Chapter 10

(Some material taken from McGraw-Hill/College Physics/Giambattista, the majority from your Katz textbook)

Conservation laws are powerful tools.

When a **vector is conserved**, both the **magnitude** and the **direction** of the vector must be constant.

Linear momentum (or sometimes just ***momentum***) is a vector quantity having the same direction as the velocity.

Definition of linear momentum:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

In any interaction between two objects, momentum can be transferred from one object to the other.

Reminder again: momentum is a vector just like velocity!

The momentum changes of the two objects are always equal in size and opposite in direction, so the total momentum of the two objects is unchanged by the interaction.

(By *total momentum* we mean the vector sum of the individual momenta of the objects.)

Makes intuitive sense in our common usage:
larger mass = larger momentum
increased velocity = increased momentum

My sports car, with a mass of 1400 kg, is traveling at 78 mph due north. What is its momentum? (Useful to know: 1 m/s = 2.2 mph)

My sports car, with a mass of 1400 kg, is traveling at 78 mph due north. What is its momentum? (Useful to know: 1 m/s = 2.2 mph)

p = m**v** = 1400 kg * 78 mph N * (1 m/s / 2.2 mph)
= 50,000 kg m/s North

A car weighing 12 kN is driving due north at 30.0 m/s. After driving around a sharp curve, the car is moving east at 13.6 m/s.

What is the change in momentum of the car?

Being careful!

There are two potential pitfalls:

1. momentum depends not on weight but on mass, and
2. momentum is a vector, so we must take its direction into consideration as well as its magnitude. To find the change in momentum, we need to do a ***vector*** subtraction.

Solution

$$m = \frac{W}{g} = \frac{1.2 \times 10^4\ \mathrm{N}}{9.8\ \mathrm{m/s^2}} = 1220\ \mathrm{kg}$$

$$\vec{\mathbf{v}}_\mathrm{i} = 30.0\ \mathrm{m/s,\ north}$$

$$\vec{\mathbf{p}}_\mathrm{i} = m\vec{\mathbf{v}}_\mathrm{i} = 1220\ \mathrm{kg} \times 30.0\ \mathrm{m/s\ north}$$

$$= 3.66 \times 10^4\ \mathrm{kg{\cdot}m/s\ north}$$

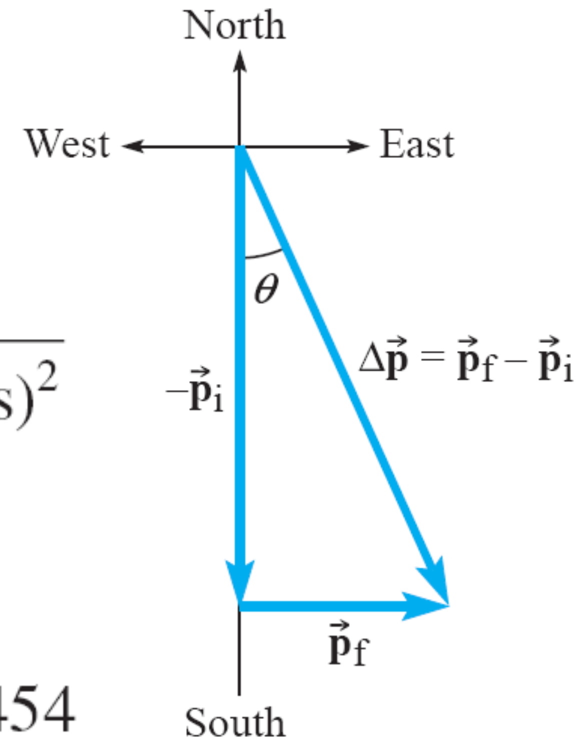
$$\vec{\mathbf{v}}_\mathrm{f} = 13.6\ \mathrm{m/s,\ east}$$

$$\vec{\mathbf{p}}_\mathrm{f} = m\vec{\mathbf{v}}_\mathrm{f} = 1220\ \mathrm{kg} \times 13.6\ \mathrm{m/s\ east}$$

$$= 1.66 \times 10^4\ \mathrm{kg{\cdot}m/s\ east}$$

Solution

As always, "change in" is (final - initial), and f-i is equivalent to f+(-i)



$$|\Delta\vec{\mathbf{p}}| = \sqrt{p_i^2 + p_f^2}$$

$$= \sqrt{(3.66 \times 10^4\ \mathrm{kg{\cdot}m/s})^2 + (1.66 \times 10^4\ \mathrm{kg{\cdot}m/s})^2}$$

$$= 4.02 \times 10^4\ \mathrm{kg{\cdot}m/s}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{p_f}{p_i} = \frac{1.66 \times 10^4\ \mathrm{kg{\cdot}m/s}}{3.66 \times 10^4\ \mathrm{kg{\cdot}m/s}} = 0.454$$

$$\theta = \tan^{-1} 0.454 = 24.4°$$

The change in momentum of the car is 4.0×10^4 kg·m/s directed 24° east of south.

Alternative definition of linear momentum:

$$\vec{p} = m\vec{v}$$

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$$

So the net force on an object is related to its rate of change of momentum!

In a system composed of more than two objects, interactions between objects inside the system do not change the total momentum of the system—they just transfer some momentum from one part of the system to another.

Only external interactions can change the total momentum of the system.

$$\vec{p}_{tot} = \sum_{j=1}^{n} \vec{p}_j$$

If the net external force acting on a system is zero, then the momentum of the system is conserved.

$$\text{If } \sum \vec{\mathbf{F}}_{\text{ext}} = 0, \quad \vec{\mathbf{p}}_{\text{i}} = \vec{\mathbf{p}}_{\text{f}}$$

Momentum as a vector is conserved. This means that each component (x, y, z) must also stay the same!

A squid propels itself by filling an internal cavity with water. Then the *mantle,* a powerful muscle, squeezes the cavity and expels water through a narrow opening (the *siphon*) at high speed.

Suppose a squid of mass 182 g (including the water that will be expelled) is initially at rest. It then expels 54 g of water at an average speed of 62 cm/s (relative to the surrounding water). Ignoring drag forces, how fast is the squid moving immediately after expelling the water?

Solution

’s’ = squid, ‘w’ = water

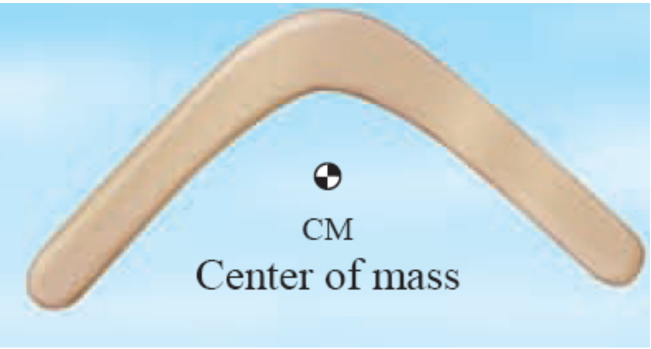
$$\vec{\mathbf{p}}_{\mathrm{i}} = \vec{\mathbf{p}}_{\mathrm{f}} \quad \Rightarrow \quad 0 = m_{\mathrm{s}}\vec{\mathbf{v}}_{\mathrm{s}} + m_{\mathrm{w}}\vec{\mathbf{v}}_{\mathrm{w}}$$

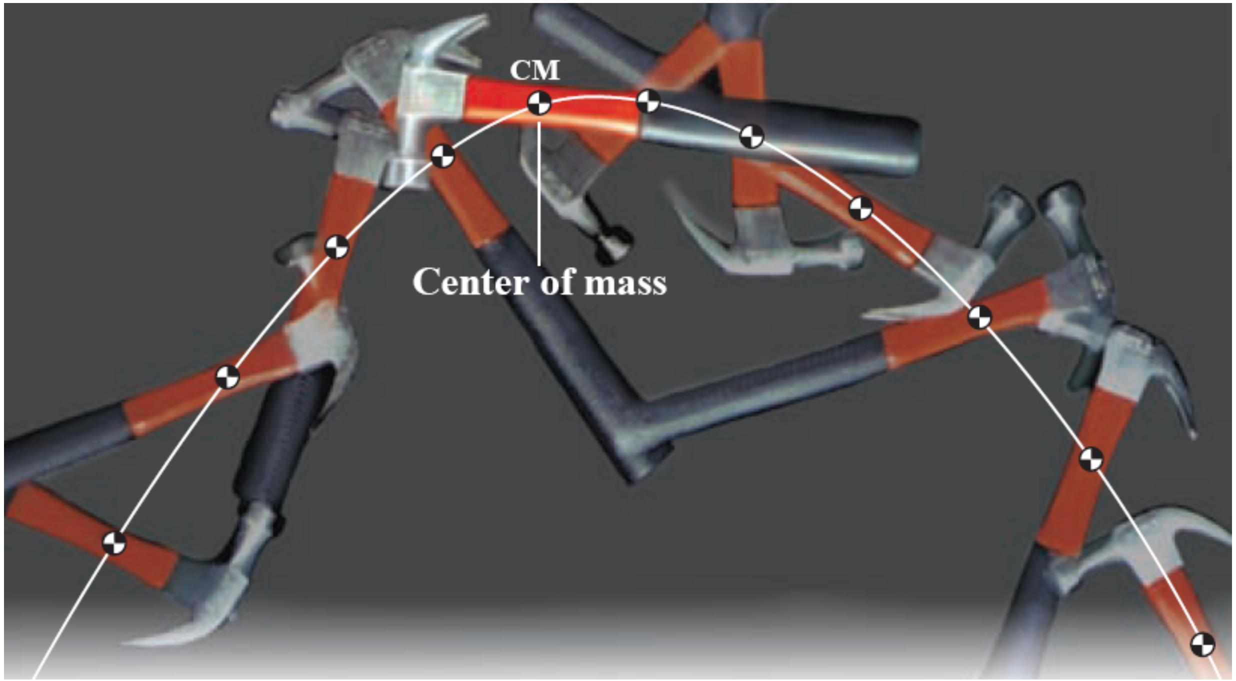
$$m_{\mathrm{s}} = 182\ \mathrm{g} - 54\ \mathrm{g} = 128\ \mathrm{g}$$

$$\vec{\mathbf{v}}_{\mathrm{s}} = -\frac{m_{\mathrm{w}}\vec{\mathbf{v}}_{\mathrm{w}}}{m_{\mathrm{s}}}$$

$$v_{\mathrm{s}} = \frac{m_{\mathrm{w}}v_{\mathrm{w}}}{m_{\mathrm{s}}} = \frac{(54\ \mathrm{g}) \times (62\ \mathrm{cm/s})}{128\ \mathrm{g}} = 26\ \mathrm{cm/s}$$

We can define a point called the **center of mass** (CM) that serves as an average location of the system.





Definition of center of mass:

Vector form: $$\vec{\mathbf{r}}_{CM} = \frac{\sum m_i \vec{\mathbf{r}}_i}{M}$$

Component form: $$x_{CM} = \frac{\sum m_i x_i}{M} \qquad y_{CM} = \frac{\sum m_i y_i}{M} \qquad z_{CM} = \frac{\sum m_i z_i}{M}$$

where $i = 1, 2, 3, \ldots, N$ and $M = \sum m_i$

There are the **weighted average** (objects with more mass count more)

Definition of center of mass for extended objects:

$$x_{CM} = \frac{1}{M}\int x dm$$

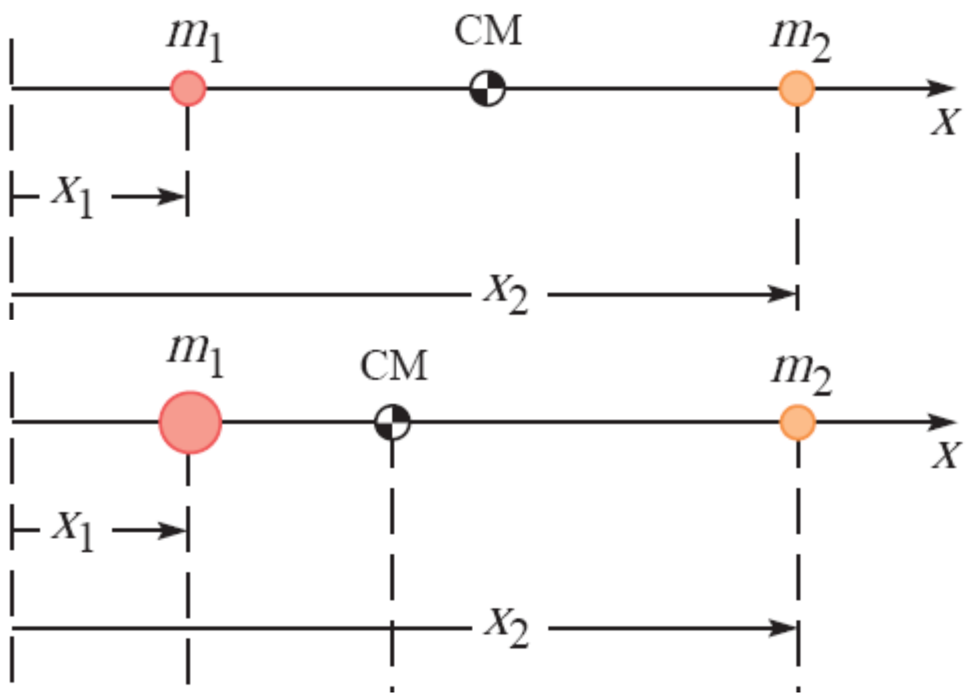
$$y_{CM} = \frac{1}{M}\int y dm$$

$$z_{CM} = \frac{1}{M}\int z dm$$

$$\vec{r}_{CM} = \frac{1}{M}\int \vec{r} dm$$

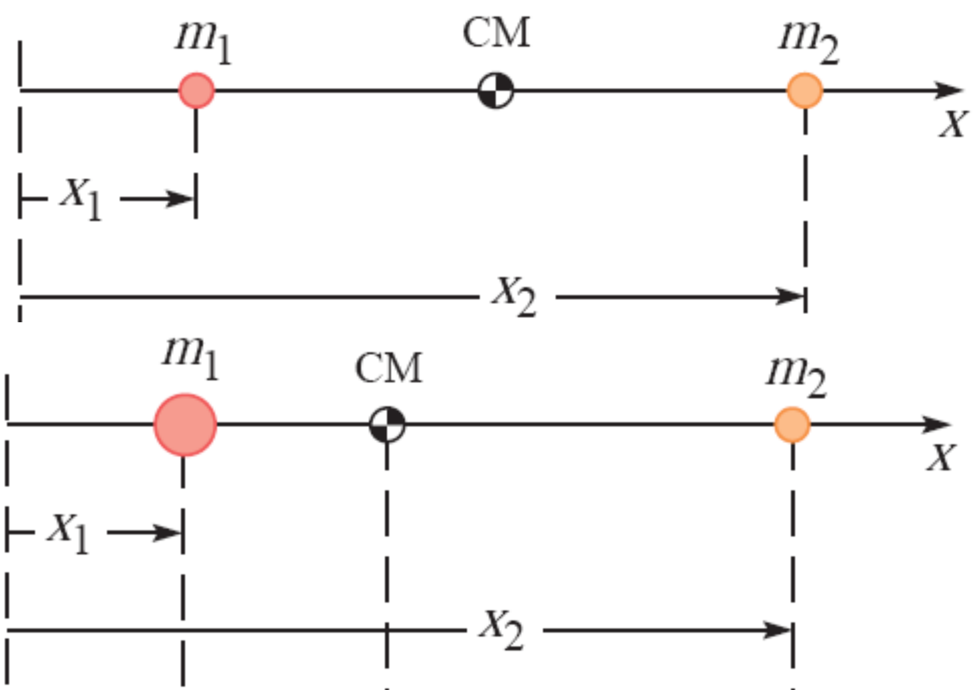
The components of the center of mass are *weighted averages* of the position components of the particles.

Objects with larger mass 'pull' the center of mass towards them



Note that our answer will look like it depends on our choice of origin…. but the physical location will not, regardless of choice! Some choices are easier to calculate than others, though

Question: What is the center of mass of the system of you and the sun? **(Hint: No math required, though we can double-check with the appropriate formulas)**

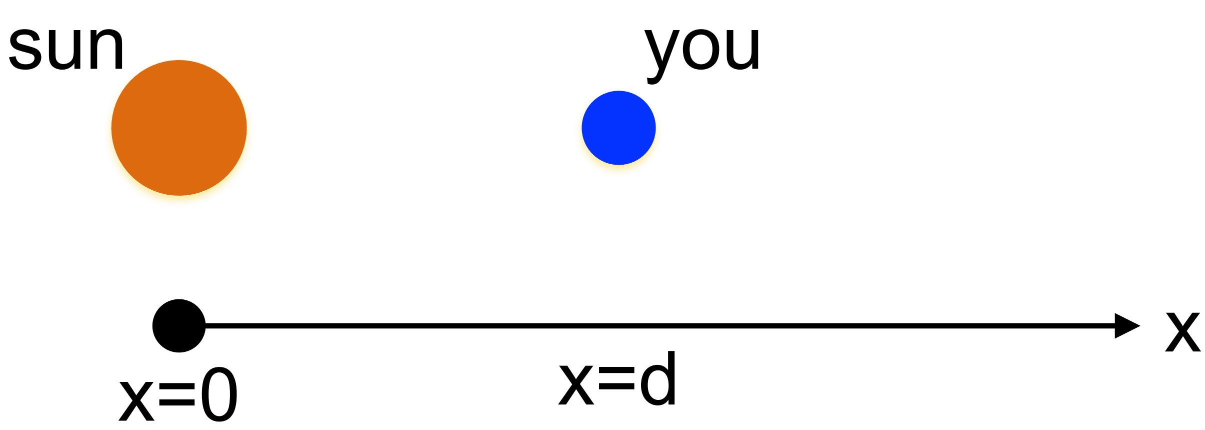


Question: What is the center of mass of the system of you and the sun? **(Hint: No math required, though we can double-check with the appropriate formulas)**

Earth-sun distance = d = 150 billion meters

Mass of the sun = Ms = $2x10^{30}$ kg

Your mass = M ~ 100 kg

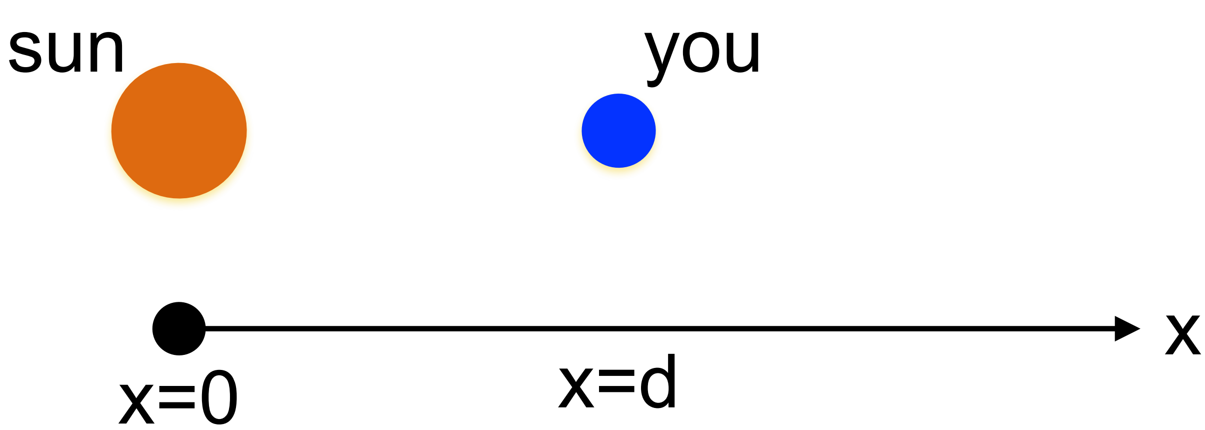


Earth-sun distance = d = 150 billion meters

Mass of the sun = Ms = 2×10^{30} kg

Your mass = m ~ 100 kg

Component form: $x_{\rm CM} = \frac{\sum m_i x_i}{M}$



x_{CM} = $(Ms*x_s)/(m+M_s)$ + $(m*x_{you})/(m+M_s)$ but

$m+M_s$ = 2×10^{30} kg + 100 kg ~ 2×10^{30} kg = M_s

x_{CM} = $(Ms*x_s)/(M_s)$ + $(m*x_{you})/(M_s)$ and $x_s = 0$

x_{CM} =$(m*x_{you})/(M_s)$ = 7.5×10^{-18} m! What does this mean?

The motion of the center of mass obeys the following statement of Newton's second law:

$$\sum \vec{\mathbf{F}}_{\text{ext}} = M\vec{\mathbf{a}}_{\text{CM}}$$

Note: This doesn't mean that particles in the system aren't accelerating, only that the center of mass is not!

The motion of the center of mass obeys the following statement of Newton's second law:

$$\sum \vec{\mathbf{F}}_{\text{ext}} = M\vec{\mathbf{a}}_{\text{CM}}$$

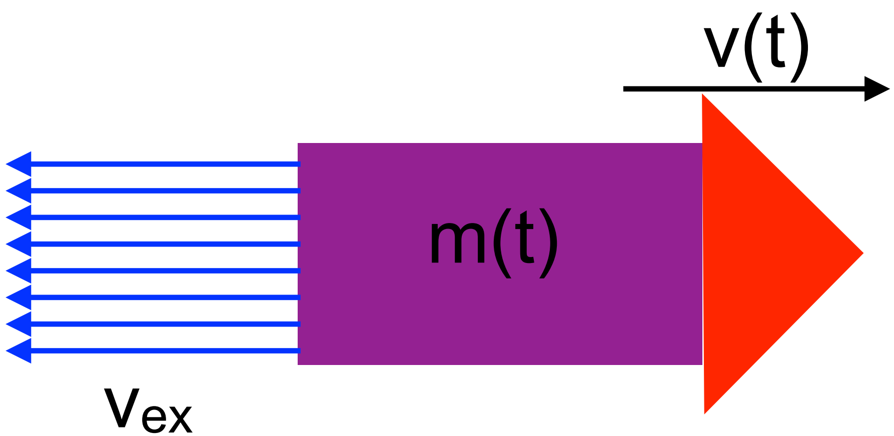
Can relate the total external forces to the total change in momentum…. or to the change in momentum of the Center of Mass!

$$\vec{F}_{ext} = \frac{d\vec{p}_{CM}}{dt} = \frac{d\vec{p}_{tot}}{dt}$$

If a net force is only along one direction, ONLY that component of the momentum changes. In other words, if the net external force is in the y direction, the x and z directions of the CM momentum and total momentum don't change

Rocket equation

Rocket has mass m(t) and speed v(t) at time t. In time dt, it fires rocket propellant dm backwards with constant speed (in its reference frame) v_{ex}. The rocket starts with initial velocity v_0 and initial mass m_0 (which includes the propellant). What is its speed at a later time, when it has a mass m?



ADVANCED WARNING: CALCULUS AHEAD :)

Rocket equation

$p(t) = m(t)v(t)$ **Momentum before ejecting fuel**

$p(t+dt) = m(t+dt)v(t+dt)-dm(v-v_{ex})$ **Momentum after ejecting fuel**

$p(t+dt) = (m+dm)(v+dv)-dm(v-v_{ex})$ **Rewriting**

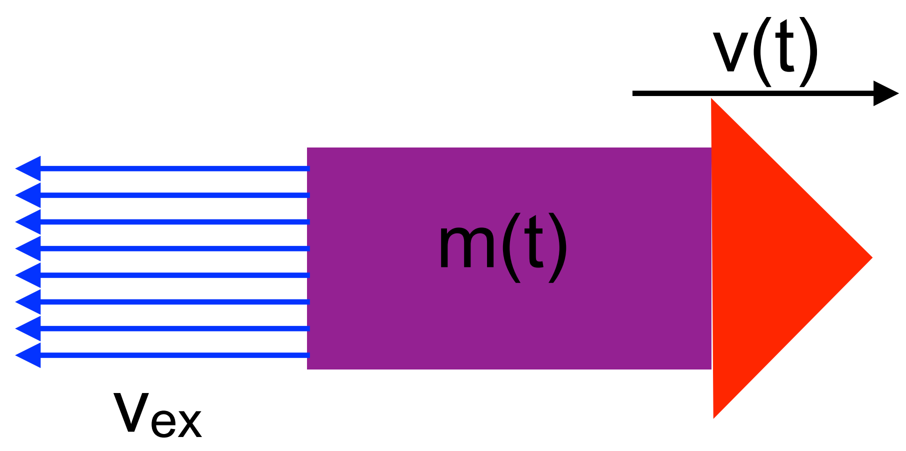
$p(t+dt) = mv+mdv+vdm+dmdv-vdm+dmv_{ex}$ **Expand out**

$p(t+dt) = mv+mdv+dmdv+dmv_{ex}$ **Cancel terms out**

$p(t+dt) = mv+mdv+dmv_{ex}$ **Remove double infinitesimal**

$p(t+dt) = mv+mdv+dmv_{ex} = p(t) = mv$ **Momentum conserved!**

$mdv = -dmv_{ex}$ **Equation to solve**



Rocket equation

$$dv = -v_{ex}\frac{dm}{m}$$

$$\int_{v_0}^{v} dv' = -v_{ex}\int_{m_0}^{m}\frac{dm'}{m'}$$

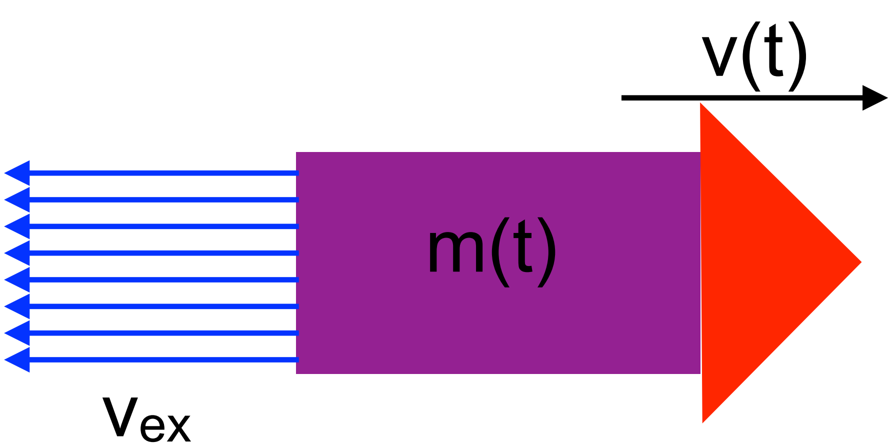
$$v - v_0 = -v_{ex}(\ln m - \ln m_0)$$

$$v - v_0 = v_{ex}(\ln m_0 - \ln m)$$

$$v - v_0 = v_{ex}\ln\frac{m_0}{m}$$

Not on your exams, but … this is pretty cool stuff, no?

$m < m_0$ so log is positive and velocity increases while there is fuel to eject



Propelling a bike with a fire extinguisher :)

https://www.youtube.com/watch?v=TkFky-gKRRI

Group work time!

https://forms.gle/bawkKSLAmSRzN8sq6

Two objects with different masses have the same kinetic energy. Which has the larger magnitude of momentum?

Due to the gravitational interaction between the two stars in a binary star system, each moves in a circular orbit around their center of mass.

One star has a mass of 15.0×10^{30} kg; its center is located at x = 1.0 AU and y = 5.0 AU. The other has a mass of 3.0×10^{30} kg; its center is at
x = 4.0 AU and y = 2.0 AU.

Find the CM of the system composed of the two stars.

(AU stands for *astronomical unit.* 1 AU = the average distance between the Earth and the Sun = 1.5×10^{8} km)

A sports car traveling along a straight line increases its speed from 20.0 mi/h to 60.0 mi/h.

a) What is the ratio of the final to the initial magnitude of its momentum?

b) What is the ratio of the final to the initial kinetic energy?

A uranium nucleus (mass 238 u) initially at rest undergoes radioactive decay. After an alpha particle (mass 4.0 u) is emitted, the remaining nucleus is thorium (mass 234 u). If the alpha particle is moving at 0.050 times the speed of light, what is the recoil speed of the thorium nucleus?

Particle A has a mass of 5.0 g and particle B has a mass of 1.0g. Particle A is located at the origin and particle B is at the point (x,y) = (25 cm, 0). What is the location of the center of mass?

A mother pushes her son in a stroller at a constant speed of 1.52 m/s. The boy tosses a 56.7-g tennis ball straight up at 1.75 m/s and catches it. The boy's father sits on a bench and watches.

a) According to the mother, what are the ball's initial and final momenta?
b) According to the father, what are the ball's initial and final momenta?
c) According to the mother, is the ball's momentum ever zero? If so, when? If not, why not?
d) According to the father, is the ball's momentum ever zero? If so, when? If not, why not?

A boy of mass 25.0 kg is sitting on one side of a seesaw while his older sister, who has a mass of 35.0 kg, is sitting on the other side. Each child is 1.50 m from the center of the seesaw.

a) Where is the center of mass of the two children relative to the center of the seesaw?
b) The older sister moves so that the center of mass of the two children is directly above the center of the seesaw. How far did she move, and in what direction? Did she move closer to or further from the center?

The distance between Jupiter and one of its moons, Io, is 4.22×10^8 m. What is the location of the center of mass of the Jupiter-Io system if Jupiter's mass if 1.9×10^{27} kg and Io's mass is 8.9×10^{22} kg?

A rugby player with a mass of 65.0 kg is running to the right at a speed of 6.00 m/s toward another player of mass 90.0 kg, who is running in the opposite direction at a speed of 5.00 m/s. What is the total momentum of the two players (both magnitude and direction)?

Nicholas, with mass 75.0 kg, jumps vertically upward to a maximum height of 45.0 cm. With what speed does the Earth recoil because of his jump? Assume the mass of the Earth is approximately 6.0x10^24 kg.

The space shuttle uses its thrusters with an exhaust velocity of 4440 m/s. The shuttle is initially at rest in space and accelerates to a final speed of 1.00 km/s.

a) What percentage of the initial mass of the shuttle, including the fuel tank, must be ejected to reach that speed?
b) If the mass of the shuttle and fuel is initially 1.85 x 10^6 kg, how much fuel is expelled?

A comet is traveling through space with speed 3.33 x 10^4 m/s when it encounters an asteroid that was at rest. The comet and asteroid stick together, becoming a single object with a single velocity. If the mass of the comet is 1.11 x 10^14 kg and the mass of the asteroid is 6.66 x 10^20 kg, what is the final velocity of their combination?

A particle has a momentum of magnitude 60.0 kg m/s and a kinetic energy of 4.40×10^2 J.

a) What is the mass of the particle?
b) What is the speed of the particle?

Particle A has a mass of 7.0 g and particle B has a mass of 2.0g. Particle A is located at the origin and particle B is at the point (x,y) = (0 cm, 25 cm). What is the location of the center of mass?

John has a mass of 75 kg, and is sitting in a rubber boat at rest which has mass 20 kg and contains a bucket of water of mass 10 kg. He throws the water out of the boat horizontally with a speed of 10 m/s. What is the resulting motion of the boat?

A fisherman has a tuna biting on his fishing line, which is then connected by the rod to his boat. The fish isn't struggling and is initially 200 meters from the boat when the fisherman tugs on the line, causing the boat to move 20 meters and the tuna to move the remaining 180 meters towards the boat. The mass of the boat is 2700 kg. What is the mass of the tuna? (Assume no resistance or friction)

What is the center of mass of a system with three particles of equal mass located at the vertices of an equilateral triangle?

What is the center of mass of a uniform thin shell of mass?