Chapter 11

(Some material taken from McGraw-Hill/College Physics/Giambattista, the majority from your Katz textbook)

Analyzing Collisions Using Momentum Conservation

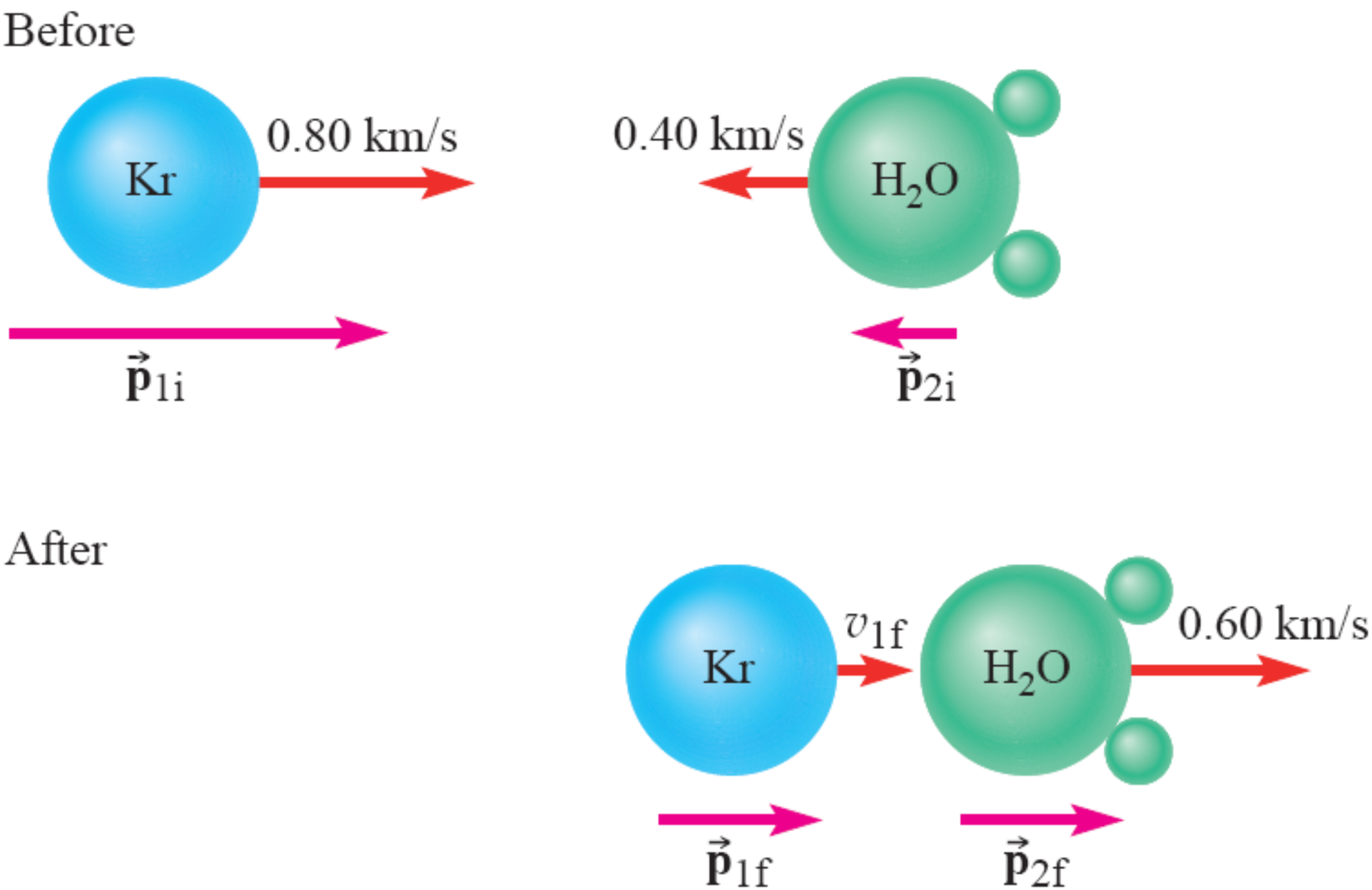
We can often use conservation of momentum to analyze collisions even when external forces act on the colliding objects.

In these cases, the total momentum after the collision is *approximately* the same as it was before the collision, and we will treat it as if it is the same. This is particularly true if we analyze things **immediately before** the collision and **immediately after** it

A krypton atom (mass 83.9 u) moving with a velocity of 0.80 km/s to the right and a water molecule (mass 18.0 u) moving with a velocity of 0.40 km/s to the left collide head-on.

The water molecule has a velocity of 0.60 km/s to the right after the collision. What is the velocity of the krypton atom after the collision? (The symbol “u” stands for the atomic mass unit.)

Drawing a picture



Solution

$$\vec{\mathbf{p}}_{1\mathrm{f}} + \vec{\mathbf{p}}_{2\mathrm{f}} = \vec{\mathbf{p}}_{1\mathrm{i}} + \vec{\mathbf{p}}_{2\mathrm{i}}$$

For simplicity we drop the "*x*" subscripts from the *x*-components, remembering that all quantities refer to *x* -components: $m_1 v_{1\mathrm{f}} + m_2 v_{2\mathrm{f}} = m_1 v_{1\mathrm{i}} + m_2 v_{2\mathrm{i}}$

As we'll see in a few slides - we can't always do this!

Since $m_1/m_2 = 83.9/18.0 = 4.661$, we can substitute $m_1 = 4.661 m_2$:

$$4.661\, m_2 v_{1\mathrm{f}} + m_2 v_{2\mathrm{f}} = 4.661\, m_2 v_{1\mathrm{i}} + m_2 v_{2\mathrm{i}}$$

This isn't required for the algebra, but is one option. It's up to you if you prefer to do this or not. There are other ways to solve this!

Solution

$$v_{1f} = \frac{4.661 v_{1i} + v_{2i} - v_{2f}}{4.661}$$

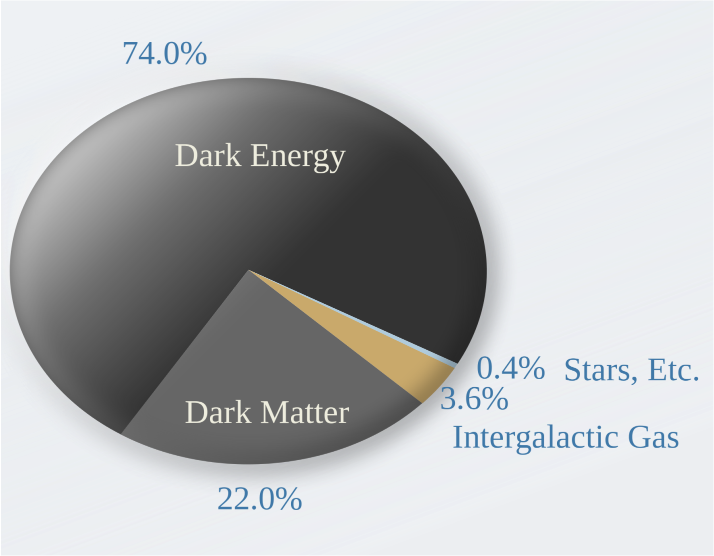
$$= \frac{4.661 \times 0.80 \text{ km/s} + (-0.40 \text{ km/s}) - 0.60 \text{ km/s}}{4.661}$$

$$= 0.59 \text{ km/s}$$

After the collision, the krypton atom moves to the right with a speed of 0.59 km/s.

You can put your pens down :)

Remember that we learned that the majority of the Universe is made up of ‘stuff’ that we cannot see! What other proof do we have for dark matter? (There’s a lot, but let’s give one more example)



Two colliding galaxies

If they are only made up of normal matter (ie stuff that bangs into one another), what does conservation of momentum tell us?

What about dark matter, which doesn't interact with light or regular matter? What happens to it?

Bullet Cluster

Elastic and Inelastic Collisions

A collision in which the *total* kinetic energy is the same before and after is called **elastic.**

There is no *conservation law* for kinetic energy by itself (we knew this already!). The elastic collision is just a special kind of collision in which no kinetic energy is changed into other forms of energy.

When the final kinetic energy is less than the initial kinetic energy, the collision is said to be **inelastic.**

Elastic and Inelastic Collisions

When a collision results in two objects sticking together, the collision is **perfectly inelastic**.

The decrease of kinetic energy in a perfectly inelastic collision is as large as *possible* (consistent with the conservation of momentum).

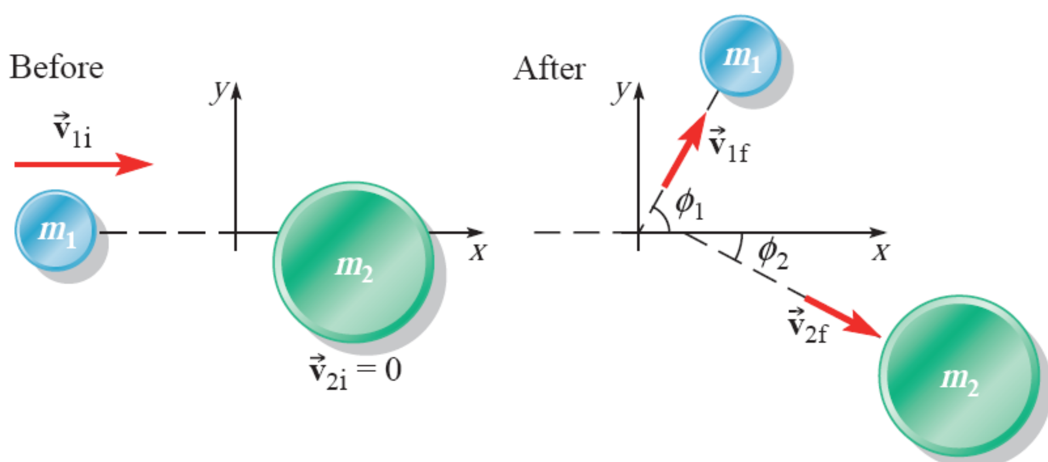
Most collisions are not limited to motion in one dimension in the absence of a track or other device to constrain motion to a single line.

In a two-dimensional collision, we use the same techniques we used for one-dimensional collisions, as long as we remember that **momentum is a vector**.

To apply conservation of momentum, it is usually easiest to work components (such as *x*- and *y*-components).

As always, I recommend drawing a picture and coordinate axes before doing anything else!

A small puck (mass m_1 = 0.10 kg) is sliding to the right with an initial speed of 8.0 m/s on an air table. An air table has many tiny holes through which air is blown; the resulting air cushion allows objects to slide with very little friction. The puck collides with a larger puck (mass m_2 = 0.40 kg), which is initially at rest. The pucks move off at angles ϕ_1 = 60.0° above and ϕ_2 = 30.0° below the initial direction of motion of the small puck.



(a) What are the final speeds of the pucks?

(b) Is this an elastic collision or an inelastic collision?

(c) If inelastic, what fraction of the initial kinetic energy is converted to other forms of energy in the collision?

What we know and don't know

Masses:	$m_1 = 0.10 \text{ kg}; m_2 = 0.40 \text{ kg}$
Before collision:	$v_{1ix} = 8.0 \text{ m/s}; v_{1iy} = v_{2ix} = v_{2iy} = 0$
After collision:	$v_{1fx} = v_{1f} \cos \phi_1; v_{1fy} = v_{1f} \sin \phi_1;$ $v_{2fx} = v_{2f} \cos \phi_2; v_{2fy} = -v_{2f} \sin \phi_2$ $(\phi_1 = 60.0° \text{ and } \phi_2 = 30.0°)$
To find:	v_{1f} and v_{2f}; total kinetic energy before and after the collision

Promised that you wouldn't get away from SOH-CAH-TOA.

Solution

(a)

$$p_{1fx} + p_{2fx} = p_{1ix} + p_{2ix}$$

$$m_1 v_{1f} \cos \phi_1 + m_2 v_{2f} \cos \phi_2 = m_1 v_{1ix} + 0$$

$$m_2 = 4m_1$$

$$m_1 v_{1f} \cos 60.0^\circ + 4m_1 v_{2f} \cos 30.0^\circ = m_1 v_{1ix}$$

$$0.500 v_{1f} + 3.46 v_{2f} = 8.0 \text{ m/s}$$

Solution

(a)

$$p_{1\mathrm{fy}} + p_{2\mathrm{fy}} = p_{1\mathrm{iy}} + p_{2\mathrm{iy}} = 0$$

$$m_1 v_{1\mathrm{f}} \sin \phi_1 + (-4 m_1 v_{2\mathrm{f}} \sin \phi_2) = 0$$

$$v_{1\mathrm{f}} \sin 60.0° - 4 v_{2\mathrm{f}} \sin 30.0° = 0$$

$$v_{2\mathrm{f}} = \frac{\sin 60.0°}{4 \sin 30.0°} v_{1\mathrm{f}} = 0.433 v_{1\mathrm{f}}$$

Solution

(a)

$$0.500v_{1f} + 3.46v_{2f} = 8.0\text{ m/s}$$

$$v_{2f} = \frac{\sin 60.0°}{4\sin 30.0°}\,v_{1f} = 0.433v_{1f}$$

$$0.500v_{1f} + 3.46(0.433v_{1f}) = 2.00v_{1f} = 8.0\text{ m/s}$$

$$v_{1f} = 4.0\text{ m/s}$$

$$v_{2f} = 0.433v_{1f} = 1.73\text{ m/s} \rightarrow 1.7\text{ m/s}$$

Solution

(b)

$$K_i = \frac{1}{2} m_1 v_{1i}^2$$

$$K_i = \frac{1}{2}(0.10 \text{ kg}) \times (8.0 \text{ m/s})^2 = 3.2 \text{ J}$$

$$\begin{aligned} K_f &= \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 \\ &= \tfrac{1}{2}(0.10 \text{ kg}) \times (4.0 \text{ m/s})^2 + \tfrac{1}{2}(0.40 \text{ kg}) \times (1.73 \text{ m/s})^2 \\ &= 0.80 \text{ J} + 0.60 \text{ J} = 1.40 \text{ J} \end{aligned}$$

Elastic ? Or inelastic?

Solution

(c)

$$3.2\ \text{J} - 1.40\ \text{J} = 1.8\ \text{J}$$

$$\frac{1.8\ \text{J}}{3.2\ \text{J}} = 0.56$$

Where did remaining energy go?

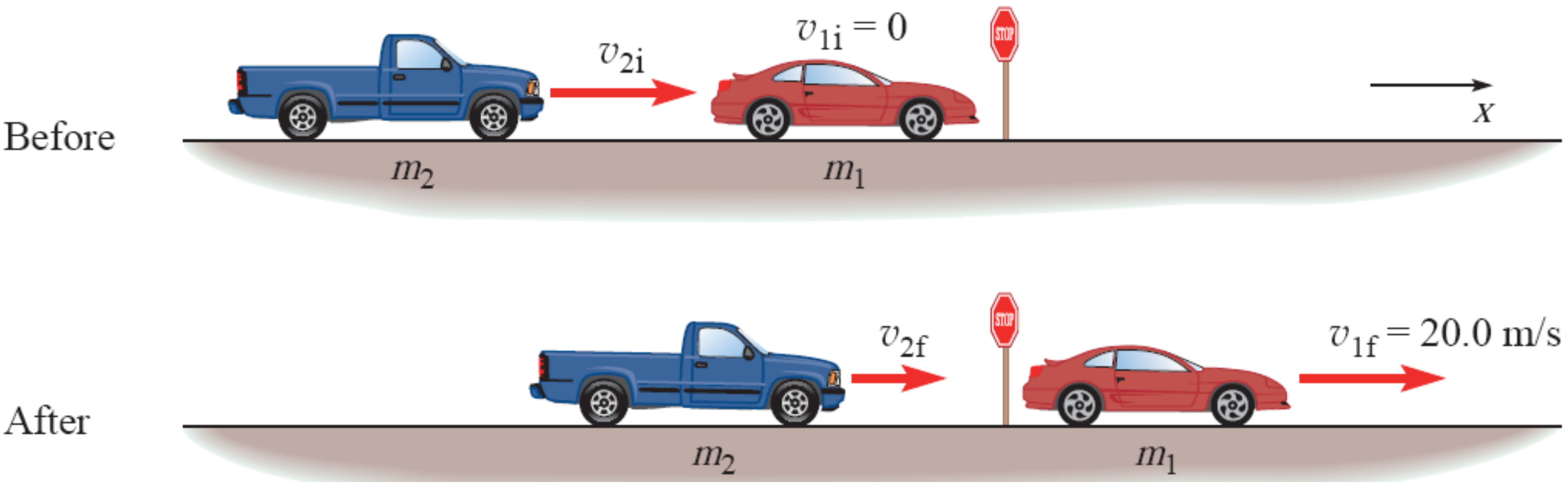
At a Route 3 highway on-ramp, a car of mass 1.50×10^3 kg is stopped at a stop sign, waiting for a break in traffic before merging with the cars on the highway.

A pickup of mass 2.00×10^3 kg comes up from behind and hits the stopped car. Assuming the collision is elastic, how fast was the pickup going just before the collision if the car is pushed straight ahead onto the highway at 20.0 m/s just after the collision?

Some pictures and coordinate axes

Given: $m_1 = 1.50 \times 10^3$ kg; $m_2 = 2.00 \times 10^3$ kg; before the collision, $v_{1i} = 0$; after the collision, $v_{1f} = 20.0$ m/s

To find: v_{2i} (speed of the pickup just before the collision)



Solution

$$\cancel{m_1} v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{2i} - \cancel{v_{1i}} = -(v_{2f} - v_{1f})$$

Why? Let's check
(this is in your book)

$$\frac{1}{2} m_1 v_{i1}^2 + \frac{1}{2} m_2 v_{i2}^2 = \frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 v_{f2}^2$$

Elastic collision:
KE is conserved

$$m_1 v_{i1}^2 + m_2 v_{i2}^2 = m_1 v_{f1}^2 + m_2 v_{f2}^2$$

$$m_1 (v_{i1}^2 - v_{f1}^2) = m_2 (v_{f2}^2 - v_{i2}^2)$$

$$m_1 v_{i1} + m_2 v_{i2} = m_1 v_{f1} + m_2 v_{f2}$$

Momentum
is conserved

$$m_1 (v_{i1} - v_{f1}) = m_2 (v_{f2} - v_{i2})$$

Solution

$$m_1(v_{i1}^2 - v_{f1}^2) = m_2(v_{f2}^2 - v_{i2}^2)$$

$$m_1(v_{i1} - v_{f1}) = m_2(v_{f2} - v_{i2})$$

Divide the top by the bottom

$$(v_{i1} + v_{f1}) = (v_{f2} + v_{i2})$$

$$(v_{i1} - v_{i2}) = -(v_{f1} - v_{f2})$$

Solution

$$\cancel{m_1} v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{2i} - \cancel{v_{1i}} = -(v_{2f} - v_{1f})$$ From our derivation

$$m_2 v_{2i} = m_2 v_{1f} - m_2 v_{2f}$$ Multiply both sides by m_2

$$2m_2 v_{2i} = (m_1 + m_2) v_{1f}$$ Add first and third lines together

$$v_{2i} = \frac{m_1 + m_2}{2m_2} v_{1f} = \frac{1500\ \text{kg} + 2000\ \text{kg}}{4000\ \text{kg}} \times 20.0\ \text{m/s} = 17.5\ \text{m/s}$$

The change in momentum of an object when a single force acts on it is equal to the product of the force acting on the object and the time interval during which the force acts: $\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}\,\Delta t$

Can we see where this comes from?

The product $\vec{\mathbf{F}}\,\Delta t$ is given the name **impulse**.

$$\text{total impulse} = \vec{\mathbf{F}}_1\,\Delta t + \vec{\mathbf{F}}_2\,\Delta t + \cdots$$
$$= (\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \cdots)\,\Delta t = \sum\vec{\mathbf{F}}\,\Delta t$$

$$\Delta\vec{\mathbf{p}} = \sum\vec{\mathbf{F}}\,\Delta t$$

Impulse When Forces Are Not Constant: $\Delta\vec{\mathbf{p}} = \sum\vec{\mathbf{F}}_{\mathrm{av}}\,\Delta t$

An individual force varying in time applies an impulse:

$$\vec{I} = \int \vec{F} dt$$

Total impulse on an object: $\vec{I}_{tot} = \sum \vec{I} = \Delta\vec{p}$

A car moving at 20.0 m/s (44.7 mi/h) crashes into a tree. Find the magnitude of the average force acting on a passenger of mass 65 kg in each of the following cases.

(a) The passenger is not wearing a seat belt. He is brought to rest by a collision with the windshield and dashboard that lasts 3.0 ms.

(b) The car is equipped with a passenger-side air bag. The force due to the air bag acts for 30 ms, bringing the passenger to rest.

Strategy

We know that:

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}_{\text{av}}\,\Delta t,$$

where $\vec{\mathbf{F}}_{\text{av}}$ is the average force acting on the passenger and Δt is the time interval during which the force acts.

The change in the passenger's momentum is the same in the two cases.

What differs is the time interval during which the change occurs. It takes a larger force to change the momentum in a shorter time interval.

Solution

$$|\vec{\mathbf{p}}_{\mathrm{i}}| = |m\vec{\mathbf{v}}_{\mathrm{i}}| = 65\ \mathrm{kg} \times 20.0\ \mathrm{m/s} = 1300\ \mathrm{kg \cdot m/s}$$

His final momentum is zero, so the magnitude of the momentum change is

$$|\Delta\vec{\mathbf{p}}| = 1300\ \mathrm{kg \cdot m/s}$$

$$\text{(a) No seat belt: } |\vec{\mathbf{F}}_{\mathrm{av}}| = \frac{|\Delta\vec{\mathbf{p}}|}{\Delta t} = \frac{1300\ \mathrm{kg \cdot m/s}}{0.0030\ \mathrm{s}} = 4.3 \times 10^5\ \mathrm{N}$$

$$\text{(b) Air bag: } |\vec{\mathbf{F}}_{\mathrm{av}}| = \frac{|\Delta\vec{\mathbf{p}}|}{\Delta t} = \frac{1300\ \mathrm{kg \cdot m/s}}{0.030\ \mathrm{s}} = 4.3 \times 10^4\ \mathrm{N}$$

Airbag “spreads out” the momentum transfer

When else might you care about this?


75
BOSCH
SOLAR ENERGY

Padded outfield walls are also useful

https://www.youtube.com/watch?v=3syehHkvNoo

Egg-tossing?
Let's watch at the 2:50 mark

Group work time!
https://forms.gle/aPd7iyos1mLvJjC46

In the railroad freight yard, an empty freight car of mass m rolls along on a straight level track at 1.0 m/s and collides with an initially stationary, fully loaded boxcar of mass 4.0 m. The two cars couple together on collision

a) What is the speed of the two cars after the collision?
b) Suppose instead that the two cars are at rest after the collision. With what speed was the loaded boxcar moving before the collision if the empty one was moving at 1.0 m/s?

A car with a mass of 1700 kg is traveling directly northeast at a speed of 14 m/s and collides with a smaller car with a mass of 1300 kg that is traveling directly south at a speed of 18 m/s. The two cars stick together during the collision. With what speed and direction do the two cars move right after the collision?

A baseball player in the outfield (m=80 kg) is running at 5.5 m/s while tracking a baseball. He runs into a padded outfield wall and bounces back at 0.5 m/s. He is in contact with the wall for 20 ms. What was the magnitude of the average force from the wall acting on him during the collision ?

A 175-g billiard ball is shot towards an identical ball at velocity **v**i = 6.50 ihat m/s. The identical ball is initially at rest. After the balls hit, one of them travels with a velocity **v**f = (1.20 ihat + 2.52 jhat) m/s. What is the velocity of the second ball after the impact? Ignore effects of friction.

You are trapped on the second floor of a burning building. The stairway is impassable, but there is a balcony outside the window. Describe what happens if:

a) You jump from the balcony window to the pavement below, landing stiff-legged on your feet

b) You jump into a hedge, landing on your back and rolling

c) You jump into a firefighters' net, landing on your back. What happens to the net as you land in it? And what might the firefighters do to cushion your fall even more?

A bird (mass 31 g) is flying at 11.1 m/s when it flies into a glass window and bounces off at a speed of 4.1 m/s. The bird is in contact with the glass for 0.071 s. What is the average force on the bird during the collision?

When a person feels that he is about to fall, he will often put out his hand to try to 'break the fall.' Explain why this natural reaction usually leads to bruises or minor broken bones such as in the wrists instead of major broken bones such as in the skull

A 35.0 kg child steps off a 4.0 ft high diving boards and executes a cannonball jump into a pool (air resistance is negligible).

a) What is the impulse exerted by the water on the child?
b) Why would the child have a much higher risk of injury if she were to land on the cement edge of the pool instead of in the water?

A truck collides with a small, empty parked car.

a) Compare the force exerted by the truck on the car with the force exerted by the car on the truck
b) Compare the impulse exerted by the truck on the car with the impulse exerted by the car on the truck
c) Compare the change in the truck’s momentum with the change in the car’s momentum

A skater of mass 45.0 kg standing on ice throws a stone of mass 7.65 kg with a speed of 20.9 m/s in a horizontal direction. Find the distance over which the skater will move in the opposite direction if the coefficient of kinetic friction between the skater and the ice is 0.03.

An object of mass 2.0 kg moving with a velocity of 3.0 m/s collides with another object of mass 1.0 kg moving with a velocity of 4.0 m/s in the same direction. The two objects get stuck together in the collision. What is the velocity of the combination after the collision?

One object ($m1 = 0.200$ kg) is moving to the right with a speed of 2.00 m/s when it is struck from behind by another object ($m2 = 0.300$ kg) that is moving to the right at 6.00 m/s. If friction is negligible and the collision between these objects is elastic, find the final velocity of each.

A proton with an initial speed of 2.00 x 10^8 m/s in the x direction collides with another proton initially at rest. The first proton's velocity after the collision is 1.64 x 10^8 m/s at an angle of 35 degrees with the horizontal. What is the velocity of the second proton after the collision?

Two bumper cars at the county fair are sliding toward one another. Initially, bumper car 1 is traveling to the east at 5.62 m/s, and bumper car 2 is traveling 60.0 degrees south of west at 10.00 m/s. After they collide, bumper car 1 is observed to be traveling to the west with a speed of 3.14 m/s. Friction is negligible between the cars and the ground. If the masses of bumper cars 1 and 2 are 596 kg and 625 kg, respectively, what is the velocity of bumper car 2 immediately after the collision?

A ball of mass 50.0 g is dropped from rest from a height of 10.0 m. It rebounds after losing 75% of its kinetic energy during the collision process. If the collision with the ground took 0.010 s, find the magnitude of the impulse and force experienced by the ball

A particle of mass m is traveling with velocity **v** at an angle 30 degrees from the x axis as below. It collides with a particle of mass 2m traveling with the same speed at an angle of 30 degrees from the x axis also as shown. The collision is perfectly inelastic. Find:

1) The velocity vector of the objects after the collision
2) The fraction of kinetic energy lost in the collision