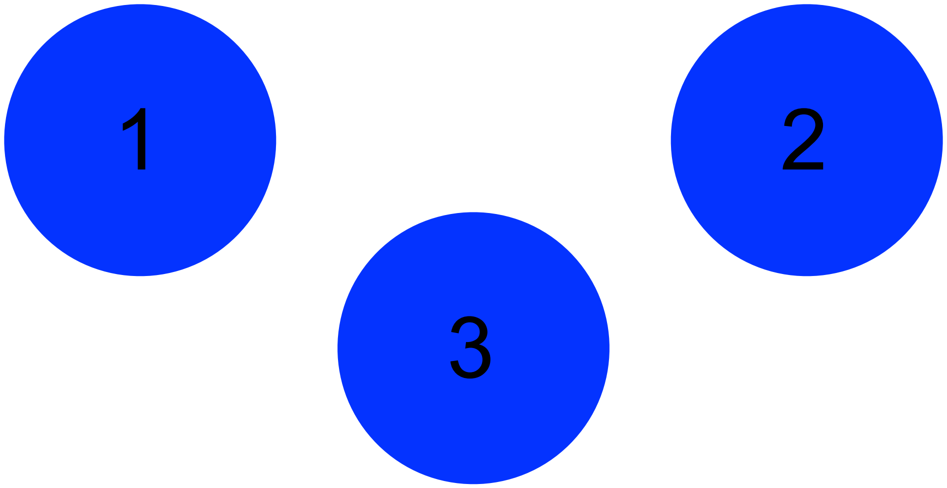
Back to our system of particles and momentum balance

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_1^{ext}$$

$$\mathbf{F}_2 = \mathbf{F}_{23} + \mathbf{F}_{21} + \mathbf{F}_2^{ext}$$

$$\mathbf{F}_3 = \mathbf{F}_{31} + \mathbf{F}_{32} + \mathbf{F}_3^{ext}$$



Found that change of total system momentum P influenced only be external forces

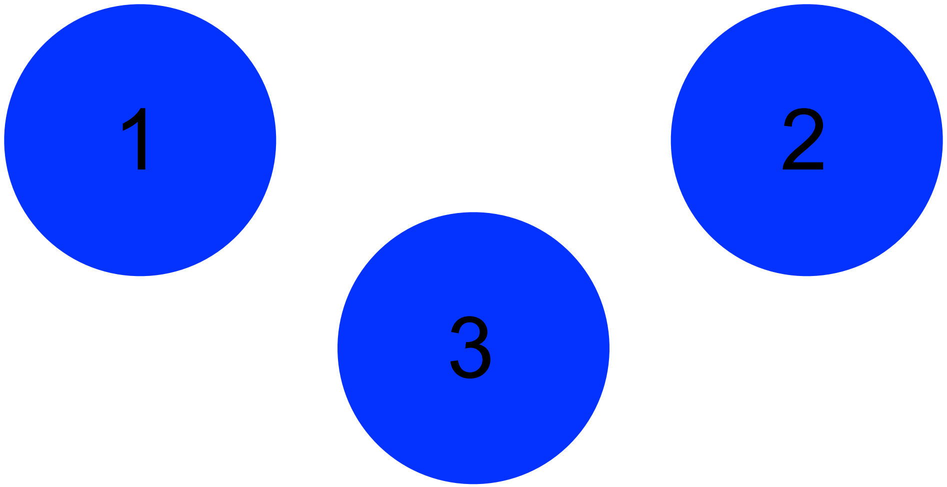
$$\dot{\mathbf{P}} = \mathbf{F}_1^{ext} + \mathbf{F}_2^{ext} + \mathbf{F}_3^{ext}$$

Back to our system of particles and momentum balance

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_1^{ext}$$

$$\mathbf{F}_2 = \mathbf{F}_{23} + \mathbf{F}_{21} + \mathbf{F}_2^{ext}$$

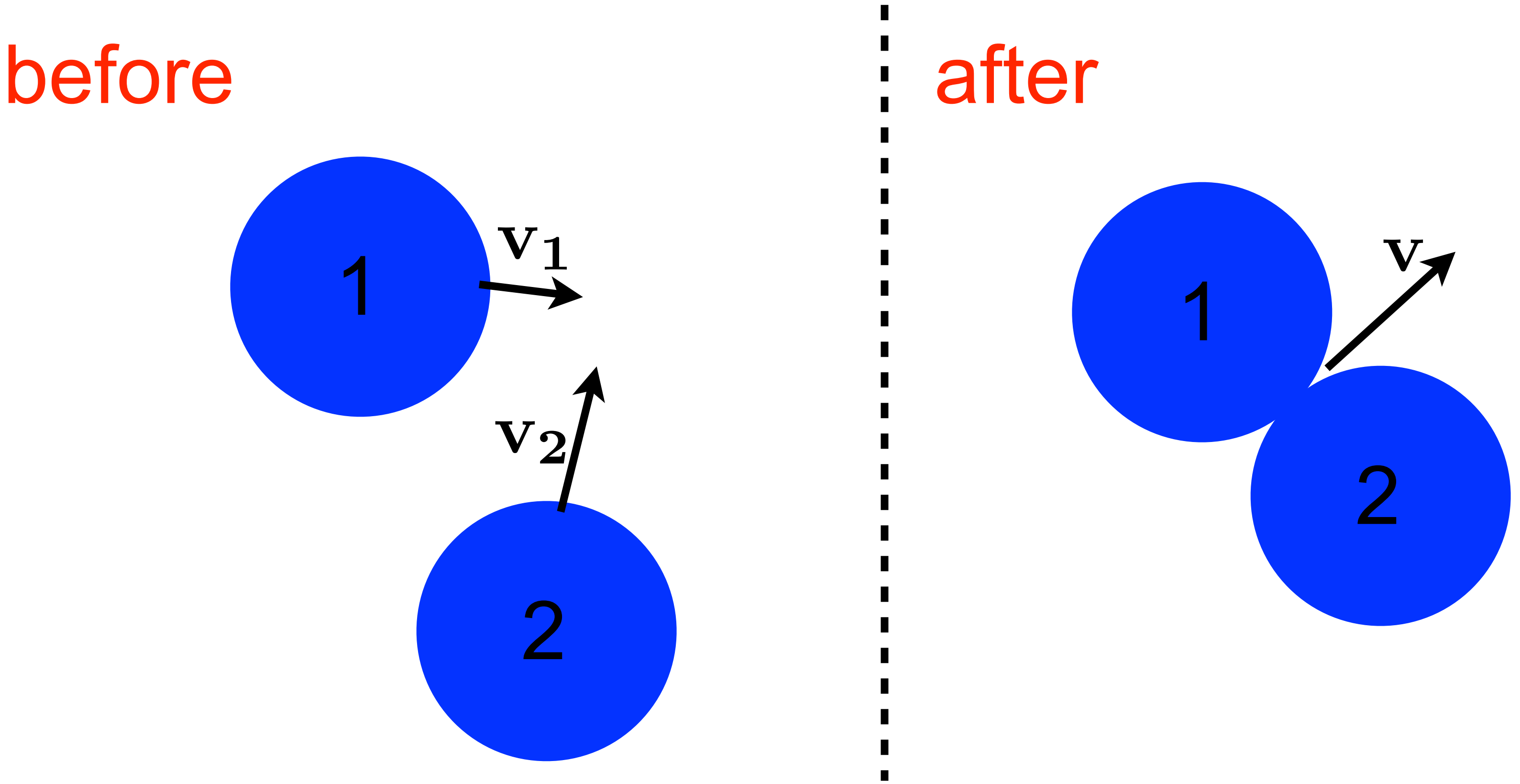
$$\mathbf{F}_3 = \mathbf{F}_{31} + \mathbf{F}_{32} + \mathbf{F}_3^{ext}$$



If no external forces, then momentum is conserved for the entire system

$$\dot{\mathbf{P}} = \mathbf{F}_1^{ext} + \mathbf{F}_2^{ext} + \mathbf{F}_3^{ext}$$

Back to our system of particles and momentum balance

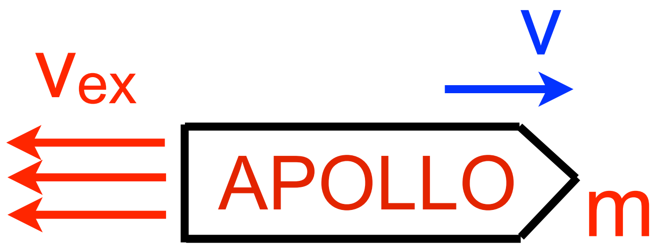


$$\mathbf{P}_{\text{init}} = m_1\mathbf{v_1} + m_2\mathbf{v_2}$$

$$\mathbf{P}_{\text{final}} = (m_1 + m_2)\mathbf{v}$$

$$\mathbf{P}_{\text{init}} = \mathbf{P}_{\text{final}} \rightarrow \mathbf{v} = \frac{m_1\mathbf{v_1} + m_2\mathbf{v_2}}{m_1 + m_2}$$

Rockets



$$P(t) = m(t)v(t)$$

$$P(t+dt) = m(t+dt)v(t+dt) - dm(v - v_{\text{ex}})$$

$$P(t+dt) = (m+dm)(v+dv) - dm(v - v_{\text{ex}})$$

$$P(t+dt) = mv + mdv + vdm + dm \cdot dv - dm(v - v_{\text{ex}})$$

$$P(t+dt) = mv + mdv + vdm - dm(v - v_{\text{ex}})$$

$$P(t+dt) = P(t)$$

$$mv = mv + mdv + vdm - dm(v - v_{\text{ex}})$$

For $F_{ext} = 0$

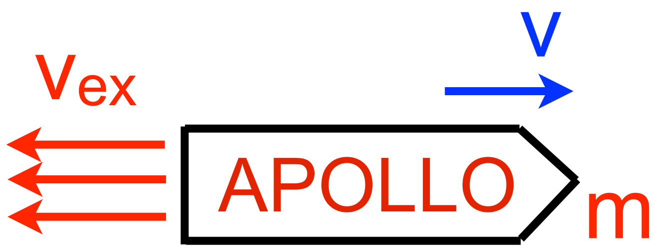
$$(v - v_{\text{ex}})dm = mdv + vdm$$

$$-v_{\text{ex}}dm = mdv$$

$$-v_{\text{ex}}dm/dt = mdv/dt$$

$$-v_{\text{ex}}\dot{m} = m\dot{v}$$

Rockets



Constant?

$$dv = -v_{\text{ex}} \frac{dm}{m}$$

$$\int_{v_0}^{v} dv' = -v_{\text{ex}} \int_{m_0}^{m} \frac{dm'}{m'}$$

$$v - v_0 = -v_{\text{ex}} \ln(m/m_0)$$

$$v - v_0 = v_{\text{ex}} \ln(m_0/m)$$

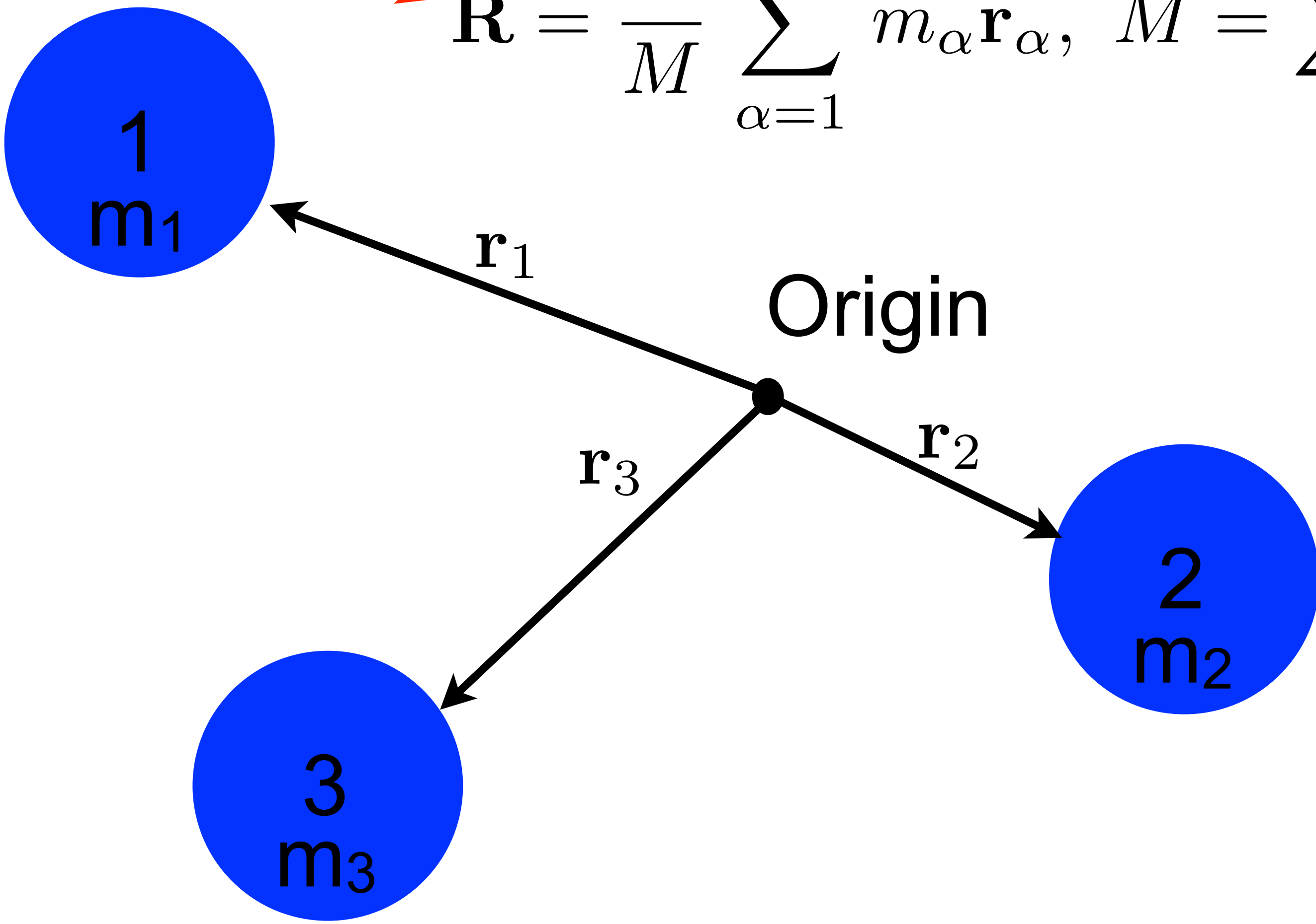
Some problems for class

For us to do together: 3.4, 3.11, 3.13 (should be fun and messy)

Center of mass

Center of Mass

$$\mathbf{R} = \frac{1}{M} \sum_{\alpha=1}^{\alpha=N} m_\alpha \mathbf{r}_\alpha, \quad M = \sum m_\alpha$$

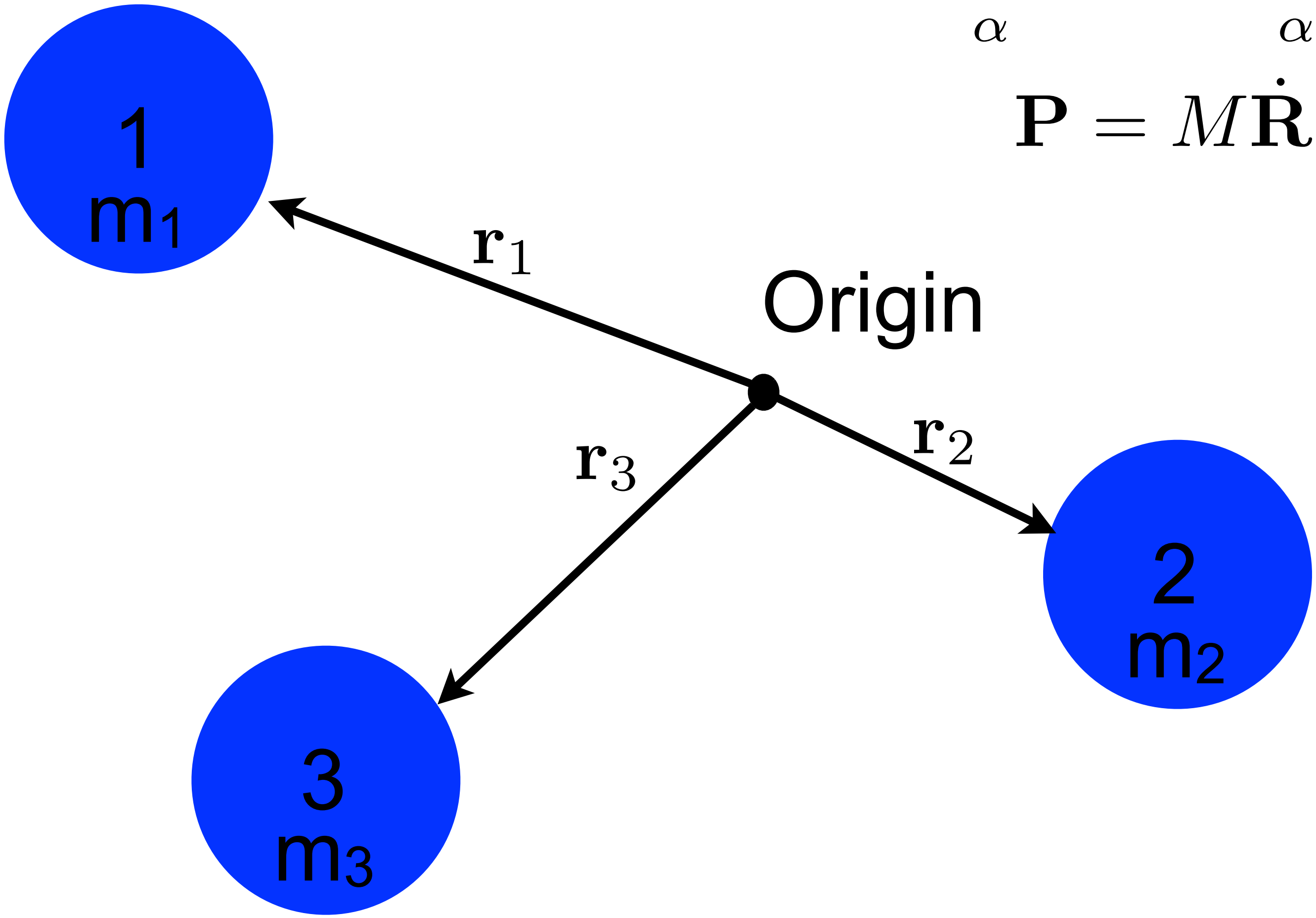


$$\mathbf{R} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3}{m_1 + m_2 + m_3}$$

Center of mass

$$\mathbf{P} = \sum_\alpha \mathbf{p}_\alpha = \sum_\alpha m_\alpha \dot{\mathbf{r}}_\alpha$$

$$\mathbf{P} = M\dot{\mathbf{R}}$$



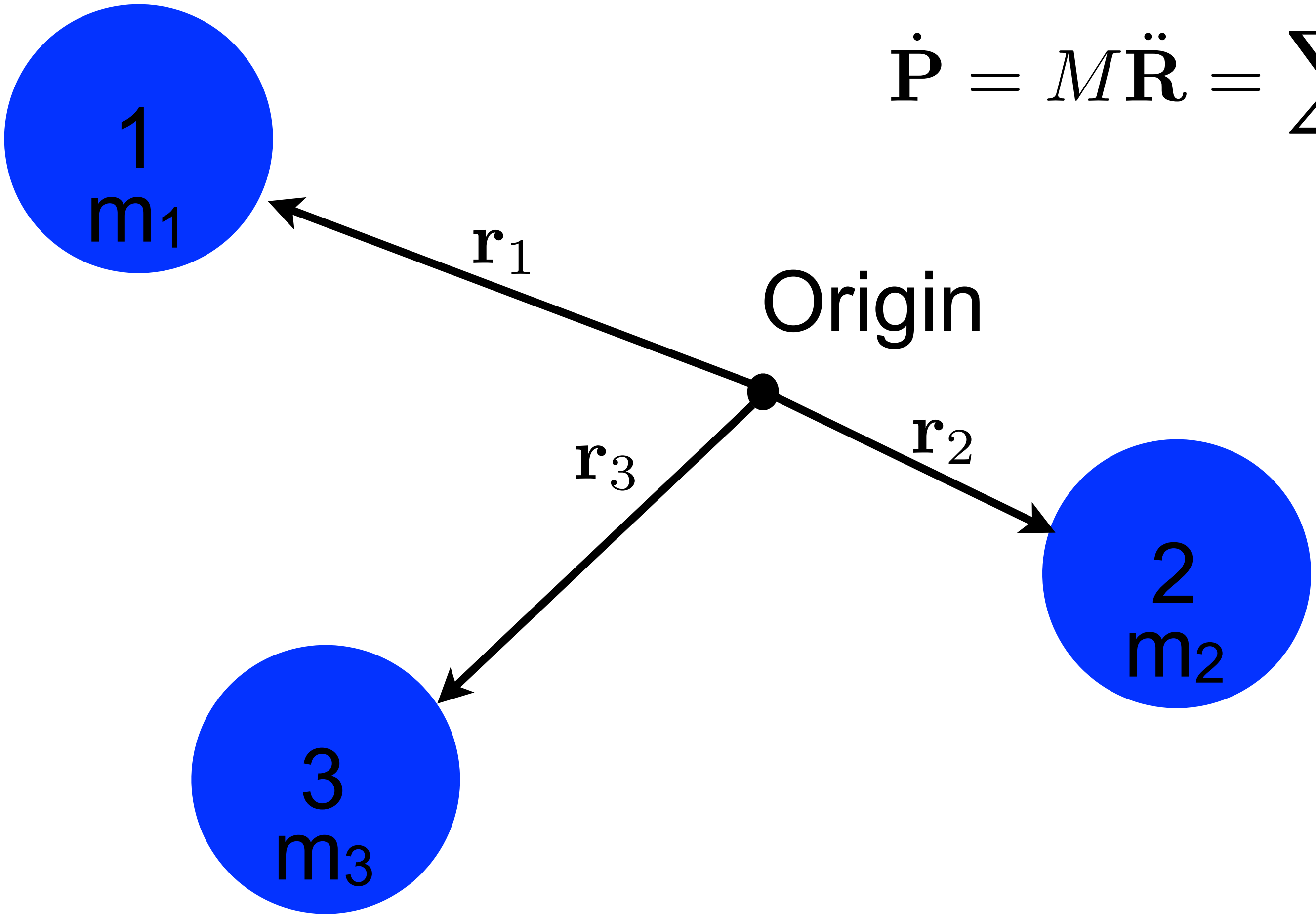
$$\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3$$

$$\mathbf{P} = m_1\dot{\mathbf{r}}_1 + m_2\dot{\mathbf{r}}_2 + m_3\dot{\mathbf{r}}_3 = (m_1 + m_2 + m_3)\dot{\mathbf{R}}$$

Center of mass

$$\mathbf{P} = M\dot{\mathbf{R}}$$

$$\dot{\mathbf{P}} = M\ddot{\mathbf{R}} = \sum \mathbf{F}^{ext}$$



$$\mathbf{P} = (m_1 + m_2 + m_3)\dot{\mathbf{R}}$$

$$\dot{\mathbf{P}} = (m_1 + m_2 + m_3)\ddot{\mathbf{R}} = \mathbf{F}_1^{ext} + \mathbf{F}_2^{ext} + \mathbf{F}_3^{ext}$$

Center of mass

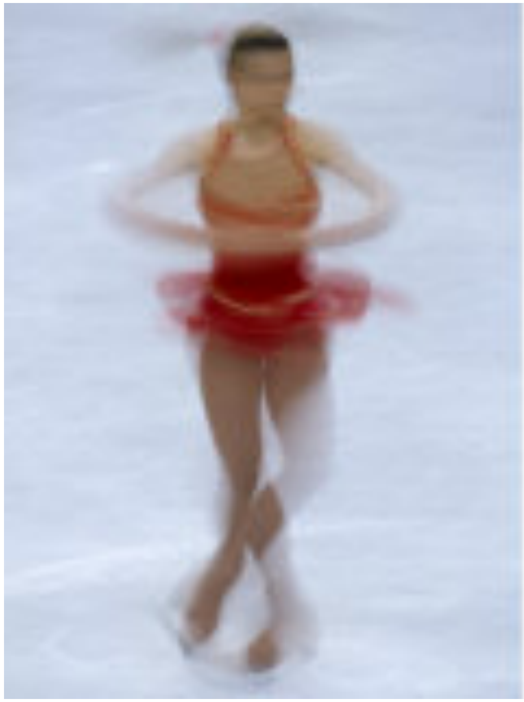
$$\mathbf{P} = M\dot{\mathbf{R}}$$

$$\dot{\mathbf{P}} = M\ddot{\mathbf{R}} = \sum \mathbf{F}^{ext}$$

Treat the center of mass as if it was a single particle!

Makes life a lot easier (this truck, stuck on a ramp, is made up of a lot of particles/objects)

Back to conservation of momentum



We looked at linear momentum, what about angular momentum?

Angular momentum

Angular momentum (a vector)

$$\ell = \mathbf{r} \times \mathbf{p}$$

$$\dot{\ell} = \frac{d}{dt}(\mathbf{r} \times \mathbf{p}) = (\dot{\mathbf{r}} \times \mathbf{p}) + (\mathbf{r} \times \dot{\mathbf{p}})$$

$$\dot{\ell} = [\dot{\mathbf{r}} \times (m\dot{\mathbf{r}}) + (\mathbf{r} \times \dot{\mathbf{p}})]$$

$$\dot{\ell} = [m(\dot{\mathbf{r}} \times \dot{\mathbf{r}}) + (\mathbf{r} \times \mathbf{F})]$$

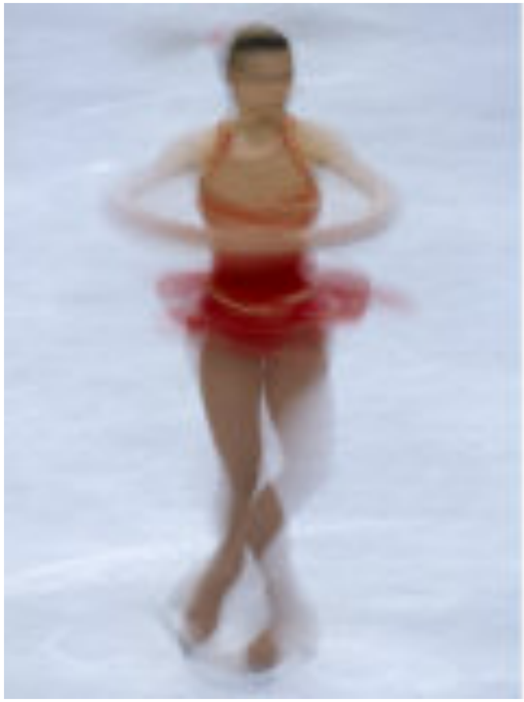
$$\dot{\ell} = (\mathbf{r} \times \mathbf{F})$$

Any vector cross itself is zero

$$\dot{\ell} = (\mathbf{r} \times \mathbf{F}) = \mathbf{\Gamma}$$

Torque (also a vector)

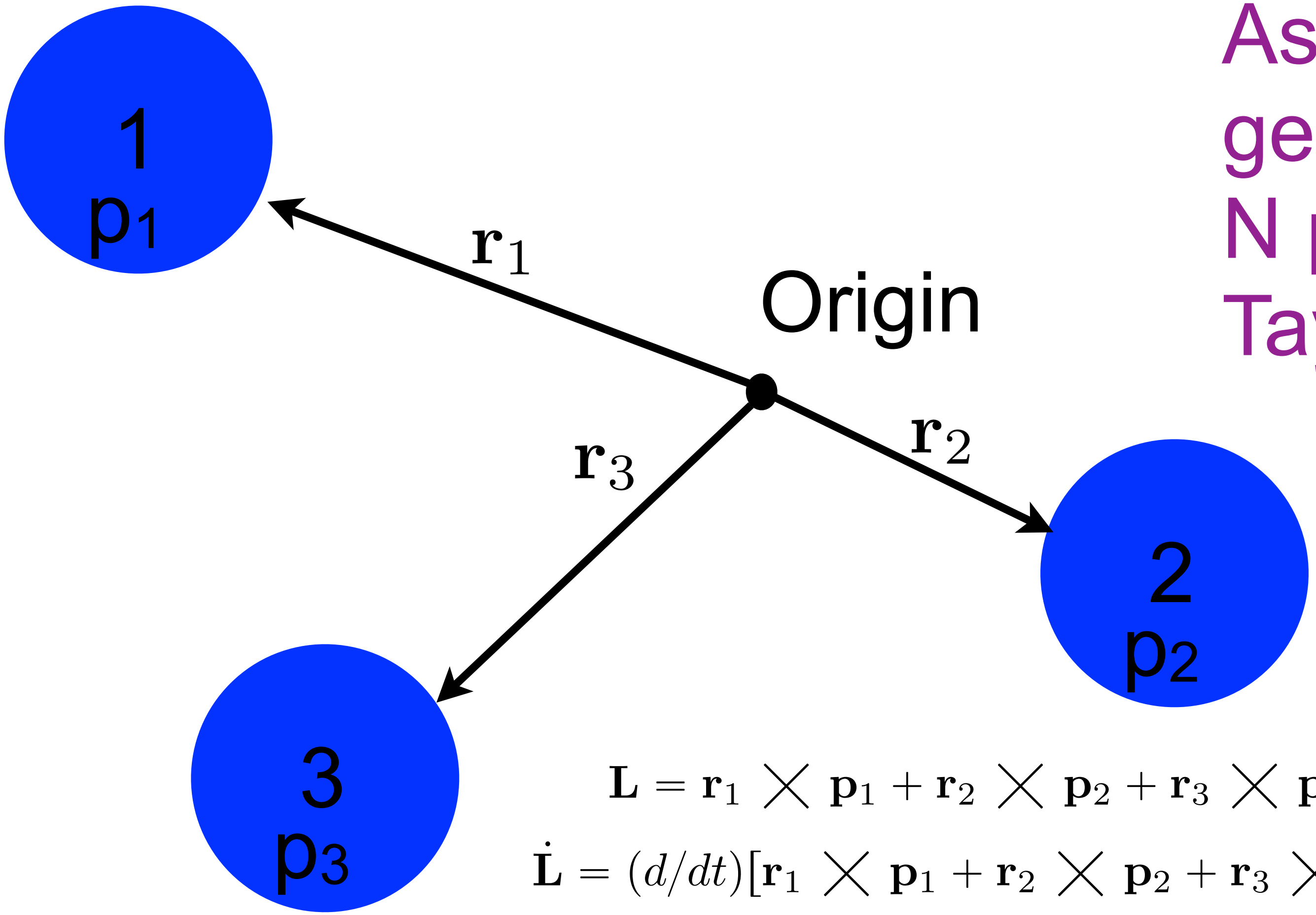
On angular momentum



Since position depends on choice of origin, so does angular momentum! Similarly, so does the torque

$$\dot{\ell} = (\mathbf{r} \times \mathbf{F}) = \mathbf{\Gamma}$$

What about a system of particles?



As before, see generalization to N particles in Taylor

$$\mathbf{L} = \mathbf{r}_1 \times \mathbf{p}_1 + \mathbf{r}_2 \times \mathbf{p}_2 + \mathbf{r}_3 \times \mathbf{p}_3$$

$$\dot{\mathbf{L}} = (d/dt)[\mathbf{r}_1 \times \mathbf{p}_1 + \mathbf{r}_2 \times \mathbf{p}_2 + \mathbf{r}_3 \times \mathbf{p}_3]$$

$$\dot{\mathbf{L}} = [\dot{\mathbf{r}}_1 \times \mathbf{p}_1 + \dot{\mathbf{r}}_2 \times \mathbf{p}_2 + \dot{\mathbf{r}}_3 \times \mathbf{p}_3 + \mathbf{r}_1 \times \dot{\mathbf{p}}_1 + \mathbf{r}_2 \times \dot{\mathbf{p}}_2 + \mathbf{r}_3 \times \dot{\mathbf{p}}_3]$$

$$\dot{\mathbf{L}} = m_1\dot{\mathbf{r}}_1 \times \dot{\mathbf{r}}_1 + m_2\dot{\mathbf{r}}_2 \times \dot{\mathbf{r}}_2 + m_3\dot{\mathbf{r}}_3 \times \dot{\mathbf{r}}_3 + \mathbf{r}_1 \times \dot{\mathbf{p}}_1 + \mathbf{r}_2 \times \dot{\mathbf{p}}_2 + \mathbf{r}_3 \times \dot{\mathbf{p}}_3$$

$$\dot{\mathbf{L}} = \mathbf{r}_1 \times \dot{\mathbf{p}}_1 + \mathbf{r}_2 \times \dot{\mathbf{p}}_2 + \mathbf{r}_3 \times \dot{\mathbf{p}}_3$$

$$\dot{\mathbf{L}} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 + \mathbf{r}_3 \times \mathbf{F}_3$$

What about a system of particles?

$$\dot{\mathbf{L}} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 + \mathbf{r}_3 \times \mathbf{F}_3$$

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_1^{ext}$$

$$\mathbf{F}_2 = \mathbf{F}_{23} + \mathbf{F}_{21} + \mathbf{F}_2^{ext}$$

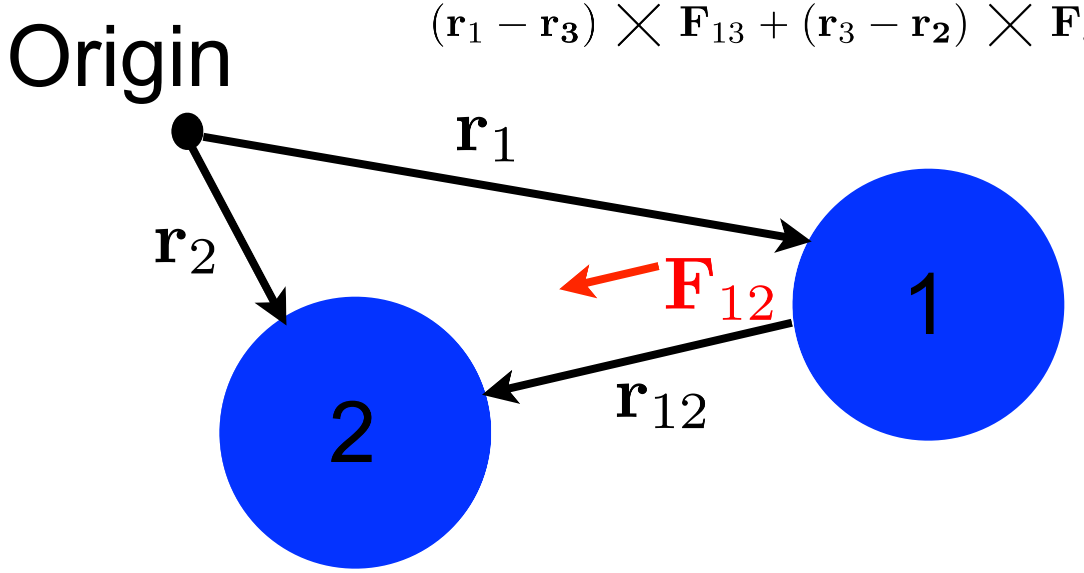
$$\mathbf{F}_3 = \mathbf{F}_{31} + \mathbf{F}_{32} + \mathbf{F}_3^{ext}$$

$$\dot{\mathbf{L}} = \mathbf{r}_1 \times (\mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_1^{ext}) + \mathbf{r}_2 \times (\mathbf{F}_{23} + \mathbf{F}_{21} + \mathbf{F}_2^{ext}) + \mathbf{r}_3 \times (\mathbf{F}_{31} + \mathbf{F}_{32} + \mathbf{F}_3^{ext})$$

Remember that $F_{ij} = -F_{ji}$

$$\dot{\mathbf{L}} = \mathbf{r}_1 \times \mathbf{F}_1^{ext} + \mathbf{r}_2 \times \mathbf{F}_2^{ext} + \mathbf{r}_3 \times \mathbf{F}_3^{ext} +$$
$$\mathbf{r}_1 \times \mathbf{F}_{13} - \mathbf{r}_3 \times \mathbf{F}_{13} + \mathbf{r}_3 \times \mathbf{F}_{32} - \mathbf{r}_2 \times \mathbf{F}_{32} + \mathbf{r}_2 \times \mathbf{F}_{21} - \mathbf{r}_1 \times \mathbf{F}_{21}$$

$$\dot{\mathbf{L}} = \mathbf{r}_1 \times \mathbf{F}_1^{ext} + \mathbf{r}_2 \times \mathbf{F}_2^{ext} + \mathbf{r}_3 \times \mathbf{F}_3^{ext} +$$
$$(\mathbf{r}_1 - \mathbf{r}_3) \times \mathbf{F}_{13} + (\mathbf{r}_3 - \mathbf{r}_2) \times \mathbf{F}_{32} + (\mathbf{r}_2 - \mathbf{r}_1) \times \mathbf{F}_{21}$$



If internal forces are along vector connecting particles, we call them **central forces**

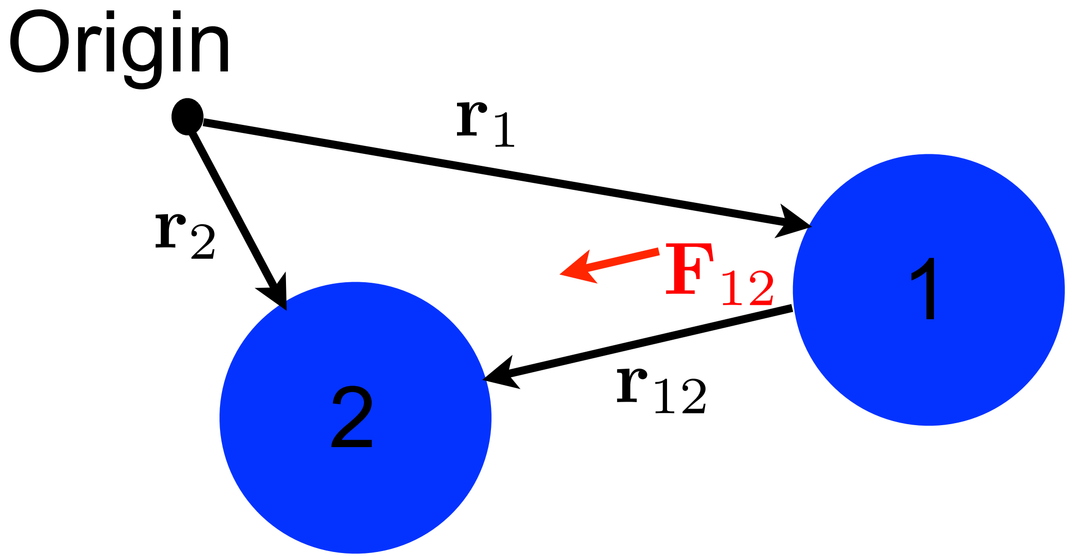
What about a system of particles?

$$\dot{\mathbf{L}} = \mathbf{r}_1 \times \mathbf{F}_1^{ext} + \mathbf{r}_2 \times \mathbf{F}_2^{ext} + \mathbf{r}_3 \times \mathbf{F}_3^{ext} +$$

$$(\mathbf{r}_1 - \mathbf{r_3}) \times \mathbf{F}_{13} + (\mathbf{r}_3 - \mathbf{r_2}) \times \mathbf{F}_{32} + (\mathbf{r}_2 - \mathbf{r_1}) \times \mathbf{F}_{21}$$

For central forces...

$$\dot{\mathbf{L}} = \mathbf{r}_1 \times \mathbf{F}_1^{ext} + \mathbf{r}_2 \times \mathbf{F}_2^{ext} + \mathbf{r}_3 \times \mathbf{F}_3^{ext} = \mathbf{\Gamma}^{\mathbf{ext}}$$



If internal forces are along vector connecting particles, we call them **central forces**

Reminder of momentum of inertia

I = moment of inertia, defined about a rotation axis

We will hopefully get back to it later in the course, but a reminder...

$$I = \sum_\alpha m_\alpha \rho_\alpha^2 = \int D dV \rho^2$$

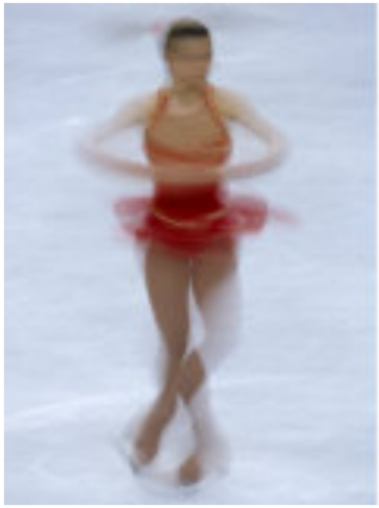
ρ is distance from mass to axis of rotation

D is density of object

Reminder of momentum of inertia

About z axis = axis of rotation → $$L_z = I\omega$$

Can simplify life quite a bit!
Let's go over Example 3.3 in textbook together
Then 3.35 to be done in class together



Do we remember how this explains the figure skater?

Homework (due in 1 week)

Taylor 3.2, 3.3, 3.8, 3.15, 3.16, 3.17, 3.29