Chapter 2

(Some material taken from McGraw-Hill/College Physics/Giambattista, the majority from your Katz textbook)

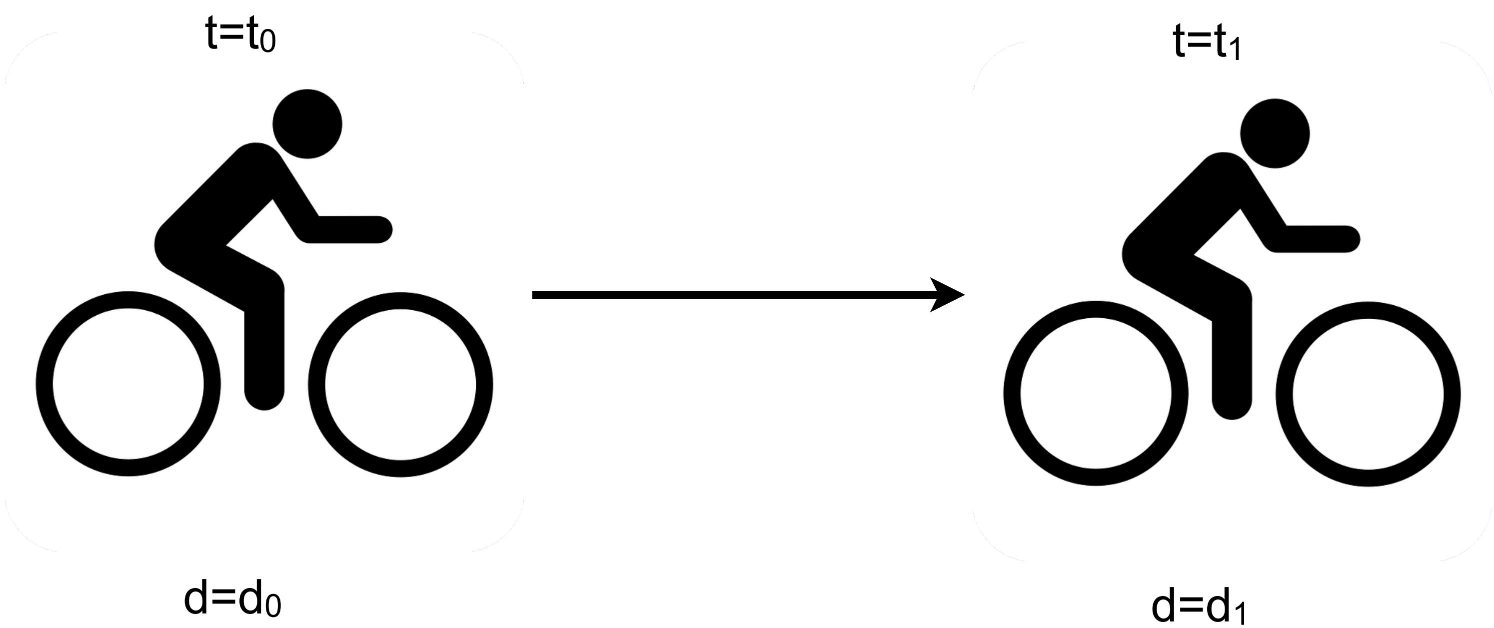
In everyday language, ***speed*** and ***velocity*** are synonyms.

In physics, there is an important distinction between the two.

In physics, *velocity* includes the ***direction*** of motion as well as the distance traveled per unit time.

When a moving object changes direction, its velocity changes even though its speed may not have changed.

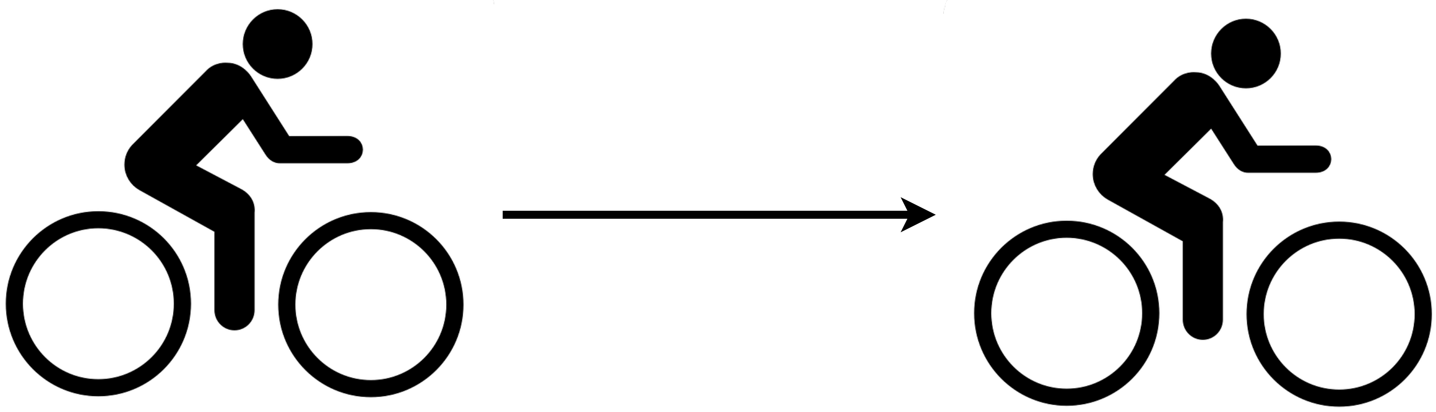
Speed = *distance divided by time* (really, distance for a given unit of time)



Let's compute the speed together

Speed = *distance divided by time* (really, distance for a given unit of time)

$t = t_0$ $t = t_1$

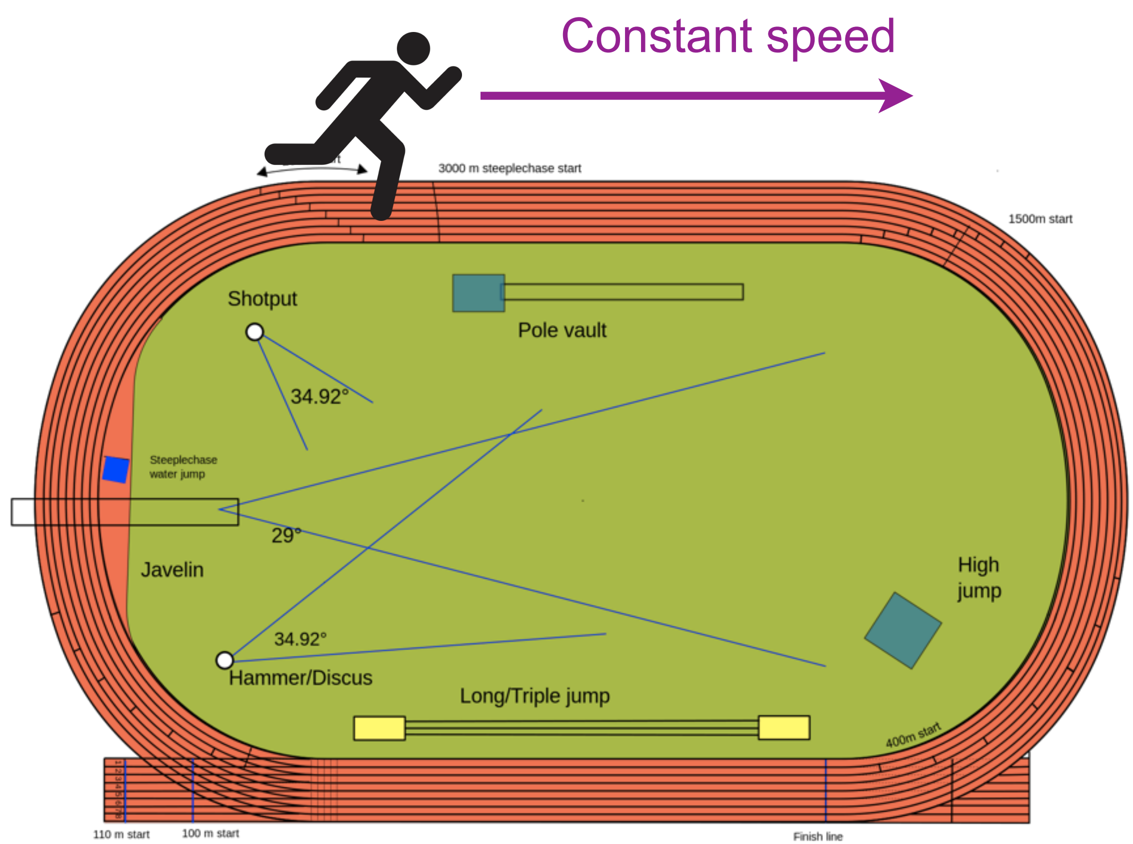


$d = d_0$ $d = d_1$

$$\text{Speed} = \frac{Distance}{Time} = \frac{d_1 - d_0}{t_1 - t_0}$$

We will often set $t_0 = 0$ (ie we start the clock or stopwatch at $t=0$)

Velocity = *directional distance per unit of time* (velocity is a **vector**, speed is not)



Even for constant speed, where is the runner's velocity changing and where is it constant?

Who here has seen/used vectors before?

So far, we've discussed only velocity, but we will see many other vectors


Have you been using only scalars all your life?
It's time for your suffering to end.


Oops, sorry, Vecna, not Vector. Carry on….

Scalars and Vectors

A **scalar quantity** can have magnitude, algebraic sign (positive or negative), and units, but not a direction in space.

When scalars are added or subtracted, they do so in the usual way: 3 kg of water plus 2 kg of water is always equal to 5 kg of water. **What sorts of scalars have we already seen in this course?**

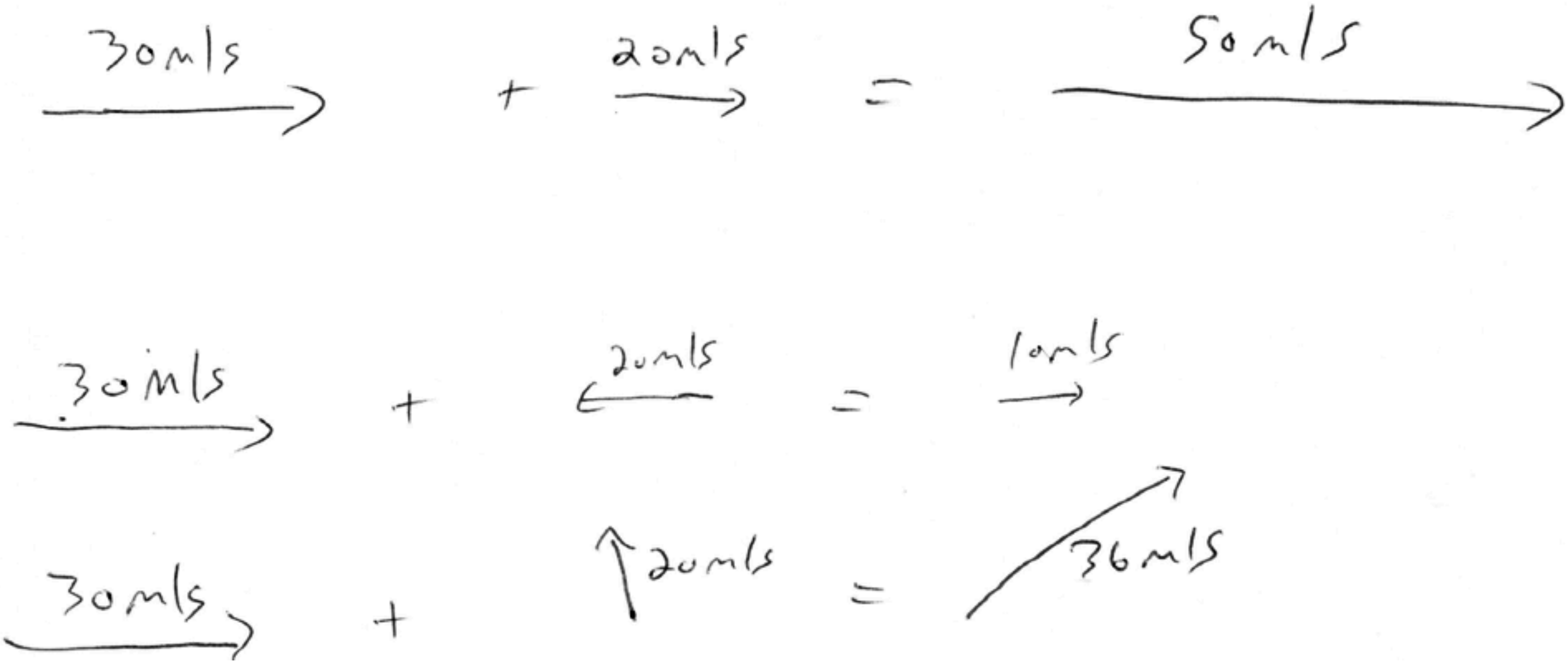
Adding vectors is different. All vectors follow a different rule of addition that takes into account the directions of the vectors being added.

A 30 m/s velocity added to a 20 m/s velocity can give different results depending on the directions of the two velocities (since velocities are vectors)

If you are traveling 30 m/s in one direction and then your velocity changes by 20 m/s in the same direction you travel, you are suddenly moving much faster.

If the velocity change is in the opposite direction, the two partially cancel, and the end speed is much smaller.

Let's look at this pictorially on the board



We'll discuss this last one soon!

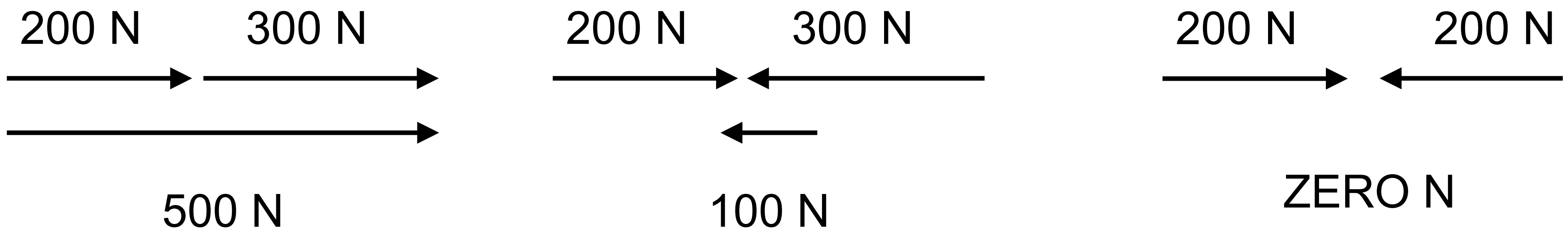
An arrow over a symbol or a **boldface** indicates a vector quantity:

When the symbol for a vector is written without the arrow and in italics rather than boldface (*v*), it stands for the *magnitude* of the vector (which is a scalar).

Absolute value bars are also used to stand for the magnitude of a vector, so $F = |\vec{\mathbf{F}}|$.

The magnitude of a vector may have units and is never negative; it can be positive or zero.

Always be careful about whether you are using scalars or vectors!



Order of addition does not matter

Unit Vector Notation

The **unit vectors** $\hat{\mathbf{x}}$ (read aloud as "x hat"), $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$

are defined as vectors of magnitude 1 that point in the +*x*, +*y*, and +*z* directions, respectively.

In some books, you may see them written as $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$, respectively.

They are called *unit* vectors because the magnitude of each is the pure number 1—they do *not* have physical units such as kilograms or meters.

We will often refer to ihat, jhat and khat :)

Unit Vector Notation

Any vector $\vec{\mathbf{F}}$ can be written as the sum of three vectors along the **coordinate axes**:

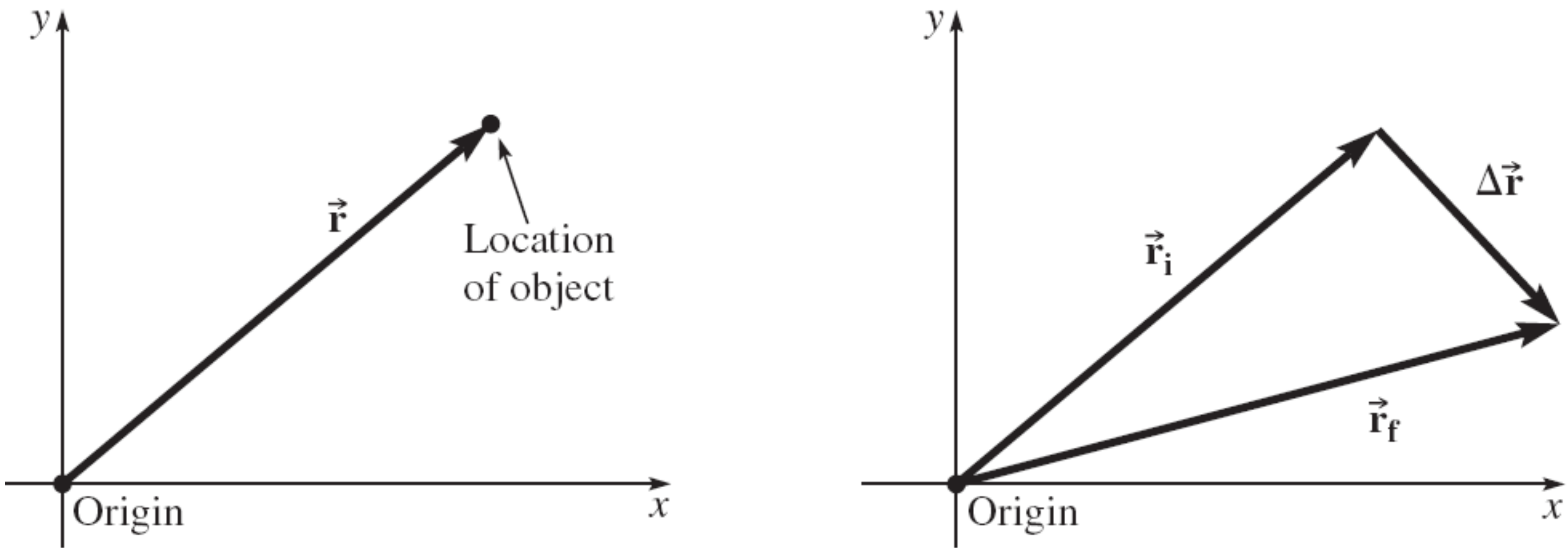
$$\vec{\mathbf{F}} = F_x\hat{\mathbf{x}} + F_y\hat{\mathbf{y}} + F_z\hat{\mathbf{z}}$$

If using two dimensions:

$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \left(F_{1x}\hat{\mathbf{x}} + F_{1y}\hat{\mathbf{y}}\right) + \left(F_{2x}\hat{\mathbf{x}} + F_{2y}\hat{\mathbf{y}}\right)$$

$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \left(F_{1x} + F_{2x}\right)\hat{\mathbf{x}} + \left(F_{1y} + F_{2y}\right)\hat{\mathbf{y}}$$

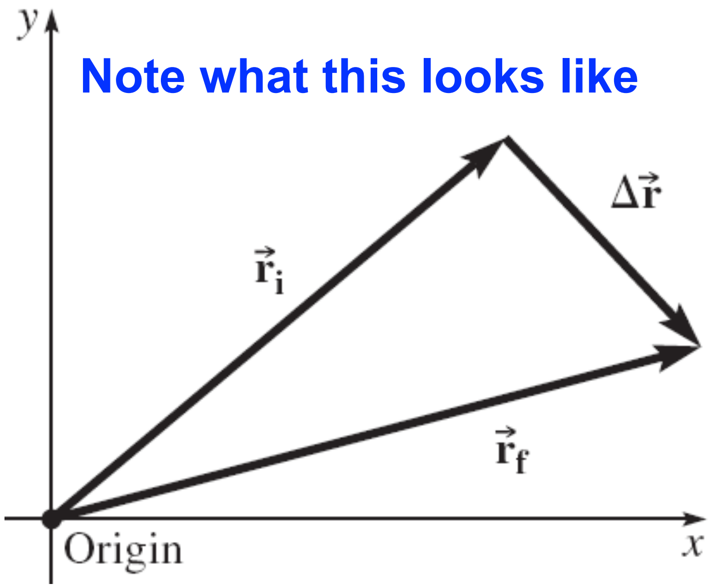
Position



To describe the location of something, we give its distance from the origin and the direction. These two quantities, direction and distance, together constitute a **vector** quantity called the **position** of the object (symbol $\vec{\mathbf{r}}$).

Displacement

Displacement is defined as the change of the ***position vector***: the final position vector minus the initial position vector.



Displacement is $\Delta\vec{\mathbf{r}}$

written $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$

What if we reverse the order?

The symbol Δ means ***the change in*** the quantity that follows. **It is ALWAYS: (final - initial)**

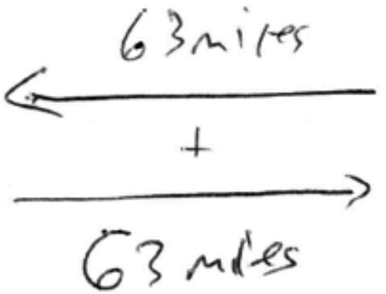
Simplest case

I drive 63 miles west on I-88. i then drive 63 miles east on I-88. What is my total displacement? Let's draw this on the board as vector addition

Simplest case

I drive 63 miles west on I-88. i then drive 63 miles east on I-88. What is my total displacement? Let's draw this on the board as vector addition

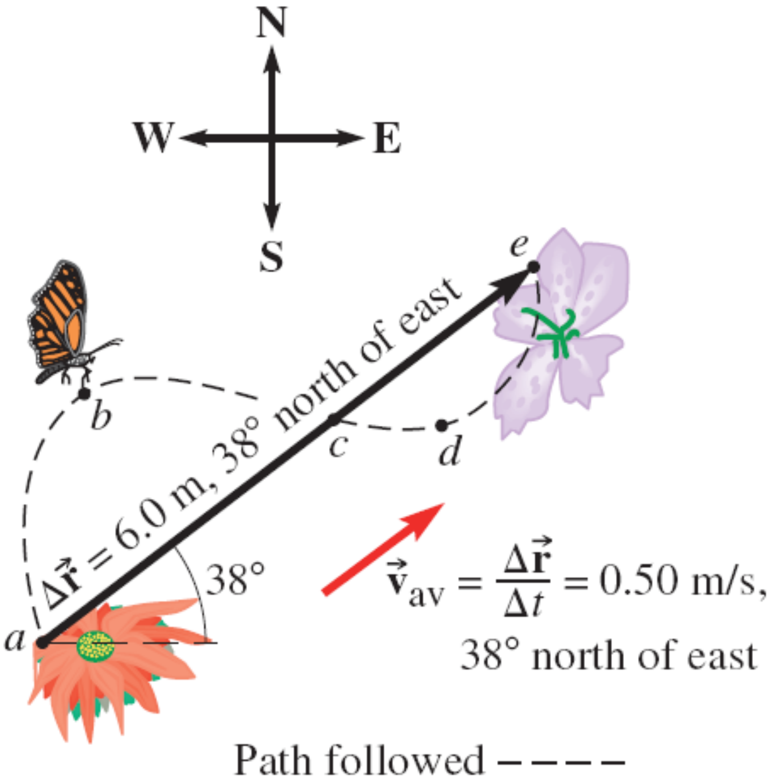
63 miles west 63 miles east = 0 displacement!



= 0, even if distance traveled is 126 miles

Average Speed Versus Average Velocity

The direction of the **average velocity** is the same as the direction of the displacement vector $\Delta\vec{\mathbf{r}}$.



The average velocity would be the same for any other path that takes the butterfly from *a* to *e* in the same amount of time Δt, because both the displacement and the time interval would be the same. However, the ***average speed*** would depend on the total *distance* traveled.

Because one is a vector and the other is not!

Average Velocity

$$\vec{\mathbf{v}}_{av} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i}{t_f - t_i}$$

$$v_{av,x} = \frac{\Delta x}{\Delta t} \quad \text{and} \quad v_{av,y} = \frac{\Delta y}{\Delta t}$$

Driving in the ‘burbs

I start out in Chicago at 4:14 pm. I drive due West 32 miles to Aurora, where I arrive at 6:30 pm (rush hour traffic!) to pick up my sister. I then drive due East for 22 miles to Oak Park, where I drop my sister off at 7:31 pm. I continue back due West another 46 miles and arrive in DeKalb at 8:53 pm. PHEW

What is my total displacement?
What is my average speed?
What is my average velocity?

Your Prof and his imaginary Tesla

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What is my total displacement?

What is my average speed?

What is my average velocity?

32 miles west + 22 miles east

+ 46 miles west = (32 + 46 − 22) miles west

= 56 miles west = displacement

Distance travelled = (32 + 22 + 46) miles = 100 miles

I start out in Chicago at 4:14 pm. I drive due West 32 miles to Aurora, where I arrive at 6:30 pm (rush hour traffic!) to pick up my sister. I then drive due East for 22 miles to Oak Park, where I drop my sister off at 7:31 pm. I continue back due West another 46 miles and arrive in DeKalb at 8:53 pm. PHEW What is my total displacement? Average speed? Average velocity?

time = 853 pm − 414 pm = 4 hours 39 minutes

$= 4\ \text{hrs} \times \frac{60\ \text{min}}{\text{hr}} + 39\ \text{mins} = \underline{279\ \text{min}}$

So $\underline{\text{avg speed}} = \frac{\text{Distance}}{\Delta t} = \frac{100\ \text{miles}}{279\ \text{min}} = 0.36\ \frac{\text{miles}}{\text{min}}$

And in more familiar units $0.36\ \frac{\text{miles}}{\text{min}} \times \frac{60\ \text{min}}{\text{hr}}$

$= 22\ \text{miles}/\text{hr}$

I start out in Chicago at 4:14 pm. I drive due West 32 miles to Aurora, where I arrive at 6:30 pm (rush hour traffic!) to pick up my sister. I then drive due East for 22 miles to Oak Park, where I drop my sister off at 7:31 pm. I continue back due West another 46 miles and arrive in DeKalb at 8:53 pm. PHEW What is my total displacement? Average speed? Average velocity?

$$\text{Avg velocity} = \frac{\text{Displacement}}{\Delta t} = \frac{56\ \text{miles west}}{279\ \text{min}} \times \frac{60\ \text{min}}{\text{hr}} = 12\ \text{mph west}$$

Instantaneous Velocity

The **instantaneous velocity** $\vec{\mathbf{v}}$ is a vector quantity whose magnitude is the speed and whose direction is the direction of motion.

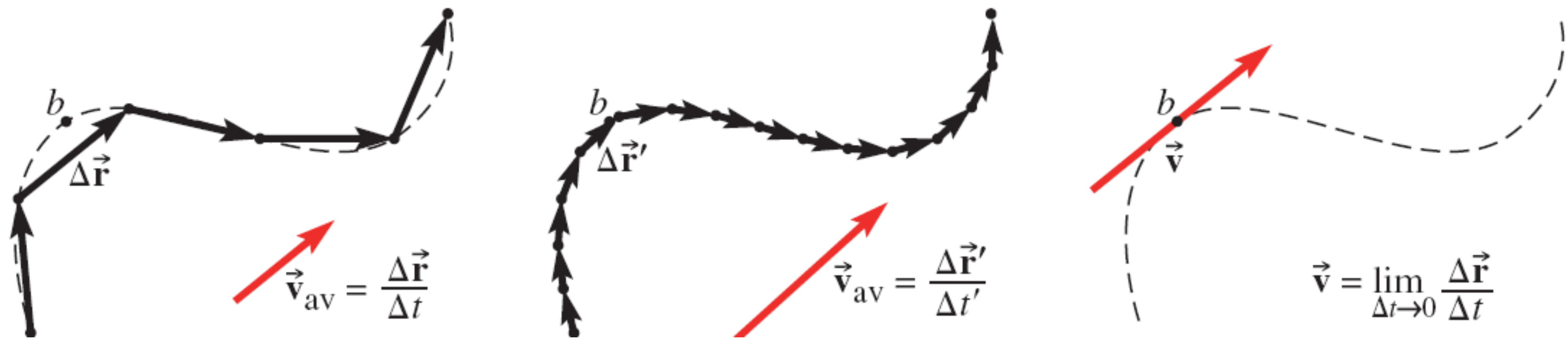
The instantaneous velocity can be used to calculate the displacement of the object during a very short time interval.

Definition of instantaneous velocity:

$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$$

($\Delta \vec{\mathbf{r}}$ is the displacement during a *very small* time interval Δt)

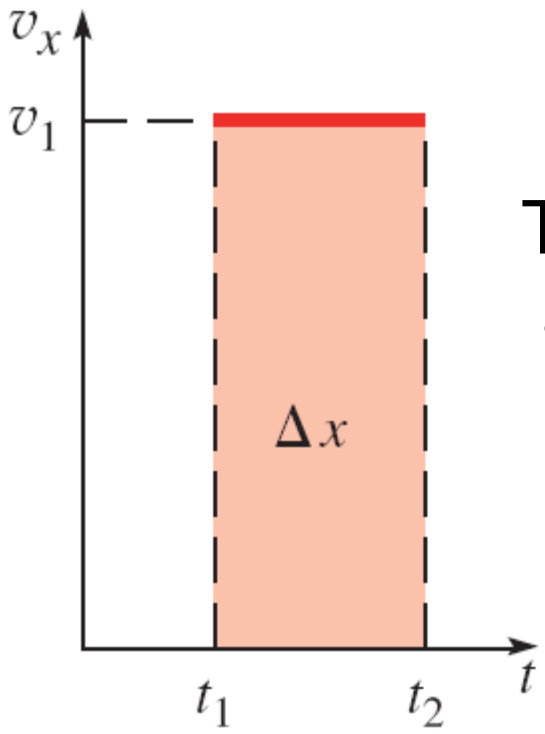
$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \quad \text{and} \quad v_y = \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}$$



Finding Displacement with Constant Velocity

$$v_x = v_{\text{av},x} = \frac{\Delta x}{\Delta t}$$

$$\Delta x = v_x \, \Delta t \quad (\text{for constant } v_x)$$

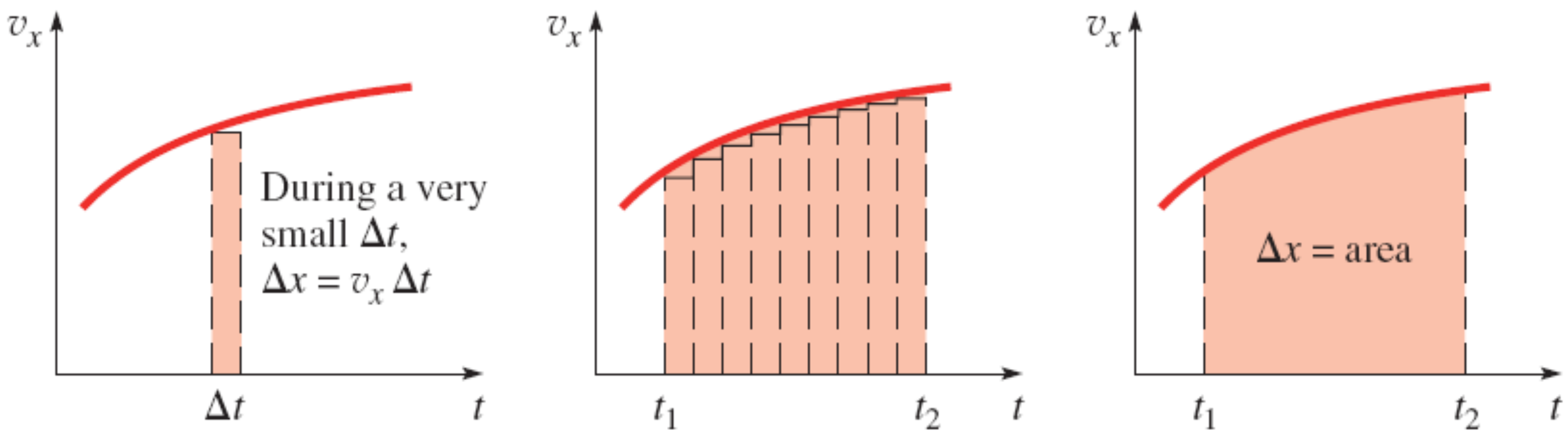


The displacement is the *area* under the curve.

Finding Displacement with Changing Velocity

Δx is the area under the graph of $v_x(t)$.

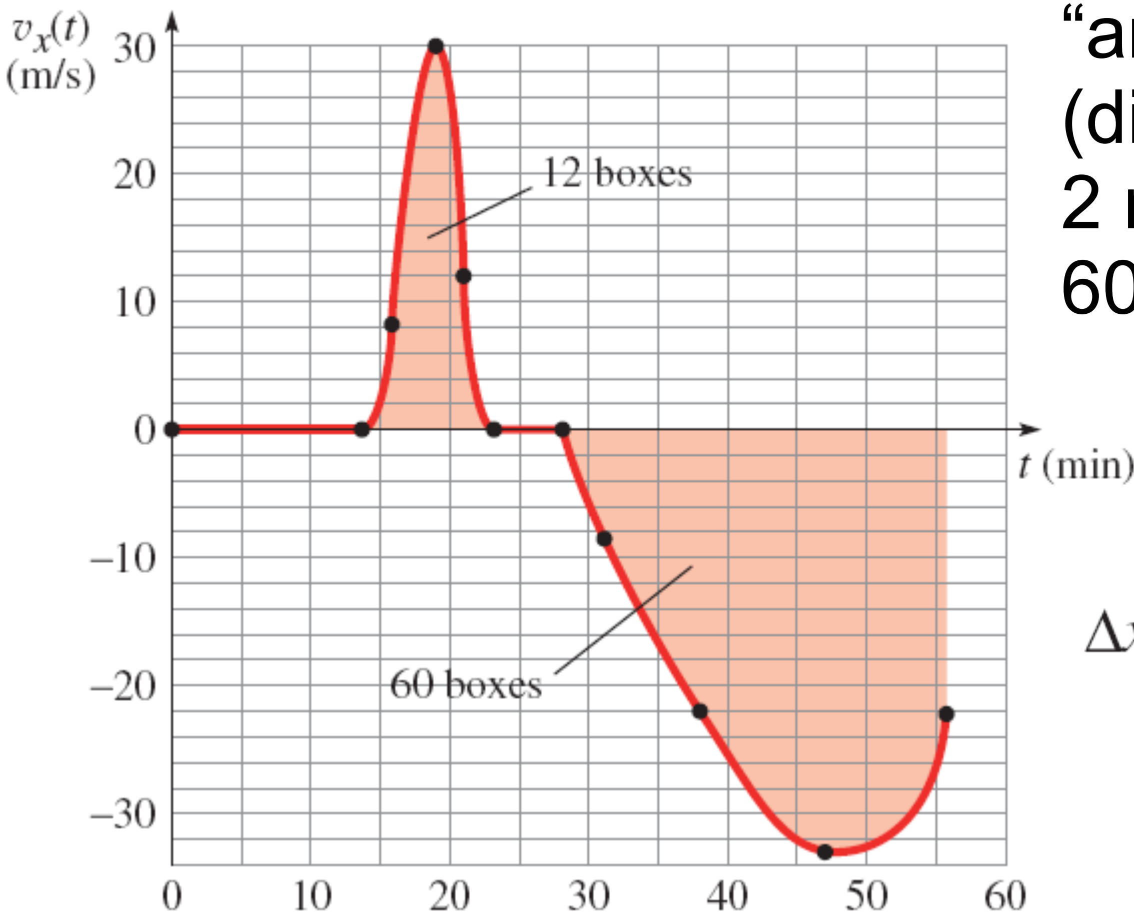
The area is negative when the graph is beneath the time axis ($v_x < 0$).



Taken calculus already? This should indicate to you that the displacement is the integral of the velocity vector! If you're taking calc for the first time this semester, please go back to this once you do integrals

Finding Displacement with Changing Velocity

$$\Delta x \approx 12 \times 0.60 \text{ km} = +7.2 \text{ km}$$

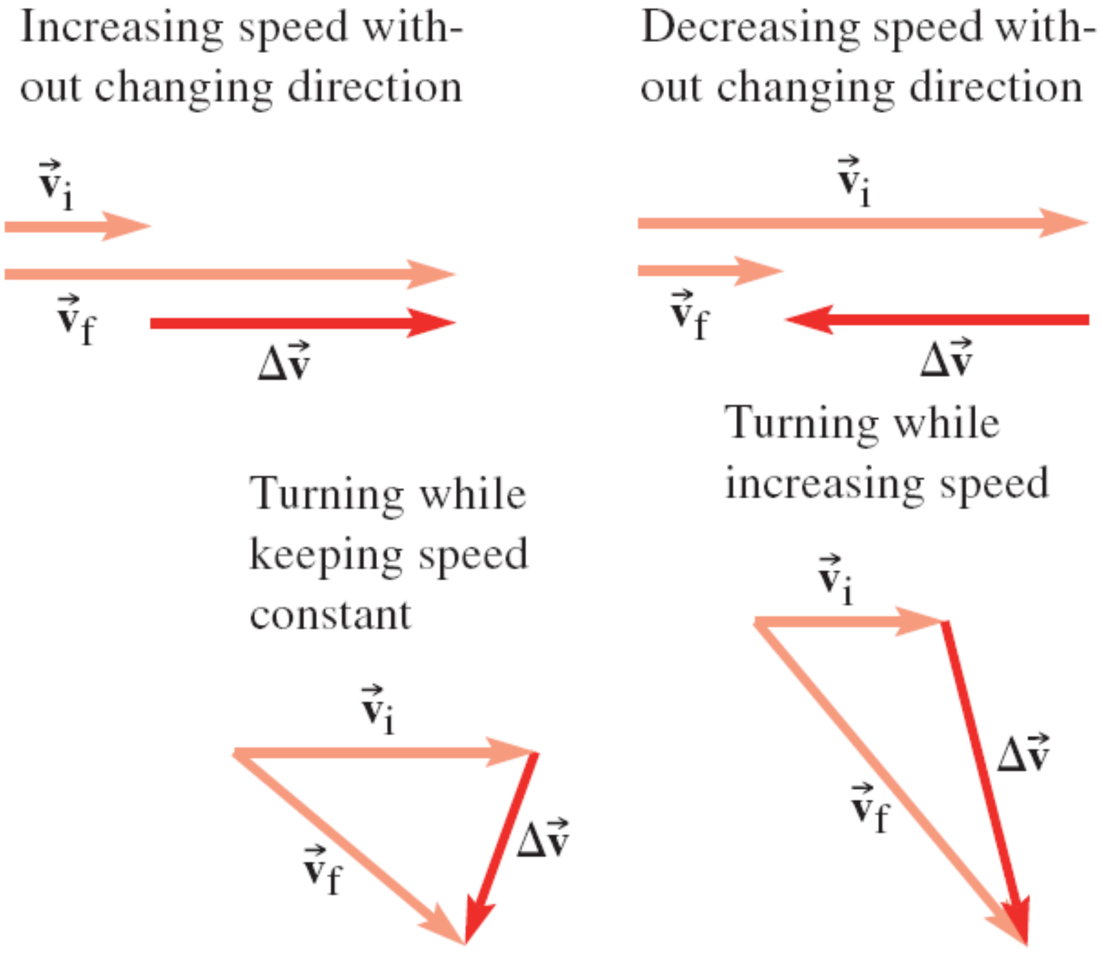


Each box represents an "area" (displacement) of 2 m/s × 300 s = 600 m = 0.60 km.

$$\Delta x \approx -(60) \times 0.60 \text{ km} = -36 \text{ km}$$

Changing velocity

The direction of the change in velocity $\Delta\vec{v}$ is **not** necessarily the same as either the initial or the final velocity direction.



Definition of Acceleration (symbol $\vec{\mathbf{a}}$)

The rate of change of the velocity - the quantity $\Delta\vec{\mathbf{v}}/\Delta t$ - is called the **acceleration**. It is ALSO a vector

Average and Instantaneous Acceleration

First we define the **average acceleration** during a time interval Δt:

$$\vec{\mathbf{a}}_{\text{av}} = \frac{\vec{\mathbf{v}}_{\text{f}} - \vec{\mathbf{v}}_{\text{i}}}{t_{\text{f}} - t_{\text{i}}} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

$$a_{\text{av},x} = \frac{\Delta v_x}{\Delta t} \quad \text{and} \quad a_{\text{av},y} = \frac{\Delta v_y}{\Delta t}$$

Definition of instantaneous acceleration:

$$\vec{\mathbf{a}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{v}}}{\Delta t}$$

($\Delta \vec{\mathbf{v}}$ is the change in velocity during a *very small* time interval Δt)

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \quad \text{and} \quad a_y = \lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t}$$

A constant acceleration means the velocity vector *changes at a constant rate*.

Now we investigate how the *position* changes in this important case when the acceleration is constant.

The *equations* we introduce next are relationships for each component direction between position, velocity, acceleration, and time that apply when something moves with constant acceleration.

Constant acceleration is a very special case, but we can always look at a tiny moment in time in which the acceleration does not change too much!

Important equation/relationship from the definition of a:

$$\Delta v_x = v_{fx} - v_{ix} = a_x \Delta t$$

(if a_x is constant during the entire time interval)

Change in position uses an **average** of the two velocities

Why the average?
Integral calculus

$$\Delta x = \tfrac{1}{2}(v_{fx} + v_{ix})\ \Delta t$$

(if a_x is constant during the entire time interval)

After some substitution and rearrangement:

$$\Delta x = v_{ix}\ \Delta t + \tfrac{1}{2}a_x(\Delta t)^2 \qquad \text{(constant } a_x\text{)}$$

$$v_{fx}^2 - v_{ix}^2 = 2a_x\ \Delta x \qquad \text{(constant } a_x\text{)}$$

Let's work this out together on the board:
First we eliminate v_{fx} and next time we eliminate dt

Important equation/relationship from the definition of a:

$$\Delta v_x = v_{fx} - v_{ix} = a_x \Delta t \qquad \Delta x = \frac{1}{2}(v_{fx} + v_{ix})\,\Delta t$$

From first, $v_f = v_i + a\Delta t$ ← equate them

From second, $\frac{2\Delta x}{\Delta t} = v_f + v_i$, $v_f = \frac{2\Delta x}{\Delta t} - v_i$

$$v_i + a\Delta t = \frac{2\Delta x}{\Delta t} - v_i$$

$$2v_i + a\Delta t = \frac{2\Delta x}{\Delta t} \qquad 2v_i\Delta t + a(\Delta t)^2 = 2\Delta x$$

$$\Delta x = v_i\Delta t + \frac{1}{2}a(\Delta t)^2$$ ✓

Important equation/relationship from the definition of a:

$$\Delta v_x = v_{fx} - v_{ix} = a_x \Delta t \qquad \Delta x = \frac{1}{2}(v_{fx} + v_{ix})\,\Delta t$$

From first, $\Delta t = \dfrac{V_f - V_i}{a}$ (equate them)

from second, $\Delta t = \dfrac{2\Delta x}{V_f + V_i}$

$$\frac{V_f - V_i}{a} = \frac{2\Delta x}{V_f + V_i}$$

cross multiply $\rightarrow (V_f - V_i)(V_f + V_i) = 2a\Delta x$

$\rightarrow V_f^2 - V_i^2 = 2a\Delta x$

Remember that knowing initial and final value of something is equivalent to knowing its initial value and it's change in value, which is equivalent to knowing the final value and it's change in value

$$\Delta v_x = v_{fx} - v_{ix} = a_x \Delta t$$

If you don't care about position this relates the acceleration, intervals of time and changes in velocity

$$\Delta x = \tfrac{1}{2}(v_{fx} + v_{ix})\ \Delta t$$

Relates the initial and final velocities, the interval of time and the change in position, and doesn't mention acceleration

$$\Delta x = v_{ix}\ \Delta t + \tfrac{1}{2} a_x (\Delta t)^2 \qquad (\text{constant } a_x)$$

Useful for relating change in position, initial velocity and acceleration in an interval of time (don't care about final velocity)

$$v_{fx}^2 - v_{ix}^2 = 2 a_x\ \Delta x \qquad (\text{constant } a_x)$$

Relates initial and final velocities, acceleration and change in position (don't care about interval of time)

In **free fall**, gravity forces an object to fall

On Earth, free fall is an idealization since there is always *some* air resistance.

Free-Fall Acceleration $\vec{a} = \vec{g}$ **(Vectors!!!)**

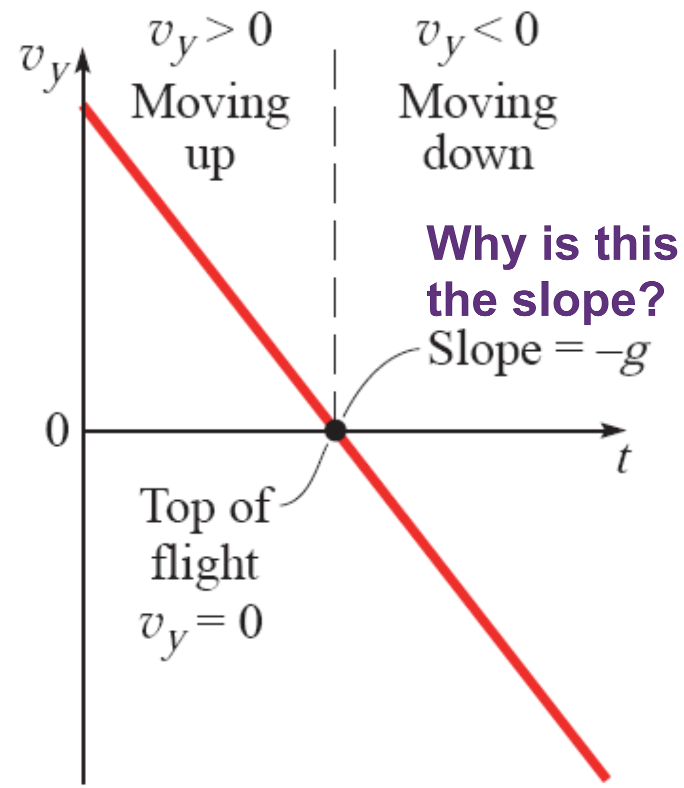
$$g = 9.8\ \text{m/s}^2$$

The vector $\vec{g}$ is sometimes called *the free-fall acceleration* because it is the acceleration of an object near the surface of the Earth

Acceleration at Highest Point

If an object is thrown straight up, its velocity is zero at the highest point of its flight.

(Pause to think about this)



Why? On the way up, the *y*-component of its velocity (v_y) is positive if the positive *y*-axis is pointing up. On the way down, v_y is negative.

Since v_y changes continuously, it must pass through zero to change sign. At the highest point, the velocity is zero but the *acceleration* is *not* zero.

Standing on a bridge, you throw a stone straight upward. The stone hits a stream, 44.1 m below the point at which you release it, 4.00 s later.

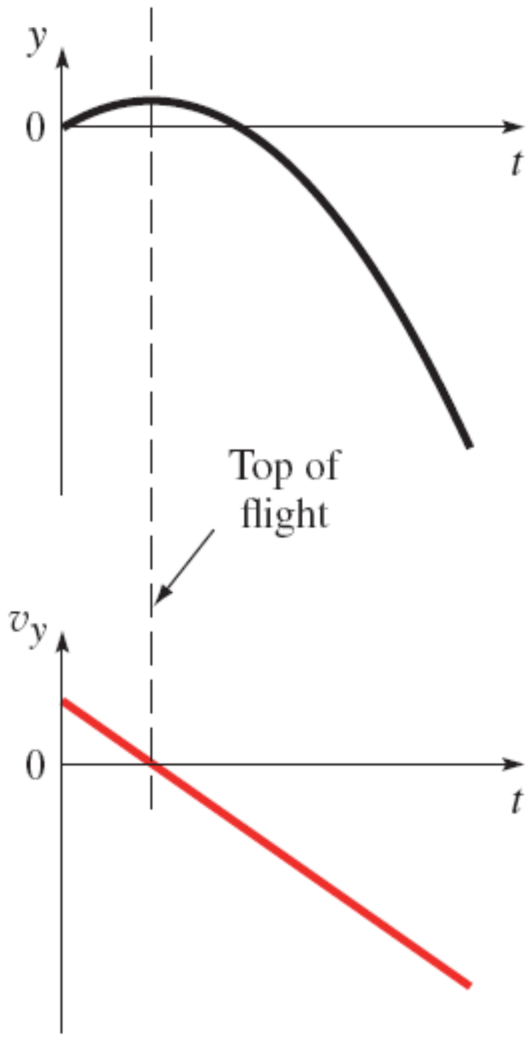
(a) Sketch graphs of $y(t)$ and $v_y(t)$. The positive y-axis points up.

(b) What is the velocity of the stone just after it leaves your hand?

(c) What is the velocity of the stone just before it hits the water?

Solution

(a)



Solution (b)

(Careful about signs!!!)

$$\Delta y = v_{iy}\,\Delta t + \frac{1}{2}a_y\,(\Delta t)^2$$

$$v_{iy} = \frac{\Delta y}{\Delta t} - \frac{1}{2}a_y\,\Delta t = \frac{-44.1\text{ m}}{4.00\text{ s}} - \frac{1}{2}(-9.80\text{ m/s}^2 \times 4.00\text{ s})$$

$$= -11.0\text{ m/s} + 19.6\text{ m/s} = 8.6\text{ m/s}$$

The initial velocity is 8.6 m/s upward.

Solution (c)

$$v_{fy}^2 = v_{iy}^2 + 2a\Delta y$$

$$v_{fy} = \sqrt{v_{iy}^2 + 2a\Delta y}$$

$$v_{fy} = \sqrt{(8.6m/s)^2 + 2(-9.8m/s^2)(-44.1m)}$$

$$v_{fy} = 30.6m/s$$

We know the direction was DOWN!

Solution (c, another way to do this, gives the same answer)

$$v_{fy} = v_{iy} + a_y t$$

$$v_{fy} = +8.6m/s - (9.8m/s)(4.0s) = -30.6m/s$$

Gives the right sign !

(Because there's no squaring of numbers to remove the sign, as in the previous solution)

A Fedex truck and a UPS truck start out in adjacent lanes moving in the (+)x direction with constant accelerations.

The FedEx truck starts with an initial velocity of 20 m/s and has an acceleration of +0.400 m/s^2. The UPS truck starts with a velocity of 30 m/s and has an acceleration of −0.400 m/s^2.

Find the time at which the FedEx truck overtakes the UPS truck

Solution

(a)

$$x_f = x_i + \Delta x = x_i + v_{ix}\,\Delta t + \frac{1}{2}a_x(\Delta t)^2$$

$$\overset{0}{x_{i,U}} + v_{i,U}t + \frac{1}{2}a_U t^2 = \overset{0}{x_{i,F}} + v_{i,F}t + \frac{1}{2}a_F t^2$$

$$t(v_{i,U} + \frac{1}{2}a_U t - v_{i,F} - \frac{1}{2}a_F t) = 0$$

Meaning?

$$\Delta t = 0$$

$$t = \frac{2(v_{i,U} - v_{i,F})}{a_F - a_U} = \frac{2(30m/s - 20m/s)}{0.4m/s^2 - (-0.4m/s^2)} = 25s$$

The FedEx truck overtakes the UPS truck 25 s after they leave the starting point.

Group work time!

https://forms.gle/YP8Cu5XZFoGYxVtp9

A mule hauls the farmer's wagon along a straight road for 6.6 km directly south to the neighboring farm where a few bushels of corn are loaded onto the wagon. Then the farmer drives the mule back along the same straight road, heading north for 7.7 km to the market.

a) Find the displacement of the mule from the starting point to the market.
b) If the entire trip took 6 hours and we ignore the time to load the corn, what was the mule's average speed?
c) If the entire trip took 6 hours and we ignore the time to load the corn, what was the magnitude of the mule's average velocity?

101 MPH!

Nolan Ryan threw a baseball pitch measured at 45.1 m/s. If he released the ball 18.4 m away from home plate, how long did it take the ball to get to the batter? (Ignore friction, air resistance and gravity)

A cheetah can accelerate from rest to 24 m/s in 2.0s. Assuming the acceleration is constant over that time interval:

a) What is the magnitude of the acceleration?
b) What is the distance traveled by the cheetah in those 2.0 seconds?
c) A runner can accelerate from rest to 6.0 m/s in 2.0 seconds. By what factor is the cheetah's average acceleration magnitude greater than that of the runner?

a) Give an example of a moment in real life when your velocity is zero but your acceleration is non-zero.
b) Give an example of a moment in real life when your velocity is non-zero but your acceleration is zero.
c) Draw position vs time and velocity vs time curves for each of these cases

A uniformly accelerating rocket is found to have a velocity of 15.0 m/s when its height is 5.00 m above the ground, and 1.50 s later the rocket is at a height of 58.0 m. What is the magnitude of its acceleration?

Grant jumps 1.3 meters straight up into the air to slam-dunk a basketball into the net. (He doesn’t rise above the net). With what speed did he leave the floor?

A race car travels 825 km around a circular track of radius 1.313 km. How many times did it go around the track?

During a relay race, you run the first leg of a race, a distance of 2.0 x 10^2 m to the north in 22.23 seconds. You then run the same distance back south in 24.15 seconds. Suppose the positive y axis points north.

a) What is your average velocity for the first leg of the race?
b) What is your average velocity for the entire race?
c) What is your average speed for the first leg of the race?
d) What is your average speed for the entire race?

An electronic line judge captures the impact of a 57.0-g tennis ball traveling at 33.0 m/s with the side line of a tennis court. The ball rebounds with a speed of 20.0 m/s and is seen to be in contact with the ground for 4.00 ms. What is the magnitude of the average acceleration of the ball during the time it is in contact with the ground? Assume one-dimensional motion.

A driver uniformly accelerates his car such that the acceleration vector **a** = 6.8 m/s^2 in the i-hat direction.

a) Assuming he starts from rest, find the velocity of the car after it has accelerated for 4.5 s.
b) If immediately after that 4.5 s the driver lays off the accelerator, slams on the brakes, and comes to a stop in the subsequent 5.6 s, what is the average acceleration he experiences during that time?
c) What is the average acceleration during the total 4.5s+5.6s = 10.1 seconds?

Accelerating uniformly to overtake a slow-moving truck, a car moving initially at 24 m/s covers 68 m in 2.5 s.

a) What is the final speed of the car?

b) What is the magnitude of the car's acceleration during that time?

A pebble is thrown downward from a 44.0-m high cliff with an initial speed of 7.70 m/s.

a) How long does it take the pebble to reach the ground?
b) What is the pebble's speed when it reaches the ground?

A rock is thrown straight up in the air with an initial speed of 24 m/s at time t=0. Ignore air resistance.

a) At what time does it move with a speed of 12 m/s? (There are two answers to the problem!)
b) For a rock launched up with initial speed v, determine an expression for the time t (in terms of g) for the time when the rock is moving with speed v/2 (there are again two answers, as there should be!)

A person climbs from a Paris metro station to the street level by walking up a stalled escalator in 94 s. It takes 66 s to ride the same distance when standing on the escalator when it is operating normally. How long would it take for him to climb from the station to the street by walking up the moving escalator

It's faster to walk on a moving escalator, from experience

PHYS253 Chapter 2

An airplane is traveling in a horizontal direction at 180 m/s when the pilot receives notice to increase her air speed. She throttles the engine such that the 747 undergoes constant acceleration at 2.2 m/s^2 in the same direction it is already moving.

1) What is the plane's new speed after traveling for an additional 3.2 km?
2) How far does the plane travel before reaching a speed of 240 m/s?

You step out of an airplane (better you than me!) and free-fall towards earth. Ignore friction and air resistance. You start from rest when you fall out of the plane, and pull your parachute after falling 200 meters. What is your velocity when you pull your chute?

You step out of an airplane (better you than me!) and free-fall towards earth. Ignore friction and air resistance. You start from rest when you fall out of the plane, and pull your parachute after falling 200 meters. How long are you in free-fall before you pull your chute?