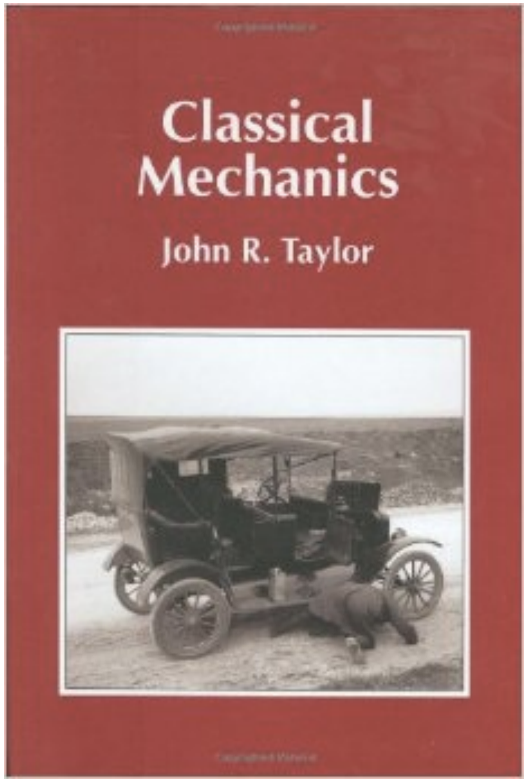
Welcome!

Physics 300: Analytical Mechanics 1

Some practical information

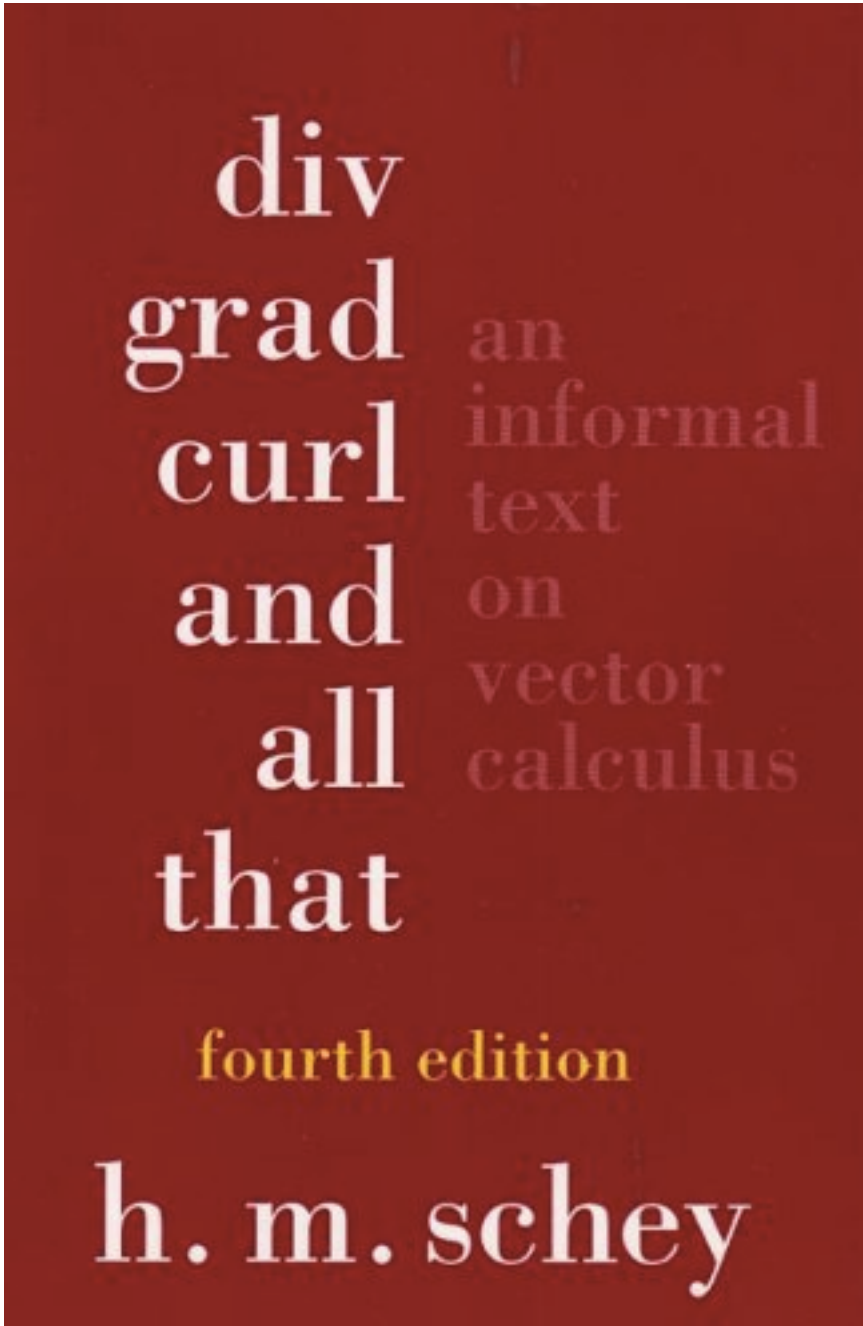
- Classes MWF 2-250 pm Faraday 237
- Taylor's Classical Mechanics is the required textbook
  - Expect a basic knowledge of Newton's laws, coordinate systems, concepts like conservation of energy (will touch on all these subjects)
  - Odd vs even problems
    - Odd (mostly) have answers in back of book - use this!
  - Plan on following the book quite closely
- Useful math formulas: book cover



front and back!

Some practical information

- For those who are not fully comfortable with vector calculus, highly, highly, highly recommend "Div, Grad, Curl, and All That"
  - A great reference for E&M, too

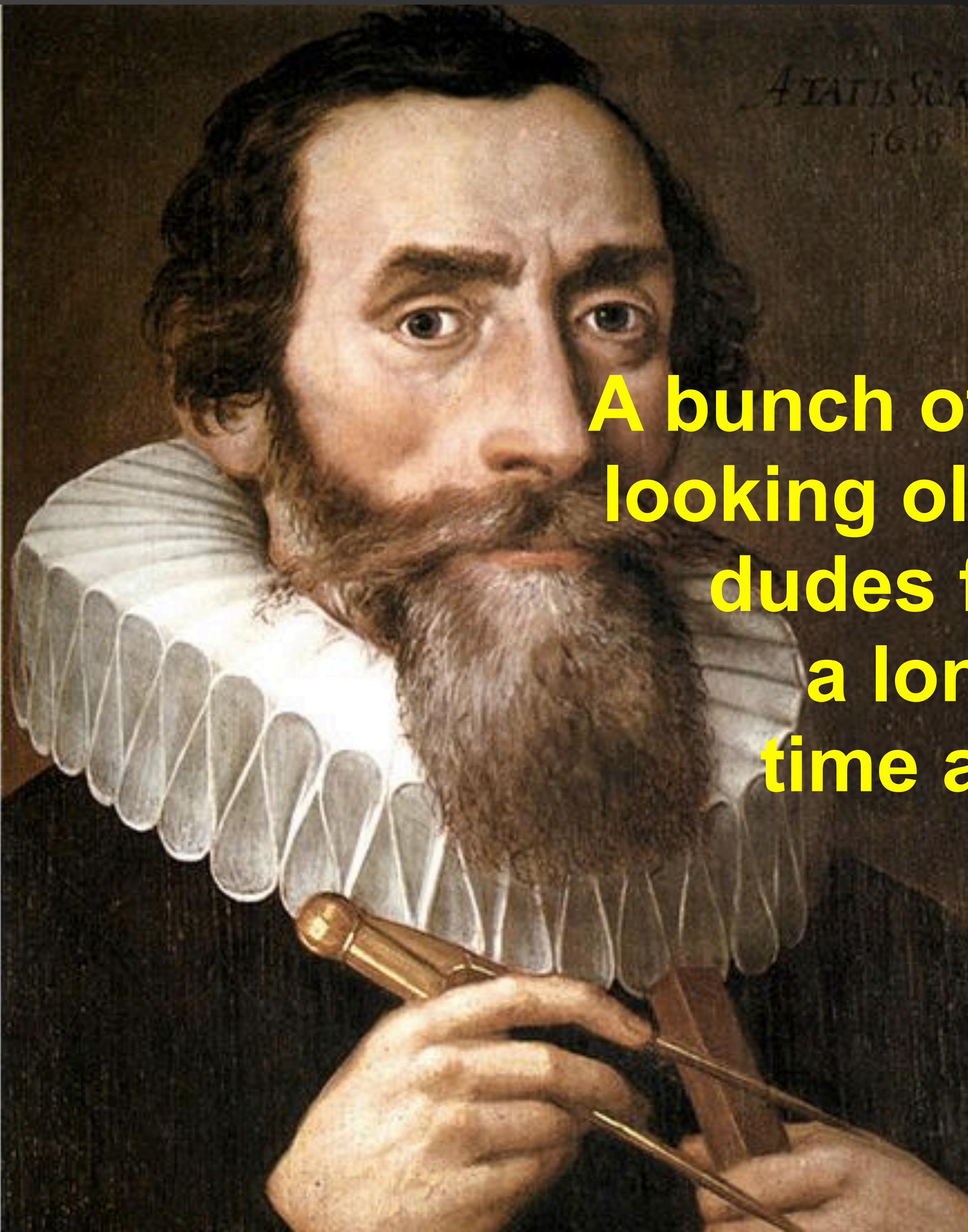


What we will cover (following Taylor)

1. Coordinate systems and Newton's laws
2. Projectiles and air resistance/drag, and charged particle motion
3. Momentum and angular momentum
4. Energy and conservative forces
5. Oscillations and Fourier analysis
6. Calculus of variations
7. Lagrange's Equations
8. Central-force problems
9. Mechanics in non-inertial frames
10. Rotational motion of rigid bodies (if there is time)

With 1 problem set per topic above

Why are we studying this?



A bunch of stuffy-looking old white dudes from a long time ago

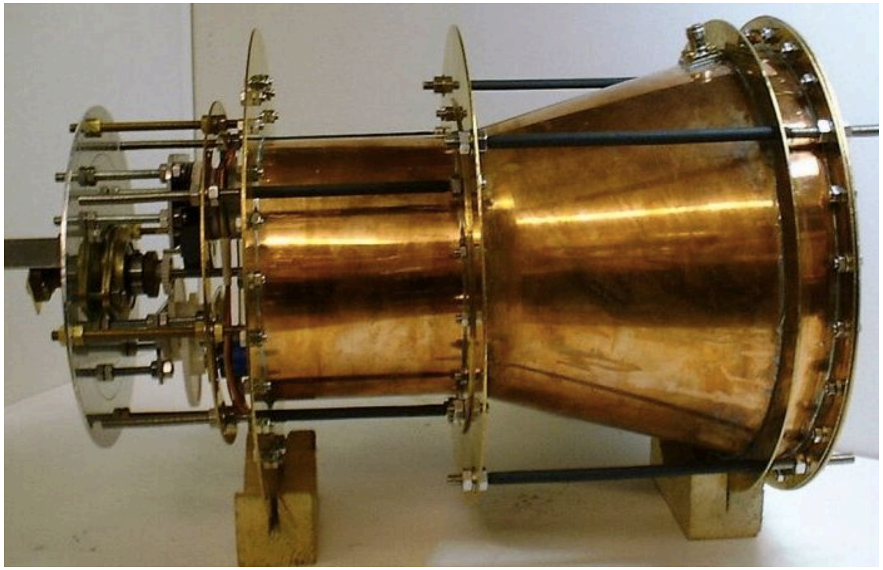
Why are we studying this?

- But... classical mechanics underlies all the newer, more modern physics
- The class will teach you key tools necessary for advanced E&M, quantum mechanics, relativity, ...
- The material here also covers the more relevant physics for our every-day lives
  - We don’t have much daily, direct interaction with the quantum world or speeds where special relativity is important

And hopefully the material can still be interesting

You have 2 minutes - with your neighbor (if you want), write down the most interesting reasons/subjects to study classical mechanics

New EMDrive for space propulsion - many scientists (myself included) are very skeptical of this “technology”



Sports

www.symmetrymagazine.org/article/august-2014/lhc-physicist-takes-on-new-type-of-collisions

Courtesy of EA SPORTS

signal to background

August 21, 2014

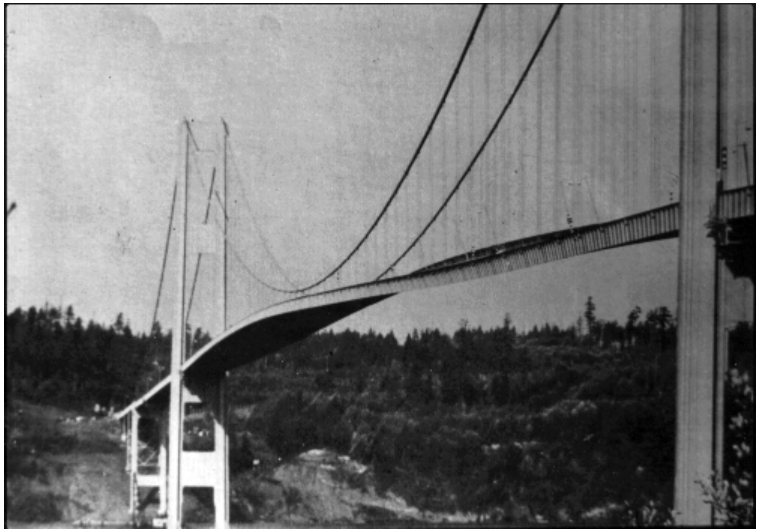
LHC physicist takes on new type of collisions

A former Large Hadron Collider researcher brings his knowledge of high-energy collisions to a new EA SPORTS NHL hockey game.

http://www.symmetrymagazine.org/article/august-2014/lhc-physicist-takes-on-new-type-of-collisions

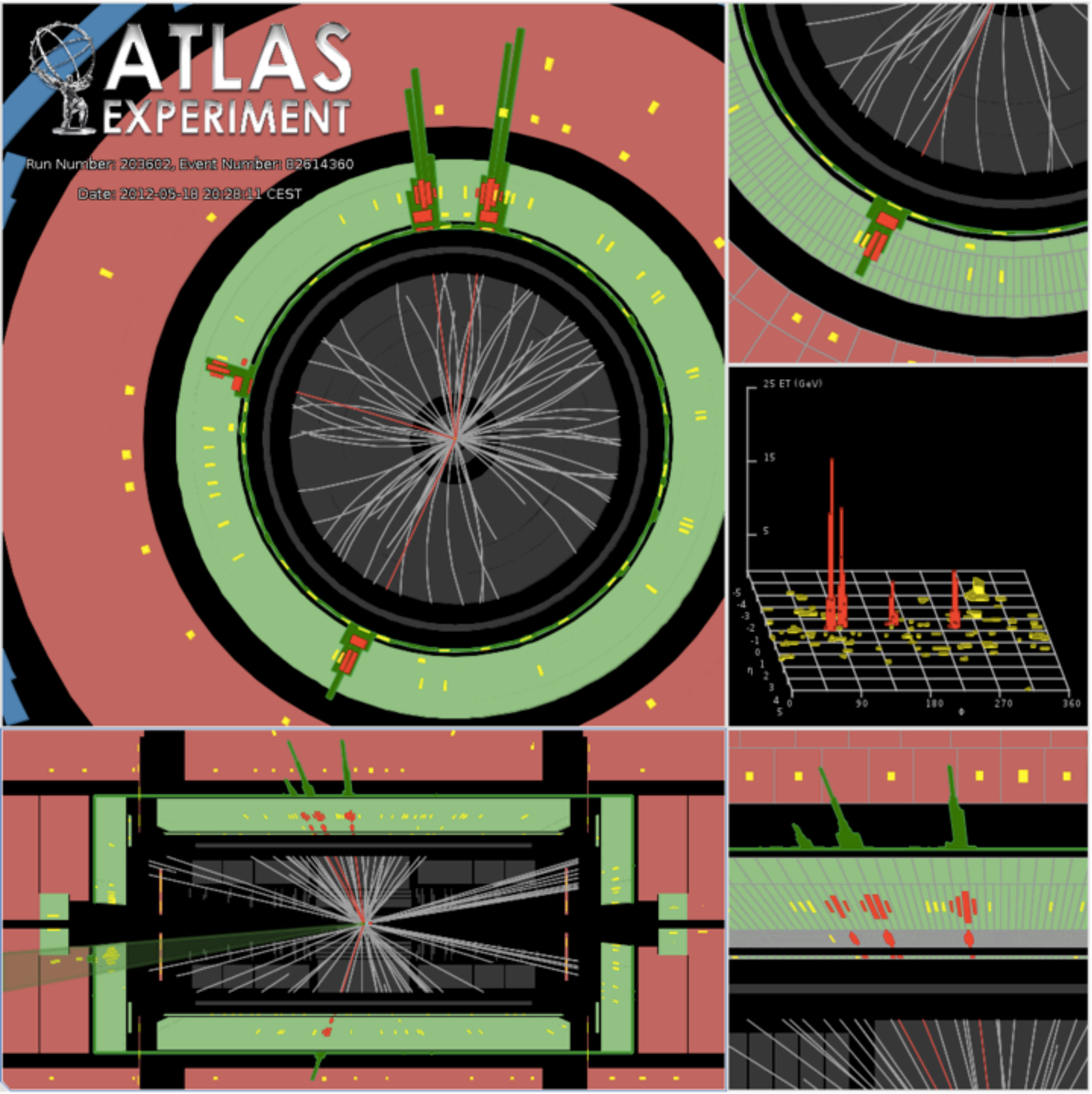
Sports

An unfortunate example for the engineers



http://upload.wikimedia.org/wikipedia/commons/1/19/Tacoma_Narrows_Bridge_destruction.ogg

And my favorite



Candidate Higgs boson event decaying to ZZ* decaying to 4 electrons

Grading

- Problem sets every 1-2 weeks, each with the same weight: combined total, 50% of grade
  - All to be due 1 week, in class, after assignment
  - To be distributed after we finish a chapter/topic
  - The HW will include computer assignments
  - Start the HW early! If you get stuck and need help, please come by during office hours
  - Please ask for help if you don't understand solutions after they are posted (we'll briefly go over them in class, but not over everything)
- Midterm: 25% of grade
  - Date TBD, but at halfway point in course. Will be **A TAKE-HOME MIDTERM (DUE 1 WEEK LATER)**
- Final: 25% of grade at nominal time (Dec 8 2-350 pm)
- Late HW/exams NOT accepted without valid Doc note or excuse

Grading

- After weighting problem sets, midterms and final as on last slide, the grades will be:
  - A: 85-100%
  - A-: 78-85%
  - B+: 70-77%
  - B: 62-69%
  - B-: 56-61%
  - C+: 48-55%
  - C: 40-47%
  - D: 25-39%
  - F: 25% or less

On homework

- I don’t want to keep you from working with others, but any work that you hand in must be your own
  - Solutions found on the web are a form of plagiarism
  - “Can I copy your solutions” are also plagiarism
  - And “Tell me the Answer” is academically dishonest, as well
    - You will anyway not get credit for answers without showing your work
  - I do want you to help your classmates, however... and don’t forget that office hours are there for those who need help

- Remember to use units (and label graph axes) where appropriate or you will not receive full credit

Office hours

- Office hours (Faraday 220): Monday and Wednesday 3-4 pm (right after class) or by appointment
  - I may spend significant time at ANL and also traveling, so if you want to meet at any time other than during the set office hours please e-mail me (jahred.adelman@niu.edu) to set up an appointment
    - You can always try and stop by, but you will have better luck if you set up an appointment
    - Can also try the phone (753-6468) but email is preferred

The class

- Roughly once a week, I will try and post previous slides on the class website for you
  - http://nicadd.niu.edu/~jahreda/phys300.html
  - Should not be considered a substitute for note-taking, but can hopefully help you in preparations for exams and homework
- Hope to split you into groups of a few people for brief (5-10 minutes, at most!) exercises somewhat regularly
  - To help you feel comfortable with material
- We will also work out some problems together in presentations here or on blackboard
  - So you can see that I can also get stuck :)

Attendance

- Please come to class (shouldn't need to ask this of you, but I state it anyway)
  - You can't hand in homework without being in here
  - The small group and other problems that we go over will be important to follow and understand
  - I am not taking attendance - but if you don't come to class, you will not do well in this course

The classroom

- Please avoid food in the classroom
  - Bottles and cans of liquid are OK (no straws!) so that we can all stay hydrated, but is otherwise disruptive to me and to others. And the class is only 1 hour long
  - Talk to me privately if this is a problem
- Cell phones need to stay in your pocket and be turned off
  - If your phone rings, we will know it was you (this class is that small). This is unfair to your fellow students
  - If you are using your phone in class, I will see you and call you out on it

About me ... and you

- For those who do not know me, I am new to the department, to the campus, to teaching at this level
- I'm a particle physicist working on searches for new physics with Higgs bosons using the ATLAS experiment at the LHC (at CERN)
  - Ask me after class or during office hours about my research. I like to talk about it :)

About me ... and you

- I'll try to update my teaching style as the semester goes on, based on my experience, observations and your feedback
  - If I am going too fast... or too slow, or if my style (or hand-writing) is incomprehensible, please speak up

And now it's your turn

- We are small enough to go around the room
  - What's your name?
  - What's your major?
  - Why are you in this class?
  - What do you like (and dislike) about physics?
  - What do you like (and dislike) about classical mechanics?

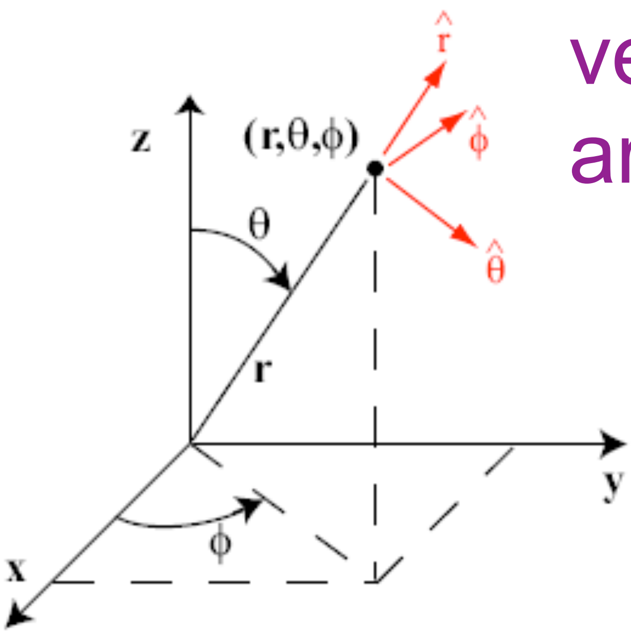
Before we get to physics...

Any questions?

Before you do anything else in classical mechanics...

SET UP A COORDINATE SYSTEM (also known as the boring stuff to get through before the interesting physics)

Ways to define our position in space



Will try and use bold-face for vectors, though sometimes arrow notation instead

Unit vectors (hats)

$$\mathbf{r} = \vec{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$$

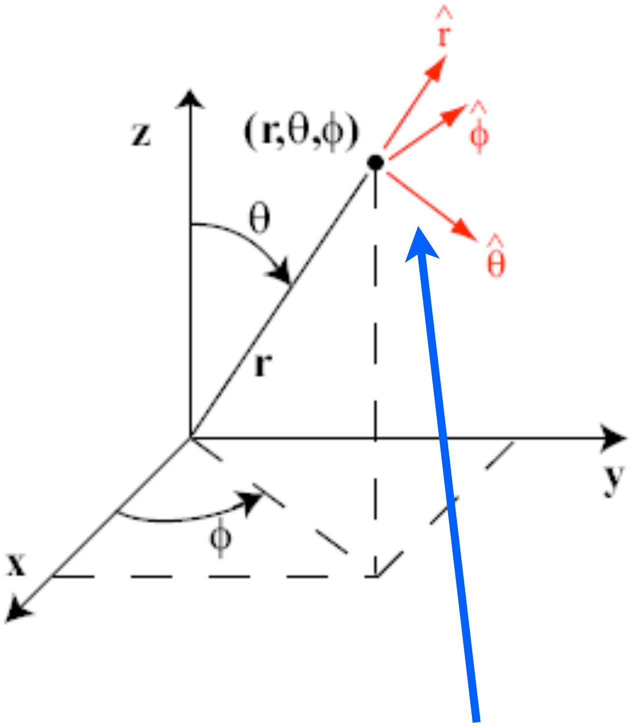
$$\mathbf{r} = (x, y, z)$$

$$\mathbf{r} = r_1\mathbf{e_1} + r_2\mathbf{e_2} + r_3\mathbf{e_3}$$

$$\mathbf{r} = \sum_{i=1}^{3} r_i\mathbf{e_i}$$

Cartesian coordinates are what we are most familiar with, but not always the easiest or best choice

Ways to define our position in space



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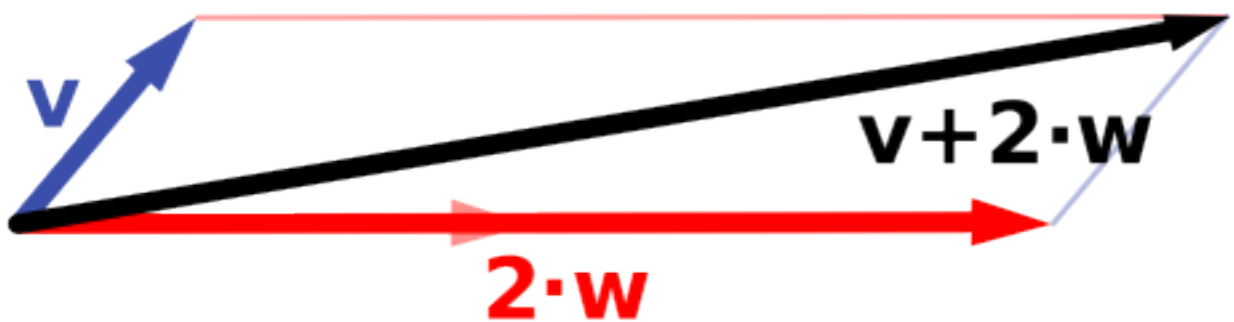
$$\mathbf{r} = r_1\mathbf{e_1} + r_2\mathbf{e_2} + r_3\mathbf{e_3}$$

$$\mathbf{r} = \sum_{i=1}^{3} r_i\mathbf{e_i}$$

Our unit vectors do not have to be constant (they can be functions of time and/or position!)

Playing with vectors

$$\mathbf{r} = (r_1, r_2, r_3), \mathbf{s} = (s_1, s_2, s_3)$$
$$\mathbf{p} = \mathbf{r} + \mathbf{s} = (p_1, p_2, p_3) = (r_1 + s_1, r_2 + s_2, r_3 + s_3)$$
$$\mathbf{s} = c\mathbf{p} = (c \cdot p_1, c \cdot p_2, c \cdot p_3)$$



Some vector operations

$$\mathbf{r} = (r_1, r_2, r_3), \mathbf{s} = (s_1, s_2, s_3)$$

$$\mathbf{r} \cdot \mathbf{s} = rs\cos\theta = r_1 s_1 + r_2 s_2 + r_3 s_3$$

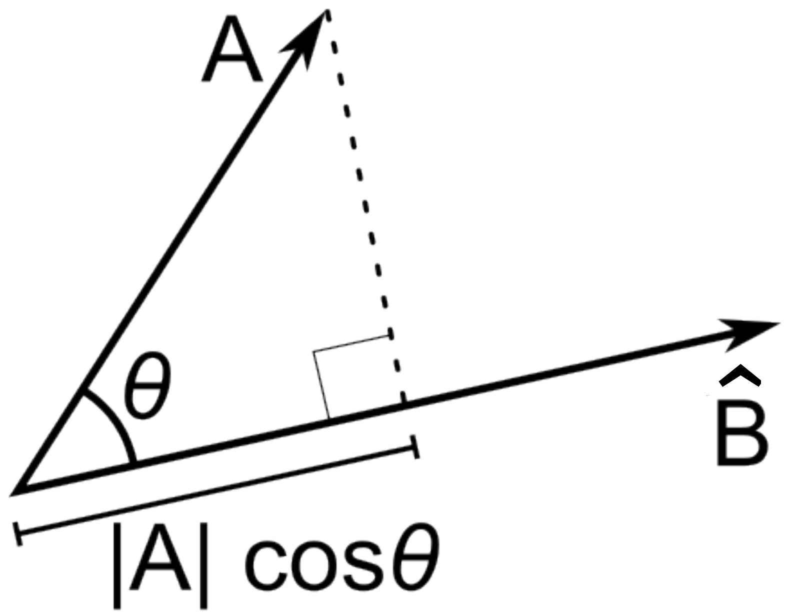
$$\mathbf{r} \cdot \mathbf{s} = \sum_{i=1}^{3} r_i s_i$$

$$r = |\mathbf{r}| = \sqrt{\mathbf{r} \cdot \mathbf{r}}$$

when $\theta = 0, \cos\theta = 1 \rightarrow \mathbf{r} \cdot \mathbf{s} = rs$ when vectors parallel

when $\theta = \pi/2, \cos\theta = 0 \rightarrow \mathbf{r} \cdot \mathbf{s} = 0$ when vectors perpendicular

Another way to think of the dot product



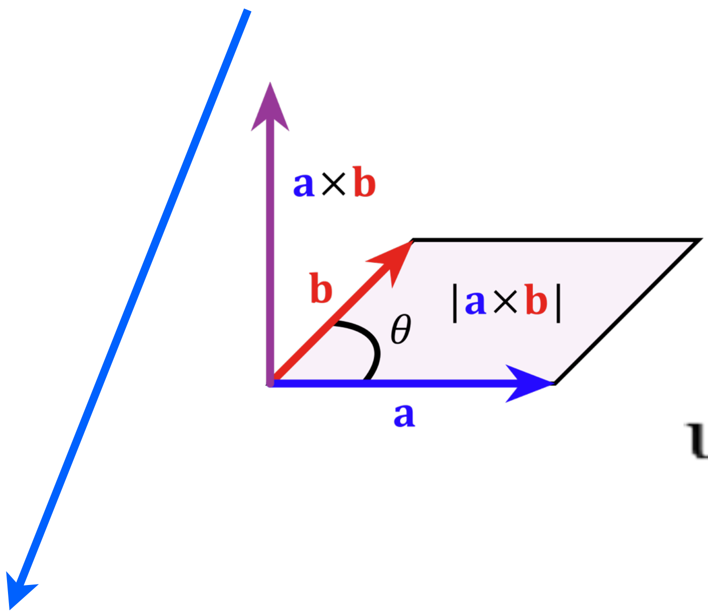
If B has unit length, A·B = |A| cosθ is the length of A when projected onto the axis given by B

More vector operations (cross product)

$$\mathbf{r} = (r_x, r_y, r_z), \mathbf{s} = (s_x, s_y, s_z)$$

$$\mathbf{p} = \mathbf{r} \times \mathbf{s} = (r_y s_z - r_z s_y, r_z s_x - r_x s_z, r_x s_y - r_y s_x)$$

$$|\mathbf{p}| = |\mathbf{r} \times \mathbf{s}| = |\mathbf{r}||\mathbf{s}| \sin\theta$$



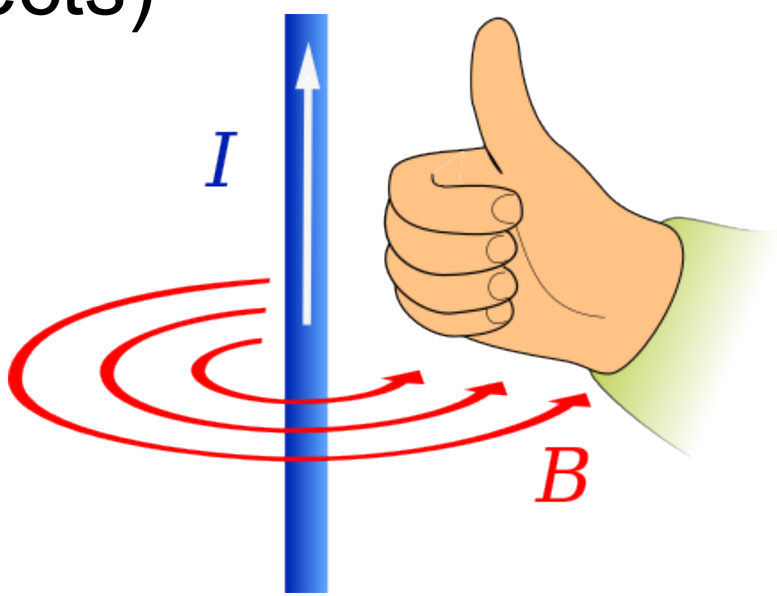
$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

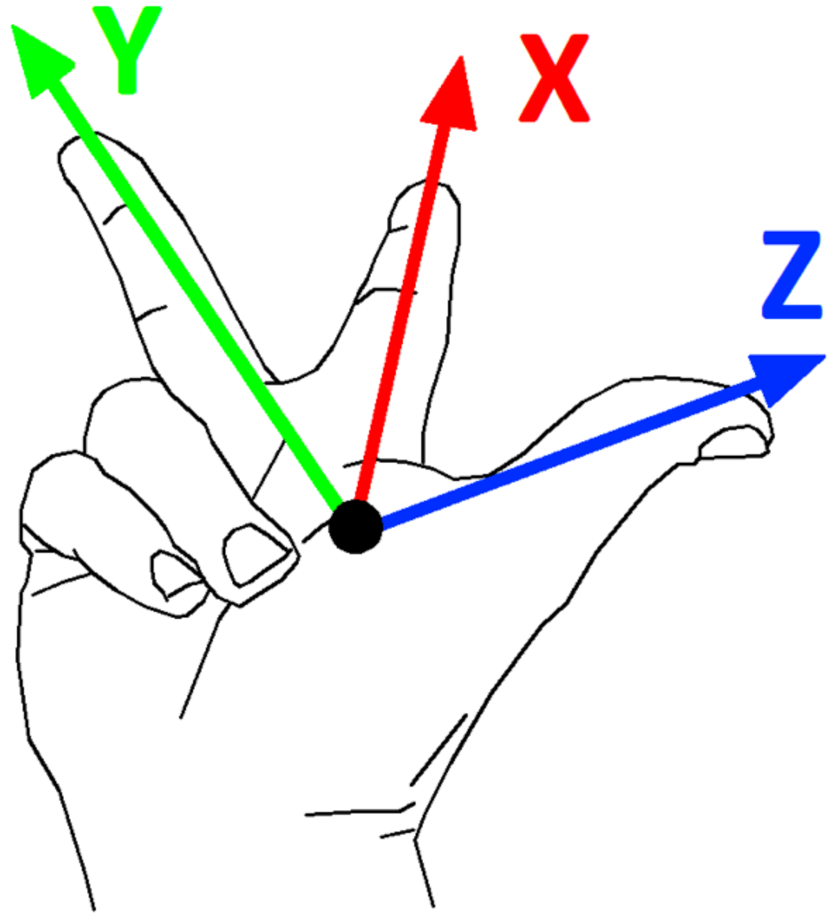
$$\text{when } \theta = 0, \sin\theta = 0 \rightarrow |\mathbf{r} \times \mathbf{s}| = 0 \text{ when vectors parallel}$$

$$\text{when } \theta = \pi/2, \sin\theta = 1 \rightarrow \mathbf{r} \times \mathbf{s} = |\mathbf{r}||\mathbf{s}| \text{ when vectors perpendicular}$$

Don't forget the right-hand rule

Find B field
given current (we'll have something similar for torque and rotating objects)





$Z = X \times Y$

Calculus with vectors?

$$\mathbf{r} = (r_1, r_2, r_3)$$

$$\frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \left(\frac{dr_1}{dt}, \frac{dr_2}{dt}, \frac{dr_3}{dt}\right)$$

$$\frac{d}{dt}(\mathbf{r} + \mathbf{s}) = \dot{\mathbf{r}} + \dot{\mathbf{s}}$$

$$\frac{d}{dt}(f\mathbf{r}) = \dot{f}\mathbf{r} + f\dot{\mathbf{r}}$$

Remember, base vectors can be functions of time

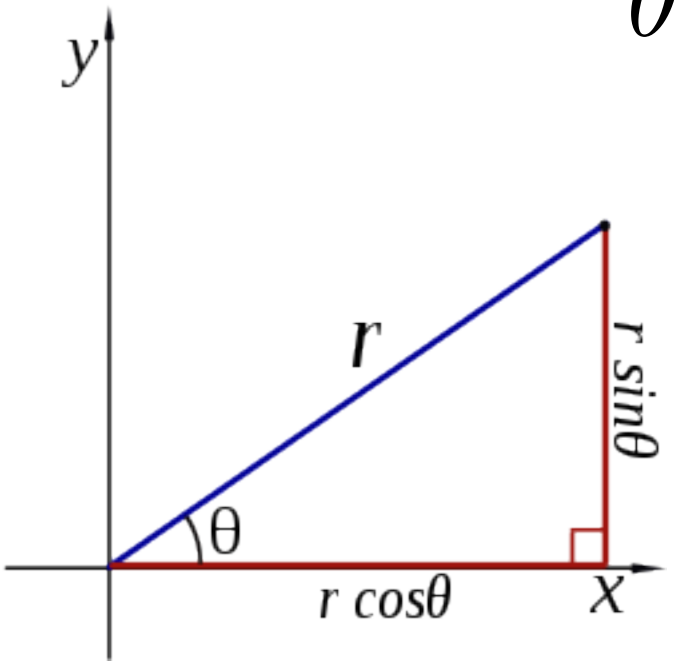
$$\mathbf{p} = r_1\mathbf{e_1} + r_2\mathbf{e_2} + r_3\mathbf{e_3}$$

$$\dot{\mathbf{p}} = \frac{d}{dt}(r_1\mathbf{e_1}) + \frac{d}{dt}(r_2\mathbf{e_2}) + \frac{d}{dt}(r_3\mathbf{e_3})$$

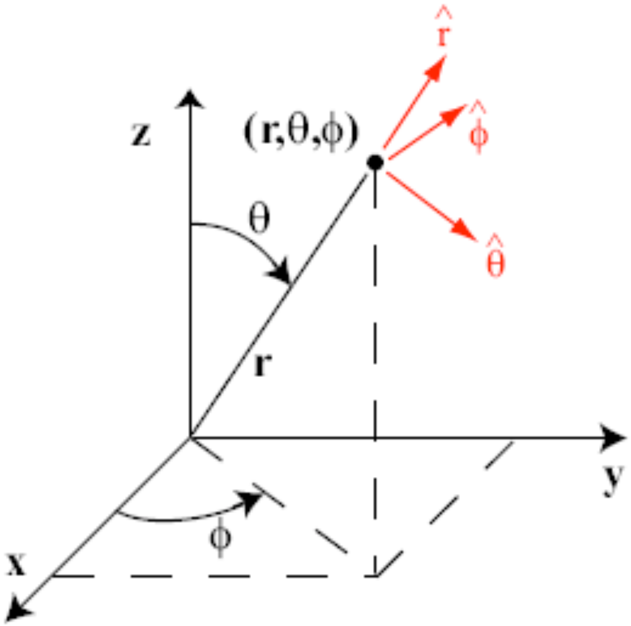
$$\dot{\mathbf{p}} = r_1\dot{\mathbf{e}}_\mathbf{1} + \dot{r}_1\mathbf{e_1} + r_2\dot{\mathbf{e}}_\mathbf{2} + \dot{r}_2\mathbf{e_2} + r_3\dot{\mathbf{e}}_\mathbf{3} + \dot{r}_3\mathbf{e_3}$$

What about polar coordinates?

$$x = r\cos\theta, y = r\sin\theta$$
$$x^2 = r^2\cos^2\theta, y^2 = r^2\sin^2\theta$$
$$x^2 + y^2 = r^2(\cos^2\theta + \sin^2\theta) = r^2$$
$$y/x = (r\sin\theta)/(r\cos\theta) = \tan\theta$$
$$\theta = \arctan(y/x)$$



How to think about unit vectors in polar coordinates?



Vector pointing
in direction of increasing u

$$\mathbf{e}_u = \frac{\partial \mathbf{r}}{\partial u}$$

$$\hat{\mathbf{e}}_u = \frac{\mathbf{e}_u}{|\mathbf{e}_u|}$$

Unit vector in any arbitrary direction (u, for example) is a normalized vector that points in the direction of increasing u

Why is that true?

$$\mathbf{e}_u = \frac{\partial \mathbf{r}}{\partial u}$$

$$\hat{\mathbf{e}}_u = \frac{\mathbf{e}_u}{|\mathbf{e}_u|}$$

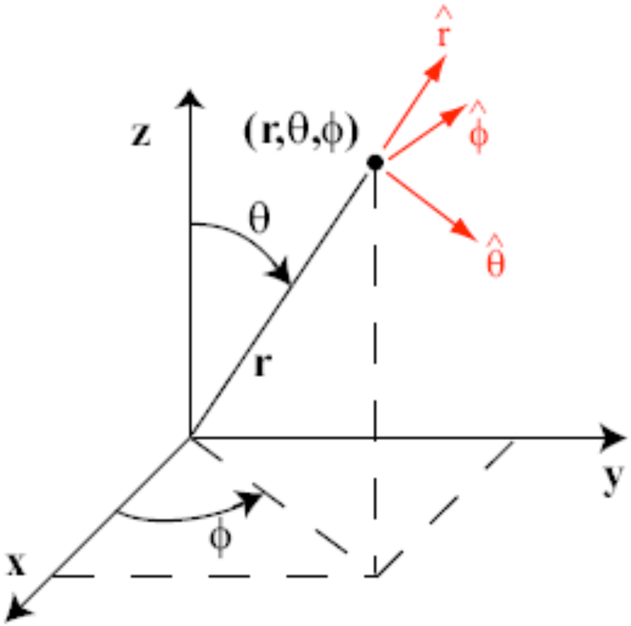
d**r** is some small displacement from point **r**

$$\mathbf{r} = u\mathbf{u} + v\mathbf{v} + w\mathbf{w}$$

$$d\mathbf{r} = \frac{\partial \mathbf{r}}{\partial u} du + \frac{\partial \mathbf{r}}{\partial v} dv + \frac{\partial \mathbf{r}}{\partial w} dw$$

$$\mathbf{e}_u \cdot du$$

Start by thinking about unit vectors in rectangular coordinates



$$\mathbf{r} = \vec{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$$

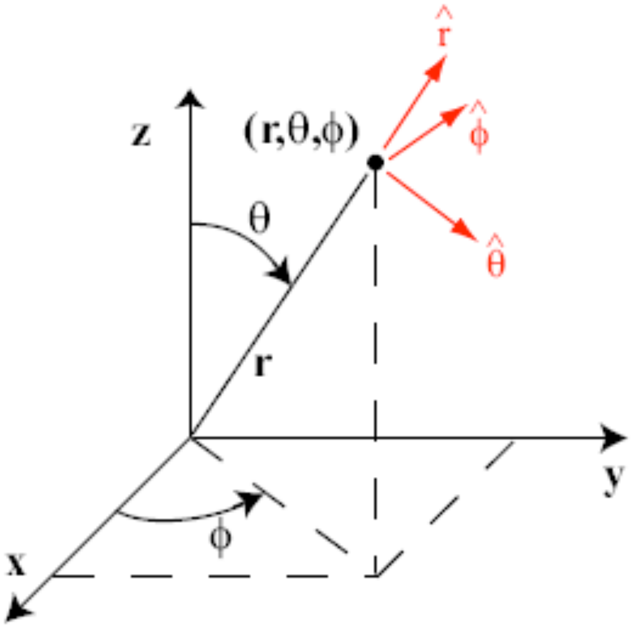
$$\mathbf{e_x} = \frac{\partial \mathbf{r}}{\partial x} = \hat{\mathbf{x}}$$

$$\mathbf{e_y} = \frac{\partial \mathbf{r}}{\partial y} = \hat{\mathbf{y}}$$

$$\mathbf{e_z} = \frac{\partial \mathbf{r}}{\partial z} = \hat{\mathbf{z}}$$

This is simple because xhat, yhat, zhat do not vary and are constant

How to think about unit vectors in polar coordinates?



$$x = r\cos\phi,\, y = r\sin\phi,\, z = z$$

$$\mathbf{r} = r\hat{\mathbf{r}} + \phi\hat{\phi} + z\hat{\mathbf{z}}$$

$$\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$$

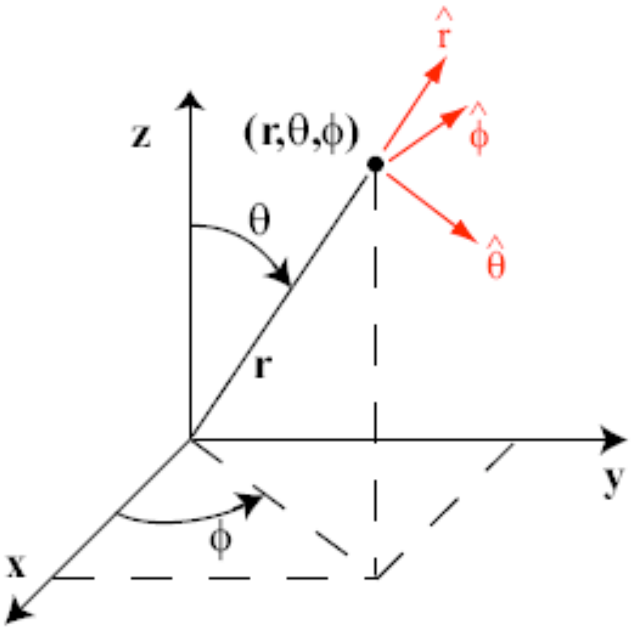
$$\mathbf{r} = r\cos\phi\hat{\mathbf{x}} + r\sin\phi\hat{\mathbf{y}} + z\hat{\mathbf{z}}$$

$$\mathbf{e}_r = \frac{\partial\mathbf{r}}{\partial r} = \cos\phi\hat{\mathbf{x}} + \sin\phi\hat{\mathbf{y}}$$

$$\mathbf{e}_\phi = \frac{\partial\mathbf{r}}{\partial\phi} = -r\sin\phi\hat{\mathbf{x}} + r\cos\phi\hat{\mathbf{y}}$$

$$\mathbf{e}_z = \frac{\partial\mathbf{r}}{\partial z} = \hat{\mathbf{z}}$$

How to think about unit vectors in polar coordinates?



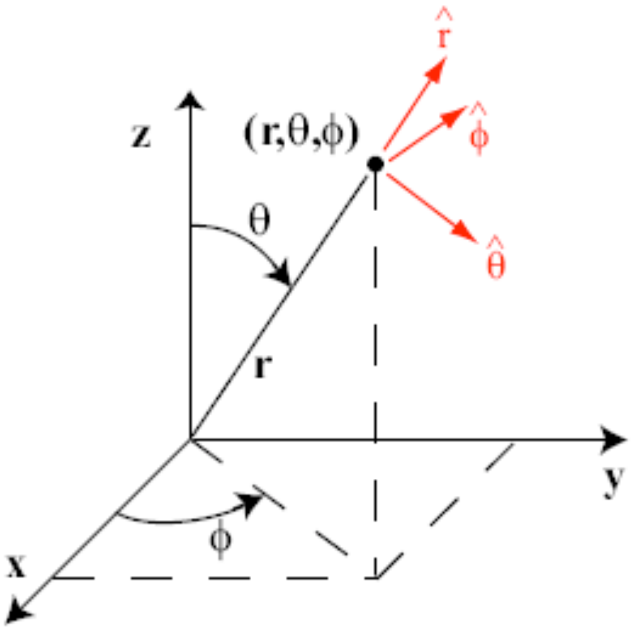
$$\mathbf{e}_r = \frac{\partial \mathbf{r}}{\partial r} = \cos\phi\hat{\mathbf{x}} + \sin\phi\hat{\mathbf{y}}$$

$$\hat{\mathbf{r}} = \hat{\mathbf{e}}_r = \frac{\mathbf{e}_r}{|\hat{\mathbf{e}}_r|} = \frac{\cos\phi\hat{\mathbf{x}} + \sin\phi\hat{\mathbf{y}}}{\sqrt{\cos^2\phi + \sin^2\phi}} = \cos\phi\hat{\mathbf{x}} + \sin\phi\hat{\mathbf{y}}$$

$$\mathbf{e}_\phi = \frac{\partial \mathbf{r}}{\partial \phi} = -r\sin\phi\hat{\mathbf{x}} + r\cos\phi\hat{\mathbf{y}}$$

$$\hat{\phi} = \hat{\mathbf{e}}_\phi = \frac{\mathbf{e}_\phi}{|\hat{\mathbf{e}}_\phi|} = \frac{-r\sin\phi\hat{\mathbf{x}} + r\cos\phi\hat{\mathbf{y}}}{r\sqrt{\cos^2\phi + \sin^2\phi}} = \mathbf{e}_\phi/r = -\sin\phi\hat{\mathbf{x}} + \cos\phi\hat{\mathbf{y}}$$

How to think about time derivatives in polar coordinates?



Also see derivation in book!

$$\hat{\mathbf{r}} = \cos\phi\hat{\mathbf{x}} + \sin\phi\hat{\mathbf{y}}$$

$$\hat{\phi} = -\sin\phi\hat{\mathbf{x}} + \cos\phi\hat{\mathbf{y}}$$

$$\frac{d\hat{\mathbf{r}}}{dt} = -\sin\phi\frac{d\phi}{dt}\hat{\mathbf{x}} + \cos\phi\frac{d\phi}{dt}\hat{\mathbf{y}} = \hat{\phi}\dot{\phi}$$

$$\frac{d\hat{\phi}}{dt} = -\cos\phi\frac{d\phi}{dt}\hat{\mathbf{x}} - \sin\phi\frac{d\phi}{dt}\hat{\mathbf{y}} = -\hat{\mathbf{r}}\dot{\phi}$$

Putting it all together

Let's work on 1.1, 1.2, 1.6 and 1.14 in Taylor

We'll do the first two together on the board, and then you work on 1.6 and 1.14 in small groups

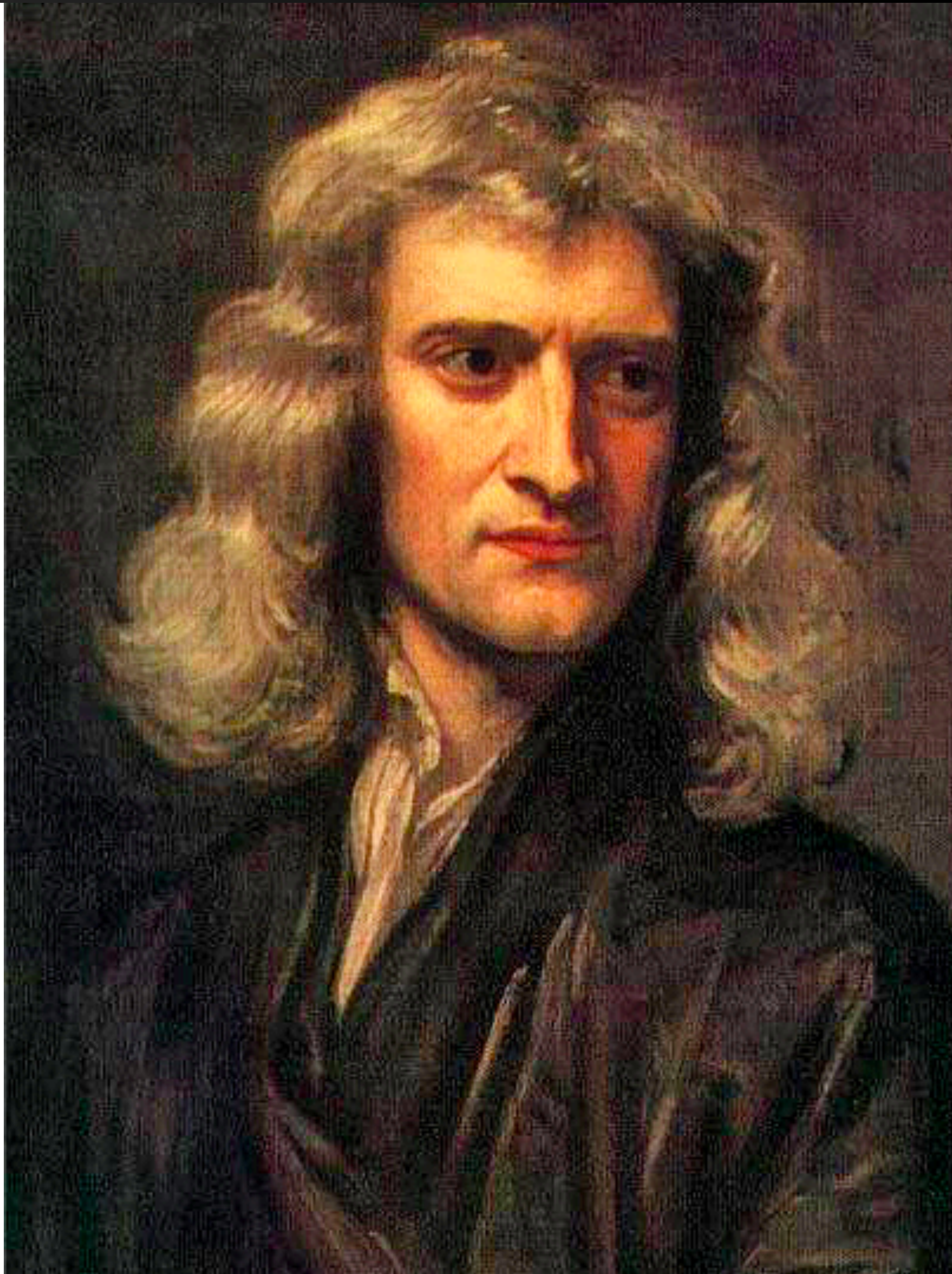
So ....

How will we use any of this? Why did we bother?

Well, let's go back to our favorite physical laws ...

Our good 'ol friend Newton and his laws

1. An object in motion stays in motion, and an object at rest stays at rest (law of inertia)
2. Our favorite equation, **F**=m**a**, remembering, though, that **F** and **a** are really vectors
3. The law of equal and opposite reaction, $\mathbf{F}_{12} = -\mathbf{F}_{21}$, again remembering that forces are vectors



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Lift force generated by wings overcomes gravity, and plane accelerates up

This law is the one that will help us to solve equations of motion...

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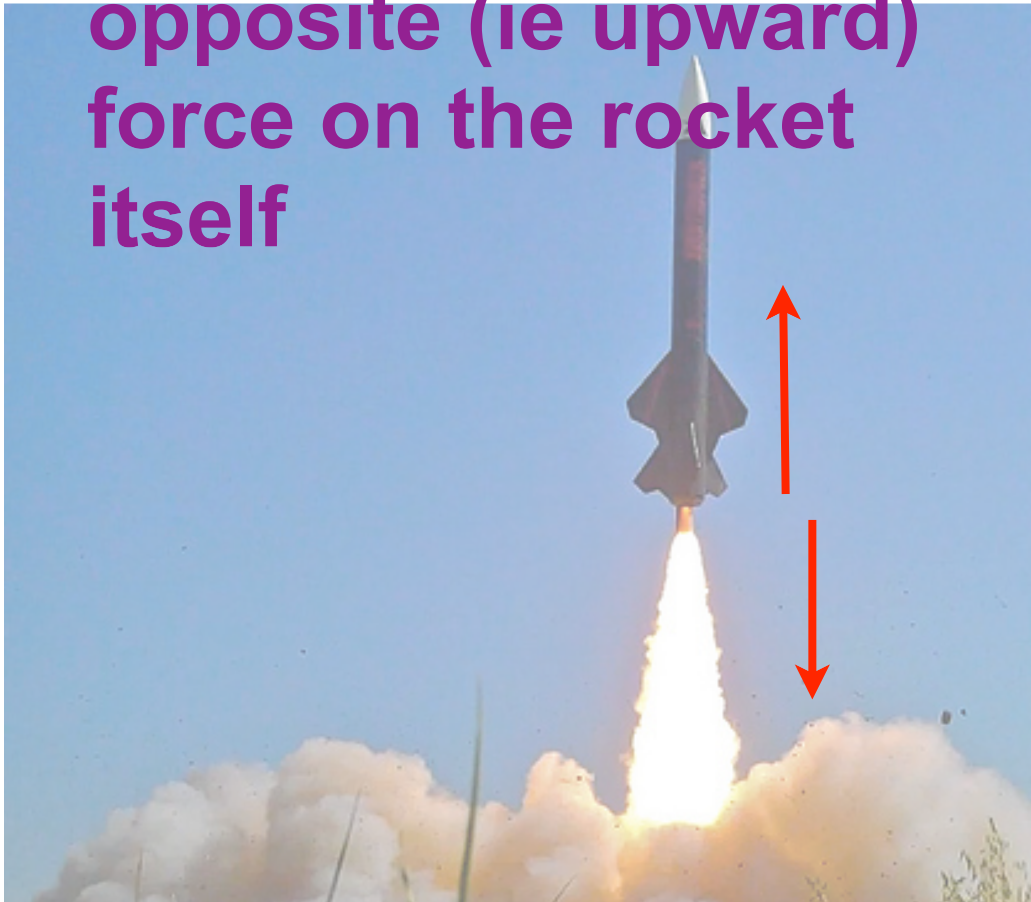
Rocket exhaust given a strong force downward... which means a strong opposite (ie upward) force on the rocket itself



The second law

$$\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt} = m\dot{\mathbf{v}} = m\frac{d^2\mathbf{r}}{dt^2} = m\ddot{\mathbf{r}}$$

$$\mathbf{p} = m\mathbf{v} \rightarrow \dot{\mathbf{p}} = m\dot{\mathbf{v}} = m\mathbf{a}$$

$$\rightarrow \dot{\mathbf{p}} = \mathbf{F}$$

A reminder of the simplest use case

One-dimensional motion subject to constant force

$$\mathbf{r}(t) = y(t), \mathbf{F}(\mathbf{t}) = F_0$$

$$\ddot{y}(t) = \frac{F_0}{m}$$

$$\dot{y}(t) = \int \ddot{y}(t)dt = \int \frac{F_0}{m}dt = v_0 + \frac{F_0}{m}t$$

$$y(t) = \int \dot{y}(t)dt = \int (v_0 + \frac{F_0}{m}t)dt = y_0 + v_0 t + \frac{F_0}{2m}t^2$$

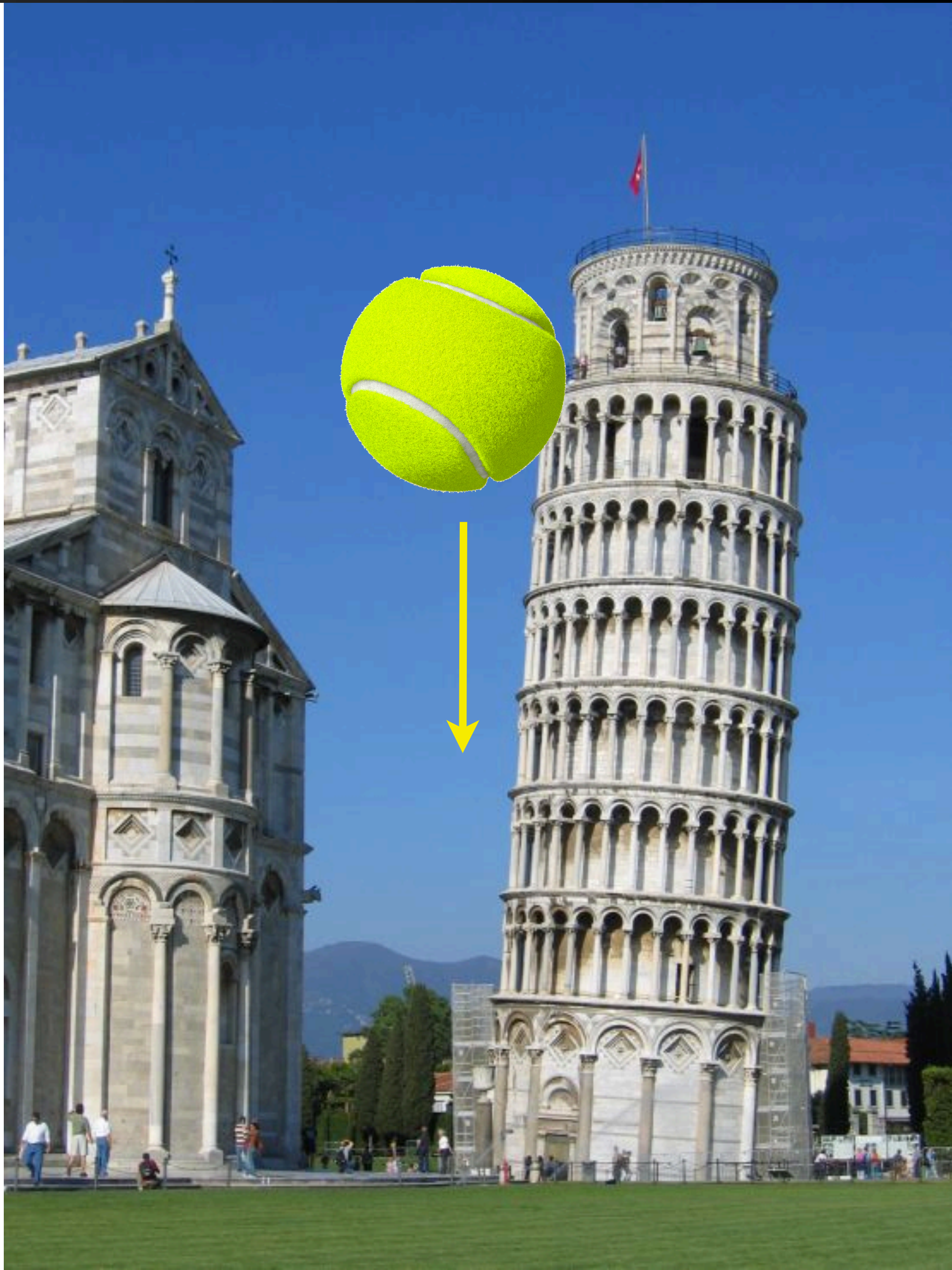
Recall from into mechanics:

$$d = d_0 + v_o t + \frac{1}{2}at^2$$

constants of integration (initial position and speed)

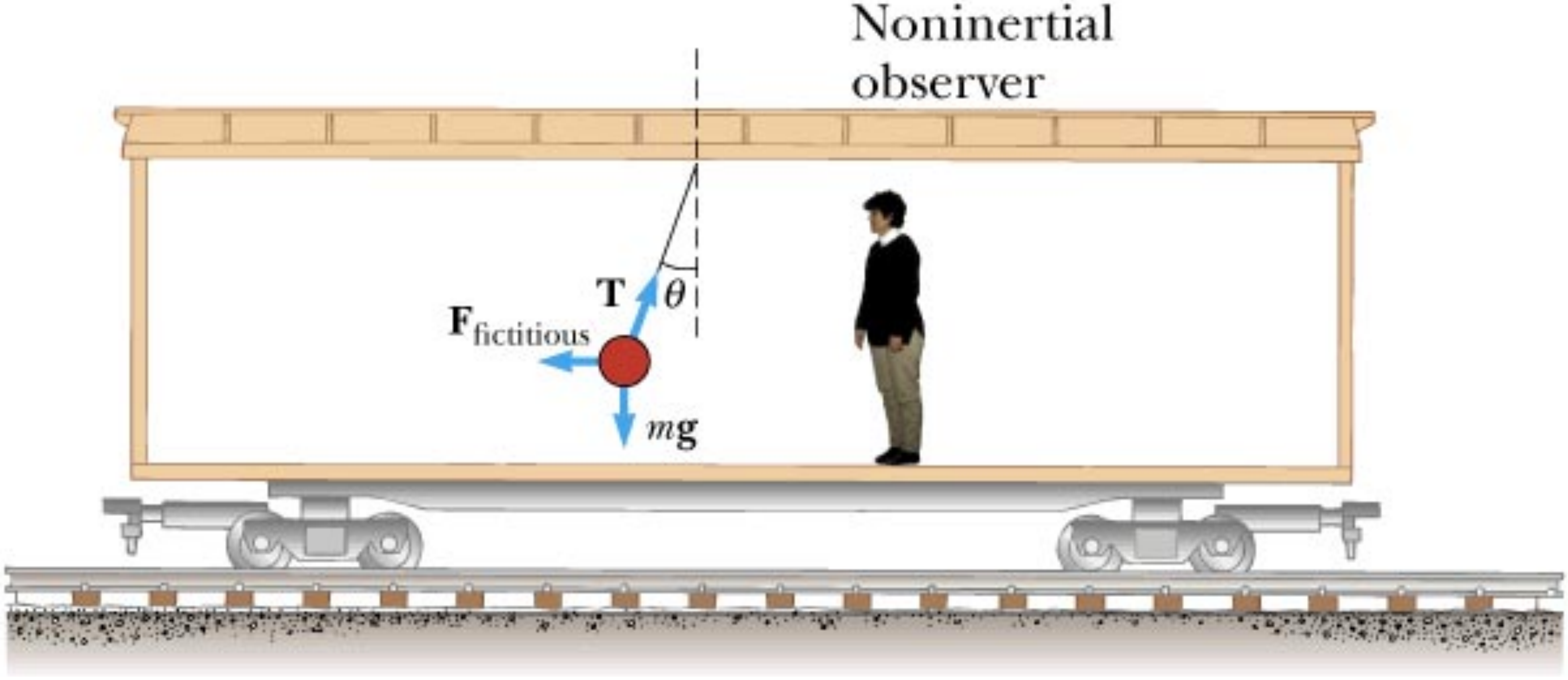
A reminder of the simplest use case

One-dimensional motion subject to constant force = gravity (with no air resistance or friction ... we'll get to that soon)



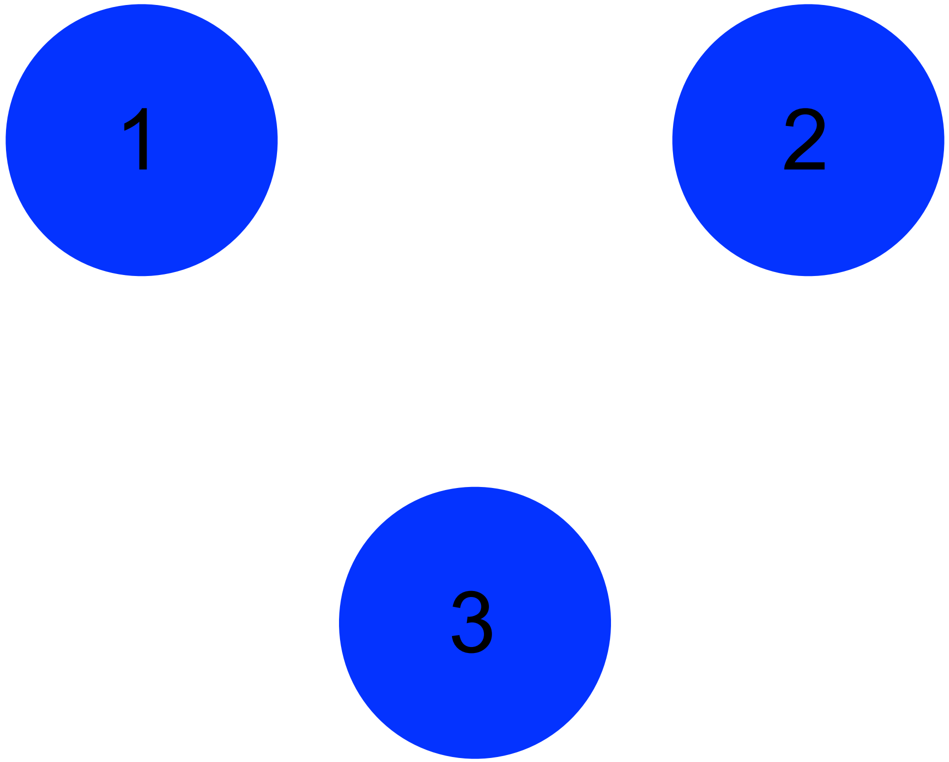
And a reminder on frames of reference

For now, consider only motion in inertial frames (will come back to noninertial frames at the end of the course). What do we mean by this?



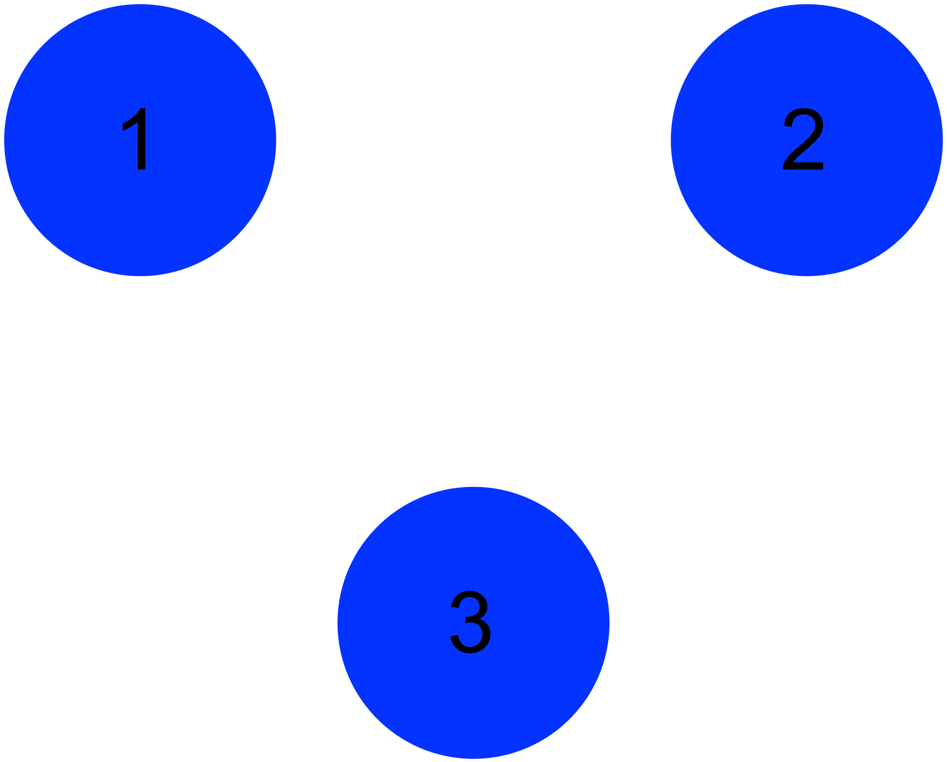
Newton's second law for collections of particles

Let's start with 3 particles (read Taylor Section 1.5 for expansion to N particles)



Newton’s second law for collections of particles

Each of 3 particles is subject to potential external forces, and also to forces on each other

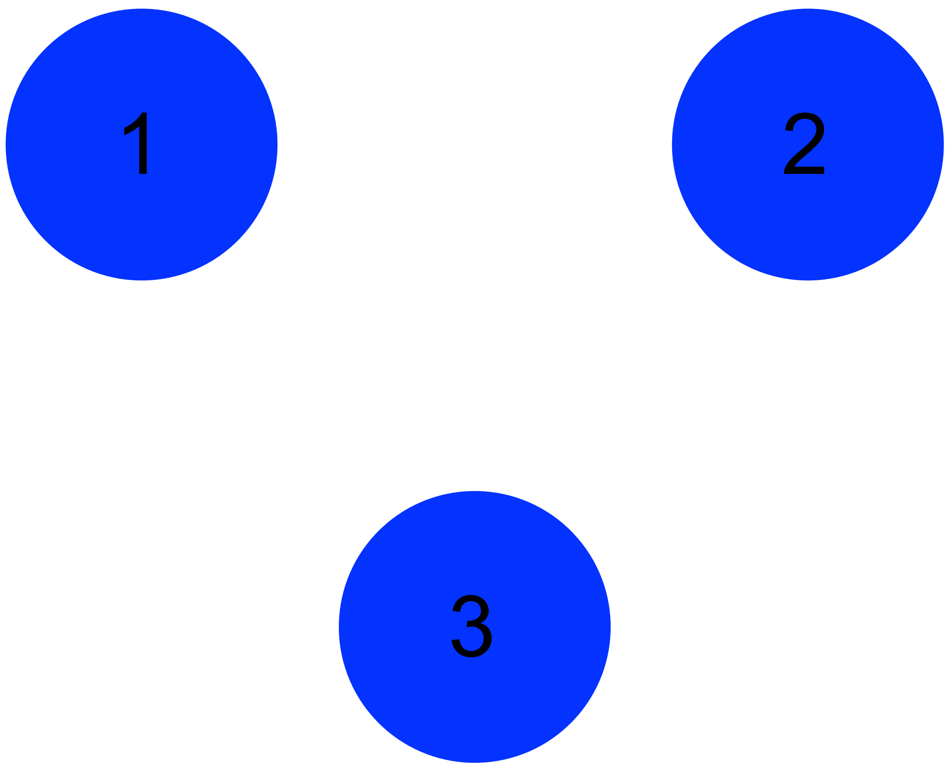


Newton's second law for collections of particles

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_1^{ext}$$

$$\mathbf{F}_2 = \mathbf{F}_{23} + \mathbf{F}_{21} + \mathbf{F}_2^{ext}$$

$$\mathbf{F}_3 = \mathbf{F}_{31} + \mathbf{F}_{32} + \mathbf{F}_3^{ext}$$



Newton's second law for collections of particles

$$\dot{\mathbf{p}}_1 = \mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_1^{ext}$$

$$\dot{\mathbf{p}}_2 = \mathbf{F}_2 = \mathbf{F}_{23} + \mathbf{F}_{21} + \mathbf{F}_2^{ext}$$

$$\dot{\mathbf{p}}_3 = \mathbf{F}_3 = \mathbf{F}_{31} + \mathbf{F}_{32} + \mathbf{F}_3^{ext}$$

Momentum of total system →

$$\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3$$

$$\dot{\mathbf{P}} = \dot{\mathbf{p}}_1 + \dot{\mathbf{p}}_2 + \dot{\mathbf{p}}_3$$

$$\dot{\mathbf{P}} = \mathbf{F}_{23} + \mathbf{F}_{32} + \mathbf{F}_1^{ext} + \mathbf{F}_{13} + \mathbf{F}_{31} + \mathbf{F}_2^{ext} + \mathbf{F}_{12} + \mathbf{F}_{21} + \mathbf{F}_3^{ext}$$

0 0 0

$$\dot{\mathbf{P}} = \mathbf{F}_1^{ext} + \mathbf{F}_2^{ext} + \mathbf{F}_3^{ext}$$

Newton's second law in polar coordinates

A nice form to have, but only works if given $F_r(t)$ and F_Φ (t), which can be trivial ... or non-trivial

$$\mathbf{F} = m\ddot{\mathbf{r}} = F_r\hat{\mathbf{r}} + F_\phi\hat{\phi}$$

$$\mathbf{r} = r\hat{\mathbf{r}}$$

So maybe we need to use the second line and differentiate that...

$$\dot{\mathbf{r}} = \dot{r}\hat{\mathbf{r}} + r\dot{\hat{\mathbf{r}}}$$

$$\frac{d\hat{\mathbf{r}}}{dt} = \dot{\hat{\mathbf{r}}} = \hat{\phi}\dot{\phi}$$

Luckily, we derived this recently

Newton's second law in polar coordinates

$$\dot{\mathbf{r}} = \dot{r}\hat{\mathbf{r}} + r\dot{\hat{\mathbf{r}}}$$

$$\frac{d\hat{\mathbf{r}}}{dt} = \dot{\hat{\mathbf{r}}} = \hat{\phi}\dot{\phi}$$

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{r}\hat{\mathbf{r}} + r\hat{\phi}\dot{\phi}$$

$$v_r = \dot{r}, v_\phi = r\dot{\phi} = r\omega$$

Radial velocity is as expected, angular velocity is what we learned in intro mechanics

Newton's second law in polar coordinates

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{r}\hat{\mathbf{r}} + r\hat{\boldsymbol{\phi}}\dot{\phi}$$

$$\mathbf{a} = \ddot{\mathbf{r}} = \frac{d}{dt}(\dot{r}\hat{\mathbf{r}} + r\hat{\boldsymbol{\phi}}\dot{\phi})$$

Apply chain rule...

$$\ddot{\mathbf{r}} = \ddot{r}\hat{\mathbf{r}} + \dot{r}\dot{\hat{\mathbf{r}}} + \dot{r}\hat{\boldsymbol{\phi}}\dot{\phi} + r\dot{\hat{\phi}}\dot{\phi} + r\hat{\phi}\ddot{\phi}$$

$$\frac{d\hat{\boldsymbol{\phi}}}{dt} = \dot{\hat{\phi}} = -\hat{\mathbf{r}}\dot{\phi}$$

Luckily, we also derived this recently

Working out the algebra ...

$$\mathbf{a} = \ddot{\mathbf{r}} = \frac{d}{dt}\left(\dot{r}\hat{\mathbf{r}} + r\hat{\boldsymbol{\phi}}\dot{\phi}\right)$$

$$\ddot{\mathbf{r}} = \ddot{r}\hat{\mathbf{r}} + \dot{r}\dot{\hat{\mathbf{r}}} + \dot{r}\hat{\boldsymbol{\phi}}\dot{\phi} + r\dot{\hat{\phi}}\dot{\phi} + r\hat{\phi}\ddot{\phi}$$

from class ↔ from class

$$\ddot{\mathbf{r}} = \ddot{r}\hat{\mathbf{r}} + \dot{r}\dot{\phi}\hat{\phi} + \dot{r}\hat{\boldsymbol{\phi}}\dot{\phi} - r\hat{\mathbf{r}}\dot{\phi}^2 + r\hat{\phi}\ddot{\phi}$$

$$\ddot{\mathbf{r}} = \hat{\mathbf{r}}\left(\ddot{r} - r\dot{\phi}^2\right) + \hat{\phi}\left(2\dot{r}\dot{\phi} + r\ddot{\phi}\right)$$

F_r F_ϕ

Putting it all together

$$\boxed{\ddot{\mathbf{r}} = \hat{\mathbf{r}}(\ddot{r} - r\dot{\phi}^2) + \hat{\phi}(2\dot{r}\dot{\phi} + r\ddot{\phi})} = \mathbf{F}/m$$

$$F_r = m(\ddot{r} - r\dot{\phi}^2), F_\phi = m(2\dot{r}\dot{\phi} + r\ddot{\phi})$$

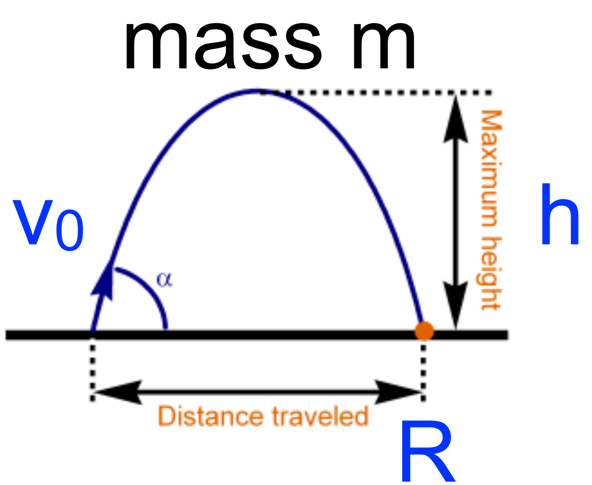
IF and only if r constant:

$$-r\omega^2 \qquad\qquad r\ddot{\phi}$$

$$F = -mv^2/r$$

Let's work out some problems

- Taylor Example 1.2 let's work it out together
- Taylor problems 1.36, 1.41 to be done together
- On your own in small groups: Solve for R and h below!



Homework due back in 1 week!

- Taylor 1.8,1.10,1.22,1.26,1.30,1.38,1.39