Mixing in Weak Decays

• Charged Weak Current (exchange of Ws) causes one member of a weak doublet to change into the other

\[ \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \]

• Taus and muons therefore decay into the lightest member of the doublet (their neutrinos)

• electrons are stable as the (e,nu) doublet is the lightest doublet. The virtual W can’t convert to anything
Mixing in Weak Decays

• In the same context, the heavier quark doublets decay via as c, t are heavier

\[
\begin{pmatrix}
  c \\
  t \\
  s \\
  b
\end{pmatrix} \quad c \rightarrow s + "W"; \quad t \rightarrow b + "W"
\]

• s and b quarks should then be stable (their lightest baryons) as the lightest member of their doublets

• But they aren’t……due to mixing between the 3 generations

• for quarks the mass eigenstates are not the same as the decay eigenstates

• b mass eigenstate: what has \( m_b \)

• b decay eigenstate: what interacts with W-t
Quark Mixing: 2 Generations

• If assume only 2 generations. Mixing matrix

\[ M = \begin{pmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} \]

• where \( \theta_C \) is the Cabibbo angle

• \( M \) then rotates from the mass eigenstates \((d,s)\) to the decay eigenstates \((d',s')\) (usually deal with mixing of charge 1/3 quarks but both mix)

\[ \begin{pmatrix} d' \\ s' \end{pmatrix} = M \begin{pmatrix} d \\ s \end{pmatrix} = M \begin{pmatrix} u' \\ c' \end{pmatrix} = M \begin{pmatrix} u \\ c \end{pmatrix} \]

• look at weak vertices (2 identical ways)
Charm Decay

- Charmed meson can Beta decay to lighter mesons which have either s or d quarks

\[ D^+ (cd) \rightarrow K^0 (sd) + \mu^+ + \nu_\mu \quad BF = .07 \]

- \[ D^+ \rightarrow \pi^0 \text{ or } \rho^0 (dd) + \mu^+ + \nu_\mu \quad BF = .003 \]

\[ \frac{.003}{.07} = .043 \approx \frac{V_{cd}^2}{V_{cs}^2} = \frac{\sin^2 \theta_C}{\cos^2 \theta_C} = .05 \]

- Modulo slightly different phase space, the ratio of these decays depends only on the different mixing. Direct measurement of the mixing angle.
Kaon Decays

- Historically first place mixing observed
- decay rates depend on same phase space and spin factors as charged pion decay
- Observed rates only 5% of what they “should” be and Cabibbo proposed a mixing angle whose source was unknown at the time
- This (partially) lead to a prediction that the c quark must exist

\[ m_K = 494\text{MeV} \quad \tau = 1.2 \times 10^{-8} \text{ sec} \]

- \[ K^+(u\bar{s}) \rightarrow \mu^+\nu \quad BF = 64\% \]
- \[ K^+ \rightarrow \pi^+\pi^0 \quad BF = 21\% \]
- \[ K^+ \rightarrow \pi^+\pi^0\pi^0 \text{ or } \pi^+\pi^-\pi^+ \quad BF = 7\% \]
- \[ K^+ \rightarrow \pi^0\mu^+\nu \text{ or } \pi^0 e^+\nu \quad BF = 8\% \]
3 Quark Generations

- For 3 generations need 3X3 matrix. It is unitary and has some phases which don’t matter and can be defined by 3 angles and 1 phase (phase gives particle antiparticle differences….antiparticles use M* Hermitian adjoint)
- called Cabibbo-Kobyashi-Maskawa (CKM) matrix and was predicted by K-M before the third generation was discovered

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\cong
\begin{pmatrix}
.97 & .22 & .004 \\
.22 & .97 & .04 \\
.01 & .04 & .997
\end{pmatrix}
\]

- Each $V_{ij}$ tells what factor needed for W vertex. Shown are experimental values. No theory predicts the amount of mixing,
CKM Matrix Numerology

- For N generations need NXN unitary matrix. Matrix has $2N^2$ terms (real and complex) and it has $N^2$ constraints (rows x columns $= 0, 1$). For 2N quarks have 2N-1 arbitrary phases
- \[ \begin{array}{c}
N = 2 \\
N^2 = 4 \\
2N-1 = 3 \\
\end{array} \]
- \[ \begin{array}{c}
\text{parameters needed} = 1 \\
\text{parameters needed} = 4 \\
\end{array} \]
- 3 generations $\rightarrow$ 3 angles (Euler angles) + 1 phase
- CKM* acts on antiquarks. phase causes a small particle-antiparticle difference. Need at least 3 generations to have CP violation/matter-dominated Universe

\[
CKM = \begin{pmatrix}
1 & 0 & 0 \\
0 & c1 & s1 \\
0 & -s1 & c1
\end{pmatrix} \times \begin{pmatrix}
c2 & 0 & -s2e^{i\phi} \\
0 & 1 & 0 \\
s2e^{-i\phi} & 0 & c2
\end{pmatrix} \times \begin{pmatrix}
c3 & s3 & 0 \\
-s3 & c3 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]
**B mesons**

- B mesons contain b quarks (D mesons contain c quarks)

\[
\begin{align*}
B^+ &= u\bar{b} & B^- &= \bar{u}b & \tau &= 1.7 \times 10^{-12} \text{s} \\
B^0 &= d\bar{b} & \bar{B}^0 &= \bar{d}b & \tau &= 1.6 \times 10^{-12} \text{s} \\
B_c^+ &= c\bar{b} & B_c^- &= \bar{c}b & \tau &= 0.5 \times 10^{-12} \text{s} \\
B_s^0 &= s\bar{b} & \bar{B}_s^0 &= \bar{s}b & \tau &= 1.5 \times 10^{-12} \text{s}
\end{align*}
\]

- B, D and τ lifetimes are just long enough so their path lengths can be detected

- use to measure B properties and identify B,D,τ in ee,pp collisions. For B mesons

<table>
<thead>
<tr>
<th>E (GeV)</th>
<th>p (GeV)</th>
<th>γ</th>
<th>v c</th>
<th>βγcτ</th>
<th>Path Length</th>
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<tr>
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<td>20</td>
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<tr>
<td>50.3</td>
<td>50</td>
<td>10</td>
<td>.995</td>
<td>3</td>
<td>mm</td>
</tr>
</tbody>
</table>

\[
H \rightarrow b\bar{b}, \tau\bar{\tau} \quad t \rightarrow bW
\]

\[
Z \rightarrow b\bar{b}, \tau\bar{\tau}
\]
B Decays

- B mesons are dominated by the decay of the b quark. As large mass, phase space differences are small and can get branching fractions by just counting

\[
\begin{align*}
\frac{d\bar{c}, d\bar{u}}{D^{-}, \pi^{-}} & \rightarrow W \\
\bar{b} \rightarrow \bar{c} & \approx \frac{V_{cb}^2}{|V_{ub}|^2} = \frac{0.04^2}{0.004^2} \approx 100 \\
\Gamma(B \rightarrow D\mu\nu) & = \frac{0.04^2 m_B}{2 m_K^5} \approx 1.7 \times 10^4
\end{align*}
\]
Particle-antiparticle mixing and CP violation

- There is another type of “mixing” which is related to quark mixing. This can lead to observation and studies of CP violation.
- Consider the mesons which are neutral and composed of different types of quarks:
  \[ K^0 (d\bar{s}) \quad D^0 (u\bar{c}) \quad B^0_d (d\bar{b}) \quad B^0_s (s\bar{b}) \]
  \[ \bar{K}^0 (\bar{d}s) \quad \bar{D}^0 (\bar{u}c) \quad \bar{B}^0_d (\bar{d}b) \quad \bar{B}^0_s (\bar{s}b) \]
- Weak interactions can change particle into antiparticle as charge and other quantum numbers are the same. The “strangeness” etc are changing through CKM mixing.
• Depends on $V_{ij}$ at each W vertex
• as $V$ and $V^*$ are different due to phase, gives particle-antiparticle difference and CP violation (any term with t-quark especially)
• the states which decay are admixtures of the “strong” state (a rotation). They can have different masses and different lifetimes

$$|K_1\rangle = \alpha|K^0\rangle + \beta|\bar{K}^0\rangle$$

$$|K_2\rangle = \alpha|K^0\rangle - \beta|\bar{K}^0\rangle$$

• #particle vs #antiparticle will have a time dependence. Eg. If all particle at $t=0$, will be a mixture at a later time
• the phenomenology of K’s is slightly different than B/D’s and we’ll just do K’s in detail. Kaons rotate and give long-lived and short-lived decays. B/D also rotate but lifetimes are ~same.
Neutral Kaon Semi-leptonic Decay

- Properties for “long” and “short” lived

\[ K^0 : \text{mass} = 498\, \text{MeV}, m_{K_L} - m_{K_S} = 3 \times 10^{-12}\, \text{MeV} \]

\[ \tau_{K_S} = 10^{-10}\, \text{sec} \quad \tau_{K_L} = 5 \times 10^{-8}\, \text{sec} \]

- Semi-leptonic (Beta) decays. Positive or negative lepton tells if K or anti-K decayed

\[ K^0 (d\bar{s}) \rightarrow \pi^- (d\bar{u}) + e^+ \text{ or } \mu^+ + \nu \]

\[ \bar{K}^0 (\bar{d}s) \rightarrow \pi^+ (\bar{d}u) + e^- \text{ or } \mu^- + \bar{\nu} \]

- Partial width is exactly the same as charged K decay (though smaller BF for Short and larger for Long).

\[ BF = 7 \times 10^{-4} K_S \Rightarrow \Gamma_\beta = \frac{BF}{\tau} \approx 0.7 \times 10^7\, \text{sec}^{-1} \]

\[ BF = 0.3 K_L \Rightarrow \Gamma_\beta = \frac{BF}{\tau} \approx 0.6 \times 10^7\, \text{sec}^{-1} \]
Neutral Kaon Hadronic Decays

- Also decay hadronically
  \[ K^0 (d\bar{s}) \rightarrow \pi^+ + \pi^- \text{ or } \pi^0 + \pi^0 \]
  \[ \bar{K}^0 (d\bar{s}) \rightarrow \pi^+ + \pi^- \text{ or } \pi^0 + \pi^0 \]
  \[ K^0 (d\bar{s}) \rightarrow \pi^+ + \pi^- + \pi^0 \text{ or } \pi^0 + \pi^0 + \pi^0 \]
  \[ \bar{K}^0 (d\bar{s}) \rightarrow \pi^+ + \pi^- + \pi^0 \text{ or } \pi^0 + \pi^0 + \pi^0 \]

- Both decay to same final states which means the mixed states \( K_1 \) and \( K_2 \) also decay to these 2\( \pi \) and 3\( \pi \) modes. Means initial states can mix and have interference

\[ K^0 \leftrightarrow \bar{K}^0 \]
Sidenote C+P for Pions

- Parity operator Pf(x,y,z)=f(-x,-y,-z). Intrinsic parity for psuedoscaler mesons (like K,πi) is -1
- Charge conjugation operator C. Changes particle to antiparticle.
  \[ C|\pi^+\rangle = \lambda |\pi^-\rangle \quad C|\pi^0\rangle = \lambda |\pi^0\rangle \]
  \[ C|K^+\rangle = \lambda |K^-\rangle \quad C|K^0\rangle = \lambda |\bar{K}^0\rangle \]
  \[ C(C|\pi^0\rangle) = C(\lambda |\pi^0\rangle) = \lambda^2 |\pi^0\rangle \implies \lambda = \pm 1 \]
- Can work out eigenvalue. As C changes charge, C=-1 for photon

\[ \begin{array}{ccc}
C & e^- & = \\
& & e^+ \\
\end{array} \]

- given its decay, pion has C= +1

\[ \pi^0 \rightarrow \gamma\gamma \quad \frac{BF(\pi^0 \rightarrow \gamma\gamma)}{BF(\pi^0 \rightarrow \gamma\gamma)} < 4 \times 10^{-7} \]
Neutral Kaon Hadronic Decays

• 2 pion and 3 pion are CP eigenstates with eigenvalue +1 for 2pi and -1 for 3pi

\[ CP|\pi^+\pi^-\rangle = +|\pi^-\pi^+\rangle \quad CP|\pi^0\pi^0\rangle = +|\pi^0\pi^0\rangle \]

\[ CP|\pi^+\pi^-\pi^0\rangle = -|\pi^-\pi^+\pi^0\rangle \quad CP|3\pi^0\rangle = -|3\pi^0\rangle \]

• \( K_1 \) and \( K_2 \) also CP eigenstates

\[ |K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) \approx K_S \quad CP = +1 \]

\[ |K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - \beta|\overline{K}^0\rangle) \approx K_L \quad CP = -1 \]

\[ \tau(K_S) = 0.9 \times 10^{-10} \text{s} \]

\[ \tau(K^\pm) = 1.2 \times 10^{-8} \text{s} \]

\[ \tau(K_L) = 5.2 \times 10^{-8} \text{s} \]

• different values of matrix element if initial and final states are the same CP eigenstate or if they are not CP eigenstates (like K+ or beta decays)

• if CP is conserved, \( K_1/K_S \) decays to 2 pions and \( K_2/K_L \) decays to 3 pions. More phase space for 2 pions and so faster decay, shorter lifetime.
Decay and Interference

- From Schrodinger eq. plane wave solutions

\[ \psi(K_S) = A_S(t = 0)e^{-\left(\frac{\Gamma_S}{2} + im_s\right)} \]
\[ \psi(K_L) = A_L(0)e^{-\left(\frac{\Gamma_L}{2} + im_L\right)} \]

\[ e^{iEt/\hbar}, \frac{E}{\hbar} \Rightarrow m \]
\[ |\psi|^2 = e^{-t/\tau} \Rightarrow \Gamma = \frac{1}{\tau} \]

**assume**: \( K_S \cong K_1; K_L \cong K_2 \)

- the two amplitudes have to be added and then squared. Gives interference. Example: start with pure \( K^0 \)

\[ \left| K^0 \right> = \frac{1}{\sqrt{2}} (|K_L> + |K_S>) \Rightarrow A_S(0) = A_L(0) = \frac{1}{\sqrt{2}} \]

- Intensity is this amplitude squared

\[ I(K^0) = [\psi(K_S) + \psi(K_L)][\psi^*(K_S) + \psi^*(K_L)] \]
\[ = \frac{1}{4} [e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta mt] \]

\[ \Delta m \equiv m_L - m_S \cong 10^{-5} \, eV \]

- small mass difference between the two weak decay eigenstates
Decay and Interference

• Do the same for anti-K

\[ I(\bar{K}^0) = [\psi(K_S) - \psi(K_L)] [\psi^*(K_S) - \psi^*(K_L)] \]

\[ = \frac{1}{4} \left[ e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right] \]

• get mixing. Particle<->antiparticle varying with time.

• At large time get equal mixture = 100% \( K_L \)

• the rate at which \( K \rightarrow\) anti-K depends on \( 1/\Delta m \). You need to mix \( K<->\)antiK before they decay to have \( K_S \) and \( K_L \)

\[ \Delta m \tau_S = 0.47 \Rightarrow " K_S " , " K_L " \text{ decays} \]

But

\[ (\Delta m)^{-1} \gg \tau_{K^0} \Rightarrow \text{ just } " K^0 " \text{ decays} \]
**K_S Regeneration**

- Assume pure $K_L$ beam
- strikes a target made up of particles (p,n)
- different strong interaction cross section for $K$ and anti-$K$
  \[
  \bar{K}^0(\bar{d}s) + n \rightarrow \Lambda(uds) + \pi^0
  \]
  \[
  K^0 + n \not\rightarrow \Lambda(uds) + \pi^0
  \]
- mix of $K$-anti$K$ no longer 1:1. Example, assume “lose” 0.5 anti$K$, 0.0 $K$. gives (ignoring phases and so not quite right)

\[
K^0 - \frac{\bar{K}^0}{2} = aK_L + bK_S = \\
\Rightarrow a = \frac{3}{4}, b = \frac{1}{4}
\]

- First observed by Lederman et al. measures particle/antiparticle differences. Useful experimental technique
CP Violation

- C changes particle to antiparticle
- P operator flips space (mirror image)
- T time reversal \( t \rightarrow -t \)
- fundamental axiom (theory?) of quantum mechanics CPT is conserved
- Weak interaction violate all 3. CP violation is the same as T violation. Three observations (so far) of this

1. Universe is mostly matter (Sakharov 1960s)
2. \( K_L \) decay to 2 pions (Christianson, Cronin, Fitch and Turlay, 1964)
3. neutral B decays
spark chambers and so poor mass resolution. Identify $K \rightarrow 2\pi$ as in forward direction

mostly: $K_L \rightarrow \pi^+ + \pi^- + \pi^0$

$45 \pm 9 \quad K_L \rightarrow \pi^+ + \pi^-$
CP Violation in K decays

- $K_s$ and $K_L$ (the particles which have different lifetimes) are NOT eigenstates of CP. Instead $K_1$ and $K_2$ are

$$K_{1,2} = \frac{1}{\sqrt{2}}(K^0 \pm \bar{K}^0)$$

$$K_L = \frac{1}{\sqrt{1+|\varepsilon|^2}}(K_2 + \varepsilon K_1) \quad |\varepsilon| = 2.2 \times 10^{-3}$$

$$K_S = \frac{1}{\sqrt{1+|\varepsilon|^2}}(K_1 - \varepsilon K_2)$$

- When $K_L$ decays, mostly it is decaying to a $\text{CP}=-1$ state (3 pions) but sometimes to a $\text{CP}=+1$ state (2 pions)
CP violation in K decays

• CP is then explained by having a phase in the mixing between K and anti-K
• other sources of CP violation ("fifth force") are ruled out as inconsistent with the various ways of observing CP violation

\[ K_L \rightarrow \pi^+ \pi^- \quad BF = 2.1 \times 10^{-3} \]
\[ K_L \rightarrow \pi^0 \pi^0 \quad BF = 9 \times 10^{-4} \]
\[ K_S \rightarrow \pi^+ \pi^- \pi^0 \quad BF = (3\pm) \times 10^{-7} \]

\[
\text{charge asymmetry} = \delta_L = 0.3 \times 10^{-2}
\]

\[
\frac{\Gamma(K_L \rightarrow \pi^- \mu^+ \nu) - \Gamma(K_L \rightarrow \pi^+ \mu^- \bar{\nu})}{\Gamma(K_L \rightarrow \pi^- \mu^+ \nu) + \Gamma(K_L \rightarrow \pi^+ \mu^- \bar{\nu})} \quad \text{(or e)}
\]

\[
\left| \frac{\text{amp}(K_L \rightarrow \pi^0 \pi^0)}{\text{amp}(K_L \rightarrow \pi^+ \pi^-)} \right| = 0.9950 \pm 0.0008
\]
Indirect vs Direct CP

- Indirect CP is due to the mixing (the box diagram)
- Direct is in the decay and that the charged and neutral modes are slightly different (different isospin)

\[ K_L \rightarrow \pi^+\pi^- \quad BF = 2.1 \times 10^{-3} \]
\[ K_L \rightarrow \pi^0\pi^0 \quad BF = 9 \times 10^{-4} \]
\[ \frac{|amp(K_L \rightarrow \pi^0\pi^0)|}{|amp(K_L \rightarrow \pi^+\pi^-)|} = 0.9950 \pm 0.0008 \]
A Study of Direct CP Violation in the Decay of the Neutral Kaon via a Precision Measurement of $|n_{o-}/n_{+}|$

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B. Cousins, J. Greenhalgh, and M. Schwartz
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Fermilab proposal 617 January 1979
20+ year experiment at FNAL and CERN

ABSTRACT

In this proposal, we describe an experiment to measure the ratio $R$ of the CP violating amplitudes $|n_{o-}|$ and $|n_{+}|$ to a precision of better than 1% thereby improving the present results by about one order of magnitude. If the CP violation is confined to the mass matrix, $R = 1.0$ exactly. Recent theoretical considerations which unify the CP violating interaction with the CP conserving weak and electromagnetic interactions among six quarks predict $R$ differing from 1.0 by sizable amounts.

wrong. small effect and very large hadronic factors
B’s: Mixing and CP violations

- Neutral B’s (and Ds) also mix and have CP violating decays. These depend on CKM matrix elements (and are better at determining them than K decays). Bs and Ks both oscillate a few times before they decay.
- Different than K system as many decay channels most of which are not CP eigenstates. Also no “L,S” as the lifetimes of the “1,2” states are about the same.

\[ B_{1,2} = \frac{1}{\sqrt{2}} (B^0 \pm B^0) \]

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>( \Delta m )</th>
<th>( \Delta m \tau / \hbar )</th>
</tr>
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<tbody>
<tr>
<td>( K_S )</td>
<td>( 0.9 \times 10^{-10} ) sec</td>
<td>( 4 \times 10^{-12} ) MeV, ( 0.5 \times 10^{10} ) ( \hbar / s )</td>
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<tr>
<td>( B_d )</td>
<td>( 1.5 \times 10^{-12} ) sec</td>
<td>( 3 \times 10^{-10} ) MeV, ( 0.5 \times 10^{12} ) ( \hbar / s )</td>
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<tr>
<td>( B_s )</td>
<td>( 1.5 \times 10^{-12} ) sec</td>
<td>( 1.5 \times 10^{-8} ) MeV, ( 19 \times 10^{12} ) ( \hbar / s )</td>
</tr>
</tbody>
</table>
12. CP violation in meson decays

finding CP violation, that is, \(|q/p| \neq 1\) and/or \(\sin \theta_{B} \neq 0\), would constitute evidence for new physics.

More details on theoretical and experimental aspects of \(B^0 \rightarrow B^0\) mixing can be found in [39]. Note that BABAR use \(R_{B} = \frac{\alpha^B}{\beta^B}\) and \(n_{\pi} = \frac{|q/p|}{\alpha_{B}^{\pi}}\). Belie \(R_{\eta} = \frac{\alpha^{\eta}}{\beta^{\eta}}\) and \(\eta_{\pi} = \frac{|q/p|}{\alpha_{\eta}^{\pi}}\).

12.6. \(B^0\) and \(B^0\), Decays

The upper bound on the CP asymmetry in semileptonic \(B^0\) decays [41] implies that CP violation in \(B^0 \rightarrow B^0\) mixing is a small effect [see \(\lambda_{B} \approx 2 \times 10^{-3}\)] where \(\lambda_{B} \approx 2 \times 10^{-3}\) and \(\rho_{B} \approx \alpha_{B}^{\pi} \alpha_{\eta}^{\pi} \beta_{B}^{\eta} \beta_{\eta}^{\pi}\).

The Standard Model prediction is

\[\lambda_{B} \approx 2 \times 10^{-3}\]

In models where \(\Gamma_{\pi}/M_{B}\) is approximately real, such as the Standard Model, an upper bound on \(\Delta \lambda_{B}/\Delta m_{B} \approx 2 \times 10^{-3}\) provides yet another upper bound on the deviation of \(\eta_{\pi}/\eta_{\eta}\) from one. This constraint does not hold if \(\Gamma_{\pi}/M_{B}\) is approximately imaginary. [An alternative parameterization uses \(q/p = 1 - i \epsilon_{B}/(1 + \epsilon_{B})\), leading to \(\lambda_{B} \approx 2 \times 10^{-3}\).]

The small deviation [less than one percent] of \(\eta_{\pi}/\eta_{\eta}\) from 1 implies that, at the present level of experimental precision, CP violation in \(B^0\) mixing is a negligible effect. Thus, for the purpose of analyzing CP asymmetries in hadronic \(B^0\) decays, we can use:

\[\lambda_{B} \approx 2 \times 10^{-3}\]

where \(\phi_{B}(\beta)\) refers to the phase of \(M_{B}\) appearing in Eq. (12.39) that is appropriate for \(B^0 \rightarrow B^0\) oscillations. Within the Standard Model, the corresponding phase factor is given by

\[e^{\frac{-i \phi_{B}(\beta)}{\Delta m_{B} \Delta t}} = \frac{(V_{ud}) B}{(V_{ub}) C_{B}}\]

Some of the most interesting decays involve final states that are common to \(B^0\) and \(\bar{B}^0\) [42,43]. It is convenient to rewrite Eq. (12.37) for \(B^0\) decays as [44,45,46]

\[A_{f}(t) = S_{f}(\Delta m_{B} t) \cdot C_{f}(\Delta m_{B} t)\]

\[S_{f} \approx \frac{2 \sin |\lambda_{f}|}{|\lambda_{f}|} \cdot C_{f} \approx \frac{1}{1 + |\lambda_{f}|^2}\]

where we assume that \(\Delta m_{B} = 0\) and \(|q/p| = 1\). An alternative notation in use is \(A_{f} \approx C_{f}\), but this \(A_{f}\) should not be confused with the \(A_{f}\) of Eq. (12.10).

A large class of interesting processes proceed via quark transitions of the form \(d \rightarrow u q q'\) with \(q' = s\) or \(d\). For \(q = c\) or \(u\), there are contributions from both tree \([t]\) and penguin \([p]\) amplitudes. \(q_{u} = u, c, t\) is the quark in the loop) diagrams (Fig. 12.2) which carry different weak phases:

\[A_{f} = \left(V_{ud}^{*} V_{tq}ight) A_{f} + \sum_{q_{u}, s, c, t} \left(V_{us}^{*} V_{tsq}ight) p_{tq}^{s, c, t}\]

where \(p_{tq}^{s, c, t} = p_{tq}^{s, c, t}\) and \(p_{tq}^{s, c, t} = p_{tq}^{s, c, t}\). CP-violating phases in Eq. (12.74) appear only in the CKM elements, so that

\[A_{f} = \frac{(V_{ud}) B}{(V_{ub}) C_{B}} A_{f} + \left(V_{us}^{*} V_{tsq}\right) p_{tq}^{s, c, t}\]

For \(f = J/\psi K\), which proceeds via \(b \rightarrow c u s\) transition, we can write

\[A_{f} = \frac{(V_{ud}^{*} V_{tq}) A_{f} + (V_{us}^{*} V_{tsq}) p_{tq}^{s, c, t}}{(V_{ub}) C_{B}}\]

where \(T_{tq} = t_{tq} + \rho_{tq}^{s, c, t}\) and \(p_{tq}^{s, c, t} = p_{tq}^{s, c, t}\). A subtlety arises in this decays that is related to the fact that \(B^0\) decays into a final \(J/\psi K^0\) state while \(B^0\) decays into a final \(J/\psi K^0\) state. A common final state, e.g., \(J/\psi K^0\), is reached only via \(b \rightarrow s K\), mixing. Consequently, the phase factor [defined in Eq. (12.39)] corresponding to neutral \(K\) mixing, \(e^{-i \phi_{B}(\beta)} = (V_{ud}^{*} V_{tq})/V_{ub}\), plays a role.

\[A_{f} = \frac{(V_{ud}^{*} V_{tq}) A_{f} + (V_{us}^{*} V_{tsq}) p_{tq}^{s, c, t}}{(V_{ub}) C_{B}}\]

For \(q = s\) or \(d\), there are only penguin contributions to \(A_{f}\), that is, \(t_{f} = 0\) in Eq. (12.73). [The tree \(b \rightarrow u q q'\) transition followed by \(u \rightarrow q q'\) recattering is included below in the \(p_{tq}^{s, c, t}\) terms.] Again, CKM unitarity allows us to write \(A_{f}\) in terms of two CKM combinations. For example, for \(f = \phi K_{S}\), which proceeds via \(b \rightarrow s K\), transition, we can write:

\[A_{f} = \frac{(V_{ud}^{*} V_{tq}) A_{f} + (V_{us}^{*} V_{tsq}) p_{tq}^{s, c, t}}{(V_{ub}) C_{B}}\]

where \(p_{tq}^{s, c, t} = p_{tq}^{s, c, t}\) and \(p_{tq}^{s, c, t} = p_{tq}^{s, c, t}\). Figure 12.2: Feynman diagrams for [a] tree and [b] penguin amplitudes contributing to \(B^0 \rightarrow f\) or \(B_s \rightarrow f\) via a \(b \rightarrow c u s\) quark decay process.
**Glashow-Weinberg-Salam Model**

- EM and weak forces mix...or just EW force. Before mixing Bosons are massless:

- **Group** | **Boson Coupling** | **Quantum No.**
  - SU(2)$_L$ | $W^{1,2,3}$ | $g$ | T weak isospin
  - U(1) | $B$ | $g'$ | Y leptonic hypercharge

\[ T_3 + Y/2 = Q \] (elec.ch arg e)

- Interaction Lagrangian is

\[ L_{\text{int}} = g \mathbf{T} \cdot \mathbf{W} + g' \frac{YB}{2} \]

- convert to physical fields. Neutrals mix (B,$W^3 \rightarrow Z$, photon). $W,Z$ acquire mass. Force photon mass=0. Higgs Boson introduced to break mass symmetry (A is same field as in EM....4 vector)

\[ A = \sin \theta_w W^3 + \cos \theta_w B \]
\[ Z = \cos \theta_w W^3 - \sin \theta_w B \]

$\theta_w = \text{weak mixing angle} \Rightarrow \tan \theta_w \equiv \frac{g'}{g}$
Higgs Boson

• breaking electroweak symmetry gives: massive W+Z Bosons
  mass=0 photon
  1 or more scalar particles (Higgs)
  minimal SUSY, 2 charged and 3 neutral

• Higgs couples to mass and simplistically decays to the most massive available particles

• “easy” to produce in conjunction with heavy objects (helps to discover??)

\[
\frac{\sigma(HZ) + \sigma(HW)}{\sigma(H)_{production}} \cong 20\% \quad m_H = 100\text{GeV}
\]
Standard Model Higgs Boson

- Branching fraction depends on mass
- Use $ZH, WH$ for $m < 135$ GeV
- Use $WW$ for $m > 135$ GeV
- Current limits use 1-2 fb$^{-1}$
- D0: 12 Higgs decay channels + 20 analyses combined

$WH \rightarrow l^\pm v\bar{b}b$

$ZH \rightarrow l^+l^-b\bar{b}$

$ZH \rightarrow \nu\bar{\nu}b\bar{b}$

$H \rightarrow WW \rightarrow l\nu l\nu$

D0+CDF

Limits 1.4-8 times SM (2007)
• Look at EM and 2 weak “currents”

\[ L_{EM} = g \sin \theta_w T_3 W^3 + g' \cos \theta_w \frac{Y B_\lambda}{2} \]

\[ L_{EM} = g \sin \theta_w (T_3 + \frac{Y}{2}) A_\lambda \]

\[ g' \cos \theta_w = g \sin \theta_w \equiv e \]

\[ T_3 + \frac{Y}{2} = Q \]

\[ L_{EM} = g \sin \theta_w A Q = e A_\lambda J^{\lambda}_{em} \]

• charged current. Compare to mu/beta decay (have measured weak force, eg. weak mixing only “new” free parameter)

\[ L_{cc} = g \left[ T_\lambda^+ W_\lambda^+ + T_\lambda^- W_\lambda^- \right] \]

\[ \frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}} \Rightarrow M_W = \frac{g}{2^{5/4} \sqrt{G_F}} = \frac{e / \sin \theta_W}{2^{5/4} \sqrt{G_F}} = \frac{37.3 GeV}{\sin \theta_W} \]

• weak neutral current

\[ L_{NC} = \frac{g}{\cos \theta_W} Z (T_3 - \sin^2 \theta_w Q) = \frac{g}{\cos \theta_W} Z \lambda J^{\lambda}_{NC} \]

\[ \frac{g^2 / \cos^2 \theta_W}{8M_Z^2} = \frac{G_F}{\sqrt{2}} \Rightarrow M_Z = \frac{M_W}{\cos \theta_W} \]
**W and Z couplings**

- EW model has left-handed doublets right handed singlets
- W couplings to left-handed component and always essentially the same

\[ g_{W^\pm} \cdot T^3 = \pm \frac{1}{2} g \text{ doublet; } = 0 \text{ sin glet} \]

- Z to left-handed doublet
- Z to right-handed singlet

\[ Z = \frac{e}{\sin \theta_W \cos \theta_W} [T^3 - Q \sin^2 \theta_W] \]

- redefine as Vector and Axial parts of V-A

\[ c_V = g_L + g_R = T^3 - 2Q \sin^2 \theta_W \]
\[ c_A = g_L - g_R = T^3 \]
# Z decays/vertices

\[
\begin{array}{cccccc}
Z \rightarrow & e^+ e^- & \nu_e \bar{\nu}_e & u\bar{u} & d\bar{d} \\
\mu^+ \mu^- & \nu_\mu \bar{\nu}_\mu & c\bar{c} & s\bar{s} \\
\tau^+ \tau^- & \nu_\tau \bar{\nu}_\tau & b\bar{b}
\end{array}
\]

\[
\begin{array}{cccc}
T_3 & - \frac{1}{2} & + \frac{1}{2} & + \frac{1}{2} & - \frac{1}{2} \\
Q & -1 & 0 & + \frac{2}{3} & - \frac{1}{3}
\end{array}
\]

\[
\begin{array}{cccc}
c_V & - .08 & + \frac{1}{2} & .19 & - .34 \\
c_A & - \frac{1}{2} & + \frac{1}{2} & + \frac{1}{2} & - \frac{1}{2} \\
c_A^2 + c_V^2 & .26 & .50 & .29 & .37
\end{array}
\]

## Color factor of 3 for quarks

\[
c_V = g_L + g_R = T^3 - 2Q\sin^2 \theta_W \\
c_A = g_L - g_R = T^3 \\
\sin^2 \theta_W \approx 0.21
\]
Z Branching Fraction

- Can use couplings to get branching modes
- PDG measured values in ()

\[
\begin{align*}
\frac{Z \to ee}{Z \to all} &= \frac{.26}{3*.26+3*.50+2*3*.29+3*3*.37} = \frac{.26}{7.3} = .036 \ (0.034) \\
\frac{Z \to \nu\nu}{Z \to all} &= \frac{3*.50}{7.3} = 0.21 \ (0.20) \\
\frac{Z \to b\bar{b}}{Z \to all} &= \frac{3*.37}{7.3} = 0.15 \ (0.15) \\
\frac{Z \to q\bar{q}}{Z \to all} &= \frac{3*3*.37 + 2*3*.29}{7.3} = 0.70 \ (0.70)
\end{align*}
\]
Neutrino Physics

- Three “active” neutrino flavors (from Z width measurements). Mass limit from beta decay

\[
\begin{align*}
    m_{\nu_e} &< 3 \text{ eV} \\
    m_{\nu_\mu} &< 0.2 \text{ MeV} \\
    m_{\nu_\tau} &< 18 \text{ MeV}
\end{align*}
\]

\[
\Delta m^2_{ex} \approx 10^{-4} \text{ eV}^2 \quad x = \mu \text{ or } \tau
\]

\[
\Delta m^2_{\mu\tau} \approx 10^{-3} \text{ eV}^2 \quad x = \tau \text{ (or inactively)}
\]

- Probably have non-zero masses as they oscillate (right-handed neutrinos? messes up electroweak)
- Only have weak interactions and can be either charged or neutral currents

![Diagram of neutrino interactions and oscillations](image-url)
Neutrino Cross Sections

• Use Fermi Golden Rule

\[ Rate = \frac{2\pi}{\hbar} |M|^2 \text{ *phasespace} \]

• M (matrix element) has weak interaction physics...W, Z exchange \( \sim \) constant at modest neutrino energies. Same G factor as beta decay

\[ \frac{G}{\sqrt{2}} = \frac{g^2}{8M_w^2} \]

\[ \frac{1}{q^2 + M_w^2} \Rightarrow \frac{1}{M_w^2} \quad E_\nu < M_w \]

• cross section depends on phase space and spin terms. Look at phase space first for charged current. Momentum conservation integrates out one particle

\[ \nu_e e \rightarrow e \nu_e \quad (CC) \]

\[ \text{phase space } \propto p_e dp_e p_\nu dp_\nu \]

\[ p_{cm} = p_e = p_\nu \Rightarrow \sigma = \frac{4G^2}{\pi} p_{cm}^2 \]
Neutrino Cross Sections II

- Look in center-of-momentum frame

\[
s = M^2 = E_{tot}^2 - p_{tot}^2
\]
\[\vec{p}_e = -\vec{p}_\nu \Rightarrow p_{tot} = 0 \quad E_{tot} = E_\nu + E_e \approx 2p
\]
\[\Rightarrow s = (2p)^2 \quad \sigma = \frac{4G^2 p^2}{\pi} = \frac{G^2 s}{\pi}
\]

- \(s\) is an invariant and can also determine in the lab frame

\[
p_{tot} = p_\nu = E_\nu \quad E_{tot} = E_\nu + m_e
\]
\[s = E_\nu^2 + 2m_e E_\nu + m_e^2 - p_\nu^2 \approx 2m_e E_\nu
\]
\[\sigma = \frac{G^2 2mE_\nu}{\pi}
\]

- Cross section grows with phase space (either neutrino energy or target mass)

\[
\frac{\sigma(vp)}{\sigma(ve)} \approx \frac{m_p}{m_e} = 2000
\]
Neutral Currents

- The detection of some reactions proved that neutral current (and the Z) exist

\[ \nu_\mu + e \rightarrow \nu_\mu + e \]
\[ \nu_\mu + p \rightarrow \nu_\mu + p \]

- The cross section depends on the different couplings at each vertex and measure the weak mixing angle

\[ \sigma^{ve} = \frac{G^2 m_e E^v}{\pi} (1 - 4 \sin^2 \theta_w - \frac{16}{3} \sin^4 \theta_w) \]
\[ \sigma^{\bar{v}e} = \frac{G^2 m_e E^{\bar{v}}}{\pi} \left( \frac{1}{3} - \frac{4}{3} \sin^2 \theta_w - \frac{16}{3} \sin^4 \theta_w \right) \]

- About 40% of the charged current cross section, due to Z-e-e coupling compared to W-e-nu coupling
Neutrino Oscillations

• Different eigenstates for weak and mass

\[
\text{weak} : \nu_e, \nu_\mu, \nu_\tau \leftrightarrow \nu_1, \nu_2, \nu_3 : \text{mass}
\]

• can mix with a CKM-like 3x3 matrix with (probably) different angles and phases then quarks. The neutrino lifetime is \(\sim\)infinite and so mix due to having mass and mass differences (like KL and KS)

• example. Assume just 2 generations (1 angle)

\[
\begin{align*}
\nu_\mu &= \nu_1 \cos \theta + \nu_2 \sin \theta \\
\nu_e &= -\nu_1 \sin \theta + \nu_2 \cos \theta
\end{align*}
\]

• assume that at \(t=0\) 100% muon-type

\[
\begin{align*}
\nu_\mu (t = 0) &= 1 & \nu_e (t = 0) &= 0 \\
\Rightarrow \nu_1 (t = 0) &= \cos \theta & \nu_2 (t = 0) &= \sin \theta
\end{align*}
\]
Neutrino Oscillations II

• Can now look at the time evolution
• from the Scrod. Eq. And assuming that the energy is much larger than the mass

\[ \nu_{1,2}(t) = \nu_{1,2}(0)e^{iE_{1,2}t} \quad E_i \approx p + \frac{m_i^2}{2p} \quad \hbar = c = 1 \]

• probability of e/mu type vs time (or length \( L \) the neutrino has traveled) is then

\[
|\nu_\mu(t)|^2 = \left| \cos^2 \theta e^{iE_1t} + \sin^2 \theta e^{iE_2t} \right|^2 \\
= 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L c^4}{4E\hbar c}
\]

• where we now put back in the missing constants and used 2 trig identities

\[
t = \frac{L}{c} \frac{E}{p} \\
2 \sin \theta \cos \theta = \sin 2\theta \\
\cos(E_2 - E_1)t = 1 - 2 \sin^2 \frac{(E_2 - E_1)t}{2}
\]
Neutrino Oscillations III

- Oscillation depends on mixing angle and mass difference (but need non-zero mass or no time propagation)

\[ |\nu_\mu(t)|^2 = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L c^4}{4 E \hbar c} \]

\[ |\nu_e(t)|^2 = 1 - |\nu_\mu(t)|^2 \]

- so some muon-type neutrinos are converted to electron type. Rate depends on neutrino energy and distance neutrino travels L/E

- go to 3 neutrino types and will have terms with more than one mixing angle. Plus neutrinos can oscillate into either of the other two (or to a fourth “sterile” type of neutrino which has different couplings to the W/Z than the known 3 types)
Neutrino Oscillations IV

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

\[|\nu_\alpha\rangle = U |\nu_\beta\rangle, \quad \text{where} \quad (c_{ij} \equiv \cos \theta_{ij}, \quad s_{ij} \equiv \sin \theta_{ij})\]

\[
U = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13}e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{i\delta} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

\[\begin{array}{c}
\text{Atmospheric} \\
\nu_\mu \leftrightarrow \nu_\tau
\end{array}
\begin{array}{c}
\text{Atmospheric} \\
\nu_e \leftrightarrow \nu_\mu \nu_\tau
\end{array}
\begin{array}{c}
\text{Solar}
\end{array}
\]

\[
= \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{13} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}
\]

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Neutrino Oscillations V

With three generations of neutrinos the change of one neutrino type into another depends on many terms.

You can understand the terms by measuring at different energies and lengths.

There is another effect (interactions in matter) which we will skip that comes into play.

Oscillations can also violate CP – be different if neutrino or antineutrino beam.

\[
P(\nu_\mu \rightarrow \nu_\epsilon) = P_1 + P_2 + P_3 + P_4
\]

- \(P_1 = \sin^2(\theta_{23}) \sin^2(2\theta_{13}) \sin^2(1.27 \Delta m_{12}^2 L/E)\) “Atmospheric”
- \(P_2 = \cos^2(\theta_{23}) \sin^2(2\theta_{12}) \sin^2(1.27 \Delta m_{12}^2 L/E)\) “Solar”
- \(P_3 = \mp J \sin(\delta) \sin(1.27 \Delta m_{13}^2 L/E)\) \(\text{Atmospheric-solar interference}\)
- \(P_4 = J \cos(\delta) \cos(1.27 \Delta m_{12}^2 L/E)\)

where \(J = \cos(\theta_{13}) \sin(2\theta_{12}) \sin(2\theta_{13}) \sin(2\theta_{23})\) \(\times \sin(1.27 \Delta m_{13}^2 L/E) \sin(1.27 \Delta m_{12}^2 L/E)\)
Detecting Neutrino Oscillations

- **Disappearance**: flux reduction larger L/E
- **Solar Neutrinos**: Measure rate for both electron neutrinos and all neutrinos (using neutral current). Low energies (few MeV) cause experimental thresholds for some techniques. Compare to solar models.

\[
\frac{\text{rate}(\nu_e + n \rightarrow e^- + p)}{\text{Rate}(\nu_{e,\mu,\tau} + pn \rightarrow \nu_{e,\mu,\tau} + p + n)}
\]

- **Atmospheric neutrinos**: Measure rate as a function of energy and length (from angle)

\[
\pi \rightarrow \mu \nu_\mu \quad \mu \rightarrow e \nu_e \nu_\mu
\]

\[
\frac{\# \nu_e}{\# \nu_\mu} = \frac{1}{2} \quad \text{production}
\]

- also electron or muon neutrinos produced at reactors or accelerators. Compare flux near production to far away L/E >> 1
Neutrinos from Sum

- from Particle Data Group

Solar Neutrinos Review (p. 486)

\[ p + p \rightarrow ^2H + e^+ + \nu \]

\[ ^8B \rightarrow ^8Be + e^+ + \nu \]

\[ ^7Be + e \rightarrow ^7Li + \nu \]
Detecting Neutrino Oscillations

• Appearance: start with one flavor detect another

• Ideal. Tag nu production by detecting the lepton. Then detect neutrino interaction. Poor rates (considered pi/K beams and muon storage rings)

• Real. Tau neutrino very difficult to detect sources of pure electron neutrinos (reactors) are below muon/tau threshold

• use mostly muon neutrino beam

\[
\frac{\nu_e}{\nu_\mu} \approx 0.003 \quad K \rightarrow \pi e \nu_e \\
\pi \rightarrow \mu \nu_\mu
\]

• can measure neutrino energy in detector (if above 1 GeV. Below hurt by Fermi gas effects). Can usually separate electron from muon events with a very good \(\approx 100\%\) active detector
Nova detector will be mostly liquid scintillator (like BNL neutrino experiment of the 1980s). Greater than 80% active.

Longitudinal sampling is 0.15 $X_0$, which gives excellent $\mu$-e separation.

A 2-GeV muon is 60 planes long.

$$\nu_\mu + n \rightarrow \nu_\mu + \pi^- + \pi^+ + \pi^0 + n$$

$$(1 \text{ GeV}) \pi^0 \rightarrow \gamma + \gamma$$

3.5% $X_0$ samples in all 3 views

4 cm gap
High Priority Items in Particle Physics

• Quark Mixing and CP violation
• Neutrino Mixing and maybe CP violation
• are Quark and Neutrino mixing related?
• Source of Electro-Weak symmetry breaking (Higgs?)
• Precision measurements of current parameters (top, W, Z mass) (g-2)
• what is dark matter? dark energy?
• Searches for New Phenomena – Supersymmetry, Extra Dimensions, Leptoquarks, new quarks/leptons/bosons, compositeness, why spin \( \frac{1}{2} \) vs spin 1
• some NP can explain other questions (source of CP, dark matter, etc)
Extra Dimensions

• Possible solution to the Hierarchy Problem
  \( M_H \sim 100 \text{ GeV} \quad \quad \quad \quad M_{\text{GUT/Planck}} \sim 10^{16}-10^{19} \text{ GeV} \)

• model of Arkani-Hamed, Dimopoulos and Dvali
  \( \rightarrow \) gravity propagates to \( n \) extra spatial dimensions
  \( \rightarrow \) gives massive stable Kaluza-Klein gravitons \( G_{\text{KK}} \)
  \( \rightarrow \) effective Planck scale \( M_{\text{Pl}} \) related to fundamental Plank scale in \( (n+4) \) dim \( M_D \)

• also model of Randall-Sundrum
  \( \rightarrow \) 1 extra dim but large curvature

\[
M_{\text{Pl}}^2 \sim R^n M_D^{n+2}
\]

SIGNAL (real graviton)
- high \( E_T \) single photon + missing \( E_T \)
- monojet + missing \( E_T \)

SIGNAL (virtual graviton)
- high mass pair: \( ee, \mu\mu, \gamma \gamma \)
Supersymmetry

- add superpartner to quarks, leptons, and bosons
- Solves the Hierarchy Problem
  \[ \Delta M^2_H = \frac{|\lambda_f|^2}{8\pi^2} \times (m_f^2 - m_S^2) \log \left( \frac{\Lambda}{m_S} \right) + ... \]
- lightest supersymmetric particle (LSP) candidate for dark matter
- Unification of the gauge couplings

<table>
<thead>
<tr>
<th>Names</th>
<th>spin</th>
<th>$R_P$</th>
<th>Gauge eigenstates</th>
<th>Mass eigenstates</th>
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<td>0</td>
<td>+1</td>
<td>$H_u^0$ $H_d^0$ $H_u^+$ $H_d^-$</td>
<td>$H^0$ $A^0$ $H^\pm$</td>
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<tr>
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<td></td>
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<td>-1</td>
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<td>1/2</td>
<td>-1</td>
<td>$\tilde{G}$</td>
<td>same</td>
</tr>
</tbody>
</table>

R-Parity:

if conserved: LSP is stable, SUSY particles produced in pairs
not conserved: may generate $\nu$ masses/mixing
SUSY:

\[ \tilde{\chi}^0_2 \tilde{\chi}^\pm_1 \rightarrow \text{Trileptons} \]

- number of possible decay chains
- Very clean mode
  - 3 isolated leptons
  - MET from ν or \( \chi^0 \)
- low \( \sigma \cdot \text{BF} \) (< 0.5 pb)
- leptons can be soft and depend on \( \Delta m \)

\[ \tilde{\chi}^0_1 \Rightarrow \text{LSP} \]