change the columns/rows used for plotting



Brings up this (which helps you select data)

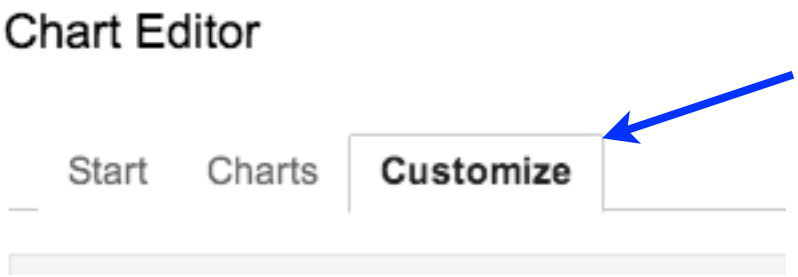


Making a plot



Many possible charts available, depending on what you want. For our purposes, we want scatter plots, usually

Making a plot



Customize labels, colors, axis titles (always very important), etc. Can of course edit later on

The result!