Chapter 13+

(Some material taken from McGraw-Hill/College Physics/Giambattista, the majority from your Katz textbook)

Rotational kinetic energy:

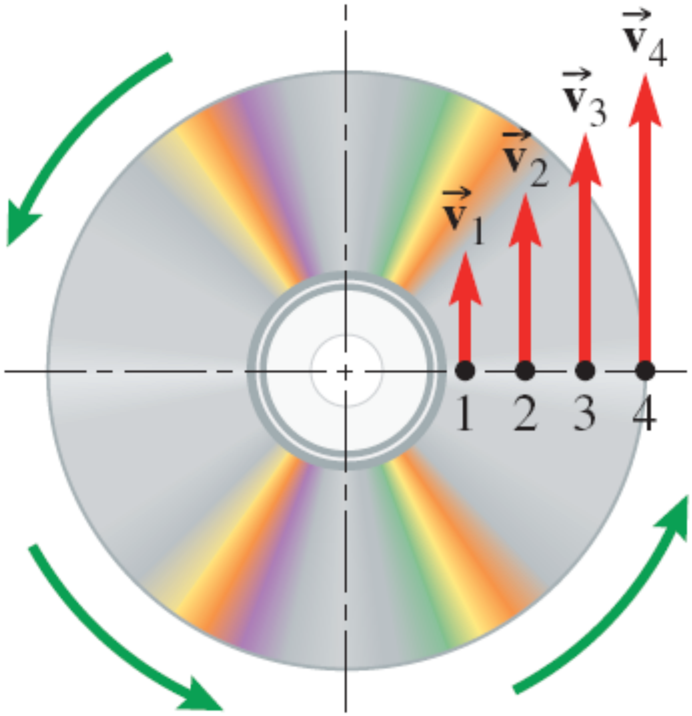
$$K_{rot} = \tfrac{1}{2}m_1 v_1^2 + \tfrac{1}{2}m_2 v_2^2 + \cdots + \tfrac{1}{2}m_N v_N^2 = \sum_{i=1}^{N} \tfrac{1}{2}m_i v_i^2$$

From last chapter and earlier!

$$v = r\omega$$

$$K_{rot} = \sum_{i=1}^{N} \tfrac{1}{2}m_i r_i^2 \omega^2$$

$$K_{rot} = \frac{1}{2}\left(\sum_{i=1}^{N} m_i r_i^2\right)\omega^2$$



Let's give the quantity in the parentheses the symbol I. Note that it depends on the object, not how fast it is rotating

Rotational kinetic energy:

$$K_{\text{rot}} = \tfrac{1}{2} I \omega^2$$

The quantity *I* is called the **rotational inertia**. It is also called the **moment of inertia**:

$$I = \sum_{i=1}^{N} m_i r_i^2 \qquad (\text{SI unit: kg}\cdot\text{m}^2)$$

Note that it **does** depend on the **axis of rotation**! (In other words, which way you rotate the object)

Rotational kinetic energy of this frisbee depends on which way you spin it! (AKA the choice of axis of rotation!)

vs

Calculating rotational moment of inertia:

1. If the object consists of a *small* number of particles, calculate the sum directly.

$$I = \sum_{i=1}^{N} m_i r_i^2$$

2. Since the rotational inertia is a sum, if possible, you can imagine breaking the object into several parts. Find the rotational inertia of each part, and then add them like any other number.

3. For symmetrical objects with simple geometric shapes, you can calculate them with calculus

$$I = \sum_{i=1}^{N} m_i r_i^2 \longrightarrow$$

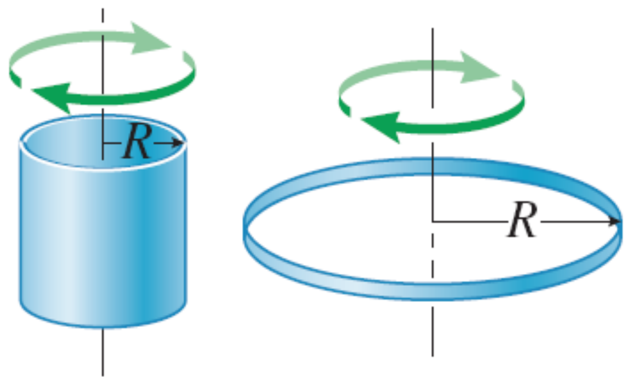
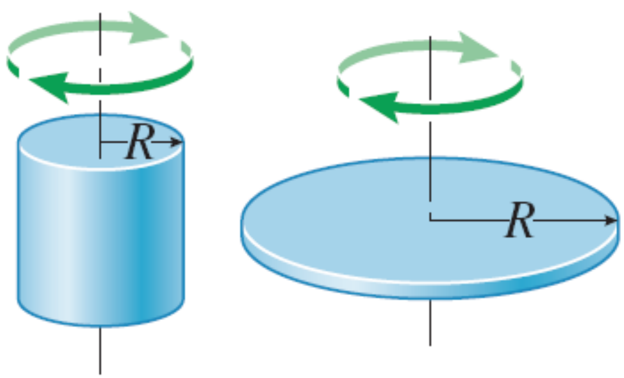
$$dI = r^2 dm$$

$$I = \int r^2 dm$$

NOT THIS SEMESTER!

Rotational Inertia for Uniform Objects with Various Geometrical Shapes

Good to check units, but you don't have to remember these!

Shape		Axis of Rotation	Rotational Inertia
Thin hollow cylindrical shell (or hoop)		Central axis of cylinder	MR^2
Solid cylinder (or disk)		Central axis of cylinder	$\frac{1}{2}MR^2$

Rotational Inertia for Uniform Objects with Various Geometrical Shapes

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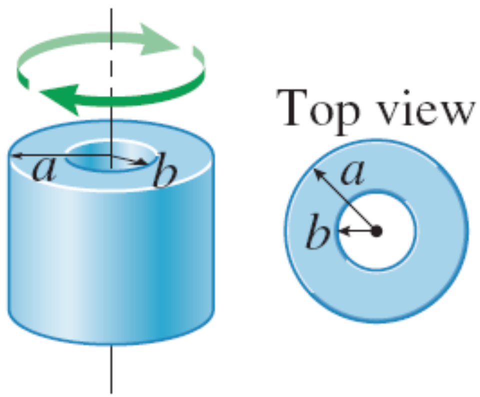
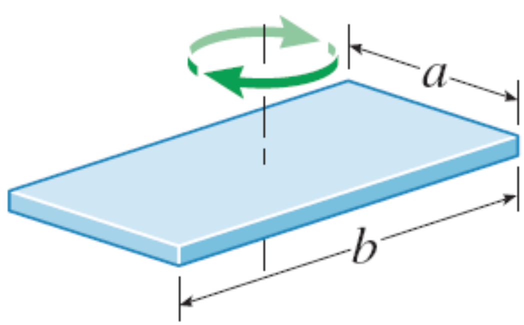
Shape		Axis of Rotation	Rotational Inertia
Hollow cylindrical shell or disk		Central axis of cylinder	$\frac{1}{2}M(a^2 + b^2)$
Rectangular plate		Perpendicular to plate through center	$\frac{1}{12}M(a^2 + b^2)$

Rotational Inertia for Uniform Objects with Various Geometrical Shapes

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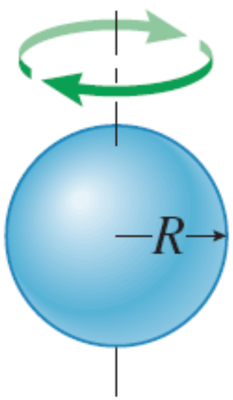
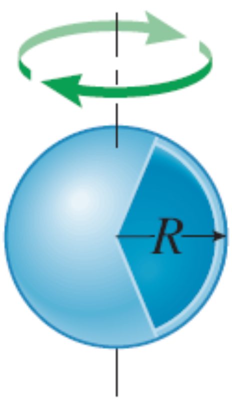
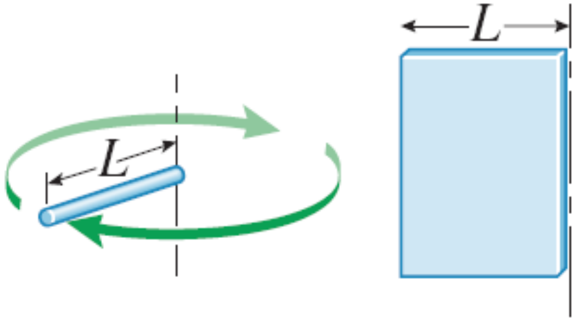
Shape		Axis of Rotation	Rotational Inertia
Solid sphere		Through center	$\frac{2}{5}MR^2$
Thin hollow spherical shell		Through center	$\frac{2}{3}MR^2$

Rotational Inertia for Uniform Objects with Various Geometrical Shapes

Good to check units, but you don't have to remember these!

Shape		Axis of Rotation	Rotational Inertia
Thin rod (or rectangular plate)		Perpendicular to rod through end (or along edge of plate)	$\frac{1}{3}ML^2$

Rotational Inertia for Uniform Objects with Various Geometrical Shapes

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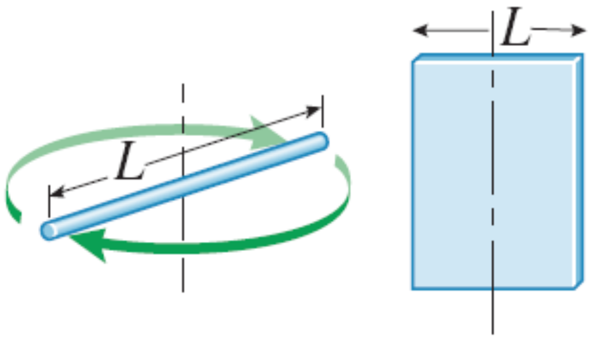
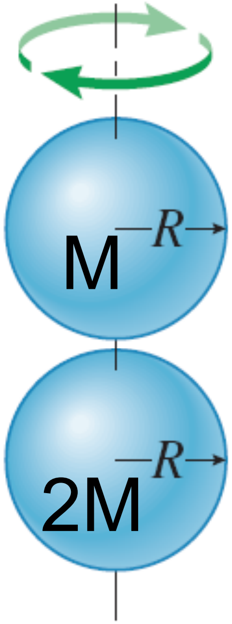
Shape		Axis of Rotation	Rotational Inertia
Thin rod (or rectangular plate)		Perpendicular to rod through center (or parallel to edge of plate through center)	$\frac{1}{12}ML^2$

Two spheres of Radius R are connected as in the picture below. If one sphere has a mass M and the other sphere has a mass 2M, what is the total moment of inertia for rotation about an axis through the center of both spheres?



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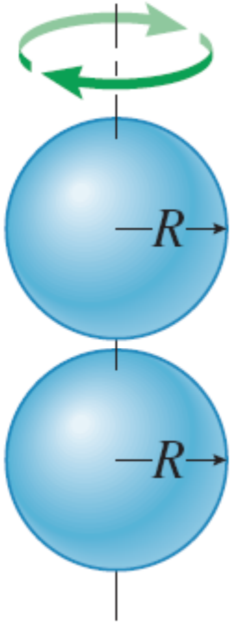


$$I = \frac{2}{5}MR^2 + \frac{2}{5}(2M)R^2 = MR^2\left(\frac{2}{5} + \frac{4}{5}\right) = \frac{6}{5}MR^2$$

Moments of inertia about a common rotation axis simply add!

When applying conservation of energy to objects that rotate, the rotational kinetic energy is included in the mechanical energy. When we say

$$W_{\text{nc}} = \Delta K + \Delta U$$

just as *U* stands for the sum of the elastic and gravitational potential energies, *K* stands for the sum of the translational and rotational kinetic energies:

$$K = K_{\text{tr}} + K_{\text{rot}}$$

Work done by a torque

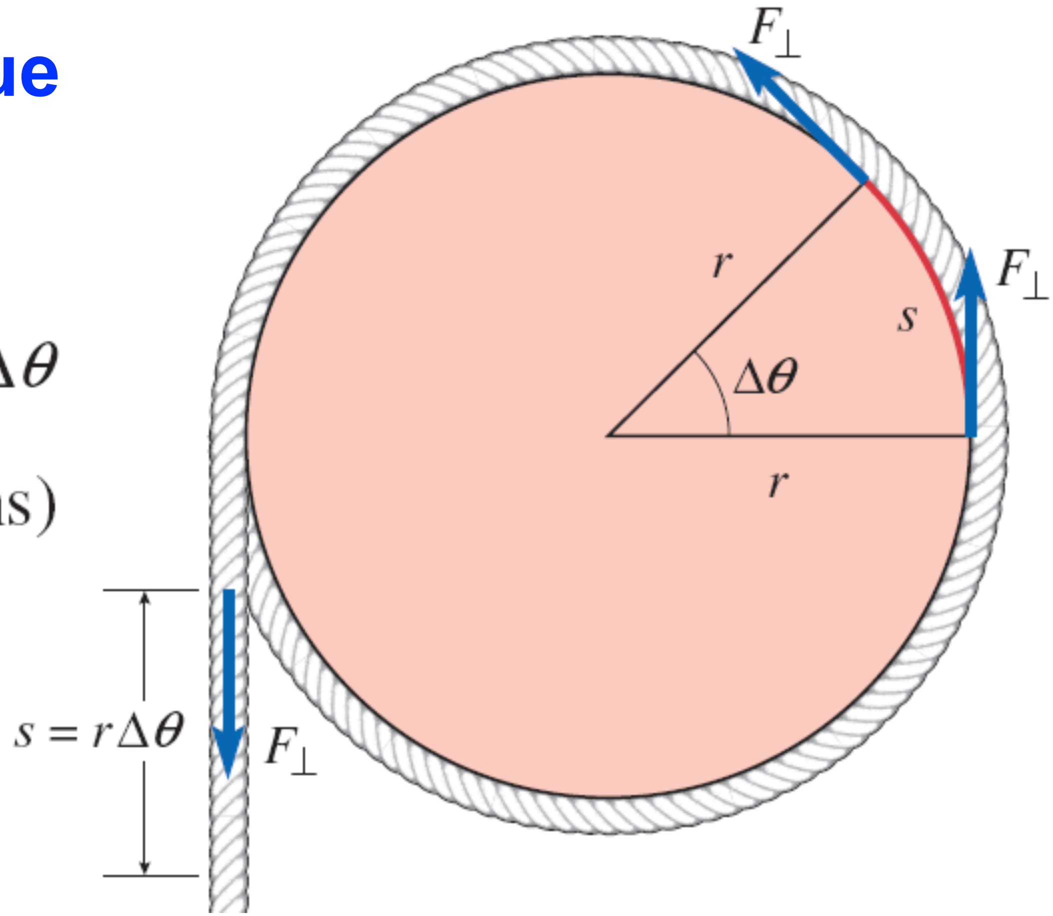
$$W = F_{\perp} s$$

$$W = F_{\perp} s = \frac{\tau}{r} \times r\Delta\theta = \tau\Delta\theta$$

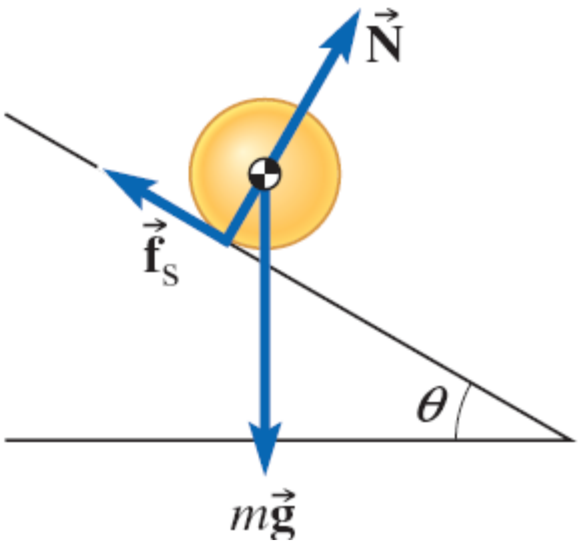
$$W = \tau\Delta\theta \quad (\Delta\theta \text{ in radians})$$

The power is:

$$P = \tau\omega$$



Acceleration of Rolling Objects



Friction is the only force giving nonzero torque about the rotation axis through the ball's center of mass.

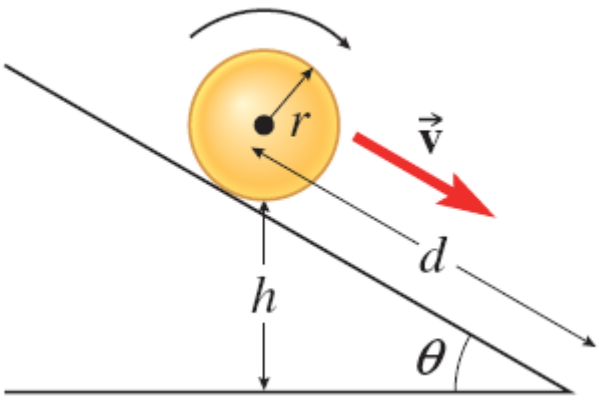
The frictional force $\vec{\mathbf{f}}$ provides a torque

$$\tau = rf$$

What about the other forces?

where *r* is the ball's radius.

Calculate the acceleration of a solid ball rolling down a slope inclined at an angle θ to the horizontal.



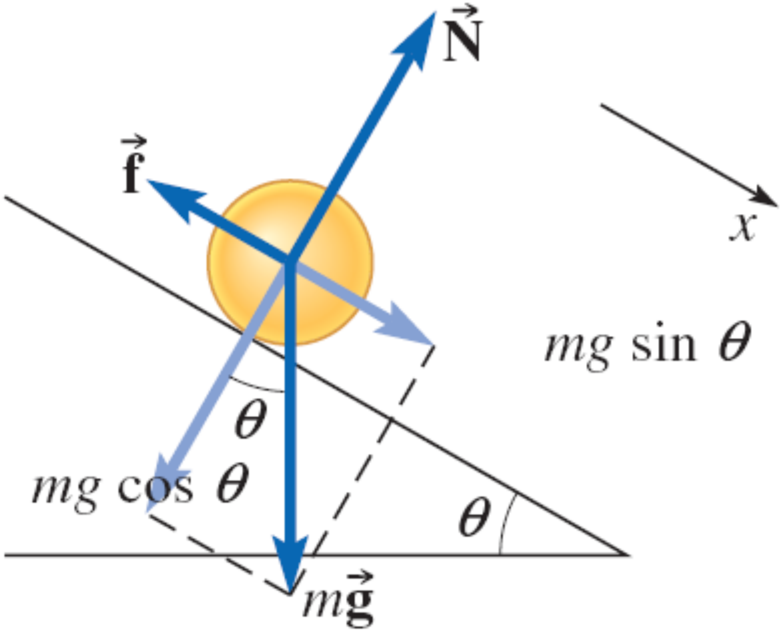
Some thoughts before calculation…

There is no reason to assume that static friction has its maximum possible magnitude. We do know that the two accelerations, translational and rotational, are related.

Solution

$$\sum \tau = rf$$

$$\alpha = \frac{\sum \tau}{I} = \frac{rf}{I}$$



$$\sum F_x = mg \sin \theta - f$$

Let's check why it's sin:

$$\sum F_x = mg \sin \theta - f = ma_{CM}$$

Recall relationship between linear and rotational acceleration

$$a_{CM} = \alpha r$$

From above $f = \dfrac{I\alpha}{r}$

$$mg \sin \theta - \frac{I\alpha}{r} = ma_{CM}$$

$$mg \sin \theta - \frac{Ia_{CM}}{r^2} = ma_{CM}$$

Solution

From previous slide

$$mg \sin \theta - \frac{I a_{CM}}{r^2} = m a_{CM}$$

$$a_{CM} = \frac{g \sin \theta}{1 + I/(mr^2)}$$

For a sold, uniform sphere (remember that you are given this or look it up… or calculate it!)

$$I = \tfrac{2}{5} m r^2$$

$$a_{CM} = \frac{g \sin \theta}{1 + \frac{2}{5}} = \frac{5}{7} g \sin \theta$$

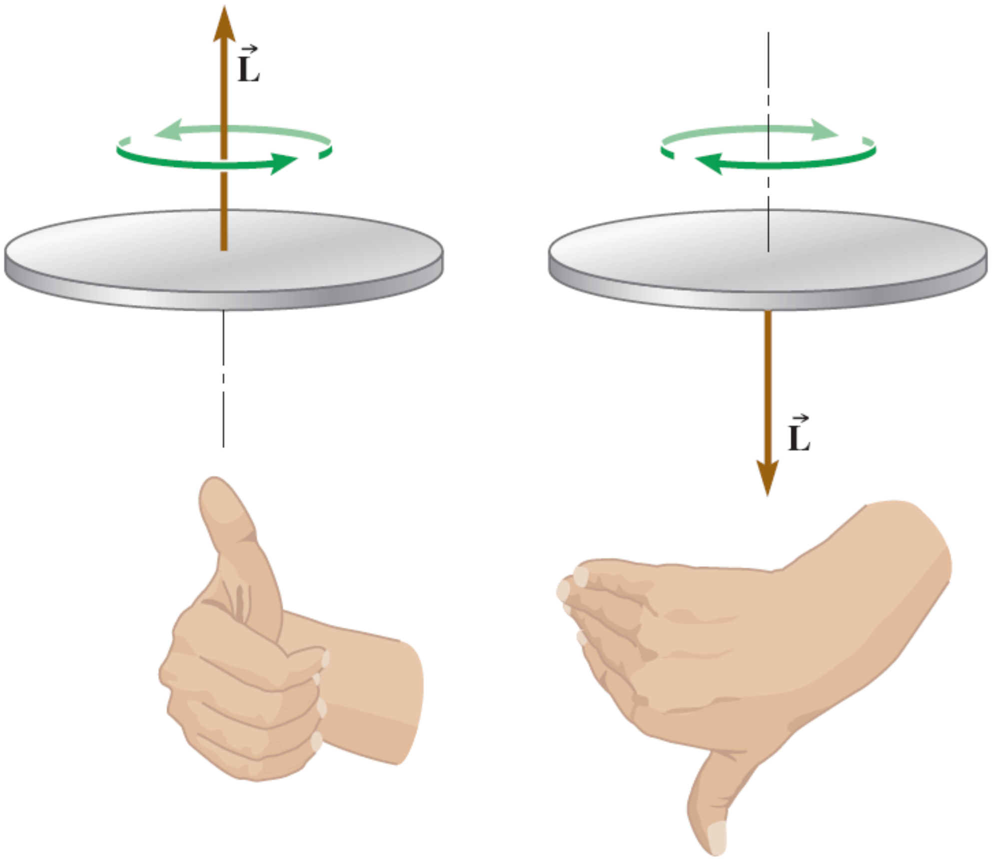
The **angular momentum (L)** of a single particle of mass m and linear momentum **p** is given by:

$$\vec{L} = \vec{r} \times \vec{p}$$

$$|\vec{L}| = |\vec{r} \times \vec{p}| = rp\sin\theta$$

If an object moves in the same direction as its position vector, is has no angular momentum!

Right-hand rule for finding the direction of the angular momentum of a spinning disk:



Newton's second law for translational motion can be written in two ways:

$$\sum \vec{\mathbf{F}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{p}}}{\Delta t} \text{ (general form)} \qquad \sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} \text{ (constant mass)}$$

We wrote Newton's second law for rotation as $\sum \tau = I\alpha$, which applies only when *I* is constant—that is, for a rigid body rotating about a fixed axis.

A more general form of Newton's second law for rotation uses the concept of **angular momentum** (symbol ***L***).

The net external torque acting on a system is equal to the rate of change of the angular momentum of the system.

$$\sum \vec{\tau} = \frac{d\vec{L}}{dt}$$

Angular momentum: $\vec{L} = I\vec{\omega}$ (rigid body, fixed axis)

For a rigid body rotating around a fixed axis,

$$\sum \vec{\tau} = \frac{d\vec{L}}{dt} = \frac{d(I\vec{\omega})}{dt} = I\frac{d\vec{\omega}}{dt} = I\vec{\alpha}$$

Conservation of angular momentum:

$$\text{If } \sum \vec{\tau} = 0,\ \vec{L}_i = \vec{L}_f$$

Linear momentum and angular momentum *cannot* be added to find the "total momentum." They are entirely different quantities (really, two different vectors!), not two forms of the same quantity.

Conservation of linear momentum and conservation of angular momentum are *separate* laws of physics.

Any figure skaters here?

What happens when a spinning skater moves legs and body from "out" to an "in" position?

Has anyone seen the spinning-chair-with-weights trick? We can try that here and then view the video, too

https://www.youtube.com/watch?v=fwAjPOR28J0

Zero net force is sufficient to ensure *translational* equilibrium (recall **F**=m**a**); if an object is also in *rotational* equilibrium, then the net torque acting on it must also be zero.

Conditions for equilibrium (both translational and rotational):

$$\sum \vec{\mathbf{F}} = 0 \quad \text{and} \quad \sum \tau = 0$$

The best place to choose the axis for calculating the torque is usually at the point of application of an unknown force so that the unknown force does not appear in the torque equation.

And why would it then not appear?

Calculating cross products?

So what we want is the condition below - but what if we only know the components of the vectors for computing torque (**r** and **F**)?

$$\sum F_x = \sum F_y = \sum F_z = \sum \tau_x = \sum \tau_y = \sum \tau_z = 0$$

Applying equilibrium condition

What are the components of the cross-product? Can multiply it out to find…

$$\vec{C} = \vec{A} \times \vec{B} \rightarrow$$

$$C_x = (A_y B_z - A_z B_y)$$

$$C_y = (A_z B_x - A_x B_z)$$

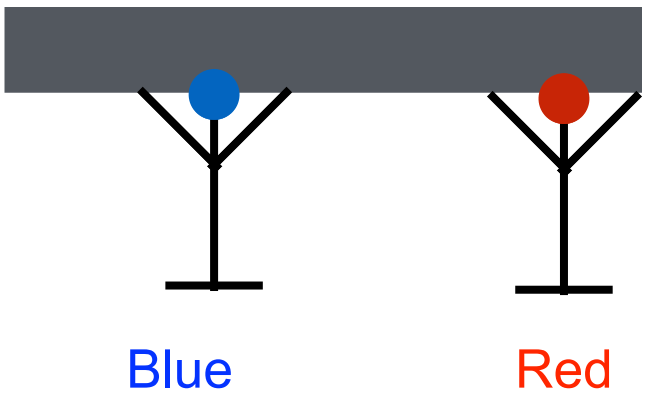
$$C_z = (A_x B_y - A_y B_x)$$

Careful here - don't want to mix up the order or the signs!

Group work time!

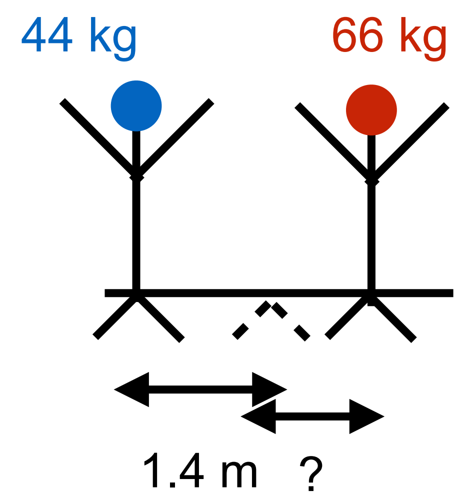
https://forms.gle/otWR6JTyddWWF2eYA

Two carpenters are carrying a uniform beam. The beam is 8.00 m long and weighs 425 N (95.5 lb). One of the carpenters, being a bit stronger than the other, agrees to carry the beam 1.00 m in from the end (he is a former PHYS253 student); the other carries the beam at its opposite end. What force does each apply to carry the beam if it is not being raised or lowered or rotating?



Who has the harder time here? Before we solve this, any guesses??
Red or blue?

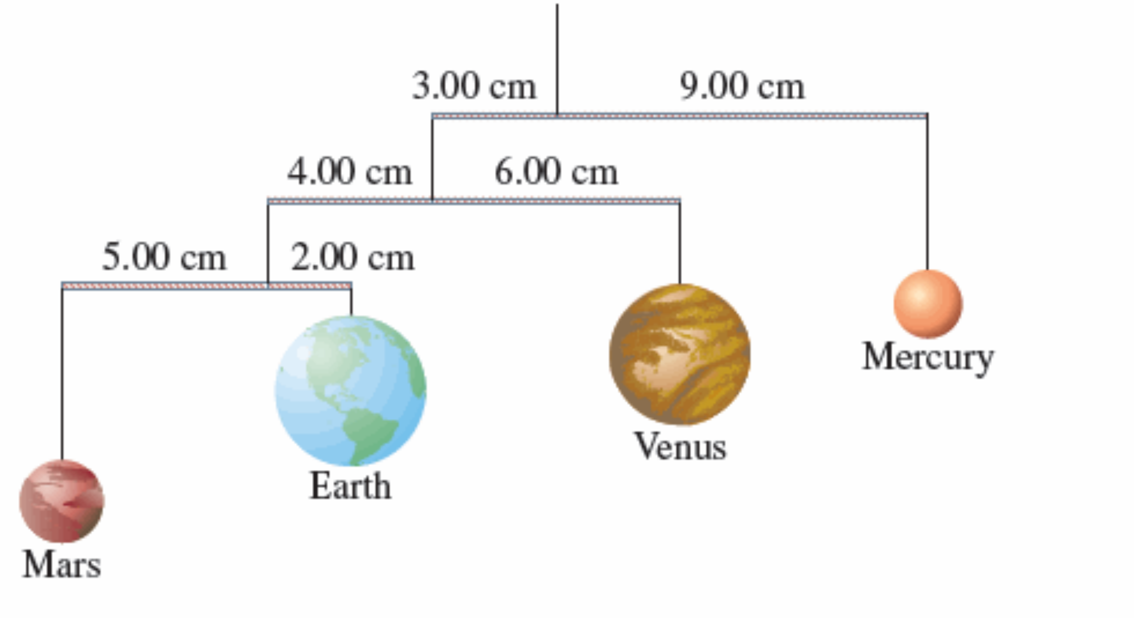
You and your sister are trying to balance on a seesaw. She has a mass of 44 kg, and you have a mass of 66 kg. She is sitting 1.4 m from the fulcrum (pivot). How far from it should you sit to balance the seesaw?



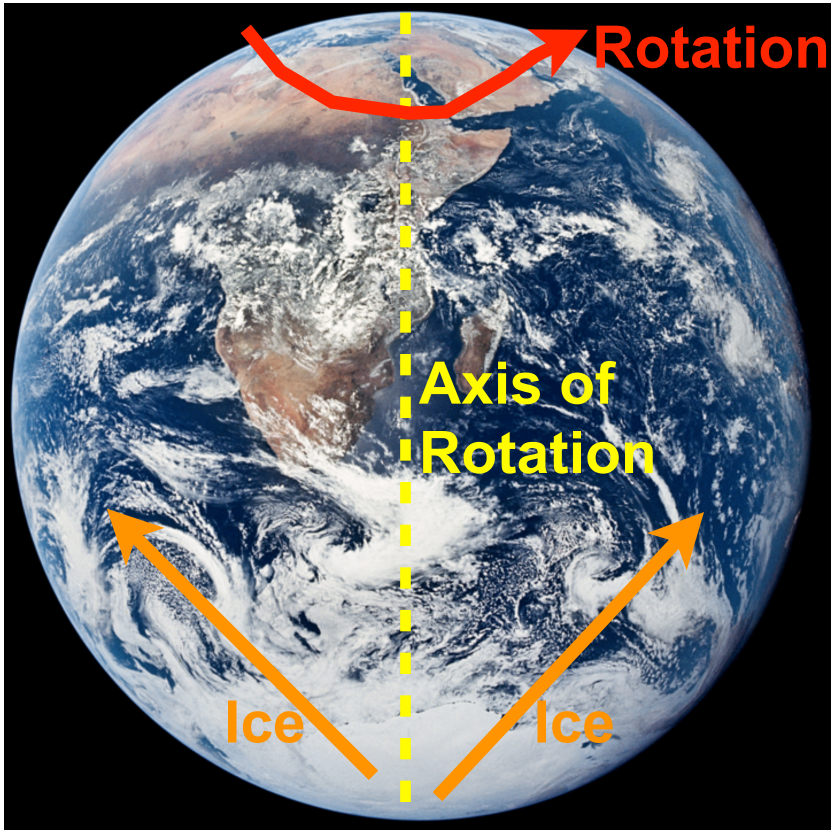
From experience:
should you sit a distance larger or smaller than 1.4 m away?

The inner planes of our solar system are represented on a mobile constructed from drinking straws and light strings for a school project, as shown in the figure. The mass of the piece representing the Earth is 25.0 g and the mass of the straws can be ignored. The system is in equilibrium. What is the mass of the piece representing:

a) Mars ?
b) Venus ?
c) Mercury ?



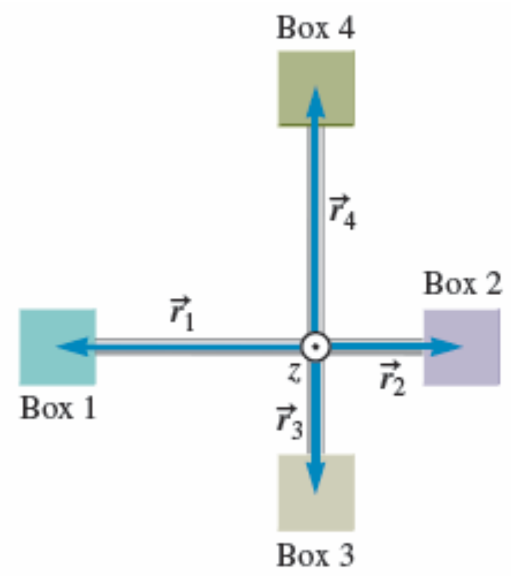
One of the effects of significant global warming will be the melting of the polar ice caps. This will change the length of the day (the period of Earth's rotation). Why? Would the day get longer or shorter?



A stone used to grind wheat into flour is turned through 12 revolutions by a constant force of 20.0 N applied to the rim of a radius-10.0 cm wheel. How much work is done on the stone during the 12 revolutions?

A skater is initially spinning at a rate of 10.0 rad/s with a rotational inertia of 2.50 kg m^2 when her arms are extended. What is her angular velocity after she pulls her arms in and reduces her rotational inertia to 1.60 kg m^2 ?

A system consists of four boxes modeled as particles connected by very lightweight, stiff rods, as shown in the figure. The system rotates around the z axis, which points out of the page. Each particle has mass 5.00 kg. The distances from the z axis to each particle are r1=32.0 cm, r2 = 16.0 cm, r3=17.0 cm, r4=34.0 cm. Find the rotational inertia of the system around the z axis.



The net torque of 50.0 Nm on a wheel rotating around an axis through its center is due to an applied force and a frictional torque at the axle. Starting from rest, the wheel reaches an angular speed of 12.0 rad/s in 5.00 s. At t = 5.00 s, the applied force is removed, and the frictional torque brings the wheel to a stop in 30.0 s.

a) What is the rotational inertia of the wheel?
b) What is the magnitude of the frictional torque acting on the wheel?
c) What is the total number of revolutions the wheel undergoes during this 35.0-s interval?

A 10.0 kg disk of radius 2.0 m rotates from rest as a result of a 20.0-N tangential force applied at the edge of the disk. What is the kinetic energy of the disk 4.00 s after the force is applied?

Model the Earth as a solid sphere rotating around an axis through its center of mass. Find the rotational kinetic energy of the Earth. The radius of the Earth is approximately 6400 km, and its mass is ~6x10^24 kg

A merry-go-round at a park is subject to a constant torque with a magnitude of 645 Nm as a parent pushes the ride.

a) How much work is performed by the torque as the merry-go-round rotates through 1.75 revolutions?
b) If it takes the merry-go-round 4.51 s to go through the 1.75 revolutions, what is the power transferred by the parent to the ride?

A frisbee has a radius of 11 cm and a mass of 175 grams, and is spinning along its central axis with an angular speed of 120 rad/s. What is its rotational kinetic energy?

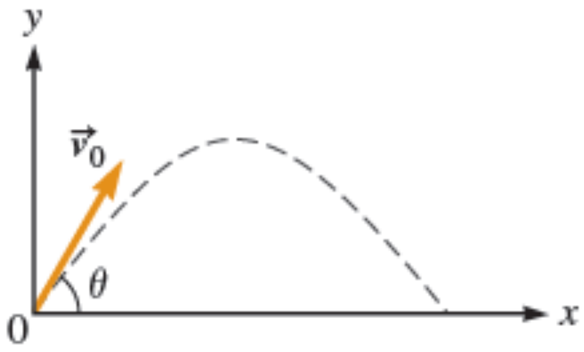
A thin rod of length 2.65 m and mass 13.7 kg is rotated an an angular speed of 3.89 rad/s around an axis perpendicular to the rod and through its center of mass. Find the magnitude of the rod’s angular momentum

Two children (m = 30.0 kg each) stand opposite each other on the edge of a merry-go-round. The merry-go-round, which has a mass of 180kg and a radius of 1.5 m, is spinning at a constant rate of 0.50 rev/s. Treat the two children and the merry-go-round as a system

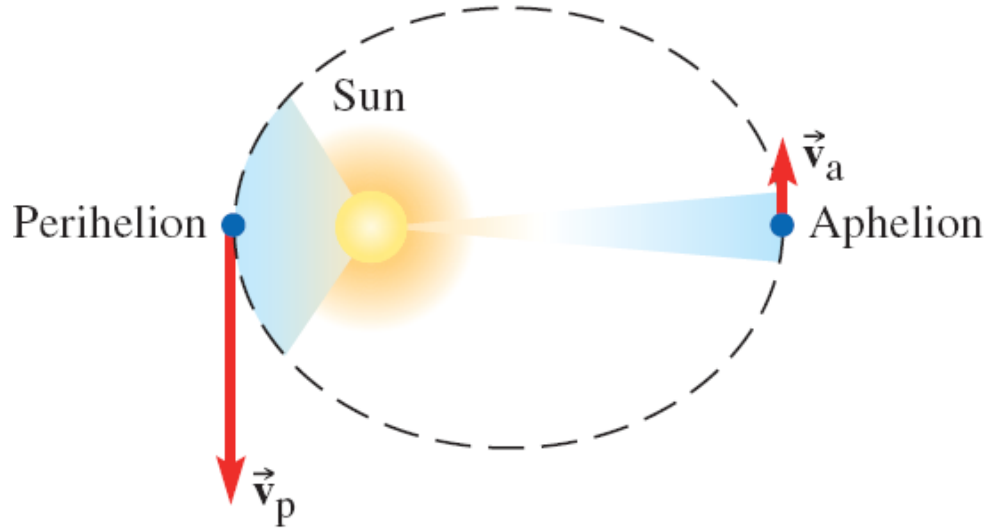
a) Calculate the angular momentum of the system, treating each child as a particle
b) Calculate the total kinetic energy of the system
c) Both children walk half the distance toward the center of the merry-go-round. Calculate the final angular speed of the system

A disk of mass m1 is rotating freely with constant angular speed ω. Another disk of mass m2 that has the same radius is gently placed on the first disk. If the surfaces are rough so that there is no slipping between the disks, what is the fractional decrease in kinetic energy of the system?

An object of mass M is thrown with a speed v0 at an angle θ with respect to the horizontal. Find the angular momentum of the object around the origin when the object is at the highest point of its trajectory. Ignore drag and air resistance.



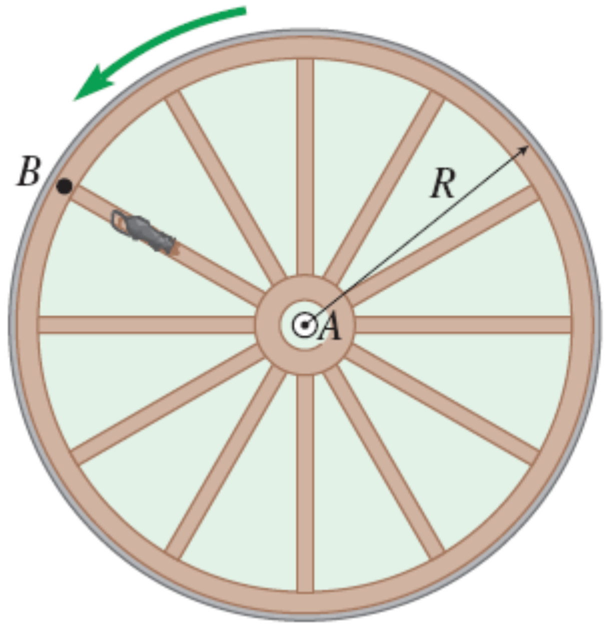
At perihelion (closest approach to the Sun), Earth is
1.47×10^8 km
from the Sun and its orbital speed is 30.3 km/s.



What is Earth's orbital speed at aphelion (greatest distance from the Sun), when it is 1.52×10^8 km from the Sun?

Note that at these two points Earth's velocity is perpendicular to a radial line from the Sun.

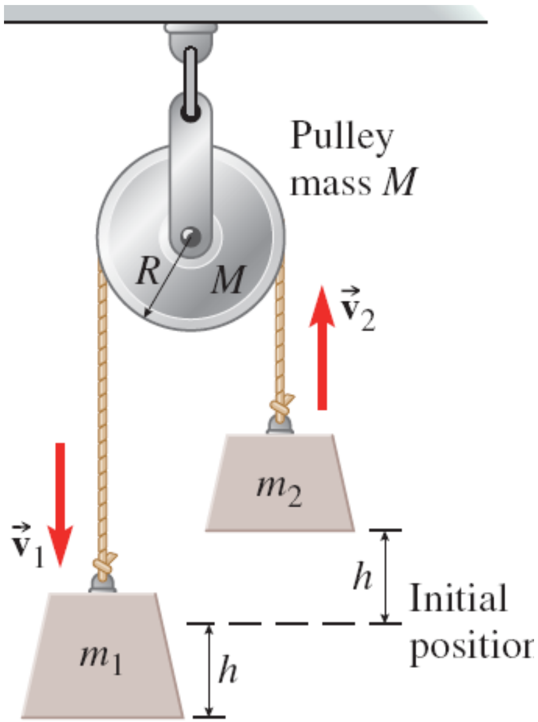
A 0.10-kg mouse is perched at point *B* on the rim of a 2.00-kg wagon wheel that rotates freely in a horizontal plane at 1.00 rev/s.



The mouse crawls to point *A* at the center. Assume the mass of the wheel is concentrated at the rim.

What is the frequency of rotation in rev/s when the mouse arrives at point *A*?

Atwood's machine consists of a cord around a pulley of rotational inertia *I*, radius *R*, and mass *M*, with two blocks (masses m_1 and m_2) hanging from the ends of the cord.



Assume that the pulley is free to turn without friction and that the cord does not slip. Ignore air resistance.

If the masses are released from rest, find how fast they are moving after they have moved a distance *h* (one up, the other down).

A potter's wheel is a heavy stone disk on which the pottery is shaped. Potter's wheels were once driven by the potter pushing on a foot treadle; today most potter's wheels are driven by electric motors.

(a) If the potter's wheel is a uniform disk of mass 40.0 kg and diameter 0.50 m, how much work must be done by the motor to bring the wheel from rest to 80.0 rpm?

(b) If the motor delivers a constant torque of 8.2 Nm during this time, through how many revolutions does the wheel turn in coming up to speed?

A grinding wheel is a solid, uniform disk of mass 2.50 kg and radius 9.00 cm.

Starting from rest, what constant torque must a motor supply so that the wheel attains a rotational speed of 126 rev/s in a time of 6.00 s?