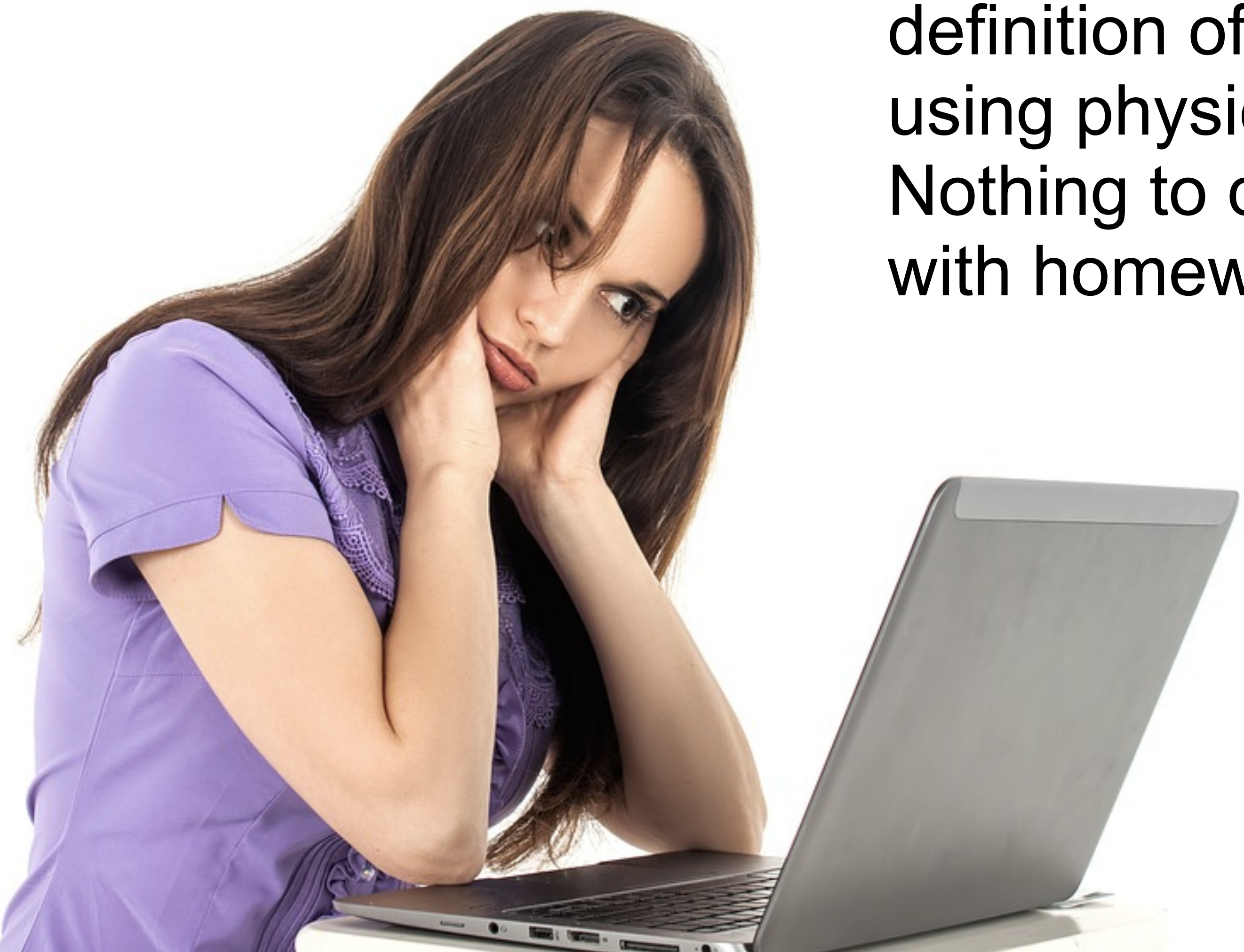
Chapter 9

(Some material taken from McGraw-Hill/College Physics/Giambattista, the majority from your Katz textbook)

Work *occurs when a force acts on an object that moves*.

Only the *component* of the force *in the direction of the displacement/movement* does work.

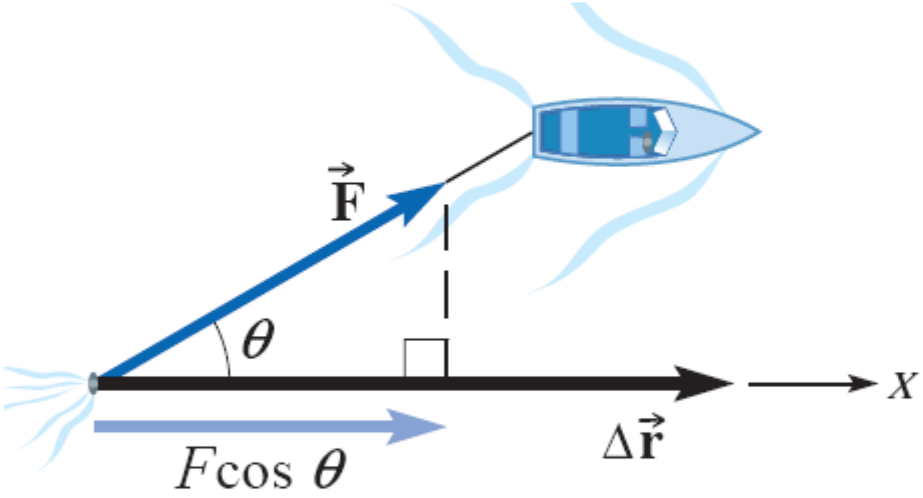


Note the very specific definition of **work** when using physics language! Nothing to do (for example) with homework

Work done by a constant force $\vec{\mathbf{F}}$ acting on an object whose displacement is $\Delta\vec{\mathbf{r}}$:

$$W = F\,\Delta r\cos\theta$$

(θ is the angle between $\vec{\mathbf{F}}$ and $\Delta\vec{\mathbf{r}}$)



Work can be positive, negative, or zero.

Work done by a constant force $\vec{\mathbf{F}}$ acting on an object whose displacement is $\Delta\vec{\mathbf{r}}$:

$$W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$$

scalar product (or **dot product**)

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB \cos\ \theta,$$

Useful form if you know the angle between the force and displacement

Work done by a constant force $\vec{\mathbf{F}}$ acting on an object whose displacement is $\Delta\vec{\mathbf{r}}$:

If we choose the *x*-axis parallel to the displacement,

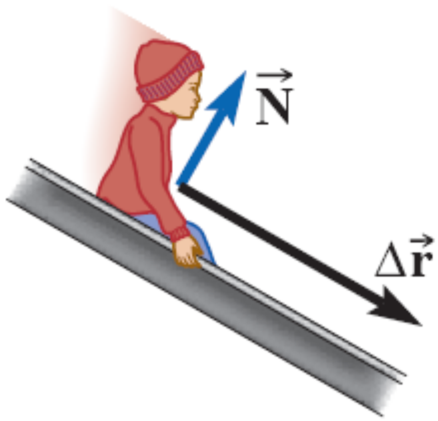
$$W = F_x \, \Delta x$$

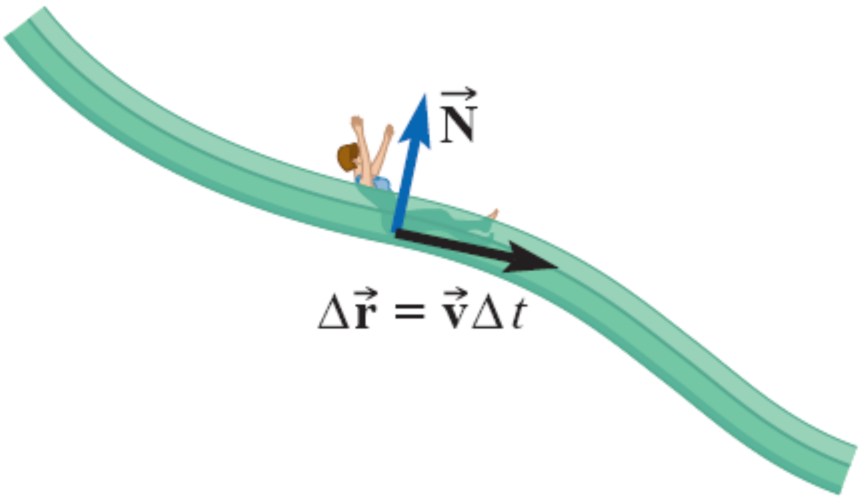
$$(\vec{\mathbf{F}} \text{ and/or } \Delta\vec{\mathbf{r}} \text{ parallel to the } x\text{-axis})$$

Useful form if you know the components of the force and displacement (should give the same answer, of course)

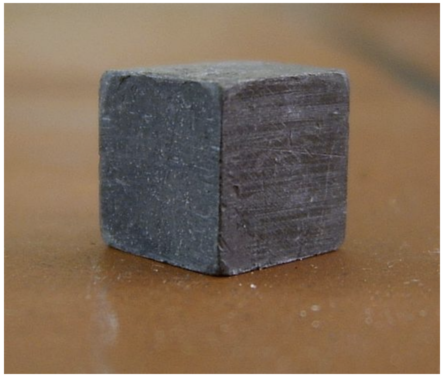
More generally:

$$W = F_x \Delta x + F_y \Delta y + F_z \Delta z$$

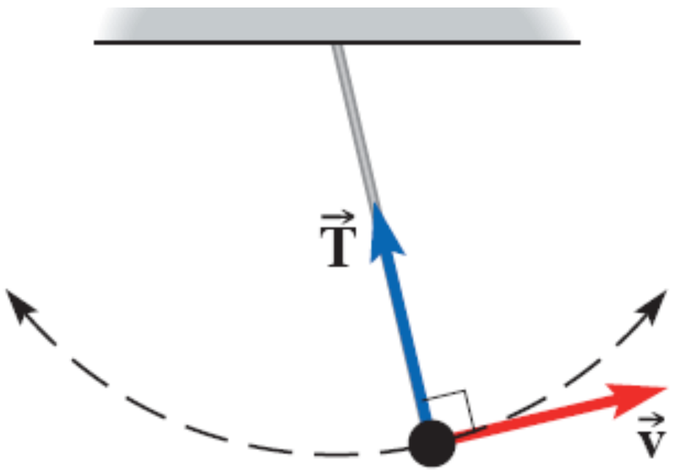




The normal force does no work when it is perpendicular to the displacement.



Try to lift a heavy lead brick - if it doesn't move, you do **no** work!



The tension in the string of a pendulum is always perpendicular to the velocity of the pendulum bob, so the string does no work on the bob.

Change in potential energy:

The change in potential energy of an object is equal in magnitude but opposite in sign to the work done by the force associated with the potential energy, regardless of the path it takes:

$$\Delta U = -W$$

Change in gravitational potential energy:

$$\Delta U_{\mathrm{grav}} = -W_{\mathrm{grav}}$$

If the gravitational field is uniform, the work done by gravity is

gravity points down!

$$W_{\mathrm{grav}} = F_y \Delta y = -mg\,\Delta y$$

where the *y* -axis points up. As we have seen (just a reminder),

And this fits together with last chapter:

$$\Delta U_{\mathrm{grav}} = mg\,\Delta y \qquad (\text{uniform } \vec{\mathbf{g}},\ y\text{-axis up})$$

(This holds *even if the object does not move in a straight-line path*.)

Friction

Not every force has an associated potential energy. These are **nonconservative forces**. For instance, there is no such thing as "frictional potential energy." When kinetic friction does work, it converts energy into a disorganized form that is not easily recoverable as kinetic energy. What is this disorganized form? **Thermal energy (heat!)**

The same is true for drag forces (which are another type of friction)

Mechanical Energy

The total work done on an object can always be written as the sum of the work done by conservative forces (W_{cons}) plus the work done by nonconservative forces (W_{nc}).

Non-conservative forces have no potentials associated with them, but we can still calculate the work that they do (W_{nc})

***W_{nc}* is equal to the change in mechanical energy**

Work-kinetic energy theorem:

$$W_{\text{total}} = W_{\text{cons}} + W_{\text{nc}} = \Delta K \quad \Rightarrow \quad W_{\text{nc}} = \Delta K - W_{\text{cons}}$$

$$\Delta U = -W_{\text{cons}}$$

Delta(K) = Kf - Ki
Delta(U) = Uf - Ui

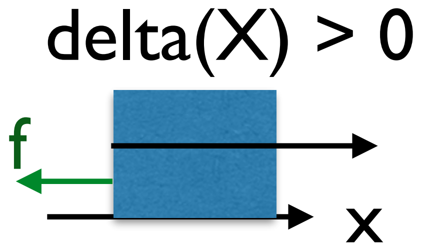
$$W_{\text{nc}} = \Delta K + \Delta U$$

or

$$(K_{\text{i}} + U_{\text{i}}) + W_{\text{nc}} = (K_{\text{f}} + U_{\text{f}})$$

When nonconservative forces do no work, mechanical energy is conserved.

$$(K_i + U_i) + W_{nc} = (K_f + U_f)$$



In other words, the total available final mechanical energy is equal to the initial value, with the addition of the work done by non-conservative forces. If this is negative, then the final mechanical energy is reduced from the initial value!

Let's think about the sign of the work done by friction

We give the name **power** (symbol *P*) to the rate of energy transfer or of energy conversion.

The average power is the amount of energy transferred (ΔE) divided by the time the transfer takes (Δt).

Average power:

$$P_{av} = \frac{\Delta E}{\Delta t}$$

The SI unit of power, the joule per second, is given the name watt (1 W = 1 J/s)

1 hp = 746 W

The magnitude of the displacement is

$$\Delta r = v\,\Delta t$$

Hence, the power—the rate at which the force does work—can be found from the force and the velocity.

$$P = \frac{W}{\Delta t} = \frac{F\Delta r\cos\,\theta}{\Delta t} = F\,\frac{\Delta r}{\Delta t}\cos\,\theta = Fv\cos\,\theta$$

Instantaneous power (rate at which work is done):

$$P = Fv\cos\theta = \vec{F}\cdot\vec{v}$$

(θ is the angle between $\vec{\mathbf{F}}$ and $\vec{\mathbf{v}}$)

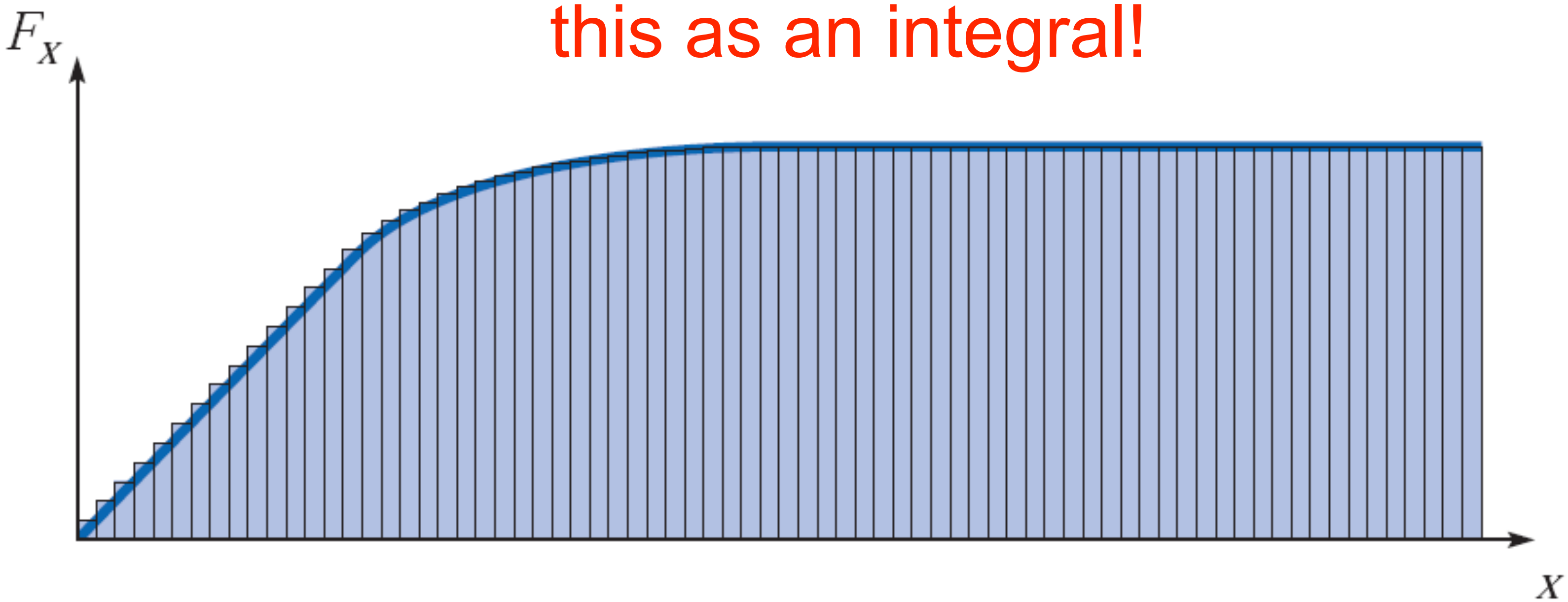
To approximate the work done by a variable force F_x, we divide the overall displacement into a series of small displacements Δx .

During each small displacement, the work done is

$$\Delta W = F_x \, \Delta x$$

On a graph of $F_x(x)$, each ΔW is the area of a rectangle of height F_x and width Δx. The total work done is the sum of the areas of these rectangles. This approximation gets better as we make the rectangles thinner and thinner, so *the total work done is the area under the graph of* $F_x(x)$ from x_i to x_f.

Of course, we should view this as an integral!



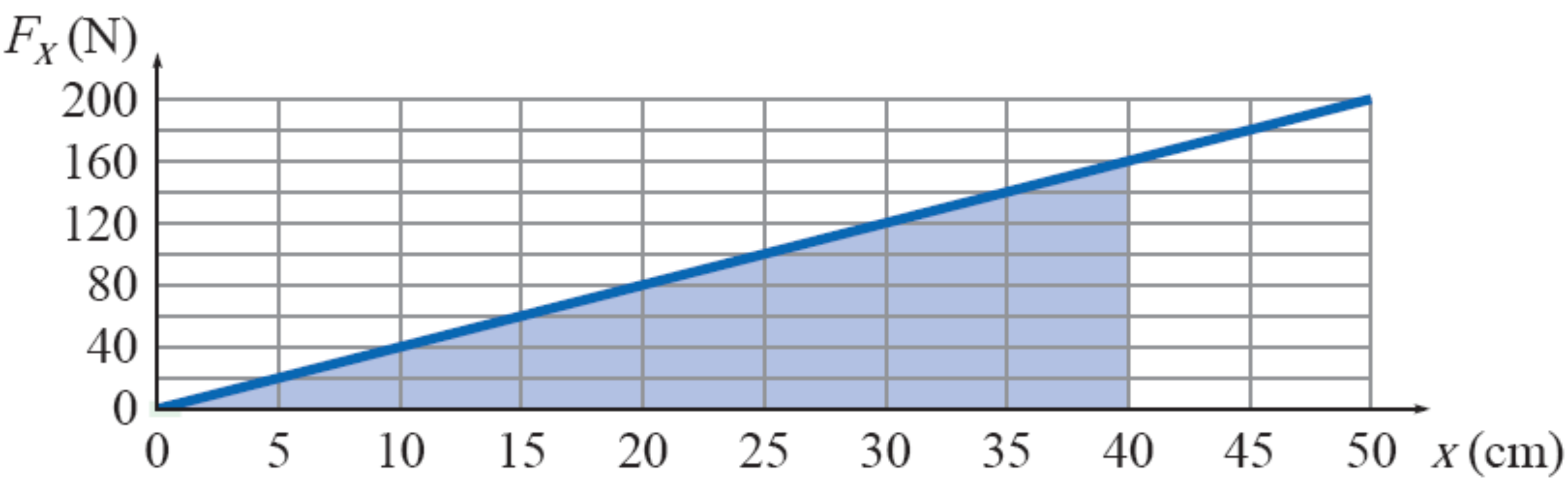
More generally, we can say that for a force in the x direction:

$$W = \int_{x_i}^{x_f} F_x dx$$

And for a force in any arbitrary direction, the work is given by a **path integral**:

$$W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r} = \int_{r_i}^{r_f} (F_x dx + F_y dy + F_z dz)$$

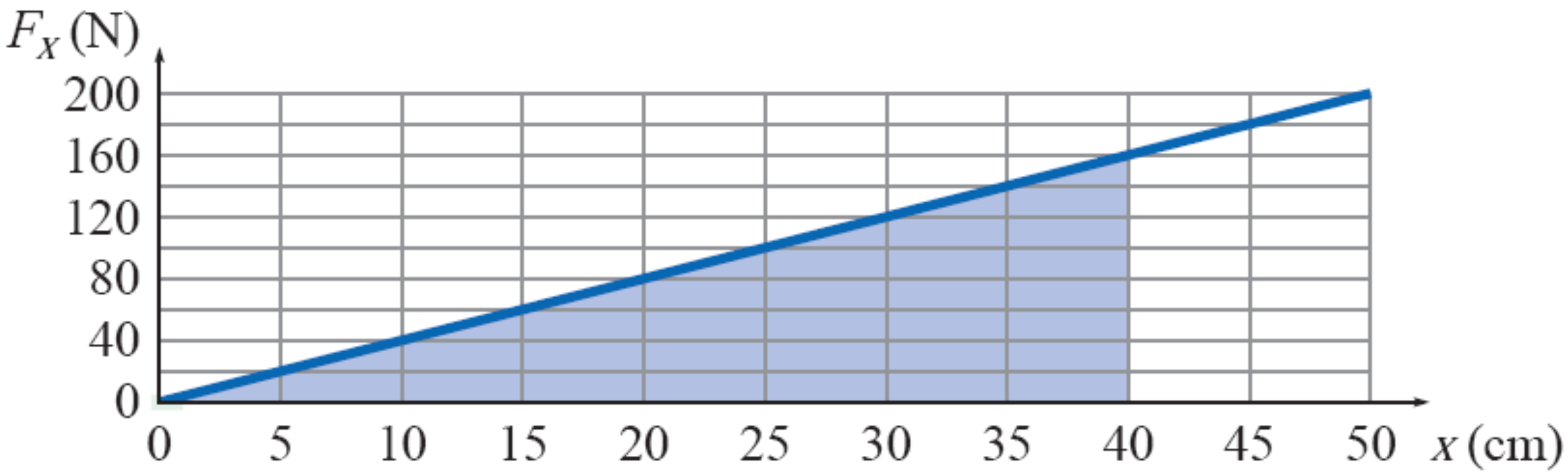
To draw back a *simple* bow, the force the archer exerts on the string continues to increase as the displacement of the string increases and the bow bends slightly. The force-versus-position graph describes such a bow. Calculate the work done by the archer on the string as he draws the string back 40.0 cm.



Solution

$$\tfrac{1}{2}(\text{base} \times \text{altitude})$$

$$W = \tfrac{1}{2}(0.400\ \text{m} \times 160\ \text{N}) = +32\ \text{J}$$



A bungee jumper makes a jump in the Gorge du Verdon in southern France. The jumping platform is 182 m above the bottom of the gorge. The jumper weighs 780 N.

If the jumper falls to within 68 m of the bottom of the gorge, how much work is done by the bungee cord on the jumper during his descent? Ignore air resistance.

Solution

$$W_{total} = W_g + W_c = \Delta K = 0$$

$$W_g = F_y\,\Delta y = -mg\,\Delta y \qquad mg = 780\text{ N}$$

+y axis = up!
gravity = down!
Angle θ ?

$$\Delta y = y_f - y_i = 68\text{ m} - 182\text{ m} = -114\text{ m}$$

$$W_g = -(780\text{ N}) \times (-114\text{ m}) = +89\text{ kJ}$$

The work done by the cord is $W_c = W_{total} - W_g = -89\text{ kJ}$.

GROUP WORK!

https://forms.gle/NRsjHQK9TJZgXAZB7

A force **F** = (7.7 i**hat** + 8.8 **jhat** - 9.9 **khat**) N is applied to a 12.4 kg object that moves 1.2 meters in the **ihat** direction and Z meters in the khat direction. If the force does +20 J of work, what is the value of Z?

A sled is dragged along a horizontal path at a constant speed of 1.5 m/s by a rope inclined at an angle of 30.0 degrees with respect to the horizontal. The total weight of the sled is 470 N. The tension in the rope is 240 N. How much work is done by the rope on the sled in a time interval of 10.0s?

PHYS253 Chapter 9

A 402 kg pile driver is raised 12 m above ground

a) How much work must be done to raise the pile driver?
b) How much work does gravity do on the driver as it is raised?
c) The driver is now dropped. How much work does gravity do on the driver as it falls?

A record company executive is on his way to a TV interview carrying his briefcase. The mass of the briefcase is 5.00 kg. The executive realizes that he is going to be late. Starting from rest, he starts to run, reaching a speed of 2.50 m/s. What is the work done by the executive on the briefcase during this time? (Ignore air resistance)

A plane weighting 220 kN lands on an aircraft carrier. The plan is moving horizontally at 67 m/s (150 mi/h) when its tailhook grabs hold of the arresting cables. The cables bring the plane to a stop in a distance of 84m.

a) How much work is done on the plane by the arresting cables?

b) What is the force (assumed to be constant) exerted on the plane by the cables?

A brick of mass 1.0 kg slides down an icy roof inclined at 30 degrees with respect to the horizontal

a) If the brick starts from rest, how fast is it moving when it reaches the edge of the roof 2.00 meters away? Ignore friction

b) What if the coefficient of friction is 0.10?

A 537 kg trailer is hitched to a truck. Find the work done by the truck on the trailer in each of the following cases, assuming rolling friction is negligible:

a) The trailer is pulled at constant speed along a level road for 2.30 km
b) The trailer is accelerated from rest to a speed of 88.8 km/h
c) The trailer is pulled at constant speed along a road inclined at 12.5 degrees for 2.30 km

Find the dot product of the vectors

A = 7.12 ihat + 2.00 jhat - 3.90 khat
and
B = 4.10 ihat - 11.00 jhat

An object is subject to a force **F** = 512 ihat - 134 jhat N such that 10,125 J of work is performed on the object. If the object travels 25.0 m in the positive x direction while this work is performed, what must be the displacement of the object in the y direction?

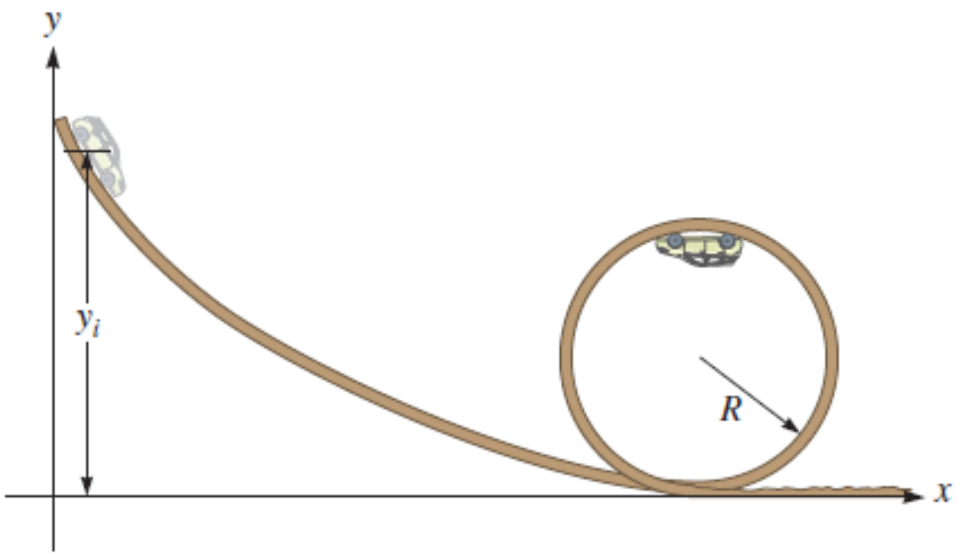
An object of mass $m = 5.8$ kg moves under the influence of one constant force. That force causes the object to move along a path given by $x = 6.0 + 5.0t + 2.0t^2$, where x is in meters and t is in seconds. Calculate the work done by the force on the object from $t = 2.0$ s to $t = 7.0$ s.

A 2.15-g hailstone, which can be modeled as a particle, falls a vertical distance of 145 m at constant speed. What is the work done on the hailstone by

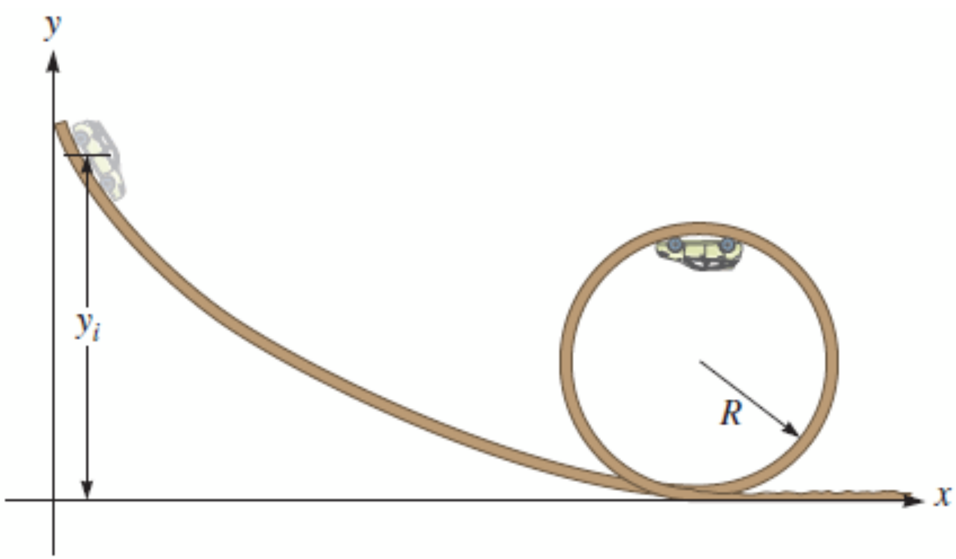
a) Gravity
b) Air resistance

A bullet flying horizontally hits a wooden block that is initially at rest on a frictionless, horizontal surface. The bullet gets stuck in the block, and the bullet-block system has a final speed v_f. Find the final speed of the bullet-block system in terms of the mass of the bullet m_b, the speed of the bullet before the collision v_b, the mass of the block m_{wb}, and the amount of thermal energy generated during the collision, E_{th}.

You designed a loop-the-loop track for a small toy car. The car starts at height y_i above the bottom of the loop, goes through the loop of radius R, and then travels along a flat, horizontal tracks. Rolling friction is negligible. What is the minimum height y_i from which the car can be released so that the car just barely makes it around the loop?



Consider the track from the previous problem. The car is released from the minimum height y_i found in the previous problem. Assume now that rolling friction is significant between the horizontal track and the car, but negligible along the rest of the tracks. If the coefficient of rolling friction between the car and the horizontal track is 0.30 and the radius of the loop is 0.45 m, how far does the car travel from the base of the loop before coming to rest?



What is the power output of a 75.0-kg student that climbs a knotted vertical rope 9.50 m in height at his high school gym at constant speed in 10.0 s?

If a man has an average useful power output of 40.0 W, what minimum time would it take him to lift fifty 10.0 kg boxes to a height of 2.00 m?

Your car's engine is capable of delivering a maximum of 350 horsepower. If we ignore air resistance and friction, what speed should the engine be able to bring your car to, from rest, in 5 seconds. Ignore air resistance and friction, and assume your car has a mass of 1400 kg. Use information from this chapter.

Why does the answer seem wrong? (Might be a few answers)